THE GENERAL ROUTING PROBLEM, *GRP*

The *General Routing Problem* (GRP) is a routing problem defined on a graph or network where a minimum cost tour is to be found and where the route must include visiting certain required vertices and traversing certain required edges. More formally, given a connected, undirected graph G with vertex set V and (undirected) edge set E, a cost c_e for traversing each edge $e \in E$, a set $V_R \subseteq V$ of *required vertices* and a set $E_R \subseteq E$ of *required edges*, the GRP is the problem of finding a minimum cost vehicle route, starting and finishing at the same vertex, passing through each $v \in V_R$ and each $e \in E_R$ at least once ([13]).

The GRP contains a number of other routing problems as special cases. When $E_R = \emptyset$, the GRP reduces to the *Steiner Graphical Travelling Salesman Problem* (SGTSP) ([6]), also called the *Road Travelling Salesman Problem* in [5]. On the other hand, when $V_R = \emptyset$, the GRP reduces to the *Rural Postman Problem* (RPP) ([13]). When $V_R = V$, the SGTSP in turn reduces to the *Graphical Travelling Salesman Problem* or GTSP ([6]). Similarly, when $E_R = E$, the RPP reduces to the *Chinese Postman Problem* or CPP ([7],[3]).

The CPP can be solved optimally in polynomial time by reduction to a matching problem ([4]), but the RPP, GTSP, SGTSP and GRP are all *NP*-hard. This means that the computational effort to solve such a problem increases exponentially with the size of the problem. Therefore exact algorithms are only practical for a GRP if it is not too large, otherwise a heuristic algorithm is appropriate. The GRP was proved to be *NP*-hard in [9].

In [2], an integer programming formulation of the GRP is given, along with several classes of valid inequalities which induce facets of the associated polyhedra under mild conditions. Another class of valid inequalities for the GRP is introduced in [11] and in [10] it is shown how to convert facets of the GTSP polyhedron into valid inequalities for the GRP polyhedron. These valid inequalities form the basis for a promising branch-and-cut style of algorithm described in [1] which can solve GRPs of moderate size to optimality.

In [8], a heuristic algorithm for the GRP is described. The author adapts Christofides' heuristic for the TSP to show that when the triangle inequality holds in the graph, the heuristic has a worst case ratio of heuristic solution value to optimum value of 1.5.

There are many vehicle routing applications of the GRP. In these cases, the edges of the graph are used to represent streets or roads and the vertices represent road junctions or particular locations on a map. In any practical application there are likely to be many additional constraints which must also be taken into account such as the capacity of the vehicles, time-window constraints for when the service may be carried out, the existence of one-way streets and prohibited turns etc.

Many applications are for the special cases when either $E_R = \emptyset$ or $V_R = \emptyset$. However, there are some types of vehicle routing applications where the problem is most naturally modelled as a GRP with both required edges and required vertices. For example, in designing routes for solid waste collection services, collecting waste from all houses along a street could be modelled as a required edge and collecting waste from the foot of a multi-story apartment block could be modelled as a required vertex. Other examples include postal delivery services where some customers with heavy demand might be modelled as required vertices, while other customers with

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General Routing Problem
required vertices
required edges
Steiner Graphical Travelling Salesman Problem
Road Travelling Salesman Problem
Rural Postman Problem
Graphical Travelling Salesman Problem
Chinese Postman Problem

homes in the same street might be modelled together as a required edge. School bus services are other examples of GRPs where a pick-up in a remote village could be modelled as a required vertex, but if the school bus must pick-up at some point along a street (and is not allowed to perform a U-turn in the street) then that may best be modelled as a required edge.

Further details about solution methods and applications for various network routing problems can be found in [12].

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References

[1] A. CORBERÁN, A.N. LETCHFORD, AND SANCHIS, J.M.: A cutting-plane algorithm for the general routing problem, Working paper.

[2] CORBERÁN, A., AND SANCHIS, J.M.: 'The general routing problem polyhedron: facets from the RPP and GTSP polyhedra', *European. J. Operational Research* **108** (1998), 538–550.

[3] EDMONDS, J.: 'The Chinese postman problem', *Operations Research* **13** (1963), B73–B77.

[4] EDMONDS, J., AND JOHNSON, E.L.: 'Matchings, Euler tours and the Chinese postman', *Math. Prog.* **5** (1973), 88–124.

[5] FLEISCHMANN, B.: 'A cutting-plane procedure for the travelling salesman problem on a road network', *European J. Operational Research* **21** (1985), 307–317.

[6] G. CORNUÉJOLS, J. FONLUPT, AND NADDEF, D.: 'The travelling salesman problem on a graph and some related integer polyhedra', *Math. Prog.* **33** (1985), 1–27.

[7] GUAN, M.: 'Graphic programming using odd or even points', *Chinese Math.* **1** (1962), 237–277.

[8] JANSEN, K.: 'An approximation algorithm for the general routing problem', *Inf. Proc. Letts* **41** (1992), 333–339.

[9] LENSTRA, J.K., AND RINNOOY-KAN, A.H.G.: 'On general routing problems', *Networks* **6** (1976), 273–280.

[10] LETCHFORD, A.N.: The general routing polyhedron: a unifying framework, To appear in European J. Operational Research.

[11] LETCHFORD, A.N.: 'New inequalities for the general routing problem', *European J. Operational Research* **96** (1997), 317–322.

[12] M.O. BALL, T.L. MAGNANTI, C.L. MONMA, AND NEMHAUSER, G.L. (eds.): *Network Routing. Handbooks of Operations Research and Management Science, 8.*, North Holland, 1995.

[13] ORLOFF, C.S.: 'A fundamental problem in vehicle routing', *Networks* **4** (1974), 35–64.