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## A R T I C L E I N F O

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Consider extreme value modelling of peaks over threshold for random variable *X*, using the generalised Pareto model form

$$Pr(X > x | X > u(\theta)) = \left(1 + \frac{\gamma(\theta)}{\sigma(\theta)}(x - u(\theta))\right)^{-1}$$

for threshold  $u(\theta)$ , shape parameter  $\gamma(\theta)$  and scale parameter  $\sigma(\theta)$ , all assumed to be functions of covariate  $\theta$  (which may itself be multivariate).

We note that, with reference to Jonathan et al. (2008):

- 1) There are conflicting modelling requirements relating to sample size. To minimise parameter uncertainty, large samples are desirable. However, a high threshold is required to justify the generalised Pareto form.
- 2) The number of degrees of freedom for covariate models is larger than for a "constant" model ignoring covariate effects (for which  $\gamma(\theta)=c_1, \sigma(\theta)=c_2$ , where  $c_1$  and  $c_2$  are constants). Adoption of a directional model must be justified over and above a constant model. Jonathan et al. (2008) used a likelihood ratio test (e.g. Fig. 8) to reject the hypothesis that the constant model was appropriate, as a function of threshold choice, and showed clearly in a number of cases that the directional model was justified. Also, they used diagnostic techniques to demonstrate clearly that the behaviour of shape and scale parameters with threshold for the constant model was inconsistent with expectation. Furthermore, estimates of extreme quantiles using an appropriate covariate model were shown to be less biased than those using a constant model (by comparing with the true quantile value, since simulated samples with known characteristics were used).

We acknowledge the following:

3) A covariate model can only be adopted when there is clear evidence to reject a simpler constant model.

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- 4) Parameter uncertainty for a covariate model will in general be higher than for a constant model, since more degrees of freedom are being fitted for a given data sample size (but see points 6 and 7).
- 5) For "judicious" (or fortunate) choice of threshold, it is likely that a constant model could be constructed which gives unbiased estimates of extreme quantiles. However, in general, there is no way to knowing which "judicious" choice of threshold is generally appropriate. See, e.g., discussion of Figs. 9 and 10 of Jonathan et al. (2008).
- 6) Even in cases where a covariate model would be appropriate, in principle the threshold can be set so large that essentially the covariate effect is eliminated. In this case, a constant model would be appropriate, but the sample size would generally be too small to estimate parameter values reliably. The purpose of using the covariate model, when justified, is to retain as large a sample as possible for modelling, thereby reducing parameter and quantile uncertainty.

We would further note, referring to Mackay et al. (2010):

7) Their use of discrete seasonal models, which require independent fits per season, rather than allowing covariate effects to vary smoothly with respect to season at modelling all data together. As a result, the models of Mackay et al. (2010) exhibit a larger number of degrees of freedom for fitting that is probably necessary (the authors acknowledge that they expect extremal characteristics to vary smoothly with season). Hence, parameter bias and uncertainty in fitting the Mackay et al. (2010) models will be higher than expected using a smooth covariate model.

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