Separating Income and Substitution Effects

Dakshina G. De Silva

Lancaster University

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Effects of a Price Decrease

Can be broken down into two components

- **Income effect**
  - When the price of one goods falls, w/ other constant
  - Effectively like increase in consumers real income
  - Since it unambiguously expands the budget set
  - Income effect on demand is positive, if normal good

- **Substitution effect**
  - Measures the effect of the change in the price ratio
  - Holding some measure of income or well being constant
  - Consumers substitute it for other now relatively more expensive commodities
  - That is, Substitution effect is always negative

- **Two decompositions:** Hicks & Slutsky.
Hicks & Slutsky Decompositions

Hicks
- **Substitution Effect**: change in demand, holding utility constant
- **Income Effect**: Remaining change in demand, due to $m$ change

Slutsky
- **Substitution Effect**: change in demand, holding real income constant
- **Income Effect**: Remaining change in demand, due to $m$ change
Mathematics of Slutsky Decomposition

We seek a way to calculate mathematically the Income and Substitution Effects

Assume:

1. Income: $m$
2. Initial prices: $p_1^0, p_2$
3. Final prices: $p_1^1, p_2$

Note that the price of good two, here, does not change

Given the demand functions, demands can be readily calculated as:

1. Initial demands: $x_i^0 = x_i(p_1^0, p_2, m)$
2. Final demands: $x_i^1 = x_i(p_1^1, p_2, m)$
We need to calculate an intermediate demand that holds buying power constant

Let \( m_s \) the income that provides exactly the same buying power as before at the new price

Thus: \( m_s = p_1^1 x_1^0 + p_2 x_2^0 \)

The demand associated with this income is:

- \( x_i^s = x_i(p_1^1, p_2, m_s) = x_i^s(p_1^1, p_2, x_1^0, x_2^0) \)

Finally we have:

- Substitution Effect: \( SE = x_i^s - x_i^0 \)
- Income Effect: \( IE = x_i^1 - x_i^s \)
Hicks’ Mathematics

The only difference is between Hicks’ and Slutsky is in the calculation of the intermediate demand

Let $m_h$ the income that provides exactly the same utility as before at the new price

If $u_0$ is initial utility level, then: $m_h$ solves

$u_0 = u(x_1(p_1^1, p_2, m_h), x_2(p_1^1, p_2, m_h))$

The demand associated with this income is:

- $x_i^h = x_i(p_1^1, p_2, m_h) = x_i^h(p_1^1, p_2, u_0)$

Finally we have:

- Substitution Effect: $SE = x_i^h - x_i^0$
- Income Effect: $IE = x_i^1 - x_i^h$
Calculating the Slutsky Decomposition

Assume that

\[ u = x^\alpha y^{1-\alpha} \]

So the demand functions are:

\[ x = \alpha \frac{m}{p_x} \]

and

\[ y = (1 - \alpha) \frac{m}{p_y} \]

Initial price is \( p_x^0 \) and final price is \( p_x^1 \)

\[ x^0 = \alpha \frac{m}{p_x^0} \]

and

\[ x^1 = \alpha \frac{m}{p_x^1} \]
Calculating the Slutsky Decomposition (cont.)

\[ y^0 = y^1 = y = (1 - \alpha) \frac{m}{p_y} \]

Now sub from \( x \) and \( y \)

\[ m_s = p_x^1 x^0 + p_y y = p_x^1 \alpha \frac{m}{p_x^0} + p_y (1 - \alpha) \frac{m}{p_y} = \left[ \alpha \frac{p_x^1}{p_x^0} + (1 - \alpha) \right] m \]
Calculating the Slutsky Decomposition (cont.)

since

\[ m_s = \left[ \alpha \frac{p_1^1}{p_0^1} + (1 - \alpha) \right] m \]

we get:

\[ x^s = \alpha m_s = \alpha m \left[ \alpha \frac{p_1^1}{p_0^1} + (1 - \alpha) \right] = \alpha^2 \frac{m}{p_0^1} + \alpha (1 - \alpha) \frac{m}{p_0^1} \]

or

\[ x^s = \alpha x^0 + (1 - \alpha) x^1 \]

Finally, we get:

\[ SE = x^s - x^0 = \alpha x^0 + (1 - \alpha) x^1 - x^0 = (1 - \alpha)(x^1 - x^0) \]

\[ IE = x^1 - x^s = x^1 - [\alpha x^0 + (1 - \alpha) x^1] = \alpha(x^1 - x^0) \]
Calculating the Hicks Decomposition

We need to calculate $m_h$

Substituting our demand functions back into utility we get:

$$u = x^\alpha y^{1-\alpha} = \left(\frac{m}{p_x}\right)^\alpha \left(\frac{(1 - \alpha)m}{p_y}\right)^{1-\alpha} = \left(\frac{\alpha}{p_x}\right)^\alpha \left(\frac{1 - \alpha}{p_y}\right)^{1-\alpha} m$$

Then $m_h$ solves:

$$\left(\frac{\alpha}{p_x^1}\right)^\alpha \left(\frac{1 - \alpha}{p_y}\right)^{1-\alpha} m_h = \left(\frac{\alpha}{p_x^0}\right)^\alpha \left(\frac{1 - \alpha}{p_y}\right)^{1-\alpha} m$$

or

$$m_h = \left(\frac{p_x^1}{p_x^0}\right)^\alpha m$$
Calculating the Hicks Decomposition (cont.)

\[ x^h = \alpha \frac{m^h}{p_x^1} = \alpha \frac{m}{p_x^1} \left( \frac{p_x^1}{p_x^0} \right)^\alpha = \alpha \frac{m}{(p_x^0)^\alpha (p_x^1)^{1-\alpha}} \]

Finally, we get:

\[ SE = x^s - x^0 = x^1 \left( \frac{p_x^1}{p_x^0} \right)^\alpha - x^0 \]

\[ IE = x^1 - x^s = x^1 - x^1 \left( \frac{p_x^1}{p_x^0} \right)^\alpha \]
Demand Curves

We have already met the Marshallian demand curve
– It was demand as price varies, holding all else constant

There are two other demand curves that are sometimes used

• Slutsky Demand
  – Change in demand holding purchasing power constant
  – The function $x_i^s = x_i(p_1^1, p_2, m_s)$ we just defined

• Hicks Demand
  – Change in demand holding utility constant
  – The function $x_i^h = x_i(p_1^1, p_2, m_h)$ we just defined
We mentioned before that with Giffen Goods, the Marshallian demand curve slopes upward.

However,

– Since the substitution effect is always negative, then

– both the Slutsky and Hicks Demands always slope downward—even with Giffen Goods