Complex interpolation of families of Orlicz sequence spaces

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It is a classical result that complex interpolation of a couple of Orlicz sequence spaces generates Orlicz sequence spaces. More precisely, if ϕ_0 and ϕ_1 are nondegenerate Orlicz functions then for $\theta \in (0, 1)$ we have

$$(\ell_{\phi_0}, \ell_{\phi_1})_\theta = \ell_{\phi_\theta}$$

with equivalence of norms, where $\phi_\theta^{-1} = (\phi_0^{-1})^{1-\theta}(\phi_1^{-1})^\theta$.

We generalize this result to families of Orlicz sequence spaces. Namely, let $\mathbb{T}$ be the unit circle, $\mathbb{D}$ be the open unit disk and $P(r, \theta - \cdot)$ be the Poisson kernel on $\mathbb{T}$ with respect to $z = re^{i\theta} \in \mathbb{D}$. If $(\phi_w)_{w\in\mathbb{T}}$ is a family of Orlicz sequence spaces satisfying certain technical conditions, then the complex interpolation method for families of Köthe function spaces of Kalton applied to the family $(\ell_{\phi_w})_{w\in\mathbb{T}}$ yields as interpolation space at z the Orlicz sequence space ℓ_{ϕ_z} where

$$\phi_z^{-1}(t) = \exp\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} P(r, \theta - s) \log \phi_{e^{is}}^{-1}(t)ds\right).$$

We apply this result to obtain a concrete example of twisted Hilbert space induced by a complex interpolation family of three spaces that cannot be obtained from a complex interpolation couple.

The Scattering Problem of Elastic Waves by an Inhomogeneous Medium with Buried Obstacles

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The scattering problem of time-harmonic elastic waves for buried obstacles inside an inhomogeneous medium, is considered. Initially, the direct scattering problem is mathematically modeled and its well-posedness via a modified variational method is established for the case of a rigid body (Dirichlet boundary condition). Further, the corresponding inverse problem is studied and the cases for a buried cavity and the mixed-type obstacle is also considered. In particular, the factorization method as an analytical mathematical tool, is used in order to reconstruct the shape and location of the boundary of the scatterer and the support of the inhomogeneous medium. Finally, useful remarks and conclusions are discussed. This is joint work with Georgios Kanakoudis, Tatiani-Foteini Mafidi and Vasilios Sevroglou.

Quasi-Fredholm spectrum under compact perturbations

Ankit Kumar - University of Delhi

Denote by $B(X)$ the Banach algebra of all bounded linear operators defined on an infinite dimensional complex Banach space X. For an operator $T \in B(X)$, let $N(T)$ and $T(X)$ denote the nullspace and the range space of T, respectively. Consider the set

$$\Delta(T) := \{n \in \mathbb{N} : m \geq n, m \in \mathbb{N} \text{ implies that } T^n(X) \cap N(T) \subset T^m(X) \cap N(T)\}.$$

The degree of stable iteration is defined by $\operatorname{dis}(T) := \inf \Delta(T)$ whenever $\Delta(T) \neq \emptyset$. If $\Delta(T) = \emptyset$, set $\operatorname{dis}(T) = \infty$. An operator $T \in B(X)$ is said to be quasi-Fredholm of degree d if there exists a $d \in \mathbb{N}$ such that

(i) $\operatorname{dis}(T) = d$,

(ii) $T^n(X)$ is a closed subspace of X for each $n \geq d$,

(iii) $T(X) + N(T^d)$ is a closed subspace of X.

For $T \in B(X)$, the quasi-Fredholm spectrum is defined by

$$\sigma_{qf}(T) := \{\lambda \in \mathbb{C} : \lambda I - T \text{ is not quasi-Fredholm}\}.$$

Let $\rho_{qf}(T) = \mathcal{C} \setminus \sigma_{qf}(T)$ be the quasi-Fredholm resolvent of T. We discuss some characteristics of quasi-Fredholm resolvent set $\rho_{qf}(T)$ for $T \in B(X)$. We give results regarding the distribution of semi B-Fredholm domain $\rho_{sbf}(T)$ in $\rho_{qf}(T)$. Also, we discuss the permanence of SVEP under(small) compact perturbations using quasi-Fredholm resolvent set and quasi-Fredholm spectrum. Also, we describe those operators for which SVEP is preserved under compact perturbations by means of quasi-Fredholm resolvent.

Compact Multiplication Operators on Semicrossed Products

Charalampos Magiatis - University of Aegean

Let $\mathcal{A}$ be a Banach algebra and $a, b \in \mathcal{A}$. The map $M_{a,b} : \mathcal{A} \to \mathcal{A}$ given by $M_{a,b}(x) = axb$ is called a multiplication operator. An element $a \in \mathcal{A}$ is called compact if the mapping $M_{a,a}$ is compact.

Properties of compact multiplication operators have been investigated first by Vala in 1964. Compactness questions for multiplication operators have also been considered in the more general framework of elementary operators.

We consider the semicrossed product $C_0(X) \times_\phi \mathbb{Z}_+$ where X is a locally compact metrizable space, and $\phi : X \to X$ a homeomorphism. We characterize the compact multiplication operators in terms of the corresponding dynamical system. As a consequence, we obtain a characterization of the compact elements. We also characterize the ideal generated by the compact elements.

The talk is based on joint work with G. Andreolas and M. Anoussis.

Toeplitz Composition Operators on the Hardy Space

Aastha Malhotra - University of Delhi

Let $\mathbb{D}$ and $\mathbb{T}$ denote the open unit disc and the unit circle in the complex plane $\mathcal{C}$, respectively. Let L^2 denote the Lebesgue (Hilbert) space on $\mathbb{T}$ and let L^∞ be the Banach space of all essentially bounded functions on $\mathbb{T}$. For a function $\psi \in L^\infty$ and a self-analytic map ϕ on $\mathbb{D}$, the *Toeplitz composition operator* $T_\psi C_\phi : \mathcal{H}^2 \to \mathcal{H}^2$ is defined as $T_\psi C_\phi f = P(\psi \cdot f \circ \phi)$ for every $f \in \mathcal{H}^2$ where $C_\phi f := f \circ \phi$ is the composition operator on the Hardy space $\mathcal{H}^2$ and $P : L^2 \to \mathcal{H}^2$ is the orthogonal projection. An anti-linear map $\mathcal{C}$ defined on a Hilbert space into itself is said to be a *conjugation* if it is both involutive and isometric. A bounded linear operator T on a Hilbert space is said to be *complex symmetric* if there exists a conjugation $\mathcal{C}$ such that $T = \mathcal{C}T^*\mathcal{C}$. We have explored some conditions under which the operator $T_\psi C_\phi$ on $\mathcal{H}^2$ becomes complex symmetric with respect to a special conjugation and also obtained various normality conditions for the operator $T_\psi C_\phi$ on $\mathcal{H}^2$.

Liouville Weighted Composition Operators over the Fock space

Himanshu Singh - University of South Florida

This presentation introduces Liouville Weighted Composition Operators, which are formally given as

$$A_{f,\phi} g = \nabla g(\phi) \cdot D\phi \cdot f,$$

over the Fock space, $F^2(\mathbb{C}^n)$, where $f : \mathbb{C}^n \mapsto \mathbb{C}^n$ and $\phi : \mathbb{C}^n \mapsto \mathbb{C}^n$ are entire functions over $\mathbb{C}^n$.

This discussion will examine various function theoretic properties of these operators, including *closability*, *boundedness* and *compactness*, as well as estimates on the *essential norm* of the operators.

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The trifecta of Hilbert spaces on Unit Disc

Himanshu Singh - University of South Florida

The Hilbert spaces are common. But the direct connection between them is rare. The aim of this paper is to establish a direct relation among the three Hilbert spaces, that are Hardy, Bergman and Dirichlet, without defining any of the Hilbert space in *weighted* sense. In order to accomplish this goal, this paper develops the Littlewood-Paley type Identities for Bergman and Dirichlet space. After defining these identities, the vision of connecting all the three Hilbert spaces via a direct connection is achieved.

Orthogonality and ideals of C*-algebras

Sushil Singla - Shiv Nadar University

In a given normed space V , an element v is said to be Birkhoff-James orthogonal to a subspace W if $\|v\| \leq \|v - w\|$ for all $w \in W$. Let $\mathcal{A}$ be a C*-algebra. A characterization of Birkhoff-James orthogonality of an element

$a \in \mathcal{A}$ to a subspace $\mathcal{B}$ of $\mathcal{A}$ in terms of state space of $\mathcal{A}$ was proved in [1]. It is well known that if $\mathcal{I}$ is a two-sided closed ideal of $\mathcal{A}$, then every state on $\mathcal{I}$ has a unique extension to a state on $\mathcal{A}$. Since $\|\cdot\|$ is a convex function, $\lim\limits_{t\to 0^+} \frac{\|v+tw\|-\|v\|}{t}$ always exists, known as Gateaux derivative of $\|\cdot\|$ at v. For $a \in \mathcal{A}$ such that $dist(a, \mathcal{I}) < \|a\|$, we shall also give an expression for the Gateaux derivative of the C*-norm in terms of states on $\mathcal{I}$. As a consequence, we will prove that if $a \in \mathcal{A}$ such that $dist(a, \mathcal{I}) < \|a\|$, then the state in the characterization of Birkhoff-James orthogonality can be chosen to be the extension of a state on $\mathcal{I}$. This will also give us alternative proofs or generalizations of various known results on the closely related notions of subdifferential sets, smooth points and Birkhoff-James orthogonality for spaces $\mathscr{B}(\mathscr{H})$ and $C_b(\Omega)$. Later, we will prove a similar expression for the Gateaux derivative of the norm function in Hilbert C*-modules.

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A generalized novel approach based on orthonormal polynomial wavelets with an application to Lane-Emden equation

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Capturing solution near the singular point of any nonlinear SBVPs is challenging because coefficients involved in the differential equation blow up near singularities. In this article, we aim to construct a general method based on orthogonal polynomials as wavelets. We discuss multiresolution analysis for wavelets generated by orthogonal polynomials, e.g., Hermite, Legendre, Chebyshev, Laguerre, and Gegenbauer. Then we use these wavelets for solving nonlinear SBVPs. These wavelets can deal with singularities easily and efficiently. To deal with the nonlinearity, we use both Newton's quasilinearization and the Newton-Raphson method. To show the importance and accuracy of the proposed methods, we solve the Lane-Emden type of problems and compare the computed solutions with the known solutions. As the resolution is increased the computed solutions converge to exact solutions or known solutions. We observe that the proposed technique performs well on a class of Lane-Emden type BVPs. As the paper deals with singularity, non-linearity significantly and different wavelets are used to compare the results, which makes the paper more important and useful for the research community.