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Allocation of School Bus Contracts By Integer Programming

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When public transport is put out to tender, the task of allocating contracts to operators can be far from trivial. In addition to vehicle constraints, there can be group bids wherein a discount is offered for certain contracts in combination. An efficient integer programming formulation is presented which is then used to solve a large-scale real-life example.

Key words: contract allocation, integer programming

INTRODUCTION

Greater Manchester Public Transport Executive (GMPTE) used to be entirely responsible for bus, tram and rail services in Greater Manchester. Since the countrywide deregulation of the bus network in 1986, however, many bus services are now successfully provided on a completely commercial basis by private operators. Despite this, there are many services which would fail to be economically viable if not given financial aid. The PTE is now required to put contracts to such services out to tender before the beginning of each year, provided that no commercial route is covering the same ground.

This paper concerns school bus contracts. There are around 300 of these each year, although this has been slowly diminishing as more routes are operated commercially. The contracts themselves cover the thirty-eight weeks of a normal school year, from Monday to Friday. At present the convention is that one contract implies one vehicle, but this could well change in future years.

Contracts specify the minimum vehicle capacity required. This effectively means that some will require double-deck buses, as opposed to single-deck or mini-buses. Conversely, height restrictions on certain roads may mean that double-deck buses are forbidden. Finally, contracts may be all-day, A.M. or P.M. only (or, very rarely, midday).

Once all bids have been submitted, the PTE has the task of allocating the contracts to the bidders in the cheapest way consistent with an acceptable quality of service. Recently, this has proven increasingly difficult to do manually, due to certain complications which are outlined below.

Number of operators

Prior to deregulation, the entire bus network in Greater Manchester was run by Greater Manchester Buses (GMB). These covered all ten districts in the region (Manchester, Trafford, Salford, Stockport, Wigan, Bolton, Rochdale, Bury, Oldham and Tameside). Now, there are well over thirty operators in the region. They are of varying size, some having only two or three vehicles, some tens of vehicles; GMB itself has several hundred. Some are localised in only one or two districts, some cover many. To further complicate the picture, at the time of the study GMB itself was due to be partitioned into two separate companies.

Vehicle constraints

Operators are free to submit more bids than they actually have the resources for. If this is the case they must specify, in conjunction with their bids, a maximum number of vehicles available.

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This can also be specified in terms of contracts, bearing in mind the present one-vehicle-per-contract rule stated above. Different constraints will apply to different types of vehicle.

Furthermore, the constraints may cover a single district, a number of adjacent districts, or the entire region, depending upon the location of depots. Constraints may also be influenced by the times of day of the contracts concerned.

Group bids

Operators are permitted to submit any number of *group bids.* In a group bid, a discount is offered if certain contracts are awarded to the operator simultaneously. At this tendering stage, it is permissible for the groups to *overlap*, that is, individual contracts may appear in more than one group. When the PTE is making the allocation, however, it may not award overlapping groups and therefore must make a decision as to which, if any, of the groups to award.

It is clear that the group bids and the vehicle constraints can interact in a non-trivial way. If a problem has only a few group bids, or a large number of them are mutually exclusive, it may be feasible to do an exhaustive search of each possible combination, solving a smaller (transportation-like) problem for each one. Until recently, this was a viable option, but the 1993–4 school year tenders contained well over fifty groups—representing a huge number of combinations. Hence, the problem was tackled using integer programming.

INTEGER PROGRAMMING FORMULATION

Assume that the operators, contracts and types of vehicle are numbered $1 \ldots M$, $1 \ldots N$ and $1 \ldots V$ respectively and indexed by i, j, and k. Contract j requires q_{jk} of vehicle type k; operator i has Q_{ik} of type k available. The costs of the non-group bids are denoted by c_{ij}; if operator i put in no bid for contract j, c_{ij} is undefined. Let there be G_i group bids from operator i. The cost of the pth group of operator i is denoted by C_{ip}. Also define the index sets $\mathbf{M} = \{1, \ldots, M\}$, $\mathbf{N} = \{1, \ldots, N\}$ and $\mathbf{V} = \{1, \ldots, V\}$ and let S_{ij} $(i \in \mathbf{M}, j \in \mathbf{N})$ denote the set of numbers p for which the pth group bid of operator i contains contract j.

The variables are as follows:

$$x_{ij} = \begin{cases} 1 & \text{if operator } i \text{ is awarded contract } j \text{ (but } \textit{not} \text{ in a group),} \\ 0 & \text{otherwise;} \end{cases}$$

$$y_{ip} = \begin{cases} 1 & \text{if the } p\text{th group of operator } i \text{ is awarded,} \\ 0 & \text{otherwise.} \end{cases}$$

(x_{ij} will only be defined if operator i put in a bid for contract j).

The IP is then:

minimise

$$\sum_i \sum_j c_{ij} x_{ij} + \sum_i \sum_p^{G_i} C_{ip} y_{ip}$$

subject to:

$$\sum_i x_{ij} + \sum_i \sum_{p \in S_{ij}} y_{ip} = 1 \quad (j \in \mathbf{N}) \tag{1}$$

$$\sum_j q_{jk} x_{ij} + \sum_j \sum_{p \in S_{ij}} q_{jk} y_{ip} \leqslant Q_{ik} \quad (i \in \mathbf{M},\ k \in \mathbf{V}) \tag{2}$$

$$x_{ij},\ y_{ip} \in \{0, 1\} \tag{3}$$

Constraints (1) ensure that each contract is allocated to either a unique single bid or a unique group bid. This also has the side-effect of preventing the awarding of mutually exclusive group bids to any operator. Constraints (2) ensure that no operator is awarded more contracts than it can cope with. Constraints (3) are the binary integrality conditions.

The formulation follows the guidelines given in Williams[1] to encourage a tight linear relaxation: the constraint matrix is sparse (i.e. has few non-zero entries), and most non-zero constraint coefficients are equal to one. It also makes the structure of the problem more apparent:

(i) if there were no group bids *and* there was only one (global) vehicle constraint for each operator, the problem would be a Generalised Assignment Problem (see the survey by Cattryse and Van Wassenhove[2]).

(ii) if there were no group bids *and* all contracts required only one vehicle, the problem would be a (polynomially solvable) Transportation Problem (TP). Note that the latter condition actually applies to the PTE's problem.

(iii) if there were no vehicle constraints, the problem would be a Set Partitioning Problem (see the early survey by Balas and Padberg[3], and also see Ryan[4]).

The GAP and SPP are both NP-hard, so the complete general problem is also NP-hard.

We must also consider some further complications which may arise:

(i) an operator may have several depots. If these are run independently, they may be regarded as operators in their own right. In practice, however, they may share some vehicle constraints. Fortunately, the adjustment to the formulation is straightforward.

(ii) there may be extra constraints limiting the total number of contracts at a particular time of day.

(iii) an operator may require a fixed-charge F_i, incurred if *one or more* contracts are awarded to it. Here we introduce a new 0/1 variable z_i, add $F_i z_i$ to the objective, and append constraints of the form

$$x_{ij} \leqslant z_i \quad (j \in N)$$

$$y_{ip} \leqslant z_i \quad (p = 1 \ldots G_i).$$

(iv) an operator might want a package of exactly E_i contracts (including those in groups), or none at all. Here we must add a 0/1 variable w_i, and the constraint

$$\sum_j x_{ij} + \sum_p^{G_i} T_{ip} y_{ip} = E_i w_i$$

where T_{ip} denotes the number of contracts covered by the pth group bid of operator i. Other complications can be dealt with similarly.

COMPUTATIONAL EXPERIMENTS

To ascertain running times, the method was first applied to a single district for the 1991-2 school year. This problem had $M = 10$, $N = 48$, 201 individual bids, only 3 group bids and 13 vehicle constraints of various types. Using Lindo Systems' HyperLindo (Schrage[5]), running on a PC386-33 with 387 maths coprocessor, this problem was solved in only a few seconds.

The whole (ten-district) problem for 1993–4 was then attempted. This had $M = 35$, $N = 298$ (although 24 of these contracts were later removed from consideration because commercial operators began serving these routes), 1180 individual bids, 57 group bids, and 35 vehicle constraints. The final IP had 1237 0/1 variables and 309 constraints, but the density was only 0.9% (only 1819 non-zeros in the constraint matrix), so the problem was extremely sparse.

Despite the complexity, the LP solution was reached after 287 pivots, and only a further 2 pivots were required in the branch-and-bound stage, representing the exploration of a single node. Total running time was only 35 seconds.

The IP result was verified by comparing it with that achieved manually by three independent, experienced bus planners at GMPTE. Two of them had actually reached the optimum; the other was only 1% above the optimum.

To facilitate *in-house* contract allocation in future, a prototype Matrix Generator was devised. The bus planners can use the familiar spreadsheet format they were already using to tabulate the prices and constraints, with only minor adjustment. The MG takes an ASCII file from this and produces another ASCII file in Lindo format.

DISCUSSION

The basic IP formulation is apparently extremely efficient (presumably because the feasible region has a large proportion of integral vertices). It should be borne in mind, however, that under current PTE practice, operators are free to impose almost any restriction they like onto the basic bid price schema. This can lead to further nonstandard constraints, such as one company offering a £1000 discount on each of the *first seven* contracts awarded them. The presence of such constraints means that the formulation (and the MG) can never be completely general. However, if certain types of constraint are observed to become more frequent, it would be worthwhile extending the MG to cope with them.

If the problem ever increased in complexity to the point where running times were prohibitive, it would be worthwhile developing specialised cutting-planes to tighten the LP relaxation and so reduce the work done in the branch-and-bound stage. For example, constraints (2) can be viewed as $\{0, 1\}$ Knapsack constraints and therefore one might replace them by a number of Lifted Cover inequalities (Balas and Zemel[6]). Alternatively, it might be worthwhile attempting some form of Lagrangean Decomposition (Campello and Maculan[7]), as the formulation naturally divides into components in a number of ways.

CONCLUSIONS

Little material has been published on mathematical programming approaches to contract allocation, although formulations have been presented for the case of materials purchasing (Gaballa[8]; Turner[9]). Given the success of the GMPTE project, it would be surprising if analogous formulations did not emerge over coming years, perhaps tailored to the needs of clients in different contexts. As more and more public services are being put out to private tender in Europe, such material is likely to appear in increasing frequency.

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