Hyper-Heuristics and Sudoku

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Contents

Introduction

Aims

- Aim of the project: to code a programme that will find a valid solution to a Sudoku grid of order n as quickly as possible.

Background

- "Sudoku" is an abbreviation of the Japanese phrase "Suuji wa dokushin ni kogiru"
- Contrary to the Japanese name, the 1st modern Sudoku was published in the US in 1979 by Howard Garns; a retired architect and puzzlemaker.

Sudoku

3			8		1			2
2		1		3		6		4
			2		4			
8		9				1		6
	6						5	
7		2				4		9
			5		9			
9		4		8		7		5
6			1		7			3

Figure: Standard Sudoku Puzzle

- A typical Sudoku puzzle, as displayed above, is "order 3"

Background

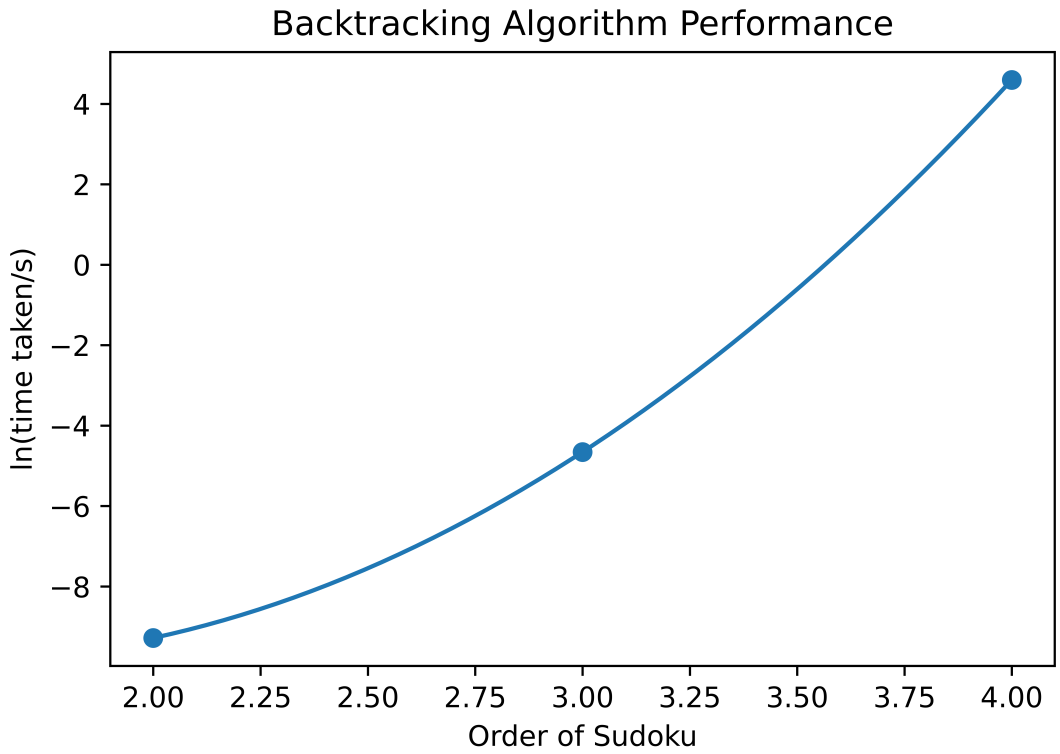
	8		14	10		12			13				2	3	
		3	4			14	16			11		15	12		8
		9						3			8		13		
	7				4	8	5			1	2		14	6	
			8	16		10	1		15		3	2		9	
11	4			9	14		12	7	5						13
9		2	10	4					1	13	16	5			
6	1			8	7	13			2	9		14		11	
	2		12		16	7			6	15	5			10	3
			13	15	10	6					11	9	8		2
1						3	8	13		16	14			12	5
	3		11	5		1		2	8		4	13			
	15	1		12	3			16	11	10				13	
		13		2			6						9		
14		12	7		1			15	4			8	3		
	11	4				16			9		13	12		2	

Figure: Order 4

Methods

Backtracking Algorithm

- Systematic Method
- Steps through grid
- Pros and Cons



Hyper-Heuristics

- What is a Heuristic?
- Which are used?
- Hyper-Heuristic?
- Cost function

1	2	3
6		4
7	8	9

(a) Sudoku Box

1	7	9	4	3	2	5	6	8
6	5	3	8	9	7	1	2	4
8	2	4	5	1	6	9	3	7
9	1	5	6	8	3	7	4	2
2	4	6	1	7	5	8	9	3
3	8	7	2	4	9	6	1	5
5	6	1	7	2	4	3	8	9
4	9	8	3	5	1	2	7	6
7	3	2	9	6	8	4	5	1

(b) Completed Sudoku

Figure: Order 3

Decisions

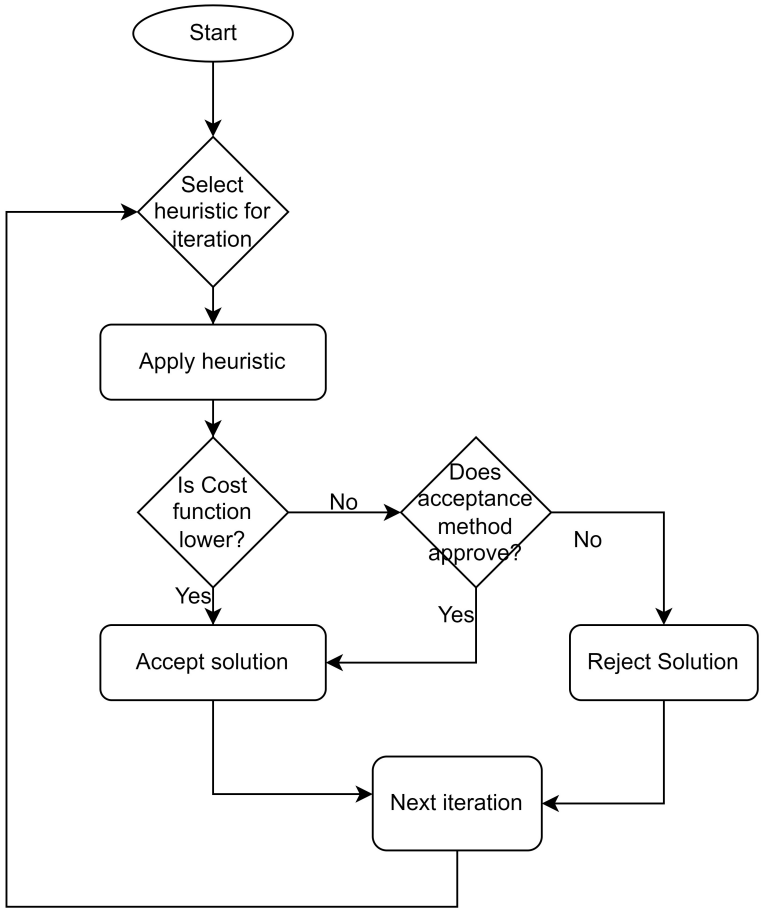


Figure: Decision Process

Results

Results

- Start with empty grid
- Random Selection, Only Improve

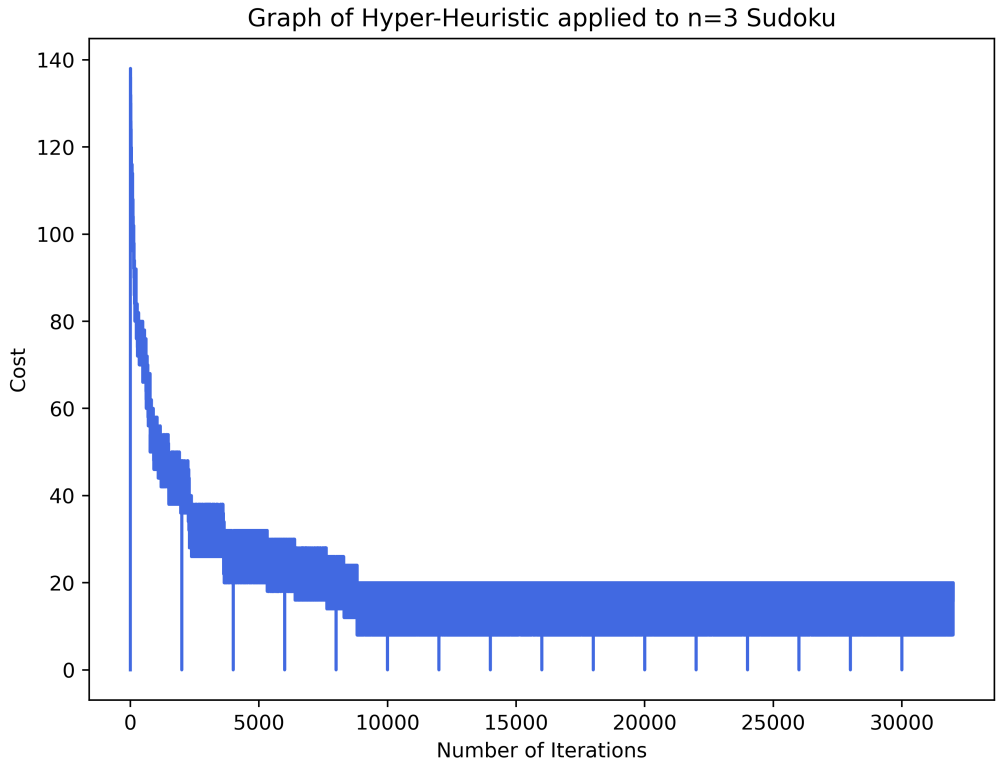


Figure: Simple Random Selection, Only Improve

Results

- Simulated Annealing:

$$p = \begin{cases} 1, & \text{if } ProposedCost < CurrentCost \\ e^{\delta/t}, & \text{if } ProposedCost \geq CurrentCost \end{cases}$$

$$,\delta = CurrentCost - ProposedCost$$

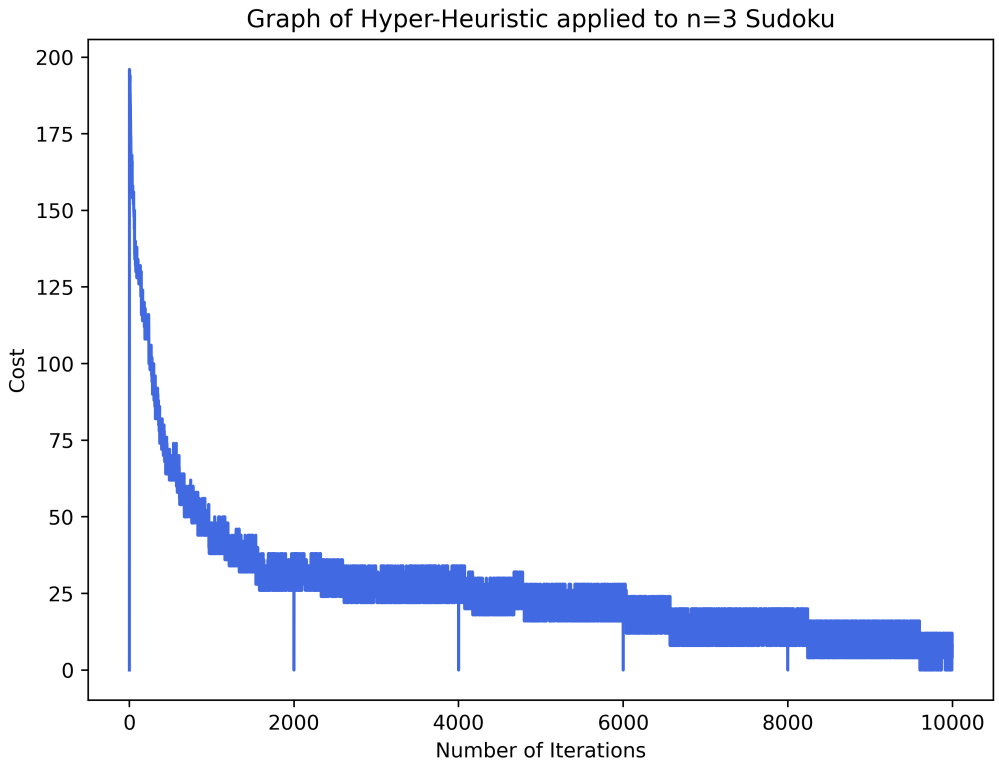


Figure: Simple Random Selection, Simulated Annealing

Results

- Beyond Simple Random?

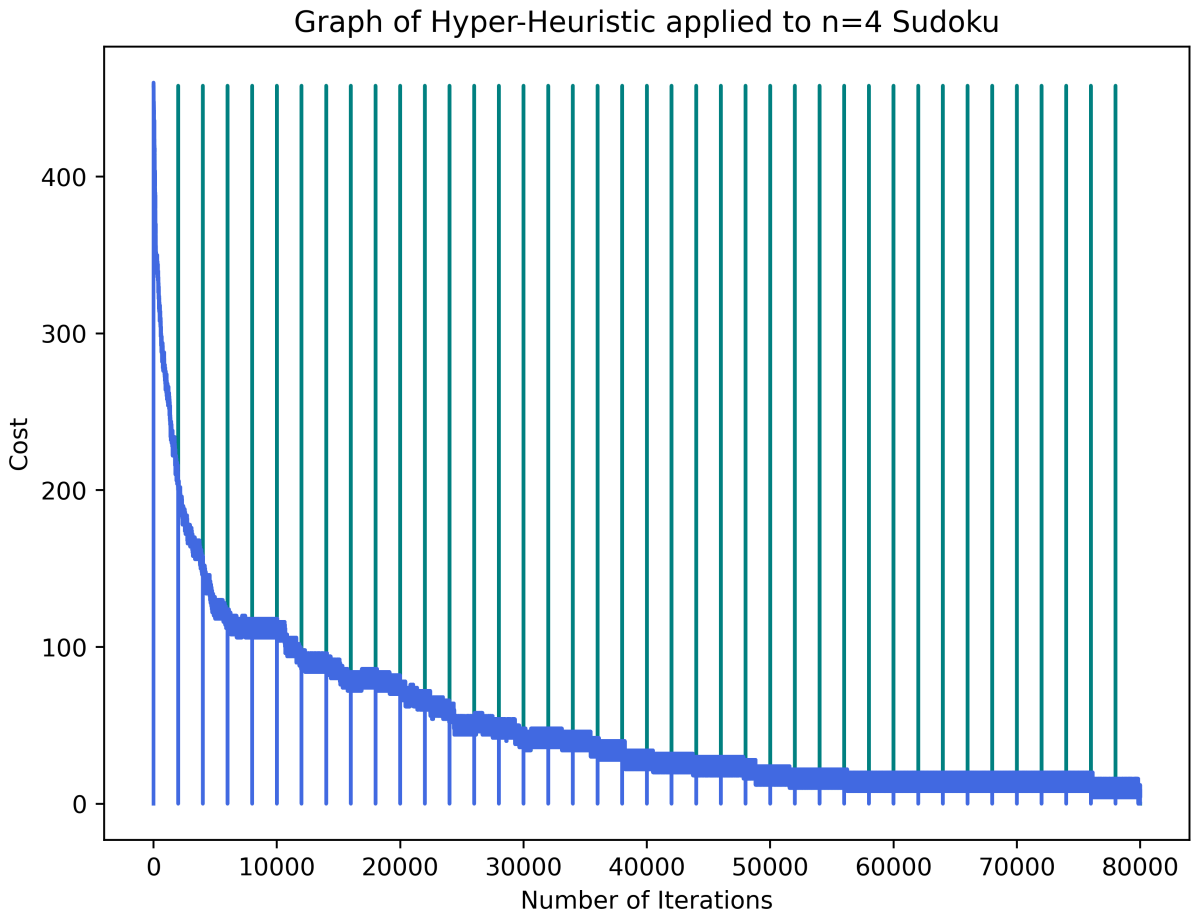


Figure: Simple Random Selection, Simulated Annealing

Results

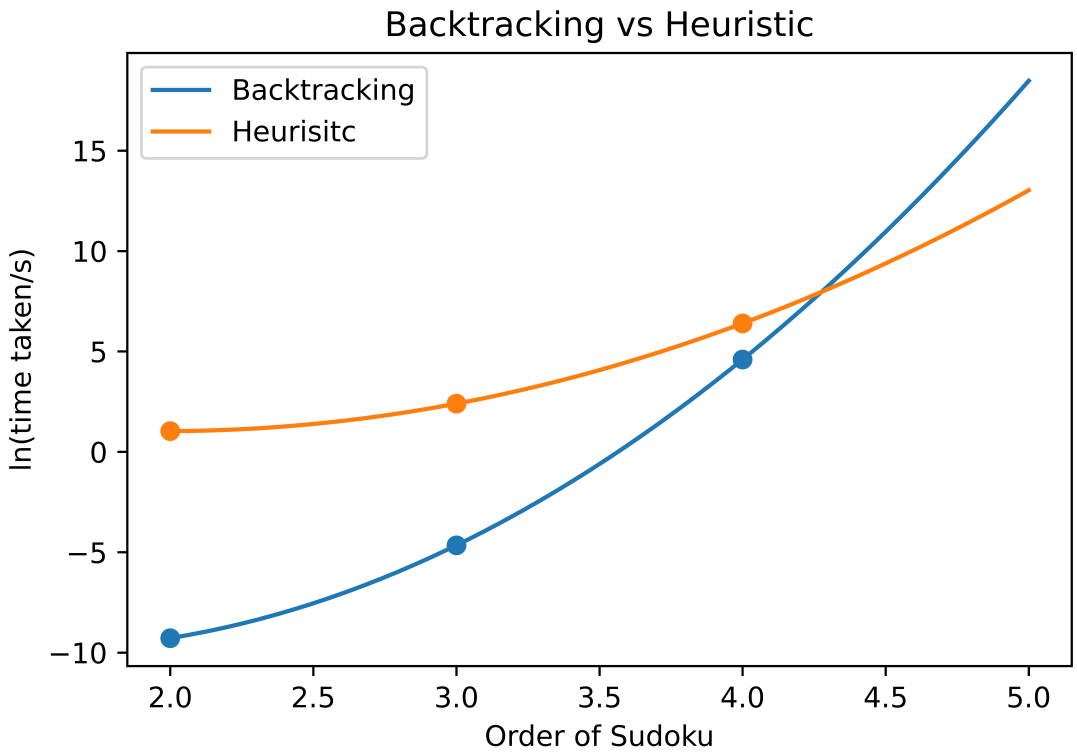


Figure: Comparison

Concluding Remarks

Summary

- Backtracking
- Hyper-Heuristic

Further Work

Improvements?

- Programme algorithm to learn
- Sequences
- Logic

- Solve partially filled grids
- Record-to-record?

References

References

[1] Broderick Crawford, Mary Aranda, Carlos Castro, and Eric Monfroy. Using constraint programming to solve sudoku puzzles. In *2008 Third International Conference on Convergence and Hybrid Information Technology*, volume 2, pages 926–931, 2008.

[2] Rhydian Lewis. Metaheuristics can solve sudoku puzzles. *J. Heuristics*, 13:387–401, 07 2007.

Thank you!

Questions?