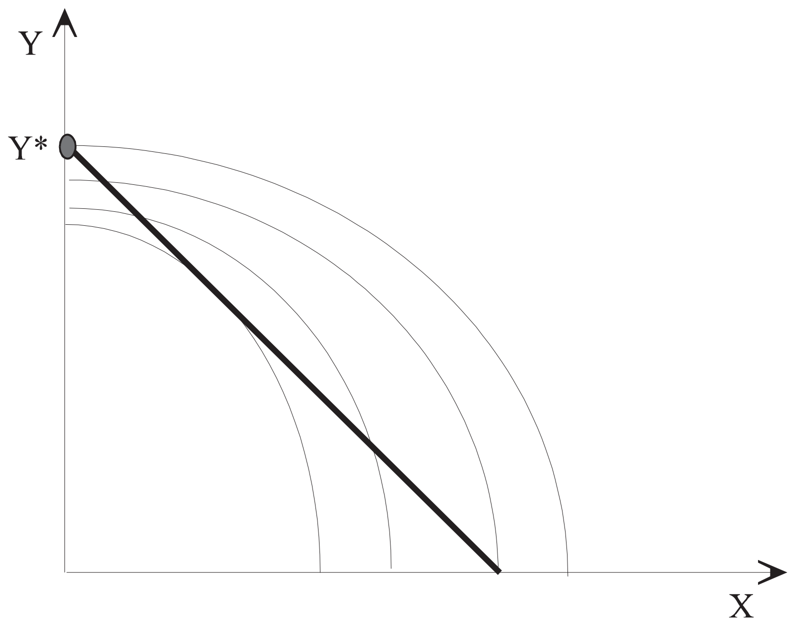
Short Questions

1. In the figure below, the budget constraint is drawn with a bold line. A set of indifference curves is drawn in regular width. Finally, utility is increasing in both *X* and *Y*, i.e., indifference curves that are further out from the origin correspond to higher utility.



On this figure, label the combination of *X* and *Y* that maximizes this consumer's utility.

It is the point marked on the figure that corresponds to a consumption of zero units of good *X* and *Y** units of good *Y*. The reason is that this point is the combination of *X* and *Y* that corresponds to the highest utility (furthest out indifference curve) that the consumer can attain while not violating his budget constraint.

2. Suppose a tourist budgets B dollars for a trip. His utility in terms of the number of days he spends on the trip, *D*, and the quality of the hotel he stays, *S*, is given by the utility function

$$U(D,S) = (\alpha + D^{\beta} + S^{\gamma})^{\sigma}$$

The daily rate in a hotel of quality *S* is $P = S^2$. There are no other expenses.

This tourist chooses *D* and *S* to maximizes his utility subject to his budget constraint.

a. Write the tourist's budget constraint.

$$B = D\, S^2$$

b. Write the Lagrangian expression associated with the consumer's utility maximization problem *in terms of the tourist's decision variables*.

$$\mathcal{L} = (\alpha + D^{\beta} + S^{\gamma})^{\sigma} + \lambda\,(B - D\, S^2)$$

Problems

1. A consumer has utility function for goods X and Y given by

$$U(X,Y) = X^{0.4}\ Y^{0.6}$$

a. What is the consumer's marginal utility for X ? What is his marginal utility for Y ?

$$MU_X = \frac{\partial\, U(X,Y)}{\partial X} = 0.4\ X^{-0.6}\ Y^{0.6}$$

$$MU_Y = \frac{\partial\, U(X,Y)}{\partial Y} = 0.6\ X^{0.4}\ Y^{-0.4}$$

b. Suppose the price of X is equal to 2 and the price of Y equal to 6. What is the utility maximizing proportion of X and Y in his consumption ? [That is, if he is a utility maximizer, how many units of X will he consume in terms of units of Y that he consumes.]

Since there is some substitutability between X and Y, utility maximization requires that the consumer increases the consumption of the two goods until the marginal utility per dollar is the same for all goods.

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} \quad \Rightarrow$$

$$\frac{0.4\ X^{-0.6}\ Y^{0.6}}{2} = \frac{0.6\ X^{0.4}\ Y^{-0.4}}{6} \quad \Rightarrow$$

$$0.2\ Y = 0.1\ X \quad \Rightarrow$$

$$X = 2\ Y$$

c. If the total amount of money he is willing to spend on the two goods is equal to 60, how much of each will he consume ?

The consumers budget constraint is:

$$60 = 2\ X + 6\ Y$$

Substituting in the consumption for X in terms of Y we get:

$$60 = 2\ 2\ Y + 6\ Y \quad \Rightarrow$$

$$60 = 10\ Y \quad \Rightarrow$$

$$Y = 6$$

The consumer will buy 6 units of good Y. Substituting this into the expression for X in part (b) of the problem we get:

$$X = 2\ 6 = 12$$

Therefore, the consumer will buy 12 units of good X.

2. Consider an individual with income *I* and utility function

$$U(X, Y) = \alpha\ X^{\frac{1}{2}} + b\ Y^{\frac{1}{2}}$$

where *X* and *Y* are two products. This utility function has indifference curves that exhibit diminishing MRS which goes to zero and infinity as they touch the x and y-axis. Therefore, one can use the standard calculus-based approaches to compute the utility maximizing choice of *X* and *Y*. The current price of *X* is 2 and the current price of *Y* is 4.

i. What is this consumer's budget constraint?

The budget constraint is:

$$2\ X + 4\ Y = I$$

ii. Calculate the optimal consumption of *X* and *Y* if the consumer has income *I*.

Since the standard calculus-based approaches are applicable, we know that the consumer's optimal choice will be such that he will equate the marginal utility per dollar of the two goods.

Therefore,

$$\frac{U_X}{P_X} = \frac{U_Y}{P_Y} \quad \Rightarrow$$

$$\frac{\frac{1}{2}\,\alpha\,X^{-\frac{1}{2}}}{2} = \frac{\frac{1}{2}\,\beta\,Y^{-\frac{1}{2}}}{4}$$

which simplifies to

$$\alpha\,X^{-\frac{1}{2}} = \frac{\beta\,Y^{-\frac{1}{2}}}{2}$$

Solving for *Y*, we obtain:

$$Y = \frac{\beta^2}{\alpha^2}\,\frac{1}{4}\,X \qquad (1)$$

This gives us the number of units of *Y* he will consume as a function of the number of units of *X* he will consume. To find the actual number of each, we need to also use the budget constraint.

Substituting the above expression into the budget constraint we get:

$$2\,X + 4\,\frac{\beta^2}{\alpha^2}\,\frac{1}{4}\,X = I$$

Solving for *X*, we get

$$X = \frac{I}{2 + \frac{\beta^2}{\alpha^2}}$$

Substituting in the equation (1) above, we get the optimal value of *Y*.

$$Y = \frac{\beta^2}{\alpha^2} \frac{1}{4} \frac{I}{2 + \frac{\beta^2}{\alpha^2}}$$

$$= \frac{\beta^2}{\alpha^2} \frac{I}{8 + 4 \frac{\beta^2}{\alpha^2}}$$

$$= \frac{I}{4 + 8 \frac{\alpha^2}{\beta^2}}$$

3. (Slightly different version of problem 1.) A consumer has utility function for goods X and Y given by

$$U(X,Y) = X^{0.2} \; Y^{0.3}$$

a. What is the consumer's marginal utility for X ? What is his marginal utility for Y ?

$$MU_X = \frac{\partial U(X,Y)}{\partial X} = 0.2 \; X^{-0.8} \; Y^{0.3}$$

$$MU_Y = \frac{\partial U(X,Y)}{\partial Y} = 0.3 \; X^{0.2} \; Y^{-0.7}$$

b. Suppose the price of X is equal to 4 and the price of Y equal to 6. What is the utility maximizing proportion of X and Y in his consumption ? [That is, if he is a utility maximizer, how many units of X will he consume in terms of units of Y that he consumes.] Use any appropriate method you like to solve the problem.

This person's utility function is Cobb-Douglas, and therefore there is some substitutability between X and Y. For this case, utility maximization requires that the consumer chooses the consumption of the two goods so that the marginal utility per dollar is the same for both of them.

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} \qquad \Rightarrow$$

$$\frac{0.2\ X^{-0.8}\ Y^{0.3}}{4} = \frac{0.3\ X^{0.2}\ Y^{-0.7}}{6} \quad \Rightarrow$$

$$\frac{0.2\ Y}{4} = \frac{0.3\ X}{6} \quad \Rightarrow$$

$$1.2\ Y = 1.2\ X \quad \Rightarrow$$

$$Y = X$$

c. If the total amount of money he is willing to spend on the two goods is equal to 60, how much of each will he consume ?

The consumers budget constraint is:

$$60 = 4\ X + 6\ Y$$

Substituting in the consumption for X in terms of Y we get:

$$60 = 4\ X + 6\ X \quad \Rightarrow$$

$$60 = 10\ X \quad \Rightarrow$$

$$X = 6$$

The consumer will buy 6 units of good X. Since X = Y, the consumer will also purchase 6 units of good Y.