

Global rebound effects viewed through the returns on investment in energy efficiency.

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Abstract

In this paper we re-examine the relationship between global Gross Domestic Product (GDP), Primary Energy Use (PEU) and Economic Energy Efficiency (EEE) to explore how investment in energy efficiency causes rebound in energy use. Assuming GDP is a measure of final useful work, we construct and fit a physical nonlinear dynamic model to global GDP, PEU and EEE data from 1900 - 2018 and use it to estimate how energy efficiency investments relate to output growth and hence rebound effects. We illustrate the effects of future deployment of enhanced energy efficiency investments using two scenarios through to 2100. The first maximizes GDP growth, requiring energy efficiency investment to rise ~ 2 fold. Here there is no decrease in PEU growth because rebound effects dominate. The second scenario minimizes PEU growth by increasing energy efficiency investment ~ 3.5 fold. Here PEU and GDP growth are fully decoupled as rebound effects are minimal, although this results in a long run zero output growth regime. We argue it is this latter regime that is compatible with the deployment of enhanced energy efficiency to meet climate objectives. However, while output growth maximising regimes prevail, all efficiency-led pledges on energy use and emissions reduction appear likely to fail.

1. Introduction

Improving energy efficiency has re-emerged as an important policy focus internationally. It comprises approximately 40 percent of the Nationally Determined Contributions (NDCs) currently pledged under the Paris climate agreement (UNFCCC, 2021; UNEP, 2021; CAT, 2021). Alongside this, energy efficiency improvements are also proposed for addressing slowing output growth in developed economies (e.g. BEIS, 2018), and the effects of war in Eastern Europe (Russian invasion into Ukraine in 2022) on the global energy supply has acted to amplified this need. Resolving how climate, growth and energy security objectives align has become central to debates surrounding the possibility of attaining so-called 'green growth' (OECD, 2011), where energy efficiency improvements are used to partially decouple economic outputs from primary energy inputs.

There are mounting concerns over the green growth narrative, largely driven by the widespread appreciation of the growth imperative at the centre of government and business, and how this agenda might undermine any desire to reduce society's environmental footprint. A significant factor in this debate is an increasing

appreciation of rebound and backfire effects, or Jevons Paradox, which emphasise that energy efficiency improvements can lead to output growth that in turn drives input growth, rather than input degrowth (Jevons, 1866). This appears to be especially true as both time and length scales increase (Bruns et al., 2021), presumably as more of the relevant feedback processes take effect. Most of the economic analyses underpinning climate scenarios implicitly assume that rebound effects are small, and hence the users of these models conclude increasing energy efficiency can play a central role in reducing greenhouse gas emissions via reductions in energy use (Riahi et al., 2017). Some go further and argue energy efficiency should be a central focus for climate policy (Grübler et al., 2018). Although numerous estimates of rebound effects suggest they are not large enough to undermine policy objectives on energy efficiency (Gillingham et al., 2013), these tend to be made on relatively short time and length scales, or with models that assume energy is a minor factor of output (Brockway et al., 2021). In contrast, recent estimates using models that attempt to capture large-scale and long-term energy feedbacks tend to indicate rebounds are significant, and that policy focused on using energy efficiency improvements to reduce energy inputs run a significant risk of failure (Bruns et al., 2021; Kander, 2020; Brockway et al., 2021). In addition, we readily observe that over time, machines have indeed become more efficient while global society has to date continually increased its rate of energy consumption.

The risks around rebound effects are probably best represented through the dynamic relationship between Primary Energy Use (PEU) inputs and Gross Domestic Product (GDP) outputs, particularly at the global scale where our collective efforts to manage markets and climate ultimately play out. However, the debate over the roles energy and energy efficiency play in determining economic output is long and contested, with no clear picture having yet emerged (Stern 2011; Kalimeris et al. 2014; Brockway et al., 2018). On the one hand there are those who point out the relatively small fraction of production costs imposed by energy mean that energy plays a minor role in growth (Dennison 1979; Newberry 2003; Grubb et al., 2018). On the other hand, there are those who emphasise how energy use necessarily underpins all activity, including that of economies (Soddy 1926; Ayres and Warr, 2009; Kümmel 2011; Garrett, 2011; Sakai et al., 2019; King 2020).

A common approach employed in energy analysis is to partition economic output between primary energy and efficiency through the identity $GDP = EEE \times PEU$, where EEE is referred to as the Economic Energy Efficiency, and its inverse the energy intensity of the economy (Kander, 2020). In addition to simply relating PEU and GDP, there is increasing evidence that this identity has a clear thermodynamic basis, placing energy front and centre in determining economic performance. Although subjective judgements are deeply embedded throughout the accounting that underpins all GDP data, because the aim is to attempt to capture the annual production of real economic value, despite all its failings, GDP data do largely capture the valued physical activity in economic systems, even if that activity is often highly dematerialised. For this physical activity to be valued it needs to be in some sense useful, even if at times we might struggle to see this utility through the deep complexity shrouding the economy. This line of arguing is consistent with the

emerging conclusion that GDP parallels the rate useful work is performed by the economy (Warr et al., 2010; Serrenho et al., 2016).

If so, EEE parallels the thermodynamic energy efficiency of the economy when translating primary energy flows into the rate at which final useful work is performed, and the identity $GDP = EEE \times PEU$ reflects the transduction of primary energy into useful work and hence the thermodynamic backstory that accompanies this view. As a result, even though $GDP = EEE \times PEU$ has previously only ever been cast as an identity, there are good theoretical and empirical reasons to assume there is a thermodynamic underpinning to this relationship. Rather than attempt to add further empirical evidence to underpin any linear scaling between GDP and the rate of final useful work, we instead assume this relationship as a central tenet of this paper. By doing so, we build on both the empirical evidence to date and the theoretical position that society necessarily values useful work, even if the accountancy process tracking this valuation might be somewhat flawed and the definition of what is useful is again shrouded in deep complexity.

The final useful work of the economy performs work on itself and its surroundings (Garrett, 2011). Although this work takes many forms, the outcome is necessarily the spatial reorganisation of matter. The reorganisation of matter by final useful work leads to the creation of ordered structures, principally networks, through encoding information in matter. This goes some way to explaining why, although we might attribute GDP to useful physical activity, this activity is not solely material, but necessarily involves a significant role for information. Because the work involved is, by definition, judged to be useful¹, these structures must also be in some sense productive. The only definition of productive afforded here is structure that is able to facilitate future flows of final useful energy and work. This view is consistent with that of creative destruction in which newly-designed productive structures replicate at the expense of existing unproductive structures (Schumpeter, 1948).

We draw two important conclusions from the preceding paragraphs that translate into two further central assumptions in this work. The first conclusion is that the investment of final useful work into the creation of productive structure can be partitioned into either increasing the flow of primary energy into the system, or increasing the internal efficiency of that system when transducing primary energy into flows of final useful work. This derives from the definition that $output = efficiency \times input$. Thus, the productive structures giving rise to output can be seen as having two traits, their ability to demand/supply inputs, and the efficiency with which these inputs are transduced from into outputs. It is this realisation that enables an analysis of rebound effects because we can now consider the impacts of incremental increases in investment in energy efficiency on final work output and primary energy use growth rates. The second conclusion is that, by definition, all output of final useful work is being invested into the creation of productive structure. Put another way, there is no room in this framework for final useful

1 Here *useful* does not mean good, but simply the physical ability to do work. Likewise, *productive* also does not mean good, but simply the ability to facilitate the future flows of final useful work.

work to be simply 'consumed,' for example by households as in conventional economic accounting, in ways that have no bearing on the future flows of final useful work. Waste is simply the fraction of the primary energy flow not translated into final useful work.

Because productive structures are necessarily space-filling networks, the economy can be viewed as inhabiting a spatial domain (Jarvis et al., 2015). If the efficiency of these networks is captured through an energy efficiency metric then, by definition, the inputs of primary energy must be a metric largely devoid of efficiency traits, therefore containing only information on the size of the domain being occupied by the networks comprising the economy. As a result, we will often refer to primary energy flow as a surrogate for size of the domain occupied by the global economy. One way of conceptualising this is through acknowledging that energy resources are distributed throughout the earth system and hence are captured by the economy growing into them. If primary energy flows relate to the size of the economy, then energy efficiency relates to the complexity of the structures within this domain.

In this paper we develop a model based on the premise that GDP measures the rate the economy performs useful work on itself and its surroundings. This useful work is invested exclusively into the creation of productive structures that affect both the rate primary energy enters the economy and the efficiency with which this is transduced into useful work. Section 2 describes this model, and Section 3 describes the data analysis that derives parameter values for the model using global GDP and PEU data 1900 - 2018. Finally, Section 4 applies the model to estimate the magnitude of rebound effects associated with differing levels of investment in energy efficiency.

2. The model

Core assumptions

A1: GDP is a proxy for the rate the economy performs final useful work on itself and its surroundings, and thus GDP and useful work are linearly related.

A2: The final useful work being performed by the economy is invested exclusively into the creation of productive structure.

A3. Productive structures have two distinct traits that can be developed independently through the investment of output: their thermodynamic efficiency, and their size and hence ability to capture primary energy.

Structure

We start describing our model by recasting the identity $GDP = EEE \times PEU$ into its energy physics equivalent. The system diagram for the model is given in Figure 1.

Let the flow of primary energy into the economy be x , and the flow of final useful work resulting from this be y such that

$$y = \eta x \quad (1),$$

where η is the overall thermodynamic energy efficiency of the economy in converting primary energy into useful work. Building on assumption A1, $\text{GDP} = \lambda y$, where λ converts from energy units into the equivalent monetary unit and is assumed constant providing GDP is expressed in constant units. Now $\text{EEE} = \lambda \eta$. For familiarity, and because we exploit output data in monetary units, we retain monetary units for both output and accumulated investments. For completeness we note $(1-\eta)x$ is the rate primary energy is dissipated when traversing the economy, although the rate the economy sheds waste heat is marginally higher than this if we also take into account the subsequent decay of its structures.

We now define two stocks which derive from investments of useful work into developing either η or x . Firstly, let us consider the stock of work attributed to the energy efficiency of productive structure, K_η , which we assume is given by the first order conservation

$$\dot{K}_\eta = wiy - d_\eta K_\eta, \quad (2a)$$

where i is the proportion of output invested overall in developing productive structure, w is the proportion of that investment that goes into developing the efficiency of productive structures, and d_η is the representative decay rate for these structures.

The stock of work attributed to filling space and hence capturing primary energy, K_x , is given by the first order conservation,

$$\dot{K}_x = (1-w)iy - d_x K_x \quad (2b)$$

where, again, d_x is the representative decay rate for this stock.

Following assumption A2, there is no such thing as household consumption or government spending as conventionally defined, rather all output of final useful work is invested in the creation of productive structure and hence $i = 1$. To adopt this perspective it is necessary to appreciate the full spectrum of service lives of productive structures in the economy, which range from seconds (e.g. the effects of lighting a room on our memory) through to centuries (e.g. the effects of blasting a tunnel to support a rail link). National account methodologies artificially impose a ~ 3 year cut-off between definitions of capital stocks and consumer goods (Chester et al., 2022), and it is this that artificially gives rise to consumption simply because it cannot be attached to the creation of short-lived productive assets. It is clear from everyday experience that the economy is awash with < 3 year productive structures that produce future returns (e.g., a short-lived toy enables child brain development that lasts a lifetime) just like their longer lived counterparts, even if we currently believe we consume these structures. From this argument $d_{\eta,x}$ represents the full spectrum of turnover timescales in the economy and not simply

the <3 year structures. As a result, we estimate $d_{\eta,x}$ as free parameters rather than pre-assuming their value as would be the norm. We also expect $d_{\eta,x}$ to be larger than would be traditionally employed in capital accounting because it necessarily includes the effects of <3 year structural turnover.

We close our framework by relating the stocks K_η and K_x to the primary energy use and energy efficiency traits for which they were invested. We anticipate return to scale effects associated with increasing the quantity of productive structure dedicated to either trait such that

$$\eta = a_\eta K_\eta^{b_\eta} \quad (3b)$$

$$x = a_x K_x^{b_x} \quad (3a),$$

where $a_{\eta,x}$ and $b_{\eta,x}$ are scaling parameters.

For primary energy use, a range of mechanisms can be invoked for return to scale effects. Firstly, there is the obvious tendency to pick the lower hanging fruit first when harvesting energy resources, hence requiring ever increasing investments to access future supply. Given energy resources are spatially distributed in the earth system, another way of seeing this effect is that distribution costs necessarily increase over time as the economy expands and energy resources become more distant from where they are converted into work (Jarvis et al., 2015; Jarvis 2018). This argument extends to energy demand since increasing the number of end-use service units in a system can also be seen to expand the domain of that system and hence the mean path length of distribution networks (Banavar et al., 1999). Finally, the metabolic scaling literature indicates that the interface across which energy flows into a system can have a lower dimensional scale to the system itself, and this necessarily creates sub-linear scaling effects (Banavar et al., 2010, Ballesteros et al., 2018).

Efficiency should also experience declining returns to scale because energetically more efficient systems tend to be structurally more complex (Ruzzenenti and Basosi 2008), requiring higher levels of investment to find, make and maintain future configurations. Specifically, finding more efficient, information rich configurations of the economy becomes progressively harder as the probability of discovering these 'better' (lower entropy, more structured) configurations declines. These diminishing returns are exacerbated by the fact that any particular configuration represents significant lock-in of investments, such that the search for better configurations is always restricted by what Kauffman (2002) would refer to as "the adjacent possible".

Through combining equations (1), (3a) and (3b) we get $y = a_\eta K_\eta^{b_\eta} a_x K_x^{b_x}$, which shares some structural similarity to an orthodox two factor production function (Solow, 1956). However, rather than having labour, capital and total factor productivity as factors driving output, the net accumulation of investments of useful work in the creation of productive structure are partitioned into the fully endogenous evolution of thermodynamic efficiency and size, which we argue are fundamental traits of the

associated networks. These networks necessarily include people who, like machines and buildings, both fill space and determine the efficiency productive structures.

Returns on investment and rebounds

The scaling relationships in equation (3a,b) are fundamental to our analysis because at any point in time they provide differential sensitivities of output returns to inputs of investment. This provides society with choices to affect output growth through how it invests in either using more primary energy, or becoming more efficient with that energy, as defined by w . The aim initially will be to find what pattern of investment is consistent with observations of GDP, PEU and EEE, 1900 - 2018, and having done so to interpret this pattern retrospectively through exploring the associated returns on these investments. We define Return On Investment (ROI) as the cumulative change in output, $\Sigma \Delta y$, from an additional increment of investment of output to increase either primary energy or efficiency, Δy_K . From equations (2) & (3) we derive the ROIs for investments in either η or x as

$$\frac{\Sigma \Delta y}{\Delta y_{K_\eta}} = \frac{b_\eta a_\eta K_\eta^{(b_\eta-1)} x}{d_\eta} \quad (4a), \text{ and}$$

$$\frac{\Sigma \Delta y}{\Delta y_{K_x}} = \frac{b_x a_x K_x^{(b_x-1)} \eta}{d_x} \quad (4b),$$

respectively². This definition for ROI predicts all future returns from a unit investment in either η or x if the current background states persist at their current level for the expected lifetime of the investment. Of course, these states will not stay constant, and so this definition of ROI is only the current view of the future performance of an investment if the economy remained at its current level. Although limited, this is probably the best any investor could hope for in practice given their inability to predict the future path of the global economy accurately. From equation (4a,b) we can see that ROIs are nonlinear in the stocks of useful work, and they depend on current levels of primary energy use and efficiency. This means the overall ROI of the economy, and hence its growth rate, can be manipulated simply by switching investment away from the lower ROI factor (i.e. varying w), and it is this switching in the model that accounts for past variations in the growth rates of PEU, GDP and EEE 1900 - 2018.

The nonlinear behaviour of the ROIs also means that switching investment from primary energy to efficiency induces nonlinear rebound effects. Immediately following a shift in investment to efficiency, primary energy inputs are starved of investment and hence its growth rate declines in what might be termed the 'direct effect' of the efficiency investment (Sorrell et al., 2009; Figure 3 inset). However, this causes efficiency to grow and hence so too do the returns to primary energy (equation 4b), in turn causing primary energy to regrow in what might be termed the indirect or rebound effect (Sanders, 1992; Borenstein, 2015). Again, this is a highly nonlinear response with the relative magnitudes of these direct and indirect

² These are both derived from the forward path gains identified in Figure 1.

growth effects depending on the background levels of primary energy and efficiency investment.

Following Bruns et al., (2021), and in line with wanting to explore rebound effects in relation to growth, we define the size of rebound effects as the ratio of the relative growth rates of primary energy, r_x , to that of efficiency, r_η . Specifically, we apply a locally linear perturbation in efficiency investment to our model, track the subsequent changes in r_x , and normalise these on the initial change in r_η the efficiency investment perturbation induced.

The remainder of the paper calibrates equations 1 – 3 using a unified 1900- 2018 global GDP, PEU and EEE dataset, and then looks to interpret the result with a specific focus on rebound effects. We end by using the calibrated model to explore two end-member scenarios through to 2100 with an eye on the implications of these forecasts for current energy, economic and climate policy. The first scenario maximises GDP growth, the second minimises PEU and hence emissions growth.

3. Model calibration and interpretation

GDP and PEU data 1900 - 2018

Given the range of available GDP and PEU observations, and the sensitivity of regression results to the particulars of these data, we have elected to produce a single, homogeneous PEU and GDP series which blends the available series listed in Table 1. The eight global GDP series used in this study are a compilation of available, reputable inflation adjusted (constant) data. To reconcile the fact that these data do not have a consistent base unit and compilation method, all GDP series were linearly scaled to the World Bank (WB) constant (2010) market exchange rate (MER) series. This only serves to homogenise units and give each series equal weight when averaging. Similarly, the four global PEU series were linearly scaled to the International Energy Agency (IEA) data, again to reconcile unit differences and methods of compilation.

All analysis is based on the annual averages of the eight GDP and four PEU series listed in Table 1. The final PEU, EEE and GDP series are shown in Figure 2i along with their associated relative growth rates in Figure 2ii. Both GDP and PEU grow throughout the period whereas EEE is somewhat stagnant up until the 1970's, after which time it grows steadily. The emergence of growth in EEE after the 1970's results in a relative decoupling of GDP and PEU growth. Prior to this, GDP and PEU growth was approximately 1:1. GDP and PEU growth peaked in the 1950's and 60's and the increases in EEE post 1970 appear to be correlated with steady declines in GDP and PEU growth. Currently, GDP growth appears to be comprised of near equal measures of EEE and PEU growth (Figure 2ii).

Fitting and model performance

Assuming λ is constant, we fit equations 1, 2, and 3 to the log of the EEE, PEU and GDP data shown in Figure 2i using a standard Levenberg-Marquardt non-

linear least squares algorithm, minimising autocorrelation in the model residuals assuming these to be AR(1).

As raised earlier, w , or the relative proportion of final useful work invested in efficiency, represents the decision variable in the analysis. Therefore, the aim is to capture how this changes over time. When fitting to the 1900 - 2018 data we find this proportion changes significantly either side of WW2 (see Figure 2.iii), with the pre World-War 2 (WW2) regime characterised by high levels of investment in efficiency, and the post WW2 regime prioritising investments in primary energy. Specifically, it is this shift in investment from efficiency to primary energy post WW2, in conjunction with the associated ROI's of these investments, that accounts for the rapid acceleration of output in the 1950's and 60's (see below). We parameterise this change to bring it within the fitting process assuming the following smooth transition

$$w(t) = w_1 + \frac{w_2 - w_1}{1 + e^{-k(t-t_{50})}} \quad (5),$$

where w_1 is the level of efficiency investment before the transition, w_2 is the level after the transition, t_{50} is the year associated with a 50 % change and k the rate constant for that change.

The raw error series appear to be non-constant variance pre v. post WW2. As a result, we weight the errors by dividing by their pre and post WW2 standard deviations prior to decorrelating and minimising. The errors being minimised are shown in Figure 3.

Despite having 12 free parameters (Table 2), four of which simply characterize variations in w , the model converges and the parameter-error space suggests uniqueness in the optimised parameter values (Figure 3). The unfiltered model residuals give a mean absolute error of just 3.46 % for GDP, 3.26 % for PEU and 2.07 % for EEE. All three series of residuals have zero mean, but significantly auto and cross-correlated (Figure 3). The AR(1) pre-filtering removes all significant short-run autocorrelation from the residual series, but some significant longer run autocorrelation was apparent suggesting the presence of longer cycles in these annual data. The weighted residuals appear to be near constant variance (Figure 3), and each passes an Anderson-Darling test for normality ($P < 0.05$). The estimated parameters are given in Table 2 along with an estimate of their 90th percentile ranges.

Depreciation rates

We estimate the depreciation rates for K_x and K_η to be 9.02 (6.27 - 11.77) %/yr and 13.00 (10.07 - 15.93) %/yr respectively. These are higher than one would expect for economy-wide capital, which is generally ascribed aggregate depreciation rates in the range 3 to 5 %/yr (Chester et al., 2022). We reconcile this difference by pointing out that all output is necessarily being invested in our framework and, as such, what would traditionally be considered as consumption is behaving as short-

lived productive structure that will lower the aggregate depreciation rate relative to the conventional assumption that investment in capital is nearer to 25% of output. If aggregate depreciation rates represent the first moment of the inverse of the turnover timescale of capital pools (Chester et al., 2022), then the turnover timescales for K_x and K_η are 11.09 (9.23 - 13.57) yrs and 7.69 (6.73 - 8.96) yrs respectively. That K_η is shorter lived on average than K_x is in line with the theory that efficiency closely aligns with the more transient information state of productive structure.

Our model predicts total capital as $\lambda(K_\eta + K_x)$, which is the outcome of the investment of final useful work into either the efficiency or size of productive structure, less decay. In comparison, orthodox wealth accounting partitions investment into produced and human capital (World Bank, 2022). Comparing the two we find our total capital estimate is 71% that of the World Bank figure (Figure 2i; World Bank 2022). We suggest that this difference is largely the product of the estimated decay rates of productive structure, which are significantly larger than what might be assumed for either the produced or human capital comprising the World Bank total. We also note human capital is a forward looking valuation over working lifetimes, and hence the product of a somewhat uncertain long range forecast.

Because the World Bank total capital is comprised of approximately one third produced capital and two thirds human capital (World Bank, 2022), if we simply take the concept of human capital literally, by considering it a stock that accumulates from an investment of output in the same manner as produced capital such as machines and buildings, then providing produced and human capital have similar representative turnover timescales, investments in human capital must be approximately twice those in produced capital. This suggests the fraction of output being invested in total capital creation is at least three times that assumed in orthodox growth modelling i.e. much closer to the $i = 1$ assumed here for productive structure creation.

Scaling relationships

Figure 4i shows the two estimated scaling relationships for equation (3). As predicted, the observed scaling is sub-linear for both primary energy use and efficiency at $\eta \propto K_\eta^{0.362}$ and $x \propto K_x^{0.617}$. However, the sum of the two scaling exponents is only marginally less than one, at 0.979 (0.956 - 1.001), indicating GDP output is close to linear in total capital investment, even if it is highly nonlinear in each factor. This estimated net linearity is not so surprising given the global economy grew consistently throughout the 118 years the model was constrained on, and that such behaviour is not surprising in a system so focused on maintaining output growth, where there must be strong selective pressure to develop constant return to scale economic structures such that growth is maintained. This form of linearity is a common theme in macroeconomic growth models, where constant aggregate returns to scale across factors of production are assumed (Krugman and Wells, 2015). However, even though the effect is small, there are important ramifications if the relationship between output and total capital is sublinear

because it implies that the economy actually has net declining returns to scale over long timescales such that growth cannot be sustained. After all, the global economy has experienced a relatively stable investment regime for approaching five decades now (Figure 2iii) and output growth has been falling throughout (Figure 2ii).

If useful work is expended to create productive structures and these structures are necessarily space filling networks, then because K_x is, by definition, devoid of efficiency, it should align with the size of the space occupied by the economy. K_x describes the size of the spatial domain of both the supply side (primary energy resources being harvested) and the demand side (the domain occupied by the sum of all units of demand). Figure 4ii shows the relationship between K_x and the mass of the global economy as estimated by Krausmann et al., (2017). This relationship is close to linear, supporting the view that K_x is a proxy measure for domain size if the mass density of the domain of the global economy is somewhat conserved. Assuming $\lambda = 1.4$ \$2010/MJ (see below), for the period 1900 - 2010 we estimate that density to be, on average, 1.65 ± 0.45 Pg/EJ. The inverse metric, 0.63 ± 0.14 EJ/Pg (kJ/g) indicates that approximately 0.63 EJ of useful work output is needed to accumulate 1 Pg of mass in economic structures (e.g., buildings, railways, people).

The scaling $x \propto K_x^{0.617}$ signals the penalties associated with increasing domain size are significant, penalties due to, among other things, increasing the mean path length of distribution networks within the economy (Jarvis et al., 2015). However, $\eta \propto K_\eta^{0.362}$ signals the penalties on increasing efficiency are approximately twice that of primary energy, underscoring the difficulties associated with finding and developing more efficient networks. Perhaps more importantly though is the fact that this scaling on efficiency appears just large enough to raise the economy to near linear scaling overall. This underscores the importance of efficiency improvements in maintaining output growth, which highlights the likelihood of rebound effects being significant (as discussed in Section 4).

Historical narrative

Despite the possibility of near constant growth, what we observe in historical data are relatively radical variations in the growth rate of GDP on a broad range of timescales (Figure 2ii). We identify three growth regimes. Pre-WW2 output growth is low and volatile. Post WW2 and pre-1970 is marked by a sustained period of increasing output growth in what has become known as the Great Acceleration (Stefan et al., 2015). Finally, the post-1970 era is characterised by modest deceleration of output trending toward the secular stagnation of developed economies (Summers, 2015). Our model offers the following account for these regimes.

Pre WW2, investment in efficiency accounts for approximately 70% of the total output (Figure 2iii). However, the returns on these efficiency investments are poor (ROI $\eta < 1$; Figure 2iv) such that total capital is actually falling or stagnant throughout the pre-WW2 period (Figure 2i). It is interesting to note that this period of shrinking or stagnating total capital and low overall returns on investment is

correlated with the era of extreme volatility in output growth, the great recessions/depressions and two world wars. Such volatility would not be helped by the higher turnover rates associated with the efficiency-orientated capital which was dominating the global economy at this time (Figure 2i).

In contrast to ROI_{η} , pre-WW2, $ROI_x \approx 2.5$ (Figure 2iv), so shifting investment into PEU represents a significant opportunity to increase output growth. Although investors appear slow to realise this, unsurprisingly investment in PEU eventually rises significantly from the 1940's onwards (Figure 2iii) and, as a result, the global economy experiences rapidly increasing output growth (Figure 2ii) in what might be referred to as a wave of globalisation given the space filling character of this investment. Here, increases in output growth are supported almost exclusively by the rate of increase in PEU investment allied to the relatively large ROI of these investments (Figures 2iii and iv). However, exploiting this opportunity also undermines the returns of this strategy, and by the 1960's these returns approach those for efficiency (Figure 2iv). Somehow this state must have been experienced in a very real way because investments in PEU stop increasing as ROI_{η} and ROI_x approach parity in the 1960's (Figure 2iii and iv).

By the 1970's, ROI_{η} exceeds that of ROI_x and, not surprisingly, the fraction of investment into PEU stops rising, prompting modest increases in efficiency growth leading to some relative decoupling of PEU and GDP (Figure 2ii and iii; Csereklyei et al., 2016). It is noteworthy that this shift coincides with when the global economy experiences a series of energy crises that are also seen to promote increased interest in energy efficiency, and we suggest that these crises derive in part from the state $ROI_{\eta} > ROI_x$ being reached. King (2022) suggests this transition to relative decoupling is a consequence of slowing growth in the face of resource (e.g. PEU) constraints.

Because of the persistently high levels of investment in PEU, ROI_x has fallen consistently since the 1940's (Figure 2iv), mirroring observed declines in the energy ROIs (EROIs) of specific energy sources (Brockway et al., 2019). These are commonly ascribed to the depletion of the more energy dense forms of these sources (Hall et al., 2014), an account mirrored here when describing the reasons behind the sub-linear scaling in equation (3b). However, EROIs are usually defined in terms of returns of cumulative primary energy per unit primary energy invested, not final output returns on a unit of final output investment, and the declines in output ROIs observed here are also driven by stagnation in efficiency improvements (equation 4b). Currently, ROI_x is close to one, a condition that has persisted for at least the last two decades (Figure 2iv). We argue this underpins the secular stagnation currently experienced in developed economies. In contrast, $ROI_{\eta} > 2$ (Figure 2iv), and it is this opportunity, we argue, that is driving the increased interest in efficiency-led investments as a means of tackling secular stagnation, rather than any desire to reduce the environmental burden of the economy.

4. Rebound effects

Because of the nonlinear dynamics of the model, we explore rebound effects through simulation, applying a small (10^{-6}) perturbation to efficiency investment and tracking the subsequent growth rate of PEU. We repeat this across a range of dynamic equilibrium states corresponding to differing levels of efficiency investment, w . Figure 5i shows the perturbation result for our estimate that $\sim 18\%$ of investment went to efficiency in 2018. Here, rebound is measured by the perturbation in relative growth of PEU normalised on the initial perturbation in relative growth of efficiency, i.e. how much PEU growth is associated with a unit disturbance in efficiency growth.

From Figure 5i we see that, despite the nonlinearity of the model, the PEU growth perturbation is locally linear and comprised of a simple two phase response. The first phase is the immediate (direct) increase in efficiency growth and reduction in PEU growth driven by the marginal shift in output investment from PEU to efficiency. The second (indirect) phase has a time constant of approximately 15 years (saturating in about 30 years) and represents a recovery (rebound) in PEU growth driven by increasing returns to PEU, in turn driven by increasing efficiency (equation 4b). For 2018 we estimate the direct effect to be a 28% reduction in PEU growth and the indirect (rebound) effect a 58% increase in equilibrium, giving an overall backfire of +30%. These estimates of present-day rebound are significantly more than the economy-wide rebounds reviewed by Brockway et al., (2021), but close to the rebound estimates of Sakai et al., (2019) using an explicit energy-economy model and Bruns et al., (2021) using a vector autoregression analysis of PEU and GDP data. They are also consistent with the theoretical result from the King (2022) biophysical stock and flow consistent macroeconomic growth model.

Figure 5ii also shows the response surface for rebound with respect to background efficiency investment is highly nonlinear, even in a relatively simple model like the one developed here. At low levels of efficiency investment, direct effects are small and rebound effects large, hence backfire dominates because the ROI for efficiency is high (equation 4a). However, above $\sim 40\%$ efficiency investment direct effects start to dominate such that there is negative net growth in PEU by efficiency investments above this level because the ROI for efficiency necessarily declines.

Future scenarios

If the global economy was under leveraged on primary energy pre-WW2, then it appears heavily over leveraged currently, given returns to efficiency appear more than twice those of returns to primary energy in 2018 (Figure 2iii). Furthermore, because $\text{ROI}_x \approx 1$ currently, all growth in the economy, including growth in PEU, must actually be derived from current levels of efficiency investments. To explore the extent efficiency can be leveraged to further support output growth, we search for a pattern of efficiency investment that maximises output growth 2018 - 2100 (the “maxGDP” scenario). From Figure 2iii we see that this requires doubling investment in efficiency, from 18% of output in 2018 to a little more than 40%. This highlights how increases in efficiency investment are likely to become a

central component to maintaining, or even boosting, output growth in the coming decades. We suggest investors are becoming increasingly aware of the growth-enhancing effect of efficiency investment, and this is the primary motive behind the so-called 'green growth' agenda. However, as Figure 5ii shows, the direct effect from any such investment is fully abolished by PEU rebound such that PEU growth continues at or near its current rate in this scenario (Figure 2ii).

Scenarios used to inform negotiations on climate change generally call for a transition to zero growth in PEU by mid-century in order to honour the Paris Agreement (e.g. Riahi et al., 2017). To explore whether this is indeed possible we search for the pattern of efficiency investment that makes PEU growth zero by mid-century (the “minPEU” scenario). For this we require radical proportions of investment in energy efficiency that transition to 75% of output by 2070 (Figure 2iii and 5ii). At this fraction of efficiency investment, additional efficiency investments result in declining, but positive, rates of growth of PEU in the short run to 2050, and rebound is significantly less because the ROI of efficiency is now low (equation 4a). As a result, this growth appears to be legitimately green with respect to long-term PEU and climate objectives (i.e. PEU stops increasing). However, it also leads to zero output growth in the long term as the ROI of efficiency evaporates, partially recreating the state that existed pre-WW2 (Figure 2i and ii).

In summary, this highly nonlinear growth effect from the fraction of investment into efficiency is very unintuitive. A modest fraction of investment toward efficiency leads to relatively large PEU growth via the rebound effect, but a large fraction of investment toward efficiency leads to relatively small PEU growth (and eventual stagnation).

The model contains no hard limits, either on domain size or thermodynamic efficiency. There are just significant diminishing returns to scale on each. In reality, the economy is subject to physical limits to growth through constraints on both the physical size of its planetary home, the resources therein, and the thermodynamic limits on efficiency any system can ultimately achieve. To explore these hard limits we attempt to reconstruct, albeit speculatively, both efficiency, η , and the fraction of available space, f , of the economy. Jarvis (2018) and Warr et al., (2010) estimate that the global economy is currently somewhere near 10% efficient at translating primary energy into final useful work, while Ritchie and Roser (2013) speculate that humans have appropriated approximately 30% of the available space on earth. We assume these as our 2018 initial conditions and apply the observed/simulated relative growth rates for EEE and PEU from 2018 to reconstruct both η and f pre/post 2018. Assuming $\eta = 0.1$ in 2018 is equivalent to assuming $\lambda = 1.4$ (2010\$)/MJ for the global economy, which compares to Serrenho et al.'s, (2016) estimate of $\lambda = 1.2$ (2010\$)/MJ for Portugal and Warr et al.'s (2010) estimate of $\lambda = 0.8$ (2010\$)/MJ for the US.

Figure 2iii indicates that, for the maxGDP scenario, η rises to ~40% by 2100 and is still growing, with the economy running out of physical space (i.e., $f > 100\%$) by around 2080. For the minPE scenario, efficiency and filled space stabilise at ~35 %

and ~50% respectively by 2100 (Figure 2iii) i.e. the spatial footprint of the economy is similar to that of today, but efficiency has risen more than three-fold. The available portfolio of energy saving technologies appears substantial (Grubler et al., 2018), as does the opportunity to exploit artificial intelligence to co-ordinate the selection and development of more efficient configurations of the economy. As a result, significant increases in present-day efficiency appear within reach, although we may also be describing an economy too complex for people to engage with. Carnot also tells us the thermodynamic limit will be substantially less than 100% and, just like running out of physical space, this represents a hard boundary. Thus, we might suspect the maxGDP result is not achievable through to 2100 due to both the spatial and efficiency constraints, whereas the minPEU scenario might only be constrained on efficiency. Any approach to hard boundaries in either size or efficiency would be experienced through additional, rapid decreases in returns to scale and hence output growth.

6. Conclusions

The question motivating this research was whether the current green growth narrative was a fallacy in relation to climate objectives. Our conclusion is that, as currently practised, it most likely is. If growth remains the objective of economic policy and practice then our analysis indicates modest efficiency improvements will become central to achieving this objective. Under the current investment regime rebound effects look likely to swallow up any of the planned climate dividends of reducing carbon emissions via efficiency investments. If we were genuinely interested in using efficiency improvements to play a credible role in our collective attempts to avoid dangerous climate change, we need to explore radically higher efficiency investment regimes, because these appear much less prone to rebound effects. This strategy, however, would also be associated with implicitly abandoning growth as a long-term objective, even though in the medium term growth is enhanced by this strategy. Such a strategy cannot be seen as problem free as it may also recreate some of the conditions that prevailed pre-WW2.

Just as it did in the 1970's, the 2022 energy crises reminds us that we are fundamentally linked to the physical realm through flows of energy from nature to our economy. Any rethinking this motivates should not simply focus on dampening turbulent energy markets, for a similar recalibration on energy is needed to help us better engage with the task of reducing energy-related emissions. If we are to rethink the role of energy in our lives it also feels appropriate that we recast the models we use to resolve the spaghetti of economic interactions that often frustrate our understanding. We take our efforts here as our first approximation of this.

Acknowledgements

We would like to thank Cormac Lynch for expert assistance with compiling the *GWP* and *PEU* data sets and Paul Brockway, Charles Hall and Peter Haff for

helpful feedback on earlier drafts of this paper. AJ is also grateful of financial support from the UK Economic and Social Research Council (ESRC) via the Rebuilding Macroeconomics Network, and the UK Natural and Environmental Research Council (NERC) via the AMDEG project. CK received no financial support for this research.

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Table 1. Global Primary Energy Use (PEU) and constant Gross Domestic Product (GDP) data sources used in this study.

variable	cover	source (as of 21/08/2019)
GDP		
World Bank <i>GWP</i> (PPP 2011 USD)	1990 - 2018	https://data.worldbank.org/indicator/NY.GDP.MKTP.PP.KD
World Bank <i>GWP</i> (MER 2011 USD)	1960 - 2018	https://data.worldbank.org/indicator/ny.GDP.mktp.kd
United Nations (2010 USD)	1970 - 2017	https://unstats.un.org/unsd/amaapi/api/file/6
Penn World Tables (Expenditure PPP 2011 USD)	1950 - 2017	http://febpwt.webhosting.rug.nl/Dmn/Templates/Execute/53
Penn World Tables (Output PPP 2011 USD)	1950 - 2017	http://febpwt.webhosting.rug.nl/Dmn/Templates/Execute/54
Penn World Tables (National-accounts 2011 USD)	1950 - 2017	http://febpwt.webhosting.rug.nl/Dmn/Templates/Execute/47
Maddison (CGWP 2011 USD)	1900 - 2016	https://www.rug.nl/ggdc/historicaldevelopment/maddison/data/mpd2018.xlsx
Maddison (RGWP 2011 USD)	1900 - 2016	https://www.rug.nl/ggdc/historicaldevelopment/maddison/data/mpd2018.xlsx
PEU		
International Energy Agency (EJ yr ⁻¹)	1970 - 2016	https://webstore.iea.org/world-energy-balances-2018
British Petroleum (Mtoe yr ⁻¹)	1965 - 2018	https://www.bp.com/content/dam/bp/business-sites/en/global/corporate/xlsx/energy-economics/statistical-review/bp-stats-review-2019-all-data.xlsx
International Institute Applied Systems Analysis (EJ yr ⁻¹)	1900 - 2014	http://www.iiasa.ac.at/web/home/research/researchPrograms/TransitionstoNewTechnologies/PFUDB.en.html
Energy Information Administration (TBtu yr ⁻¹)	1980 - 2016	https://www.eia.gov/totalenergy/data/browser/xls.php?tbl=T01.01

Table 2. Model parameter estimates. Uncertainties as 90th percentiles.

parameter	units	value
λa_η	T\$/yr/TW	0.829 (0.706 - 0.953)
a_x	TW	0.390 (0.364 - 0.415)
b_η		0.362 (0.341 - 0.382)
b_x		0.617 (0.588 - 0.646)
d_η	%/yr	13.00 (10.07 - 15.93)
d_x	%/yr	9.020 (6.273 - 11.773)
λK_{n1900}	T\$	40.21 (32.24 - 48.18)
λK_{x1900}	T\$	4.274 (3.790 - 4.757)
w_1		0.265 (0.215 - 0.316)
w_2		0.821 (0.806 - 0.835)
k	%/yr	0.176 (0.162 - 0.189)
t_{50}	yr	1954.2 (1953.5 - 1954.9)

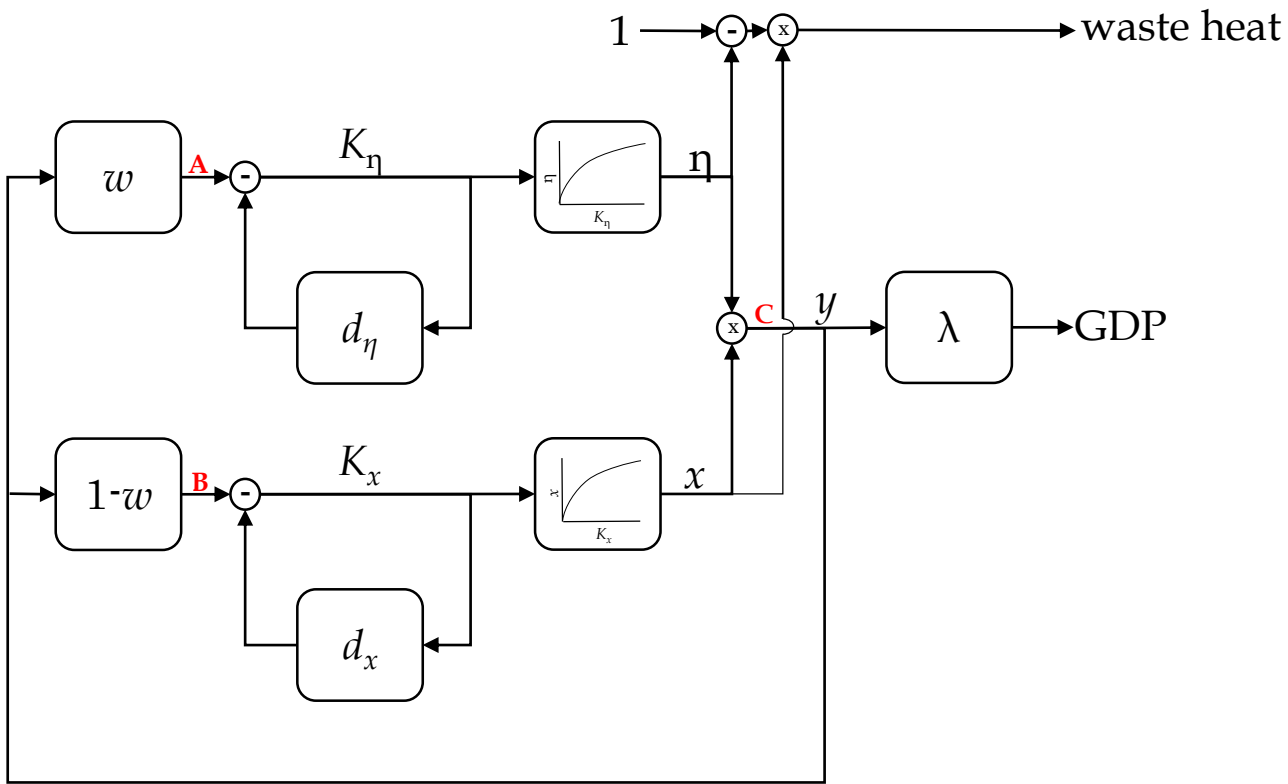


Figure 1. The system diagram for the model detailed in Section 2 (see equations 1-3 for definitions). The ROI equations (4a,b) are derived from the forward path gains A→C (ROI $_\eta$) and B→C (ROI $_x$).

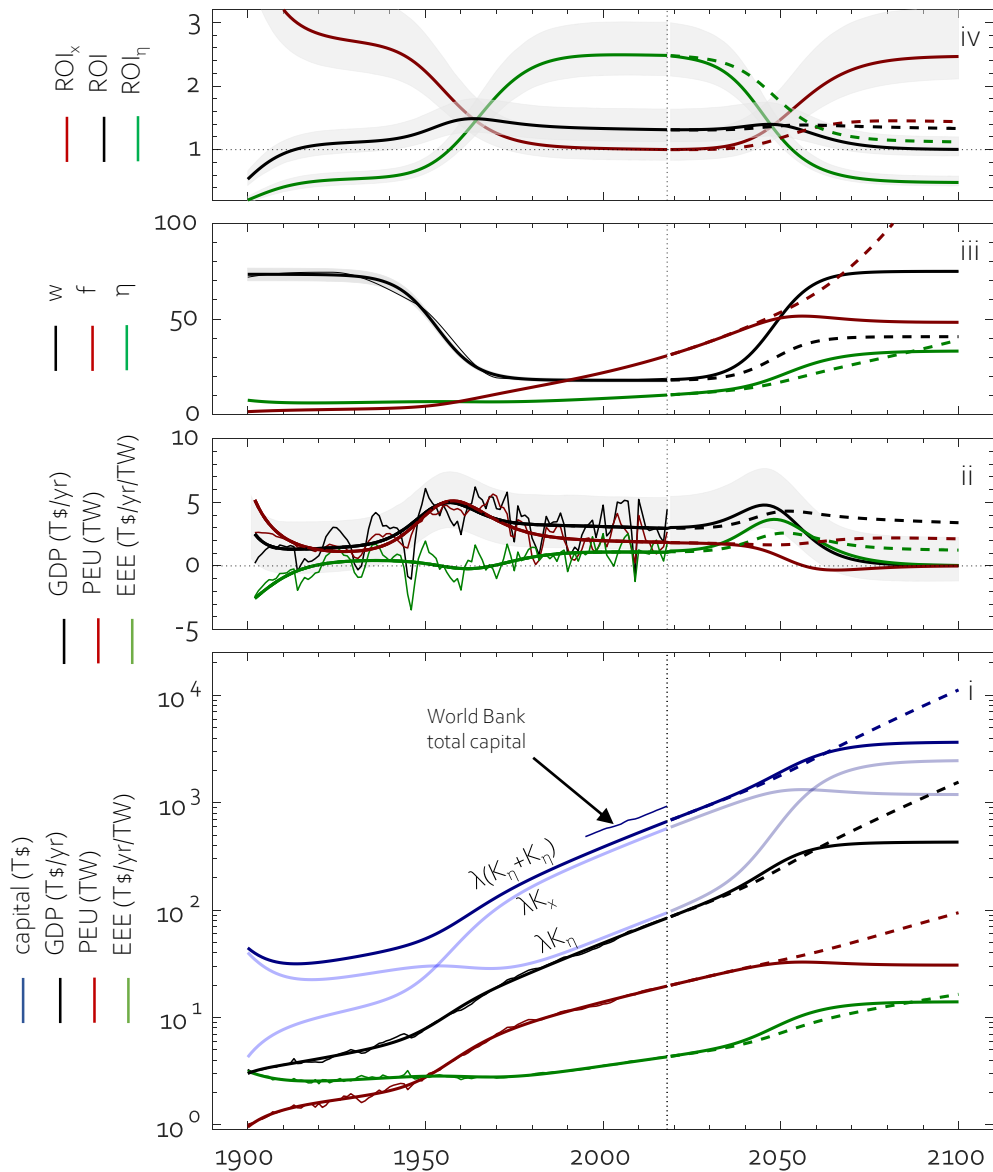


Figure 2. i. Absolute values and **ii.** relative growth rates of total capital, Gross Domestic Product (GDP), Primary Energy Use (PEU) and Economic Energy Efficiency (EEE). All dollar values expressed in constant 2010 dollars. Thin lines are data and thick are the fitted model states 1900 - 2018. Post 2018 the dashed lines are the maximum GDP growth scenario (“maxGDP”), while the unbroken line is the minimum PEU growth-scenario (“minPEU”, see text). **iii.** The estimated investment fraction of GDP into efficiency capital (w). Pre-2018 is the output of equation 5 (thick line) and its spline counterpart (thin line). Also shown are the estimates of the fraction of available space occupied by the economy (f) and thermodynamic efficiency (η) of the economy (see text). **iv.** Returns On Investment (ROIs) estimated from equation (4a and b). Uncertainty envelopes are 90th percentile ranges.

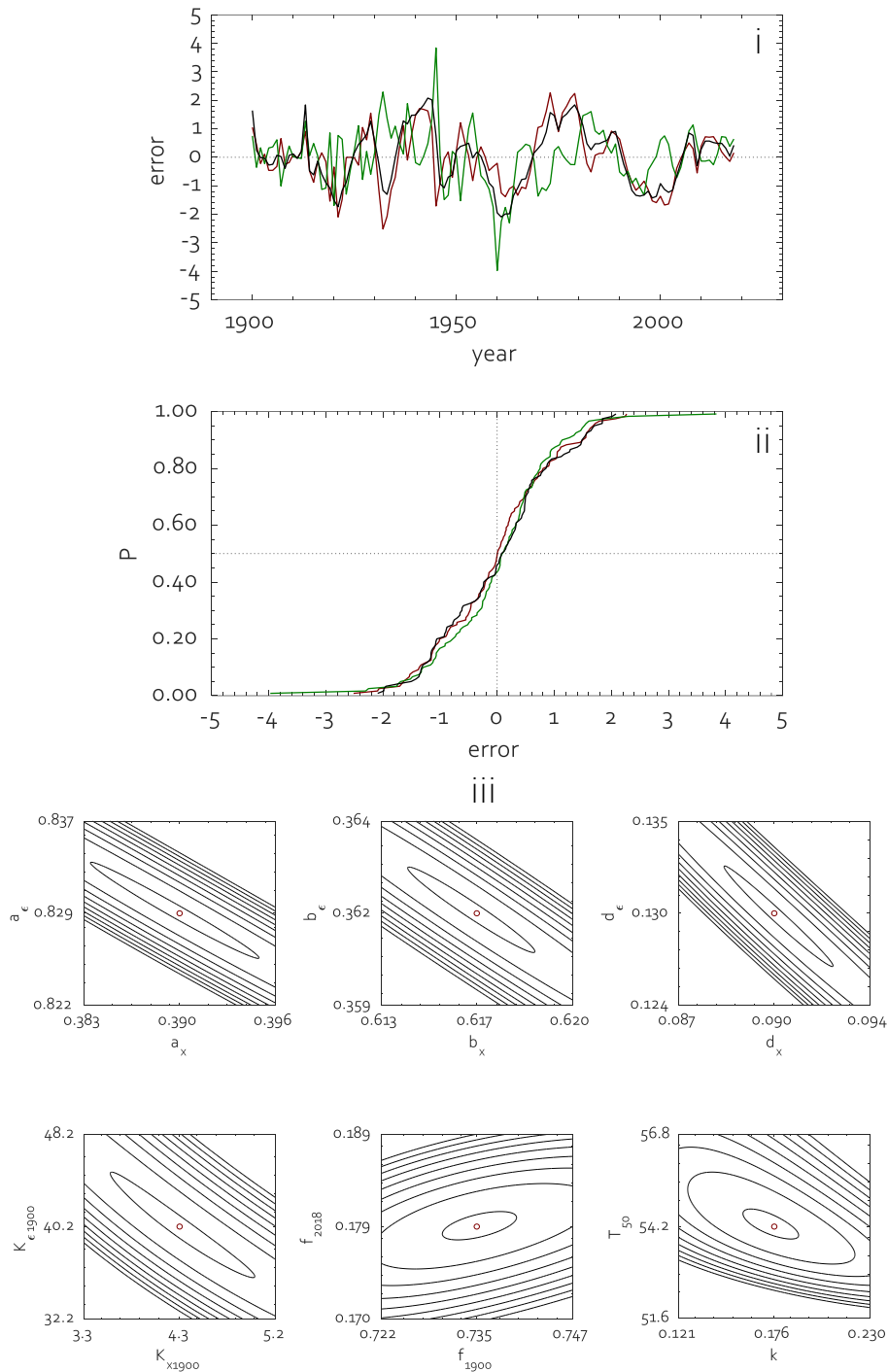


Figure 3. **i** The model error series. **ii** The cumulative probability of the model errors. **iii.** The sum of square error response surface as a function of the model parameters for selected parameter combinations.

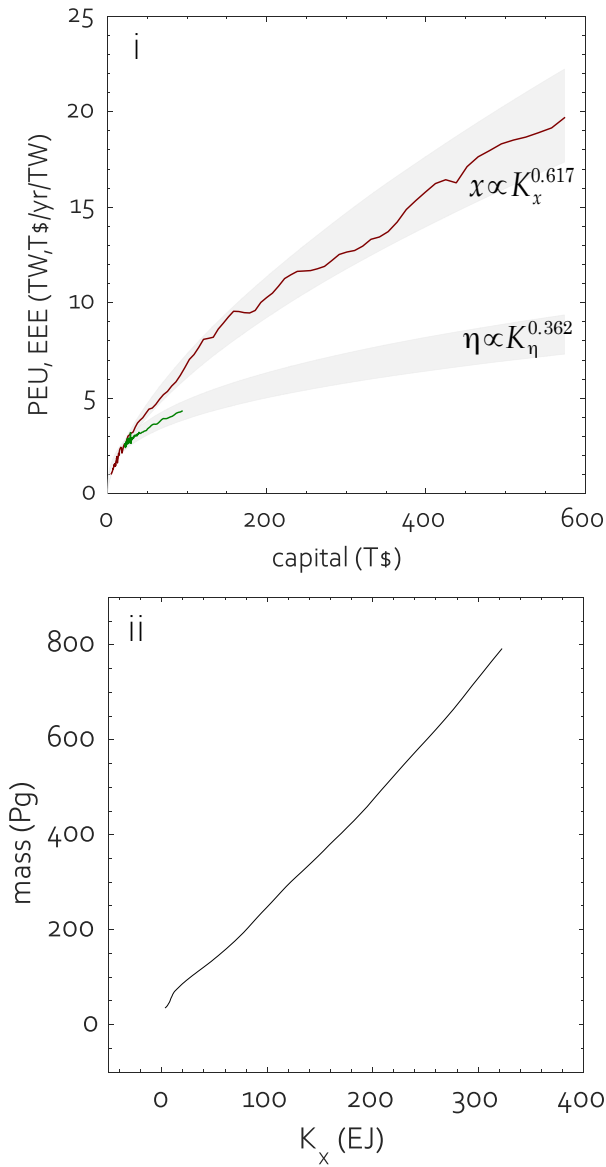


Figure 4. i. The relationships between capital and either PEU or EEE (see equations 4a & b). Uncertainty envelopes are 90th percentile ranges. **ii.** The relationship between PEU capital (K_x) and the Krausmann et al., (2017) 1900 – 2010 estimates of the total mass of the global economy. K_x is estimated assuming $\lambda = 1.4$ 2010\$/MJ to give an average slope of 1.65 ± 0.45 Pg/EJ.

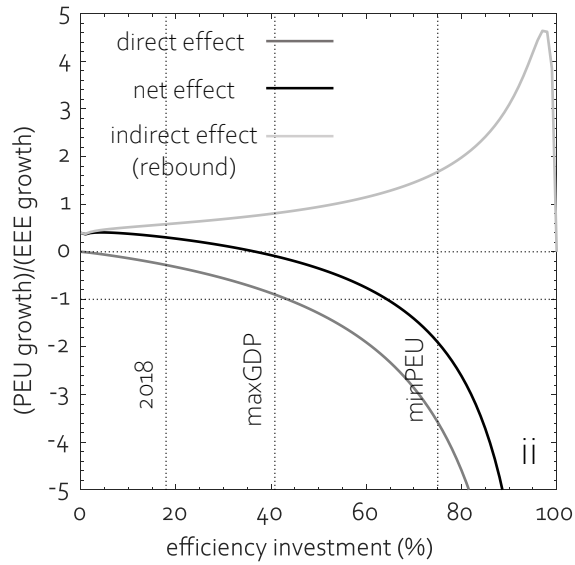
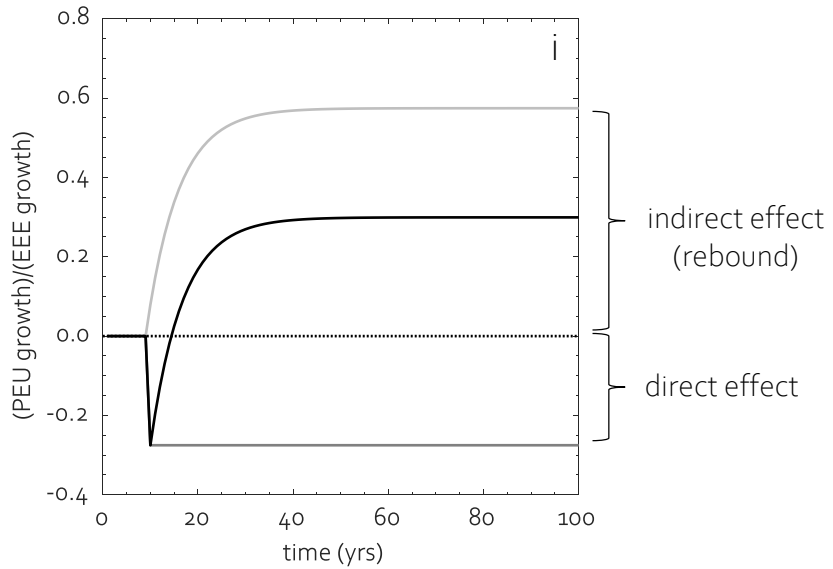


Figure 5. i. The response of the ratio of PEU to EEE relative growth rates following a small (10^{-6}) increase in investment in EEE applied to the present-day (2018) state of the economy (black). This is partitioned into its direct (dark grey) and indirect (rebound) (light grey) components. **ii.** The relationship between the direct, indirect (rebound) and net responses shown in i. given different background levels of efficiency investment. Backfire states are given by net PEU growth >0 .