Deposit Requirements in Auctions: Theory and Empirical Evidence*

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Abstract

Deposit is a non-refundable reservation fee charged by the seller, which allows the winner to reserve the object prior to the completion of the final transaction. In reality, two types of deposit requirements (inclusive or exclusive) are commonly implemented, depending on whether the deposit is in addition to the final price. In the paper, we examine both types and their impacts in auctions. For each type of deposit requirement, we first characterize equilibrium strategy for bidders, and establish the 'expected payment equivalence' which shows that the expected payment of a bidder does not depend on the particular auction form. We then characterize the optimal deposit and the associated reserve price for the seller, and further show the 'expected revenue equivalence' across both types of deposit requirements. Finally, using data from eBay car auctions in which inclusive deposit requirement is used, we estimate bidders' bidding strategy and seller revenue, providing empirical evidence to support our theoretical predictions.

Keywords: auctions, outside options, (inclusive or exclusive) deposit requirement.
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1 Introduction

For some goods purchase, buyers and sellers after selling need an additional time to make preparation for the final transaction. In this case, it is common that a buyer is required to put down a non-refundable reservation fee, also called deposit, which is used to secure the transaction prior to the full payment to the sellers. Meanwhile, this would give the buyer an opportunity to 'rethink' the purchase or to search other better outside options during the additional time. If the buyer does not complete the final transaction later on but takes an outside option, it will be interpreted as default and the seller will keep the deposit. Although in reality there exist different deposit requirements and rules imposed by sellers, it can be briefly classified into two types: *inclusive* deposit and *exclusive* deposit, according to whether the deposit paid by the buyers is in addition to the final transaction price.

In auctions, like for selling cars or houses, deposit requirements have been commonly and broadly implemented by sellers in practice. For example, sellers are allowed to set an inclusive deposit, when selling cars in eBay online car auction markets; in UK house auctions, an additional fee, which can be interpreted as an exclusive deposit, should be paid by the winners before exchange of the final selling contracts within a certain number of days. However, to date, it still remain unclear about the effects of deposit requirement on seller revenue and optimal auction design as well as buyers' responses, and surprisingly, to the best of our knowledge, no previous study has addressed similar questions in the literature.

Motivated by those observations in practice, in this paper we consider a single-object and two-stage auction game with a deposit requirement and arrival of an outside option to examine how the optimal auction design would be affected and its impact on bidding strategies. At the beginning of the auction, the seller announces the reserve price and the deposit amount a buyer needs to pay immediately after winning. In the first stage of the auction, bidders simultaneously place their bids and the bidder who submit the highest value bid wins the object. If the winner defaulted not to pay the deposit, he or she can only wait for an outside option which may or may not give a lower transaction price and will arrive in the second stage of the auction. If, however, the deposit was paid, the winner can choose either to complete the original transaction or to take the outside option at the end of the second stage.

We first examine the role of inclusive deposit requirement and characterize equilibrium strategies for bidders when the auction form takes second-price and first-price, separately.

In both auction formats, we construct an equilibrium which has a feature that the winner pays the deposit along the equilibrium path. Interestingly, we find that in the second-price auction form, if the seller does not require any deposit from the winner, bidding bidders' true values still constitutes an equilibrium, showing that the existences of a better outside option does not affect bidders' bidding strategy. However, it becomes not true if a positive deposit is required after winning; there exists a cutoff value such that if a bidder's private value is lower than the cutoff, the bidder submits an equilibrium bid lower than his/her private value; otherwise, submits no bid. A similar bidding-low feature exists in the first-price auction form; no matter whether the inclusive deposit requirement is implemented by seller, bidders submit lower equilibrium bids than in the auction without an outside option.

We further investigate the expected payments from bidders in such auction settings and establish the 'expected payment equivalence' with the inclusive deposit requirement, showing that the expected payment of a bidder in the first-price form and in the second-price form are exactly same. Note that, we here call it the 'expected payment equivalence', instead of the 'expected revenue equivalence' following the traditional way in auction theory, as the seller's expected is not always equal to the sum of the expected payments from the bidders; there exists a possibility that the winner will default to take a sufficiently low outside option and in this case the seller revenue is equal to the deposit.

We next examine how the expected revenue varies when the seller is able to charge the inclusive deposit and set a reserve price. On one hand, setting no deposit requirement would give bidders an incentive to bid high in the auction, but the seller would end up with no payment from the winner, once a sufficiently low outside option arrives. On the other hand, although charging a high deposit would force the winner to give up the opportunity of taking an outside option in the next stage, it will induce bidders to submit lower bids in the first stage. Thus, the seller needs to trade off these two effects. The characterization of the optimal auction design shows that the seller benefits from changing a certain level of the inclusive deposit and the associated reserve price should be set lower than the optimal one in a standard auction setting, which is characterized by Myerson (1981).

After having established these basic understandings about the inclusive deposit requirements in auctions, we turn to investigate another type - the exclusive deposit requirement - that the fee charged by the seller is in addition to the final auction price. We still focus on the cutoff strategies for bidders, where a bidder will submit a bid if and only if his/her private value is no less than a certain value in the second-price and first-price auction formate, separately. Similarly, the bidders submit lower equilibrium bids, after the exclusive deposit is charged by the seller. In the setting, the 'expected payment equivalence' still exists, indicating that the expected payment of a bidder does not depend on the particular auction form. Finally, we provide the characterization of the optimal inclusive deposit and the associated reserve price that maximize the expected revenue of the seller.

Further, it is of interest to study the comparison between the two types of the deposited requirements, in particular, which one would benefit more to the seller in terms of the expected revenue, or there would exist a revenue-equivalence result to the seller. It seems that the exclusive deposit requirement is more profitable, as it is an additional fee, besides the final price, charged by the seller. However, because of this additional 'cost' after winning, the bidders would strategically adjust their bids (intuitively, to bid low) in the auction stage. Thus, the comparison is not trivial and obvious. Surprisingly, our novel finding is that no matter which deposit requirement is implemented by the seller, the expected revenue is exactly the same. This can be interpreted as another version of the 'expected revenue equivalence' after considering the deposit requirement and the arrival of the outside option in the auctions. Furthermore, this interesting result provides an explanation of why both types of the deposit requirement have been commonly implemented in reality.

****** empirical evidence from eBay******

The remainder of the paper is organized as follows. In Section 2 we review previous research on auctions with outside options and the optimal auction design. In Sections 3, 4, and 5, we study the impacts of the inclusive deposit requirement on bidders' strategies, establish the 'expected payment equivalence' and in Section 6 we then characterize the optimal inclusive deposit and reserve price for the seller. In Section 7 we implement the same analysis to the exclusive deposit requirement. Section 8 contains the comparative results across the two deposit requirements. In Section 9, we provide empirical evidence from eBay car auctions. Section 10 concludes the study. Most proofs are in the Appendix.

2 Literature Review

To the best of our knowledge, our paper is the first study which examines the impacts of the deposit requirement on bidding strategies and the optimal auction design. Although no previous study has investigated the similar questions before yet, this study partially links to studies in the literature on auctions without buyer commitment and auctions with outside options . Below, we review previous research in each of these two areas and place our model in relation to the existing literature.

Our paper is related to a growing literature on auctions without buyer commitment. Resnick and Zeckhauser (2002), which is one of the first studies related to sellers' and buyers' behavior in online marketplaces, observe that the most common complain by sellers is that the winning bidder does not follow through on the transaction. Following their work, Dellarocas and Wood (2008) using a sample with over 50000 eBay auctions find that 81 percent of negative feedback on buyers results from 'bidders who back out of their commitment to buy the items they won." Engelmann, Frank, Koch, and Valente (2015) build up a simple theoretical auction model with a lab experimental evidence to consider the possibility that the seller can give a second-chance offer to the second-highest bidder if the auction winner fails to pay. Their analysis shows that the availability of the offer reduces bidders' willingness to bid in the auction and thus lowers the seller's revenue, even when no default actually happens. In addition, buyers sometime would benefit from a reputation for defaulting, counter to the idea of creating a deterrent against such behavior.

Asker (2000) studies an auction model where bidders face an uncertainty on their final valuations of the object, and this uncertainty will only be resolved after the simultaneous bidding has taken place. Asker shows that in such a case the inclusion of the withdrawal right (allowing the winner to default) raises the expected revenue to the seller. Krähmer and Strausz (2015) investigate the economic effects of withdrawal right on optimal sales contracts, involving one buyer and one seller. In their contracting environment, the buyer after having observed his private valuation has the choice between exercising his option as specified in the contract, or withdrawing from it and obtaining his outside option. Their results show that the inclusion of default rights is equivalent to introducing ex post participation constraints in the sequential screening model, and even though sequential screening is still feasible with ex post participation constraints, the seller no longer benefits from it. Instead, the optimal selling contract is static and coincides with the optimal posted price contract in the static screening model[1].

In certain aspects, the characterization of the equilibrium strategies in our theoretical analysis implies the possibility of the winner defaulting: if the price of the outside option is sufficiently low in the second stage, the winner will then decline the original transaction but choose to take the option. In this case, the seller will keep the deposit. Moreover, bidding low in our equilibria do not rely on the availability of the second-chance offer. However, the focus in our paper is completely different; we attempt to provide a rationale for the deposit requirements, which have been commonly used by sellers in the auctions, and further rank different types of the deposit requirements in terms of seller revues.

This paper is part of another growing literature on auctions with outside options. Cherry, Frykblom, Shogren, List, and Sullivan (2004) conduct a lab experiment to explore whether bidders in second-price auctions consider the existence of outside options when formulating their bidding strategies, showing that a bidder shaves his/her bid toward the price of the outside option. Kirchkamp, Poen, and Reiss (2009) consider a simple theoretical model and then run a lab experiment to study equilibrium bidding behavior of bidders in first-price and second-price auctions with outside options, showing that the first-price auctions yield more revenue to the sellers than the second-price auctions. This provides a justification of why first-price auction form is more commonly used in reality.

Lauermann and Virág (2012) study how the presence of outside options influences whether an auctioneer prefers opaque or transparent auctions, which differ according to the information that bidders receive, showing that an auctioneer might have incentives to choose opaque auctions in order to reduce the values of the bidders' outside options. Figueroa and Skreta (2007, 2009) examine revenue-maximizing auctions for multiple objects, where bidders' outside options depend on their private information and are endogenously chosen by the seller, and their results show that depending on the shape of outside options, an optimal mechanism may or may not allocate the objects efficiently.

Unlike those existing models considering either losing bidders have outside options or all bidders know their outside options before bidding, we instead consider the outside option in a different fashion that it arrives after all bidders already submitted their bids and is only available to the auction winner, in the sense that the winner can either complete the original transaction or default and take the outside option. In such an auction setting, our analysis focuses on the investigation of how the deposit requirement for the winner would

[1]See related studies by Ben-Shahar and Posner (2011), Eidenmüller (2011), and others.

affect ex-ant bidding strategies and the seller's expected revenues. As a result, charging a deposit matters in the design of optimal auction, and in our theoretical analysis, we further establish the 'expected-payment equivalence' across the different auction formats and 'the expected revenue equivalence' across different types of the deposit requirements. All those definitely contribute further understandings in the auction literature.

3 Inclusive Deposit Requirement

In this section, we develop the theoretical model to illustrate the impacts of requiring a deposit in an auction with outside options, and we then use the theoretical model to frame our empirical analysis in Section (9).

3.1 Basic settings and equilibrium concept

A single indivisible object is sold to N risk neutral bidders by employing an auction, where $1 < N < \infty$. All bidders' private values, denoted by $v_i,\ i = 1, 2, ..., N$, are independent draws from a common atomless distribution $F(.)$ over support $[\underline{v}, \bar{v}]$, with $0 < \underline{v} < \overline{v}$, $F(\underline{v}) = 0$, $F(\overline{v}) = 1$, and $f(.) \equiv F'(.)$. The game includes two stages: the first stage is auction stage, denoted by $t = 0$, and the second stage is the stage of making the payment, denoted by $t = T$. We further assume that an outside option will arrive at $t = T$, which allows bidder i to obtain the same object (utility) by paying price p. Price p is a random draw from a continuous distribution function $\Phi(.)$ with density $\varphi(.)$ over $[\underline{v}, \bar{v}]$, where $\Phi(\underline{v}) = 0$ and $\Phi(\overline{v}) = 1$. F and Φ are common knowledge in the auction.

At stage $t = 0$, the seller first, who values the object equal to zero, sets a reserve price $r \in [0, \bar{v}]$ and a non-refundable deposit policy $D \in [0, \bar{v}]$. This policy requires the auction winner to pay D immediately for holding the object before making payment to the seller at $t = T$. If the winner does not pay D, this will be interpreted as 'default' from the winner and the seller will still keep the object. The auction form takes sealed bid second-price[2]. All bidders observing r and D then decide whether to submit bids or to simply wait for the outside option with p. After all those bidders make their decisions, they submit bids simultaneously. We denote a bid from bidder i with value v_i by $b_i \in [0, +\infty)$, and b_i is

[2]We discuss the first-price auction form and then establish the expected payment equivalence in Section 4.3.

valid if and only if $b_i \geq r$. If bidder i takes the outside option directly or submit a bid less than r, we automatically set b_i as no bid from the bidder, denoted by "No", for analytical convenience. Thus, the feasible bid set of bidder i is $b_i \in [0, +\infty) \cup \{No\}$. In the auction stage, the bidder who submits the highest bid is the winner and the payment, denoted by κ, is equal to the maximum between r and the second-highest bid. If no bidder submits a 'valid' bid, the auction game then ends and the seller keeps the object.

At stage $t = T$, for notational simplicity, we take representative bidder i as the winner and denote the highest bid among all bidders except bidder i by $b_{-i}^{(1)}$, and then have $\kappa = \max\{r, b_{-i}^{(1)}\}$. Bidder i's decision on deposit is denoted by $e_i^0 \in \{0, 1\}$, where $e_i^0 = 1$ means 'D was paid' and $e_i^0 = 0$ means 'D was *not* paid' at $t = 0$. Therefore, conditional on $e_i^0 = 1$, bidder i (the auction winner) has two options at $t = T$: either completing the purchase by paying $\kappa - D$, or taking the outside option by paying p. If, however, bidder i did not pay D at $t = 0$, i.e., $e_i^0 = 0$, he simply waits for the outside option with price p at $t = T$. We here denote bidder i's decision on whether to complete the original transaction at $t = T$ by o_i^T, where $o_i^T = 0$ means 'the outside option with price p is exercised' and $o_i^T = 1$ means 'the payment $\kappa - D$ is made' at $t = T$. For characterization completeness, we denote no purchase by "No". Thus, the feasible action set of bidder i is $o_i^T \in \{No\} \cup \{0, 1\}$. Figure (1) depicts the timing of the events.

3.2 Strategies, beliefs, and equilibrium concept

Before proceeding the analysis, we state the definitions of strategies and beliefs of a bidder and strategies of the seller in the auction game with outside options described above, and the equilibrium concept we will use in the following analysis.

Strategies and belief of a bidder. Given r and D, a bidder must make three decisions: first, how much to bid in the bidding stage; second, conditional on winning the object, whether to pay the deposit required by the seller; after the arrival of outside option, whether to exercise the option or to complete the original transaction by paying $\kappa - D$, or not to purchase.

A bidding strategy for bidder i prescribes a bid b_i as a function of i's private value v_i. Without loss of generality, we focus on the symmetric bidding strategy for all bidders, and thus we can omit the subscript of the bidding strategy function. A bidding strategy at $t = 0$

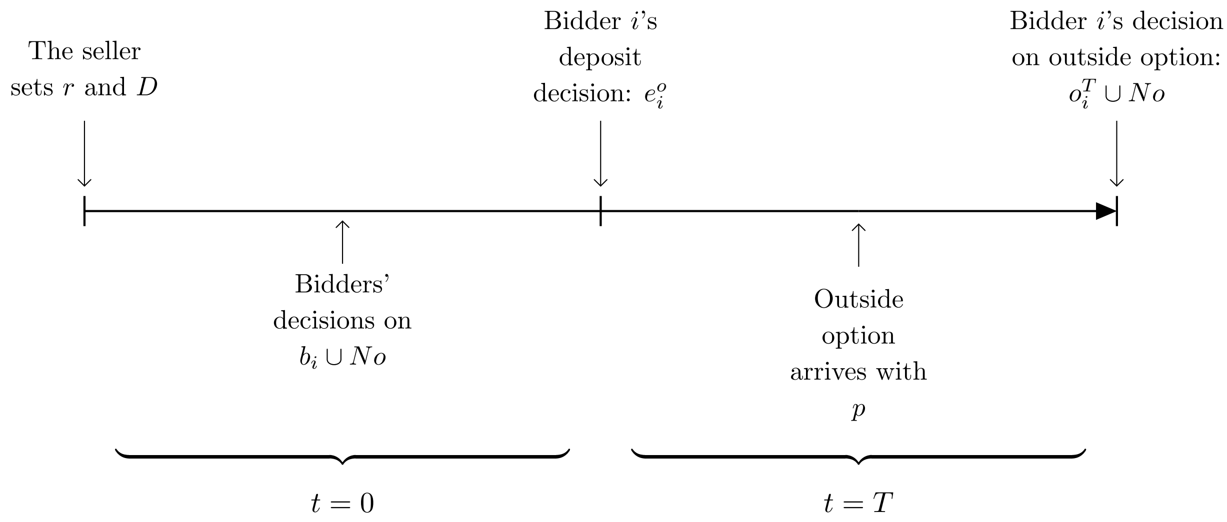

Figure 1: Timing

is a mapping

$$b : [\underline{v}, \overline{v}] \rightarrow [0, +\infty) \cup \{No\}.$$

Note that the bidding strategy complies with the restriction on bids, that is, b_i is valid if $b_i \geq r$.

Conditional on winning at the bidding stage, a strategy for bidder i on the decision of paying deposit is a mapping from his private value v_i and full payment κ to deposit payment decision at $t = 0$:

$$e_i^0 : [\underline{v}, \overline{v}] \times [\underline{v}, \overline{v}] \rightarrow \{0, 1\}.$$

Contingent on the deposit payment decision at $t = 0$, a strategy for bidder i on the decision of exercising the outside option is a mapping from his private value v_i, full payment κ, and price p to outside option decision at $t = T$:

$$o_i^T : [\underline{v}, \overline{v}] \times [\underline{v}, \overline{v}] \times [\underline{v}, \overline{v}] \rightarrow \{0, 1\} \cup \{No\}.$$

A relevant belief bidder i will entertain is about the distribution of the outside option (price p). This belief is associated with bidder i's bidding strategy $b(v_i)$ at $t = 0$ (the auction stage). Then bidder i's belief about p is a distribution

$$G_i : [\underline{v}, \overline{v}] \to [0, 1].$$

Strategies of the seller. Before the auction stage starts, the seller can set reserve price r as well as deposit policy D. Thus, the seller's strategy is a mapping of (distributions of) bidders' private values v and price p of the outside option to decisions on r and D

$$r : [\underline{v}, \overline{v}] \times [\underline{v}, \overline{v}] \to [\underline{v}, \bar{v}].$$

$$D : [\underline{v}, \overline{v}] \times [\underline{v}, \overline{v}] \to [\underline{v}, \bar{v}].$$

Equilibrium concept. The solution concept we here use is perfect Bayesian equilibrium (PBE) in the game. Let us use $-i$ denotes all other bidders, except bidder i. We then say that a profile of strategies of bidders: $(b_i, e_i^0)_{i=1,\ldots,N}$ and beliefs $G_i(\cdot)$ at $t = 0$ and $(o_i^T)_{i=1,\ldots,N}$ at $t = T$, and strategies of the seller: r and D constitutes an *equilibrium* of the auction game if the following conditions hold:

(i) Bidder i's bidding strategy $b(v_i)$ is sequentially rational, given $b(v_{-i})$ and beliefs $G_i(\cdot)$;

(ii) bidder i's deposit strategy e_i^0 is optimal, given $b(v_{-i})$;

(iii) bidder i's outside option strategy o_i^T is optimal, given $b(v_{-i})$;

(iv) the seller optimally chooses r and D, given $b(v_i)$, e_i^0, o_i^T, and $G_i(\cdot)$ for all $i = 1, \ldots, N$.

We further restrict our attention to equilibria that are in (weakly) undominated strategies. This rules out many uninteresting cases that generally arise in second-price auctions. For example, a trivial equilibrium involves one bidder *alwasy* bidding $\bar{v}$, and all other bidders not bidding but taking outside options directly. The bidder facing auction price $\underline{v}$ then optimally decides whether to pay D at $t = 0$ and to exercise the outside option at $t = T$. However, such an equilibrium is not admissible as these strategies are weakly dominated. See Blume and Heidhues (2004) for a characterization of all Nash equilibria of (static) second-price auction. In addition, in the following equilibrium analysis, whenever a strategy or belief depends on fewer variables than the ones included above, any unnecessary arguments will be dropped.

4 Bidders' Equilibrium Strategies

In this section, we solve the game by using backward induction. We first characterize bidder i's outside option strategy at $t = T$, contingent on the deposit payment decision in the auction stage. We then examine the decisions of bidder i on deposit payment and bidding strategy. Thereafter, we look at the seller's problem of optimal reserve r and deposit D.

Before proceeding further, let us first discuss a bidder's associated belief. The only rational belief bidder i should hold in the auction stage is about the distribution of price p of the outside option. But one should notice that even after the bidding stage, learning the auction payment κ would not help the winner update the belief following the Bayes' rule. In this sense, the revelent belief does not play any role in the equilibrium characterization. To keep the completeness, however, we still include the belief of bidder i at $t = 0$ given by the following equation in the bidder's equilibrium strategies.

$$G_i(p) \;=\; \Phi(p). \tag{1}$$

4.1 Decision on the outside option at $t = T$

Conditional on winning, bidder i's decision on whether to exercise the outside option with price p at $t = T$ depends on his deposit payment decision at $t = 0$. More specifically, we focus on the following outside option strategy for bidder i at $t = T$:

$$o_i^T(v_i, \kappa, p) \;=\; \begin{cases} 1 & \text{if } \big[e_i^0 = 1 \text{ and } v_i \geq \kappa - D \text{ and } p \geq \kappa - D\big]; \\ 0 & \text{if } \big[e_i^0 = 1 \text{ and } v_i \geq \kappa - D \text{ and } p < \kappa - D\big], \\ & \text{or } \big[e_i^0 = 1 \text{ and } v_i < \kappa - D \text{ and } p \leq v_i\big], \\ & \text{or } \big[e_i^0 = 0 \text{ and } v_i \geq p\big]; \\ No & \text{if } \big[e_i^0 = 1 \text{ and } v_i < \kappa - D \text{ and } p > v_i\big], \\ & \text{or } \big[e_i^0 = 0 \text{ and } v_i < p\big]. \end{cases} \tag{2}$$

In fact, (2) provides the optimal decision for bidder i. We now explain the strategy by separately considering the following two cases: Case (a.) given $e_i^0 = 0$, where bidder i (the winner) had not paid the deposit D at $t = 0$, i.e., then his decision is simply to decide whether to exercise the outside option with price p at $t = T$. If the bidder exercises the outside option, his payoff is $v_i - p$, otherwise his payoff is 0. Clearly, it is optimal for bidder

i to purchase if and only if $v_i \geq p$. Case (b.) given $e_i^0 = 1$, where bidder i (the winner) had paid the deposit D at $t = 0$, the bidder then has three choices at $t = T$: exercise the outside option with price p, or complete the original transaction with payment $\kappa - D$, or not to purchase. Clearly, it is optimal for bidder i to choose either of the first two choices if and only if $v_i \geq \min\{p, \kappa - D\}$. This implies that when $\kappa - D \leq v_i$, the bidder would definitely purchase an object: completing the original transaction with a price $\kappa - D$ if $p \geq \kappa - D$, and exercising the outside option with price p if $p < \kappa - D$. However, when $\kappa - D > v_i$ (an out of equilibrium event), the bidder would only exercise the outside option if $p \leq v_i$; otherwise, he chooses not to purchase.

4.2 Decision on the deposit payment at $t = 0$

Let us next discuss bidder i's decision on deposit payment D at $t = 0$. The discussion above implies that bidder i observing v_i and κ would never pay D at $t = 0$ if $\kappa - D > v_i$. We then look at the case in which $\kappa - D \leq v_i$. If $\kappa \in (v_i, v_i + D]$, it is clear that the bidder also has no incentive to pay the deposit D, since he would eventually pay more than his value v_i, even he purchases at time T with a price $\kappa - D$. Thus, this allows us to restrict attention to the case where $\kappa \leq v_i$ in the following analysis by separately examining bidder i's payoffs between paying D and not paying D.

Like what we have shown in (2), when the bidder has paid D, i.e., $e_i^0 = 1$, then it is optimal to exercise the outside option if $p < \kappa - D$; otherwise, the bidder should complete the original transaction by paying $\kappa - D$. We can therefore construct bidder i's payoff at $t = 0$, denoted by $\pi_D(v_i, \kappa, D)$, as follows

$$\pi_D(v_i, \kappa, D) = \int_{\underline{v}}^{\kappa - D} (v_i - p)\varphi(p)dp + \int_{\kappa - D}^{\overline{v}} [v_i - (\kappa - D)]\varphi(p)dp - D. \tag{3}$$

If bidder i does not pay D, i.e., $e_i^0 = 0$, the only option the bidder has at $t = T$ is to wait for the outside option with a price p less than v_i, and therefore, bidder i's expected payoff by not paying D at $t = 0$, denoted by $\pi_D(v_i, \kappa, D)$, is given by

$$\pi_{ND}(v_i, \kappa, D) = \int_{\underline{v}}^{v_i} (v_i - p)\varphi(p)dp. \tag{4}$$

From (3) and (4), conditional on winning, bidder i is willing to pay D if and only if

$\pi_D(v_i, \kappa, D) \geq \pi_{ND}(v_i, \kappa, D)$, in other words,

$$\int_{\underline{v}}^{\kappa-D} (v_i - p)\varphi(p)dp + \int_{\kappa-D}^{\overline{v}} [v_i - (\kappa - D)]\varphi(p)dp - D \geq \int_{\underline{v}}^{v_i} (v_i - p)\varphi(p)dp. \tag{5}$$

Note that in (5), κ is equal to r if $r \geq b_{-i}^{(1)}$, and $b_{-i}^{(1)}$ if $r < b_{-i}^{(1)}$. Moreover, if the seller does not require any deposit, i.e., $D = 0$, $\pi_D(v_i, \kappa, D)$ is strictly greater than $\pi_{ND}(v_i, \kappa, D)$, indicating that there is no cost for bidder i to default from the current auction if a better outside option arrives, and thus the bidder strictly prefers to stay in the auction before the realization of price p of the outside option.

Given the discussion above, we now give the formal characterization of bidder i's deposit strategy at $t = 0$. Let us define $L = \pi_D(v_i, \kappa, D) - \pi_{ND}(v_i, \kappa, D)$, and we then have

$$L(v_i, \kappa, D) = \int_{\underline{v}}^{\kappa-D} (v_i - p)\varphi(p)dp + \int_{\kappa-D}^{\overline{v}} [v_i - (\kappa - D)]\varphi(p)dp - D - \int_{\underline{v}}^{v_i} (v_i - p)\varphi(p)dp.$$

Simplifying the equation above shows

$$\begin{aligned} L(v_i, \kappa, D) &= \int_{\kappa-D}^{\overline{v}} [v_i - (\kappa - D)]\varphi(p)dp - \int_{\kappa-D}^{v_i} (v_i - p)\varphi(p)dp - D, \\ &= \int_{\kappa-D}^{v_i} [1 - \Phi(p)]dp - D. \end{aligned} \tag{6}$$

which implies that given κ and D, there should exist a unique threshold for bidder i's private value, denoted by $\hat{v}_i(\kappa, D)$, such that $L(\hat{v}_i, \kappa, D) = 0$, where bidder i is indifferent between paying deposit D and not paying deposit D. Moreover, it is easy to check that $\hat{v}_i(\kappa, D)$ is increasing in both κ and D. From the discussion above, the deposit strategy for bidder i at $t = 0$ can now be formally stated as follows:

$$e_i^0(v_i, \kappa) = \begin{cases} 1 & \text{if } v_i \geq \hat{v}_i(\kappa, D); \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Note that if $L(\underline{v}, \kappa, D) \geq 0$, then conditional on winning, the inequality in (5) always holds, which implies that the threshold $\hat{v}_i$ for bidder i is equal to $\underline{v}$.

4.3 Decision on the bidding at $t = 0$

From the characterizations of bidder i's strategies on the outside option and the deposit payment above, we are now already to examine equilibrium bidding strategy in the auction stage. Given the existence of the outside option, the equilibrium bid a bidder is willing to submit a valid bid, i.e., $b \geq r$, should make the bidder indifferent between 'waiting for outside option directly' and 'submitting a bid and then paying the deposit D in the auction' at $t = 0$, that is,

$$\begin{aligned}\int_{\underline{v}}^{b-D}(v_i - p)\varphi(p)dp + \int_{b-D}^{\overline{v}}[v_i - (b-D)]\varphi(p)dp - D = \int_{\underline{v}}^{v_i}(v_i - p)\varphi(p)dp. \\ \Leftrightarrow \qquad b = v_i - \int_{b-D}^{v_i}\Phi(p)dp,\end{aligned} \tag{8}$$

Note that conditional on winning, given $b_i \geq \kappa$, (5) is satisfied when (8) is satisfied. The bidder's bidding strategy for bidder i at $t = 0$ can thus be summarized as follows:

$$b(v_i) \ = \begin{cases} v_i - \int_{b(v_i)-D}^{v_i}\Phi(p)dp & \text{if } v_i \geq v^c; \\ No & \text{if } v_i < v^c. \end{cases} \tag{9}$$

In proof of Lemma (1) we will show that in fact, it is a weakly dominant strategy for bidder i with private value v_i to bid according to (9). Now let us more carefully look at the bidding strategy: First of all, if the seller chooses not to charge any deposit from the winner, i.e., $D = 0$, we can re-write the bidding function as follows: $\int_{b_i}^{v_i}(1 - \Phi(x))dx = 0$. Then, the only solution is given by $b_i = v_i$; bidders submit their bids truthfully in the equilibrium. This implies that the existence of the outside option does not affect bidders' bidding strategy in the auction. Nonetheless, if $D > 0$, truthful bidding becomes not an equilibrium.

We then look at the reserve price r. It is obvious that if $0 \leq r < b(\underline{v})$, then all bidders can submit valid bids following (8). In this case, when the seller charges no deposit, i.e., $D = 0$, it is clear that $b(\underline{v}) = \underline{v}$. If $r \geq b(\overline{v})$, no one submits a bid in the auction. If $b(\overline{v}) > r \geq b(\underline{v})$, there must exist a threshold value for bidder i, denoted by v^c, such that $b(v^c) = r$, or equivalently, $\int_{r-D}^{v^c}[1 - \Phi(p)]dp = D$. Moreover, it is easy to check that $\frac{\partial b}{\partial v_i} = \frac{1-\Phi(v_i)}{1-\Phi(b-D)} > 0$, thereby indicating that, when $D \geq 0$, v^c should be unique and strictly greater than r, i.e., $v^c > r$, and moreover, only bidders with values above v^c submit valid bids. When the seller charges no deposit, i.e., $D = 0$, we then have $v^c = r$. Clearly b is decreasing in D.

Thus, given the reserve price r, when D increases, the threshold v^c will increase until either $b(v^c = \bar{v}) = r$, or $b(\bar{v}) - D = \underline{v}$. When it is the former case, if $b(\bar{v}) - D$ is still greater than $\underline{v}$, further increase in D would result in $b(\bar{v}) < r$; no one will bid in the auction and the seller obtains zero revenue. When it is the latter case, if $b(\bar{v}) > r$, we should have $v^c < \bar{v}$ and thus bidders with private values greater than v^c submit bids; otherwise, no one will bid in the auction and the seller obtains zero revenue.

From the discussion in subsections (4.1), (4.2), and (4.3), we can then establish the following result (see Appendix for the full proof):

Lemma 1. *Given the reserve price r and the inclusive deposit D from the seller, when the auction mechanism is second-price form, there exists a PBE such that bidder i, for all $i = 1, ..., N$, uses the following strategies:*

(a) the bidding strategy b and the associated belief G_i at $t = 0$ are described by (9) *and* (1)*, respectively;*

(b) the deposit strategy e_i^0 is described by (7)*;*

(c) the outside option o_i^T is described by (2)*.*

Lemma (1) shows how bidders response to the auction game, where there exists an outside option for the winner and the seller is allowed to charge a deposit from the winner. (9) demonstrates that the deposit requirement by the seller induces bidders to submit low bids, lower than their private values.

4.4 Remarks

Before proceeding further to the optimal r and D for the seller, we now discuss the robustness of our results when relaxing some important assumptions. First, the number of bidders is fixed in the auction model. Instead, arriving process by bidders in the (internet) auctions might follow a stochastic process, that is, the number of bidders is more likely to be a random draw following some distribution (in this case, an entering bidder does not know the actual number of entering bidders, but knows the distribution of the number of potential bidders). One may then question whether bidders would still use the strategies characterized in Lemma (1) with such a stochastic entry process.

Clearly, no matter whether the participation process is stochastic, conditional on winning, it is still optimal for bidder i to adopt the deposit strategy and the outside option strategy characterized in (7) and (2). Thus, we only need to examine the impact of stochastic process of bidders' participation on the bidding strategy. As discussed above, (9) shows that the equilibrium bidding strategy still follows the nice property the second-price auction mechanism itself has, that is, a bidder's bidding strategy does not depend on the number of entering bidders in the auction game. We thus have

Remark 1. If bidders' participation is stochastic, there still exists a PBE in which each bidder adopts the strategies characterized in Lemma (1).

Note that although bidders still use the same strategies in the auction game, stochastic participation would affect each bidder's expected surplus and the seller's expected revenue, as the distribution of the second highest order statistics which determines the final price in the auction depends on the actual number of participating bidders.

Second, our construction relies on the assumption that bidders submit their bids simultaneously in the auction stage. Such a bidding process is normally interpreted as 'the second-price sealed-bid auction' form. However, in reality, most of auctions takes the English open ascending auction form, in which the current price raises continuously as long as there are at least two bidders. The auction ends until only one bidder is still active. For example, on eBay the bidding mechanism is a variant of the English auction form, called 'proxy bidding'. It is well-known in auction theory that both auction formats are equivalent. Then, it is natural to examine whether the equivalence still holds, when the winner can have a choice of default to choose a better outside option.

In this case, we can still solve the game by backward induction and clearly, the optimality of deposit strategy and outside option strategy in (7) and (2), respectively, would not change. This determines a bidder's expected payoff and further the willingness to pay (the bid). Then, in the bidding stage of the English auction, it clearly cannot be optimal for the bidder to stay in after the current price exceeds the bid, or to drop out before the current price reaches the bid. Thus, our result still holds when the bidding stage uses the English open ascending auction form.

Remark 2. The equivalence between the second-price sealed-bid auction and the English open ascending auction still hold, when the winner has a choice of default to choose a better outside option.

5 First-Price Auction and the Expected Payment Equivalence

It would be interesting to examine how the equilibrium strategies would change if the auction mechanism takes the first-price form and whether the equivalence result in terms of the expected payment of a bidder still holds across the first-price and second-price auction forms.

5.1 First-price auction

Given the first-price auction at stage $t = 0$, κ is the winning bid submitted by bidder i (the winner). It is obvious that the strategy on outside option at $t = T$ and the strategy on deposit payment at $t = 0$ are regardless of the auction form. Thus, we only need to focus on bidding strategy at $t = 0$. Note that along equilibrium path, we should have that a bidder who wins the auction will pay the deposit D.

Let us denote expected surplus by $\pi(b, v_i)$ for bidder i with private value v_i and bidding b in the auction. In the auction, we focus on the increasing, symmetric, and differentiable equilibrium bidding strategy from bidders, denoted by $\beta(v_i)$. Then, given that all other bidders use the same bidding strategy in equilibrium, bidder i's expected surplus by bidding b at $t = 0$ is given by

$$\begin{aligned}\pi(b, v_i) = {} & Q(\beta^{-1}(b))\left[\int_{\underline{v}}^{b-D}(v_i - p)\varphi(p)dp + \int_{b-D}^{\overline{v}}[v_i - (b-D)]\varphi(p)dp - D\right] \\ & + (1 - Q(\beta^{-1}(b)))\left[\int_{\underline{v}}^{v_i}(v_i - p)\varphi(p)dp\right], \\ = {} & Q(\beta^{-1}(b))\left[v_i - (b-D)\Phi(b-D) + \int_{\underline{v}}^{b-D}\Phi(p)dp - (b-D)(1-\Phi(b-D)) - D\right] \\ & + (1 - Q(\beta^{-1}(b)))\left[\int_{\underline{v}}^{v_i}\Phi(p)dp\right], \\ = {} & Q(\beta^{-1}(b))\left[v_i + \int_{\underline{v}}^{b-D}\Phi(p)dp\right] + (1 - Q(\beta^{-1}(b)))\left[\int_{\underline{v}}^{v_i}\Phi(p)dp\right] - Q(\beta^{-1}(b))b.\end{aligned} \tag{10}$$

where β is the equilibrium bidding strategy, $Q(.) \equiv F^{N-1}(.)$ and $q(.) \equiv Q'(.)$. Differentiating

(10) with respect to b, $\frac{\partial \pi(b,v_i)}{\partial b} = 0$, yields the first-order condition:

$$\frac{q(\beta^{-1}(b))}{\beta'(\beta^{-1}(b))}\Bigg[\int_{\underline{v}}^{b-D}(v_i - p)\varphi(p)dp + \int_{b-D}^{\overline{v}}[v_i - (b-D)]\varphi(p)dp - D\Bigg]$$
$$- Q(\beta^{-1}(b))\Bigg[\int_{b-D}^{\overline{v}}\varphi(p)dp\Bigg] - \frac{q(\beta^{-1}(b))}{\beta'(\beta^{-1}(b))}\Bigg[\int_{\underline{v}}^{v_i}(v_i - p)\varphi(p)dp\Bigg] = 0,$$

$$\frac{g(\beta^{-1}(b))}{\beta'(\beta^{-1}(b))}\Bigg[-\int_{b-D}^{v_i}(v_i - p)\varphi(p)dp + \int_{b-D}^{\overline{v}}[v_i - (b-D)]\varphi(p)dp - D\Bigg]$$
$$- Q(\beta^{-1}(b))\Bigg[\int_{b-D}^{\overline{v}}\varphi(p)dp\Bigg] = 0. \tag{11}$$

At a symmetric equilibrium $b = \beta(v_i)$, we re-write the equation above as follows:

$$q(v_i)\Bigg[-\int_{\beta(v_i)-D}^{v_i}(v_i - p)\varphi(p)dp + \int_{\beta(v_i)-D}^{\overline{v}}[v_i - (\beta(v_i) - D)]\varphi(p)dp - D\Bigg]$$
$$- \beta'(v_i)Q(v_i)\Bigg[\int_{\beta(v_i)-D}^{\overline{v}}\varphi(p)dp\Bigg] = 0.$$

Simplifying the equation yields

$$q(v_i)\Bigg[v_i - \beta(v_i)\Phi(\beta(v_i) - D) - \int_{\beta(v_i)-D}^{v_i}\Phi(p)dp\Bigg] - q(v_i)\beta(v_i)\Bigg[1 - \Phi(\beta(v_i) - D)\Bigg]$$
$$- \beta'(v_i)Q(v_i)\Bigg[1 - \Phi(\beta(v_i) - D)\Bigg] = 0,$$

$$q(v_i)\Bigg[v_i - \int_{\beta(v_i)-D}^{v_i}\Phi(p)dp\Bigg] + Q(v_i)\beta'(v_i)\Phi(\beta(v_i) - D) - \Bigg(q(v_i)\beta(v_i) + \beta'(v_i)Q(v_i)\Bigg) = 0,$$

$$q(v_i)v_i = \Bigg(q(v_i)\beta(v_i) + \beta'(v_i)Q(v_i)\Bigg) + \Bigg(q(v_i)\int_{\beta(v_i)-D}^{v_i}\Phi(p)dp - Q(v_i)\beta'(v_i)\Phi(\beta(v_i) - D)\Bigg).$$

or equivalently,

$$v_i q(v_i) = \frac{d}{dv_i}(Q(v_i)\beta(v_i)) + \frac{d}{dv_i}(\int_{\beta(v_i)-D}^{v_i}Q(v_i)\Phi(p)dp).$$

We then have

$$\frac{1}{Q(v_i)} \int_{\underline{v}}^{v_i} xq(x)dx = \beta(v_i) + \int_{\beta(v_i)-D}^{v_i} \Phi(p)dp.$$
$$\Leftrightarrow \beta(v_i) = \frac{1}{Q(v_i)} \int_{\underline{v}}^{v_i} xq(x)dx - \int_{\beta(v_i)-D}^{v_i} \Phi(p)dp.$$

The equilibrium bidding strategy can be then written as follows:

$$\begin{aligned} b &= \frac{1}{Q(v_i)} \int_{\underline{v}}^{v_i} xq(x)dx - \int_{b-D}^{v_i} \Phi(p)dp, \\ &= v_i - \frac{1}{Q(v_i)} \int_{\underline{v}}^{v_i} Q(x)dx - \int_{b-D}^{v_i} \Phi(p)dp. \end{aligned} \tag{12}$$

After deriving the equilibrium bidding strategy, given reserve price r from the seller, (12) can be written as follows:

$$b = v_i - \frac{1}{Q(v_i)} \int_{r}^{v_i} Q(x)dx - \int_{b-D}^{v_i} \Phi(p)dp. \tag{13}$$

There should exist a unique v^c such that $b(v^c) = r$ if $b(\overline{v}) > r \geq b(\underline{v})$. In this case, $v^c = r$ if $D = 0$[3]. If $0 \leq r < b(\underline{v})$, all bidders can submit valid bids following (13). In this case, when the seller charges no deposit, i.e., $D = 0$, it is clear that $b(\underline{v}) = \underline{v}$. If $r \geq b(\overline{v})$, no one submits a bid in the auction. Also, in order to make sure that the bidding strategy is monotone and increasing (to guarantee that the FOC of bidding strategy is positive), we require the following assumption[4]:

Assumption 1. Given reserve price r, $\frac{q(v_i)}{Q(v_i)^2} \int_r^{v_i} Q(x)dx > \Phi(v_i)$, for all $v_i \geq v^c$.

[3] With $D = 0$, (13) can be re-written as follows:

$$\frac{1}{Q(v_i)} \int_{r}^{v_i} (Q(v_i) - Q(x))dx - \int_{r}^{v_i} \Phi(p)dp = 0,$$

which implies that $v^c = r$.

[4] Differentiating b in (13) with respect to v_i yields

$$\begin{aligned} \frac{db}{dv_i} &= \frac{q(v_i)}{Q(v_i)^2} \int_{r}^{v_i} Q(x)dx - \Phi(v_i) + \Phi(b-D)\frac{db}{dv_i}, \\ &= \frac{\frac{q(v_i)}{Q(v_i)^2} \int_r^{v_i} Q(x)dx - \Phi(v_i)}{1 - \Phi(b-D)}. \end{aligned}$$

To ensure that $\frac{db}{dv_i}$ is positive, we should have Assumption (1).

Further, it is easy to check that $b(v_i) < r$ for $v_i < v^c$, in other words, bidders with values lower than v^c will not submit any valid bids, as (1.) if $v_i \leq r$, obviously the bidder will not submit a valid bid, given that his expected payoff is negative with winning; (2.) if $r < v_i < v^c$, following the bidding strategy - (13), we see that the second and the third terms in the RHS are positive, and should thus have $b(v_i) < r$.

The bidder's bidding strategy for bidder i at $t = 0$ can thus be summarized as follows:

$$b(v_i) = \begin{cases} v_i - \frac{1}{Q(v_i)} \int_r^{v_i} Q(x)dx - \int_{b(v_i)-D}^{v_i} \Phi(p)dp & \text{if } v_i \geq v^c; \\ No & \text{if } v_i < v^c. \end{cases} \tag{14}$$

where $Q(.) \equiv F^{N-1}(.)$ and $q(.) \equiv Q'(.)$. We can then establish the following results (see Appendix for the full proof):

Lemma 2. *Given the reserve price r and the inclusive deposit D from the seller, when the auction mechanism is first-price form, there exists a PBE such that bidder i, for all $i = 1, ..., N$, uses the following strategies:*

(a) the bidding strategy b and the associated belief G_i at $t = 0$ are described by (14) *and* (1)*, respectively;*

(b) the deposit strategy e_i^0 is described by (7)*;*

(c) the outside option o_i^T is described by (2)*.*

5.2 Expected payment equivalence

In the auction literature, the 'expected revenue equivalence' is one of the central findings, which states that the expected payment from a bidder is exactly the same across the standard auction forms. Equivalently, the seller's expected revenue is regardless of the specific auction form. After characterizing the bidding strategies in the second-price and the first-price auctions above, it is natural to explore whether such a property still exists in the auctions with the outside option and the deposit requirement.

Let us denote auction form by A and all bidders in the auction A use a symmetric equilibrium bidding strategy β. Let $m^A(v_i)$ be the equilibrium expected payment in auction

A by a bidder with value v_i, and $m^A(v_i = \underline{v}) = 0$. Following the last line of (10), the expected surplus of bidder i by bidding $\beta(z)$ is given by

$$\pi(z, v_i) = Q(z)\left[v_i + \int_{\underline{v}}^{\beta(z)-D} \Phi(p)dp\right] + (1 - Q(z))\left[\int_{\underline{v}}^{v_i} \Phi(p)dp\right] - m^A(z). \tag{15}$$

where $Q(.) \equiv F^{N-1}(.)$ and $q(.) \equiv Q'(.)$. Given the payoff function, we can then establish the following result (see Appendix for the full proof):

Proposition 1. *The expected payment by a bidder in any standard auction with the outside option and the inclusive deposit requirement is the same.*

This result indicates that a bidder's expected payment does not depend on the particular auction form, thereby implying that the first-price and second-price auctions should yield the same expected payment to the seller. However, one should notice that since there exists the possibility of the winner defaulting, the seller's expected revenue is not equal to the sum of all bidders' expected payments. To distinguish, we call this result the 'Expected Payment Equivalence'.

6 The Seller's Optimal Choices on r and D

In this section, we are interested in examining the optimal choices for the seller on deposit requirement D and reserve price r at the beginning of the auction stage, given bidders' equilibrium strategies in Lemma (1). The total expected payment by a bidder, including the inclusive deposit, denoted by $\Omega(v_i, v^c(r), r, D)$, is given by

$$\begin{aligned}\Omega(v_i, v^c(r), r, D) = F^{N-1}(v^c)&\Big[(1 - \Phi(r - D))r + \Phi(r - D)D\Big] \\ &+ \int_{v^c}^{v_i} \Big[(1 - \Phi(b(x) - D))b(x) + \Phi(b(x) - D)D\Big] dF^{N-1}(x),\end{aligned} \tag{16}$$

where $b(v^c) = r$ and $b(x) = x - \int_{b(x)-D}^{x} \Phi(p)dp$. The expected revenue of the seller, denoted by $\mathbb{E}[R(r, D)]$, is the sum of the *ex ante* total expected payments of all N bidders, and thus we can write $\mathbb{E}[R(r, D)]$ as follows:

$$\mathbb{E}[R(r, D)] = N \int_{v^c}^{\bar{v}} \Omega(v_i, v^c(r), r, D)dF(v_i). \tag{17}$$

The seller chooses reserve price and deposit requirement to maximize his ex-ante expected revenue in the auction game. Let us use r^* and D^* to denote the optimal reserve price and the optimal deposit for the seller, respectively, and v^{c*} denoted the threshold of submitting bids. We then have the following result (see Appendix for the full proof):

Proposition 2. *Given that all N bidders follow the equilibrium strategies, the optimal D^*, r^*, and the associated threshold v^{c*} for the seller are determined by*

$$D^* = b(v^{c*}) - \underline{v}, \tag{18}$$

$$r^* = v^{c*} - \int_{\underline{v}}^{v^{c*}} \Phi(p)dp, \tag{19}$$

$$r^* = \left[\frac{1-F(v^{c*})}{f(v^{c*})}\frac{1-\Phi(v^{c*})}{1-\Phi(r^*-D^*)}\right]\left[1-\frac{\varphi(r^*-D^*)(r^*-D^*)}{1-\Phi(r^*-D^*)}\right] - \frac{\Phi(r^*-D^*)D^*}{1-\Phi(r^*-D^*)}. \tag{20}$$

Proposition (2) provides a novel finding that when there exists an ex-post outside option for the winner, the seller can be better off by setting a certain level of the inclusive deposit in the auction game. The intuition is this: On one hand, although charging no deposit would induce bidders to bid more aggressively in the auction, this increases the possibility (the risk) that the seller may end up with no payment from the winner. On the other hand, if the seller charges a high deposit, which is equivalent to paying the full amount immediately, this will force the winner to choose to commit in the current transaction (paying the auction price); equivalently, giving up the possibility of taking outside options. Thus, charging a large deposit amount in turn lowers bids from bidders and hurts the seller[5]. As a result, the optimal D the seller should charge is an interior solution, not a corner solution, i.e., 0 or $\bar{v}$.

From the result in Proposition (2), we also observe some interesting properties about the optimal reserve price r^*. First of all, if the seller is *not* allowed to charge any deposit in the auction game, namely, $D = 0$, we then $v^c = r$ and (20) can be rewritten as follows

$$r = \left[\frac{1-F(r)}{f(r)}\right]\left[1-\frac{\varphi(r)r}{1-\Phi(r)}\right] < \left[\frac{1-F(r)}{f(r)}\right]. \tag{21}$$

[5]Since $\frac{\partial b}{\partial D} < 0$ in (8), a higher D lowers the equilibrium bids from bidders, and moreover, once the winner pays the D, he will not take the outside option, as it becomes too costly to take the outside option; the auction price minus the deposit D is negative and that is always smaller than the outside option. Equivalently, the winner always pays the auction price.

Further, when the seller is allowed to charge a deposit, (18) indicates that the optimal D^* is greater than zero. Moreover, from (8), $b(v_i)$ is decreasing in D, which implies that r^* determined in (19) is less than the reserve price with $D = 0$. Therefore, we can conclude that the existence of ex-post outside option requires the seller to lower the optimal reserve price, compared to the Myerson's one. If, however, there is no outside option, i.e., $\Phi(v) = 0$, we then have $v^{c*} = r^*$ and (20) gives the standard Myerson's optimal reserve price for the seller.

Second, let us denote the hazard rate of $\Phi(v)$ by $\lambda(v) = \frac{\varphi(v)}{1-\Phi(v)}$. Then, if $\frac{1}{\lambda(r^*-D^*)} \leq (r^* - D^*)$, it is optimal for the seller to set the reserve price r^* equal to 0. This is because

$$\frac{1}{\lambda(r^* - D^*)} \leq (r^* - D^*) \;\Leftrightarrow\; \left[1 - \frac{\varphi(r^* - D^*)(r^* - D^*)}{1 - \Phi(r^* - D^*)}\right] \leq 0, \tag{22}$$

which implies that (22) < 0, and thus the seller should sets $r^* = 0$. This result provides a new explanation for the common observation that lots of auctions on eBay set reserve price *almost* equal to 0. From the analysis, we know that the existence of outside options would drive the sellers to offer zero reserve price in the auctions.

7 Exclusive Deposit Requirement

In this section, we turn to examine the exclusive deposit requirement where the deposit is in addition to the final selling price. We then build up the expected payment equivalence across different auction forms. Thereafter, we study the optimal auction design with the exclusive deposit and the reserve price for the the seller. We keep all other notations the same but denote the exclusive deposit by D.

7.1 Decisions on outside option at $t = T$

Given that now the deposit D is in addition to the final selling price κ, bidder i's decision on whether to exercise the outside option with price p at $t = T$ can be written as follows:

$$o_i^T(v_i, \kappa, p) \;=\; \begin{cases} 1 & \text{if } \left[e_i^0 = 1 \text{ and } v_i \geq \kappa \text{ and } p \geq \kappa\right]; \\ 0 & \text{if } \left[e_i^0 = 1 \text{ and } v_i \geq \kappa \text{ and } p < \kappa\right], \\ & \text{or } \left[e_i^0 = 1 \text{ and } v_i < \kappa \text{ and } p \leq v_i\right], \\ & \text{or } \left[e_i^0 = 0 \text{ and } v_i \geq p\right]; \\ No & \text{if } \left[e_i^0 = 1 \text{ and } v_i < \kappa \text{ and } p > v_i\right], \\ & \text{or } \left[e_i^0 = 0 \text{ and } v_i < p\right]. \end{cases} \tag{23}$$

(23) is identical to the outside option strategy characterized for the inclusive deposit in (2), with one exception: instead $\kappa - D$, the winning bidder uses the auction price κ in the decision.

7.2 Decisions on the call option at $t = 0$

The analysis is very similar to what we have done in Section (4.2). We can construct $\pi_D(v_i, \kappa, D)$ and $\pi_D(v_i, \kappa, D)$ accordingly as follows:

$$\pi_D(v_i, \kappa, D) = \int_{\underline{v}}^{\kappa} (v_i - p)\varphi(p)dp + \int_{\kappa}^{\overline{v}} [v_i - \kappa]\varphi(p)dp - D. \tag{24}$$

$$\pi_{ND}(v_i, \kappa, D) = \int_{\underline{v}}^{v_i} (v_i - p)\varphi(p)dp. \tag{25}$$

Then $L = \pi_D(v_i, \kappa, D) - \pi_{ND}(v_i, \kappa, D)$ is given by

$$L(v_i, \kappa, D) = \int_{\underline{v}}^{\kappa} (v_i - p)\varphi(p)dp + \int_{\kappa}^{\overline{v}} [v_i - \kappa]\varphi(p)dp - D - \int_{\underline{v}}^{v_i} (v_i - p)\varphi(p)dp.$$

Simplifying the equation above shows

$$\begin{aligned} L(v_i, \kappa, D) &= \int_{\kappa}^{\overline{v}} [v_i - \kappa]\varphi(p)dp - \int_{\kappa}^{v_i} (v_i - p)\varphi(p)dp - D, \\ &= \int_{\kappa}^{v_i} [1 - \Phi(p)]dp - D. \end{aligned} \tag{26}$$

which implies that given κ and D, there should exist a unique threshold for bidder i's private value, denoted by $\hat{v}_i(\kappa, D)$, such that $L(\hat{v}_i, \kappa, D) = 0$, where bidder i is indifferent between paying the exclusive deposit D and not paying it. Moreover, it is easy to check that $\hat{v}_i(\kappa, D)$ is increasing in both κ and D. From the discussion above, the strategy for bidder i on the deposit at $t = 0$ can now be formally stated as follows:

$$e_i^0(v_i, \kappa) = \begin{cases} 1 & \text{if } v_i \geq \hat{v}_i(\kappa, D); \\ 0 & \text{otherwise.} \end{cases} \tag{27}$$

7.3 Auction form and decisions on bidding at $t = 0$

7.3.1 Second-price auction

Now suppose the auction form at $t = 0$ is second-price, we examine the bidding strategy. Again, the equilibrium bid a bidder is willing to submit should make the bidder indifferent between 'waiting for outside option directly' and 'submitting a bid and then paying the D in the auction' at $t = 0$, that is,

$$\begin{aligned} \int_{\underline{v}}^{b} (v_i - p)\varphi(p)dp + \int_{b}^{\overline{v}} [v_i - b]\varphi(p)dp - D = \int_{\underline{v}}^{v_i} (v_i - p)\varphi(p)dp. \\ \Leftrightarrow \qquad b = v_i - \int_{b}^{v_i} \Phi(p)dp - D, \end{aligned} \tag{28}$$

The bidder's bidding strategy for bidder i at $t = 0$ can thus be summarized as follows:

$$b(v_i) = \begin{cases} v_i - \int_{b(v_i)}^{v_i} \Phi(p)dp - D & \text{if } v_i \geq v^c; \\ No & \text{if } v_i < v^c. \end{cases} \tag{29}$$

where v^c is determined by $b(v^c) = r$, or equivalently, $\int_r^{v^c} [1 - \Phi(p)]dp = D$. Further, it is easy to show that $\frac{db}{dv_i} = \frac{1-\Phi(v_i)}{1-\Phi(b)}$. Fixed a reserve price r, when D goes beyond a threshold, then no one bids. This threshold for D is $\bar{D} = \int_r^{\bar{v}} [1 - \Phi(p)]dp$. We can then establish the following result (see Appendix for the full proof):

Lemma 3. *Given the reserve price r and the exclusive deposit D from the seller, when the auction mechanism is second-price form, there exists a PBE such that bidder i, for all $i = 1, ..., N$, uses the following strategies:*

(a) the bidding strategy b *and the associated belief* G_i *at* $t = 0$ *are described by* (29) *and* (1)*, respectively;*

(b) the deposit strategy e_i^0 *is described by* (27)*;*

(c) the outside option o_i^T *is described by* (23).

7.3.2 First-price auction

Now suppose the auction form is first-price, we focus on the increasing, symmetric, and differentiable equilibrium bidding strategy from bidders, denoted by $\beta(v_i)$. Given that all other bidders use the same bidding strategy, bidder i's expected surplus by bidding b at $t = 0$ is given by

$$\begin{aligned}\pi(b, v_i) = Q(\beta^{-1}(b))\Bigg[\int_{\underline{v}}^{b}(v_i - p)\varphi(p)dp + \int_{b}^{\overline{v}}[v_i - b]\varphi(p)dp - D\Bigg] \\ + (1 - Q(\beta^{-1}(b)))\Bigg[\int_{\underline{v}}^{v_i}(v_i - p)\varphi(p)dp\Bigg], \\ = Q(\beta^{-1}(b))\Bigg[v_i - D + \int_{\underline{v}}^{b}\Phi(p)dp\Bigg] + (1 - Q(\beta^{-1}(b)))\Bigg[\int_{\underline{v}}^{v_i}\Phi(p)dp\Bigg] - Q(\beta^{-1}(b))b.\end{aligned} \tag{30}$$

where β is the equilibrium bidding strategy, $Q(.) \equiv F^{N-1}(.)$ and $q(.) \equiv Q'(.)$. Differentiating (30) with respect to b, $\frac{\partial\pi(b,v_i)}{\partial b} = 0$, yields the first-order condition:

$$\begin{aligned}&\frac{q(\beta^{-1}(b))}{\beta'(\beta^{-1}(b))}\Bigg[\int_{\underline{v}}^{b}(v_i - p)\varphi(p)dp + \int_{b}^{\overline{v}}[v_i - b]\varphi(p)dp - D\Bigg] \\ &- Q(\beta^{-1}(b))\Bigg[\int_{b}^{\overline{v}}\varphi(p)dp\Bigg] - \frac{q(\beta^{-1}(b))}{\beta'(\beta^{-1}(b))}\Bigg[\int_{\underline{v}}^{v_i}(v_i - p)\varphi(p)dp\Bigg] = 0.\end{aligned} \tag{31}$$

At a symmetric equilibrium $b = \beta(v_i)$, we re-write the equation above as follows:

$$\begin{aligned}&q(v_i)\Bigg[-\int_{\beta(v_i)}^{v_i}(v_i - p)\varphi(p)dp + \int_{\beta(v_i)}^{\overline{v}}[v_i - \beta(v_i)]\varphi(p)dp - D\Bigg] \\ &- \beta'(v_i)Q(v_i)\Bigg[\int_{\beta(v_i)}^{\overline{v}}\varphi(p)dp\Bigg] = 0.\end{aligned}$$

Simplifying the equation yields

$$q(v_i)(v_i - D) = \Big(q(v_i)\beta(v_i) + \beta'(v_i)Q(v_i)\Big) + \Big(q(v_i)\int_{\beta(v_i)}^{v_i} \Phi(p)dp - Q(v_i)\beta'(v_i)\Phi(\beta(v_i))\Big).$$

or equivalently,

$$v_i(q(v_i) - D) = \frac{d}{dv_i}(Q(v_i)\beta(v_i)) + \frac{d}{dv_i}(\int_{\beta(v_i)}^{v_i} Q(v_i)\Phi(p)dp).$$

We then have

$$\frac{1}{Q(v_i)}\int_{\underline{v}}^{v_i}(x - D)q(x)dx = \beta(v_i) + \int_{\beta(v_i)}^{v_i}\Phi(p)dp.$$

$$\iff \beta(v_i) = \frac{1}{Q(v_i)}\int_{\underline{v}}^{v_i} xq(x)dx - \int_{\beta(v_i)}^{v_i}\Phi(p)dp - D.$$

We can thus write the equilibrium bidding strategy as follows:

$$\begin{aligned} b &= \frac{1}{Q(v_i)}\int_{\underline{v}}^{v_i} xq(x)dx - \int_{b}^{v_i}\Phi(p)dp - D. \\ &= v_i - \frac{1}{Q(v_i)}\int_{\underline{v}}^{v_i} Q(x)dx - \int_{b}^{v_i}\Phi(p)dp - D. \end{aligned} \tag{32}$$

Further, in order to make sure that the bidding strategy is monotone and increasing (to guarantee that the FOC of bidding strategy is positive), we require the following assumption[6]:

Assumption 2. Given reserve price r, $\frac{q(v_i)}{Q(v_i)^2}\int_r^{v_i} Q(x)dx > \Phi(v_i)$, for all $v_i \geq v^c$.

After deriving the equilibrium bidding strategy, given reserve price r from the seller, (32) can be written as follows:

$$b = v_i - \frac{1}{Q(v_i)}\int_{r}^{v_i} Q(x)dx - \int_{b}^{v_i}\Phi(p)dp - D. \tag{33}$$

[6]Differentiating b with respect to v_i yields

$$\begin{aligned} \frac{db}{dv_i} &= \frac{q(v_i)}{Q(v_i)^2}\int_r^{v_i} Q(x)dx - \Phi(v_i) + \Phi(b)\frac{db}{dv_i} \\ &= \frac{\frac{q(v_i)}{Q(v_i)^2}\int_r^{v_i} Q(x)dx - \Phi(v_i)}{1 - \Phi(b)}. \end{aligned}$$

To ensure that $\frac{db}{dv_i}$ is positive, we should have Assumption (2).

There should exist a unique v^c such that $b(v^c) = r$ if $r \geq b(\underline{v})$. In this case, if $D = 0$, $v^c = r$. Note that if $r < b(\underline{v})$, the all bidders can submit valid bids. In this case, when the seller charges no 'call-option' payment, i.e., $D = 0$, it is clear that $b(\underline{v}) = \underline{v}$. Further, it is easy to show that $b(v_i) < r$ for $v_t < v^c$, as (1.) if $v_i \leq r$, obviously the bidder will not submit a valid bid, given that his expected payoff is negative with winning; (2.) if $r < v_i < v^c$, following the bidding strategy, we should clearly have $b(v_i) < r$.

The bidder's bidding strategy for bidder i at $t = 0$ can thus be summarized as follows:

$$b(v_i) \;=\; \begin{cases} v_i - \frac{1}{Q(v_i)} \int_r^{v_i} Q(x)dx - \int_{b(v_i)}^{v_i} \Phi(p)dp - D & \text{if } v_i \geq v^c; \\ No & \text{if } v_i < v^c. \end{cases} \tag{34}$$

where $Q(.) \equiv F^{N-1}(.)$ and $q(.) \equiv Q'(.)$. We then have the following result (see Appendix for the full proof):

Lemma 4. *Given the reserve price r and the exclusive deposit D from the seller, when the auction mechanism is first-price form, there exists a PBE such that bidder i, for all $i = 1, ..., N$, uses the following strategies:*

(a) the bidding strategy b and the associated belief G_i at $t = 0$ are described by (34) *and* (1)*, respectively;*

(b) the deposit strategy e_i^0 is described by (27)*;*

(c) the outside option o_i^T is described by (23).

7.4 Expected payment equivalence

After establishing the bidding strategies in the second-price and first-price auctions, in this subsection we further examine whether the 'Expected Payment Equivalence' still holds with the exclusive deposit requirement. The expected surplus of bidder i by bidding $\beta(z)$ is given by

$$\pi(z, v_i) = Q(z)\left[v_i - D + \int_{\underline{v}}^{\beta(z)} \Phi(p)dp\right] + (1 - Q(z))\left[\int_{\underline{v}}^{v_i} \Phi(p)dp\right] - m^A(z). \tag{35}$$

where $Q(.) \equiv F^{N-1}(.)$ and $q(.) \equiv Q'(.)$. The result can be stated as follows (see Appendix for the full proof):

Proposition 3. *The expected payment by a bidder in any standard auction with the outside option and the exclusive deposit requirement is the same.*

7.5 The seller's optimal choices on r and D

In this section, we are interested in examining the optimal choices for the seller on the exclusive deposit payment D and reserve price r (and the associated cutoff v^c) at the beginning of the auction stage $t = 0$, given bidders' equilibrium strategies characterized above. The expected payment received by the seller from a bidder, denoted by $A(v_i, v^c(r), r, D)$, can be written as follows,

$$A(v_i, v^c(r), r, D) = F^{N-1}(v^c)\Big[(1-\Phi(r))r\Big] + \int_{v^c}^{v_i}\Big[(1-\Phi(b(x)))b(x)\Big]dF^{N-1}(x), \qquad (36)$$

where $b(v^c) = r$ and $b(x) = x - \int_{b(x)}^{x}\Phi(p)dp - D$. We know that the expected revenue of the seller, denoted by $\mathbb{E}[R(r, D)]$, is the sum of the *ex ante* total expected payments of all N bidders plus the exclusive deposit payment D, and thus we can write $\mathbb{E}[R(r, D)]$ as follows:

$$\mathbb{E}[R(r, D)] = N\int_{v^c}^{\bar{v}} A(v_i, v^c(r), r, D)dF(v_i) + (1 - F^N(v^c))D. \qquad (37)$$

Maximizing the seller's expected revenue in terms of the optimal r and D gives the following result (see Appendix for the full proof):

Proposition 4. *Given that all N bidders follow the equilibrium strategies, the optimal D^*, r^*, and the associated threshold v^{c*} for the seller are determined by*

$$b(v^{c*}) = \underline{v}, \qquad (38)$$

$$D^* = \int_{r^*}^{v^{c*}}[1 - \Phi(p)]dp, \qquad (39)$$

$$\frac{(1 - F(v^{c*}))(1 - \Phi(v^{c*}))}{f(v^{c*})} = \frac{r^* + D^*}{1 - \varphi(r^*)r^*}. \qquad (40)$$

When there is no outside option, i.e., $\Phi(.) = 0$, we clearly see that $v^{c*} = r^*$, and $D^* = 0$. This implies that the optimal reserve price is equal to the Myerson's one.

8 Expected Revenue Equivalence Between Inclusive and Exclusive Deposits

In this section, we compare the seller revenues across the inclusive and the exclusive deposit payments. Specially, we are interested in which one provides a higher seller revenue. Note that here v^c can be interpreted as the 'executive reserve' in the auction, where bidders submit valid bids if and only if their private values are no less than the threshold v^c. Let us use $\mathbb{E}[R_D(r, D)]$ and $\mathbb{E}[R_c(r, D)]$ to denote the expected seller revenues with the inclusive and the exclusive deposit requirements, respectively. We then establish the following results(see Appendix for the full proof):

Proposition 5. *The inclusive and the exclusive deposit payments yield the same expected revenue to the seller.*

Interestingly, we have seen that both settings give exactly same expected revenue to the seller. This also explains why we observe that both deposit requirements have been commonly used in reality.

9 Empirical Analysis

****** empirical evidence from eBay******

10 Conclusion

****** to be added soon ******

Appendix: Proofs

Proof of Lemma (1)

Most of the result was already shown in the text in Section (4). The only task left is to establish the optimality of the equilibrium bidding strategy, in other words, to check whether a bidder has an incentive to deviate from (9). Suppose that bidder i does not follow the equilibrium bidding strategy $b(v_i)$ but submits z in the auction, then we consider the following two cases:

Case (a.) $z < b(v_i)$. If $b(v_i) > z > b_{-i}^{(1)}$, then bidder i still wins and that the surplus is not affected. If $b_{-i}^{(1)} > b(v_i) > z$, bidder i still loses in the auction. If $b(v_i) > b_{-i}^{(1)} > z$, the bidder will lose, where, however, he would have won if he had bid $b(v_i)$. Thus, there is no incentive to bid lower than $b(v_i)$.

Case (b.) $z > b(v_i)$. If $z > b(v_i) > b_{-i}^{(1)}$, then bidder i still wins and that the surplus is not affected. If $b_{-i}^{(1)} > z > b(v_i)$, bidder i still loses in the auction. If $z > b_{-i}^{(1)} > b(v_i)$, by doing so it increases the probability of winning, but this induces RHS greater than LHS in (8), which indicates that bidding higher than $b(v_i)$ is worse than taking outside option directly. Thus, there is no incentive to bid lower than $b(v_i)$.

Given that bidders do not have incentive to deviate but follow bidding strategy $b(v_i)$, then it is optimal for bidders whose bids are less than r to not bid but take the outside options directly. Note that there exists an out of equilibrium event that a bidder with value between r and v^c, instead of submitting no bid, would submit a positive bid and then, conditional on winning, does not pay the deposit D but simply wait for the outside option. This is indifferent for the bidder to take the outside option directly at $t = 0$. Such 'off-equilibrium' behavior can be easily ruled out by the fact that an inevitable cost will be incurred when a bidder defaults from winning in online auctions. This cost is not a payment to the original seller, but is the time and money associated with the transaction cancellation. For example, in such a case eBay usually suspense the buyer's user account for 'malicious bidding'. In addition, this default cost concern reinforces the argument in Case (b.) of the proof that a bidder does not have an incentive to bid high. Therefore, we can conclude that $b(v_i)$ is a weakly dominate strategy for bidder i in the auction game. $\square$

Proof of Lemma (2)

We have established the optimality of the strategies on outside option and deposit payment. The derivation of the bidding strategy $\beta(v_i)$ in (12) gives the necessary condition for the equilibrium bidding strategy. The only task left is to establish the optimality of the equilibrium bidding strategy, in other words, to check whether a bidder has an incentive to deviate from (14), given that all other bidders follow β.

Now suppose that, instead of following the equilibrium bidding strategy to submit $\beta(v_i)$, bidder i bids $b = \beta(z)$, which is corresponding to private value z. Then bidder i's expected payoff from pretending type z and bidding $\beta(z)$ when his value is v_i can be written as follows:

$$\begin{aligned}
\pi(b, v_i) &= Q(z)\left[v_i + \int_{\underline{v}}^{\beta(z)-D} \Phi(p)dp\right] + (1 - Q(z))\left[\int_{\underline{v}}^{v_i} \Phi(p)dp\right] - Q(z)\beta(z), \\
&= Q(z)v_i + \int_{\underline{v}}^{v_i} \Phi(p)dp - Q(z)\int_{\beta(z)-D}^{v_i} \Phi(p)dp \\
&\qquad - \left[Q(z)z - \int_{\underline{v}}^{z} Q(x)dx - Q(z)\int_{\beta(z)-D}^{z} \Phi(p)dp\right], \\
&= Q(z)(v_i - z) + \int_{\underline{v}}^{z} Q(x)dx + \int_{\underline{v}}^{v_i} \Phi(p)dp + Q(z)\int_{v_i}^{z} \Phi(p)dp.
\end{aligned} \tag{41}$$

We thus obtain that

$$\begin{aligned}
\frac{\partial \pi(\beta(z), v_i)}{\partial z} &= q(z)(z - v_i) + q(z)\int_{v_i}^{z} \Phi(p)dp + Q(z)\Phi(z), \\
&= q(z)(z - v_i) + \frac{\partial}{\partial} Q(z)\int_{v_i}^{z} \Phi(p)dp.
\end{aligned} \tag{42}$$

When $z = v_i$, $\frac{\partial \pi(\beta(z), v_i)}{\partial z} = 0$; the bidder's expected payoff is maximized. Therefore, if all other bidders follow the strategy β, a bidder with private value v_i cannot benefit by bidding anything other than $\beta(v_i)$. We can then conclude that β is a symmetric equilibrium strategy. Note that there exists an out of equilibrium event that a bidder with value between r and v^c, instead of submitting no bid, would submit a positive bid and then, conditional on winning, does not pay the deposit D but simply wait for the outside option. As we discussed in proof of Lemma (1), this is indifferent for the bidder to take the outside option directly at $t = 0$. Such 'off-equilibrium' behavior can be easily ruled out by the fact that an inevitable cost will be incurred when a bidder defaults from winning in online auctions. □

Proof of Proposition (1)

Differentiating $\pi(z, v_i)$ with respect to z yields

$$\begin{aligned} \frac{\partial \pi(z, v_i)}{\partial z} &= q(z)\left[v_i + \int_{\underline{v}}^{\beta(z)-D} \Phi(p)dp\right] \\ &\qquad + Q(z)\Phi(\beta(z)-D)\beta'(z) - q(z)\left[\int_{\underline{v}}^{v_i} \Phi(p)dp\right] - \frac{d}{dz}m^A(z), \\ &= q(z)v_i + Q(z)\Phi(\beta(z)-D)\beta'(z) - q(z)\left[\int_{\beta(z)-D}^{v_i} \Phi(p)dp\right] - \frac{d}{dz}m^A(z), \\ &= q(z)v_i - \frac{\partial}{\partial z}\left(Q(z)\int_{\beta(z)-D}^{v_i} \Phi(p)dp\right) - \frac{d}{dz}m^A(z), \\ &= 0. \end{aligned} \tag{43}$$

In equilibrium, it is optimal for bidder i to report $z = v_i$, and therefore, for all v_i,

$$\frac{d}{dv_i}m^A(v_i) = q(v_i)v_i - \frac{\partial}{\partial v_i}\left(Q(v_i)\int_{\beta(v_i)-D}^{v_i} \Phi(p)dp\right). \tag{44}$$

Therefore, we obtain that

$$m^A(v_i) = \int_{\underline{v}}^{v_i} xq(x)dx - Q(v_i)\int_{\beta(v_i)-D}^{v_i} \Phi(p)dp, \tag{45}$$

which indicates that the expected payment does not depend on the particular auction form A. Therefore, we complete the proof. □

Proof of Proposition (2)

Replacing $r = b(v^c)$ and simplifying (17) yields

$$\begin{aligned} \frac{\mathbb{E}[R(r, D)]}{N} &= (1 - F(v^c))F^{N-1}(v^c)\Big[(1 - \Phi(b(v^c) - D))b(v^c) + \Phi(b(v^c) - D)D\Big] \\ &\quad + \int_{v^c}^{\bar{v}} (1 - F(x))\Big[(1 - \Phi(b(x) - D))b(x) + \Phi(b(x) - D)D\Big]dF^{N-1}(x). \end{aligned} \tag{46}$$

Step(i) Given (46), let us examine the optimal D^*. Differentiating (46) with respect to

D yields

$$\frac{\partial}{\partial D}\frac{\mathbb{E}[R(r,D)}{N} = (1-F(v^c))F^{N-1}(v^c)\Big[\varphi(b(v^c)-D)(b(v^c)-D)+\Phi(b(v^c)-D)\Big] \\ + \int_{v^c}^{\bar{v}}(1-F(x))\Big[\varphi(b(x)-D)(b(x)-D)+\Phi(b(x)-D)\Big]dF^{N-1}(x), \\ > 0. \tag{47}$$

This implies that in order to maximize $\mathbb{E}[R(r,D)]$, it is optimal for the seller to charge the deposit as large as possible. Recall v^c is uniquely determined by $b(v^c) = v^c - \int_{b(v^c)-D}^{v^c}\Phi(p)dp = r$. Thus, the maximum deposit, denoted by D^*, the seller can charge is given by

$$b(v^c) - D^* = \underline{v} \ \Leftrightarrow \ D^* = b(v^c) - \underline{v}. \tag{48}$$

Step(ii) Now let us examine the optimal r^* and v^{c*} for the seller. Differentiate (46) with respect to v^c yields

$$\frac{\partial}{\partial v^c}\frac{\mathbb{E}[R(r,D^*)}{N} = -f(v^c)F^{N-1}(v^c)\Big[(1-\Phi(b(v^c)-D^*))b(v^c)+\Phi(b(v^c)-D^*)D^*\Big] \\ + (1-F(v^c))F^{N-1}(v^c)b'(v^c)\Big[(1-\Phi(b(v^c)-D^*))-\varphi(b(v^c)-D^*)(b(v^c)-D^*)\Big], \\ = F^{N-1}(v^c)\Bigg(-f(v^c)\Big[(1-\Phi(b(v^c)-D^*))b(v^c)+\Phi(b(v^c)-D^*)D^*\Big] \\ + (1-F(v^c))b'(v^c)\Big[(1-\Phi(b(v^c)-D^*))-\varphi(b(v^c)-D^*)(b(v^c)-D^*)\Big]\Bigg), \\ = 0. \tag{49}$$

(49) implies the optimal v^{c*} must satisfy

$$-f(v^{c*})\Big[(1-\Phi(b(v^{c*})-D^*))b(v^{c*})+\Phi(b(v^{c*})-D^*)D^*\Big] \\ + (1-F(v^{c*}))b'(v^{c*})\Big[(1-\Phi(b(v^{c*})-D^*))-\varphi(b(v^{c*})-D^*)(b(v^{c*})-D^*)\Big] = 0. \tag{50}$$

Given that $\frac{\partial b(v_i)}{\partial v_i} = \frac{1-\Phi(v_i)}{1-\Phi(b(v_i)-D)}$ and $b(v^{c*}) = v^{c*} - \int_{\underline{v}}^{v^{c*}} \Phi(p)dp = r^*$, simplifying (69) gives us

$$r^* = \left[\frac{1-F(v^{c*})}{f(v^{c*})}\frac{1-\Phi(v^{c*})}{1-\Phi(r^*-D^*)}\right]\left[1 - \frac{\varphi(r^*-D^*)(r^*-D^*)}{1-\Phi(r^*-D^*)}\right] - \frac{\Phi(r^*-D^*)D^*}{1-\Phi(r^*-D^*)}. \tag{51}$$

We then complete the proof. □

Proof of Lemma (3)

****** to be added soon ******

Proof of Lemma (4)

****** to be added soon ******

Proof of Proposition (3)

Differentiating $\pi(z, v_i)$ with respect to z yields

$$\begin{aligned}
\frac{\partial \pi(z, v_i)}{\partial z} &= q(z)\left[v_i - D + \int_{\underline{v}}^{\beta(z)} \Phi(p)dp\right] \\
&\quad + Q(z)\Phi(\beta(z))\beta'(z) - q(z)\left[\int_{\underline{v}}^{v_i} \Phi(p)dp\right] - \frac{d}{dz}m^A(z) \\
&= q(z)(v_i - D) + Q(z)\Phi(\beta(z))\beta'(z) - q(z)\left[\int_{\beta(z)}^{v_i} \Phi(p)dp\right] - \frac{d}{dz}m^A(z) \\
&= q(z)(v_i - D) - \frac{\partial}{\partial z}\left(Q(z)\int_{\beta(z)}^{v_i} \Phi(p)dp\right) - \frac{d}{dz}m^A(z) \\
&= 0.
\end{aligned} \tag{52}$$

In equilibrium, it is optimal for bidder i to report $z = v_i$, and therefore, for all v_i,

$$\frac{d}{dv_i}m^A(v_i) = q(v_i)(v_i - D) - \frac{\partial}{\partial v_i}\left(Q(v_i)\int_{\beta(v_i)}^{v_i} \Phi(p)dp\right). \tag{53}$$

Therefore, we obtain that

$$m^A(v_i) = \int_{\underline{v}}^{v_i} xq(x)dx - Q(v_i)\int_{\beta(v_i)}^{v_i} \Phi(p)dp - D. \tag{54}$$

which indicates that the expected payment does not depend on the particular auction form A. □

Proof of Proposition (4)

Simplifying (37) yields

$$\begin{aligned}\mathbb{E}[R(r,D)] = N(1-F(v^c))F^{N-1}(v^c)\Big[(1-\Phi(b(v^c)))b(v^c)\Big] \\ + N\int_{v^c}^{\bar{v}}(1-F(x))\Big[(1-\Phi(b(x)))b(x)\Big]dF^{N-1}(x) + (1-F^N(v^c))D.\end{aligned} \tag{55}$$

Step(i) Given (55), let us examine the optimal D^*. Differentiating (55) with respect to D yields

$$\frac{\partial}{\partial D}\mathbb{E}[R(r,D) = (1-F^N(v^c)) > 0. \tag{56}$$

This implies that in order to maximize $\mathbb{E}[R(r,D)]$, it is optimal for the seller to charge the exclusive deposit payment as large as possible. Recall that given v^c, the reserve price r is uniquely determined by $\int_r^{v^c}[1-\Phi(p)]dp = D$. Thus, the maximum deposit, denoted by D^*, the seller can charge is given by

$$D^* = \int_{r^*=\underline{v}}^{v^c}[1-\Phi(p)]dp \iff r^* = \underline{v}. \tag{57}$$

Here, the optimal D^* is slightly different, compared to the inclusive deposit, where $r - D = \underline{v}$ determines the D^*.

Step(ii) Now let us examine the optimal v^c for the seller. Differentiate (46) with respect to v^c yields

$$\begin{aligned}
\frac{\partial}{\partial v^c}\mathbb{E}[R(r, D^*)] &= -Nf(v^c)F^{N-1}(v^c)\Big[(1-\Phi(b(v^c)))b(v^c) + D^*\Big] \\
&\qquad + N(1-F(v^c))F^{N-1}(v^c)b'(v^c)\Big[(1-\Phi(b(v^c))) - \varphi(b(v^c))b(v^c)\Big] \\
&= NF^{N-1}(v^c)\Bigg(-f(v^c)\Big[(1-\Phi(b(v^c)))b(v^c) + D^*\Big] \\
&\qquad + (1-F(v^c))b'(v^c)\Big[(1-\Phi(b(v^c))) - \varphi(b(v^c))b(v^c)\Big]\Bigg) \\
&= 0.
\end{aligned} \tag{58}$$

(49) implies the optimal v^{c*} must satisfy

$$-f(v^c)\Big[(1-\Phi(b(v^c)))b(v^c) + D^*\Big] + (1-F(v^c))b'(v^c)\Big[(1-\Phi(b(v^c))) - \varphi(b(v^c))b(v^c)\Big] = 0. \tag{59}$$

Given that $\frac{\partial b(v_i)}{\partial v_i} = \frac{1-\Phi(v_i)}{1-\Phi(b(v_i))}$ and $b(v^{c*}) = r^* = \underline{v}$, simplifying (59) gives us

$$\begin{aligned}
f(v^{c*})\Big[r^* + D^*\Big] &= (1-F(v^{c*}))(1-\Phi(v^{c*}))\Big[1-\varphi(r^*)r^*\Big] \\
\Leftrightarrow \frac{(1-F(v^{c*}))(1-\Phi(v^{c*}))}{f(v^{c*})} &= \frac{r^* + D^*}{1-\varphi(r^*)r^*}.
\end{aligned} \tag{60}$$

We then complete the proof. □

Proof of Proposition (5)

The seller's revenue with the exclusive deposit is given by

$$\begin{aligned}\mathbb{E}[R_c(r,D)] &= N(1-F(v^c))F^{N-1}(v^c)\Big[(1-\Phi(b_c(v^c)))b_c(v^c)\Big] \\ &\quad + N\int_{v^c}^{\bar{v}}(1-F(x))\Big[(1-\Phi(b_c(x)))b_c(x)\Big]dF^{N-1}(x) + (1-F^N(v^c))D. \\ &= N(1-F(v^c))F^{N-1}(v^c)\Big[(1-\Phi(b_c(v^c)))b_c(v^c)+D\Big] \\ &\quad + N\int_{v^c}^{\bar{v}}(1-F(x))\Big[(1-\Phi(b_c(x)))b_c(x)+D\Big]dF^{N-1}(x)\end{aligned} \tag{61}$$

with the bidding strategy

$$b_c(x) = x - \int_{b_c(x)}^{x}\Phi(p)dp - D \;\Leftrightarrow\; \int_{b_c(x)}^{x}[1-\Phi(p)]dp = D. \tag{62}$$

The seller's revenue with the inclusive deposit is given by

$$\begin{aligned}\mathbb{E}[R_D(r,D)] &= N(1-F(v^c))F^{N-1}(v^c)\Big[(1-\Phi(b_D(v^c)-D))b_D(v^c)+\Phi(b_D(v^c)-D)D\Big] \\ &\quad + N\int_{v^c}^{\bar{v}}(1-F(x))\Big[(1-\Phi(b_D(x)-D))b_D(x)+\Phi(b_D(x)-D)D\Big]dF^{N-1}(x).\end{aligned} \tag{63}$$

with the bidding strategy

$$b_D(x) = x - \int_{b_D(x)-D}^{x}\Phi(p)dp \;\Leftrightarrow\; \int_{b_D(x)-D}^{x}[1-\Phi(p)]dp = D. \tag{64}$$

Step (i.) Now given the same v^c and D, (62) and (64) imply that

$$b_c(x) = b_D(x) - D. \tag{65}$$

Furthermore, plugging v^c into the equation above shows

$$b_c(v^c) = b_D(v^c) - D = r = \underline{v}. \tag{66}$$

Step(ii.) Given what we have in step (i.), let us further simplify the seller revenues in (61) and (63).

$$\begin{aligned}\mathbb{E}[R_c(r, D)] = N(1 - F(v^c))F^{N-1}(v^c)\Big[(1 - \Phi(\underline{v})\underline{v} + D\Big] \\ + N\int_{v^c}^{\bar{v}}(1 - F(x))\Big[(1 - \Phi(b_c(x)))b_c(x) + D\Big]dF^{N-1}(x)\end{aligned} \tag{67}$$

$$\begin{aligned}\mathbb{E}[R_D(r, D)] &= N(1 - F(v^c))F^{N-1}(v^c)\Big[(1 - \Phi(\underline{v}))(\underline{v} + D) + \Phi(\underline{v})D\Big] \\ &\quad + N\int_{v^c}^{\bar{v}}(1 - F(x))\Big[(1 - \Phi(b_D(x) - D))b_D(x) + \Phi(b_D(x) - D)D\Big]dF^{N-1}(x) \\ &= N(1 - F(v^c))F^{N-1}(v^c)\Big[(1 - \Phi(\underline{v}))(\underline{v} + D) + \Phi(\underline{v})D\Big] \\ &\quad + N\int_{v^c}^{\bar{v}}(1 - F(x))\Big[(1 - \Phi(b_c(x)))(b_c(x) + D) + \Phi(b_c(x))D\Big]dF^{N-1}(x) \\ &= N(1 - F(v^c))F^{N-1}(v^c)\Big[(1 - \Phi(\underline{v})\underline{v} + D\Big] \\ &\quad + N\int_{v^c}^{\bar{v}}(1 - F(x))\Big[(1 - \Phi(b_c(x)))b_c(x) + D\Big]dF^{N-1}(x) \\ &= \mathbb{E}[R_c(r, D)]\end{aligned} \tag{68}$$

This implies that (68) exactly equals (67); both settings generate exactly same revenues for the seller.

Step(iii.) In the last step we show that the optimal v^{c*} and D^* are exactly the same in both settings. Recall that v^{c*} in the setting with the inclusive deposit requirement is determined by

$$
\begin{aligned}
& - f(v^{c*})\Big[(1-\Phi(b_D(v^{c*})-D^*))b_D(v^{c*}) + \Phi(b_D(v^{c*})-D^*)D^*\Big] \\
& + (1-F(v^{c*}))b'_D(v^{c*})\Big[(1-\Phi(b_D(v^{c*})-D^*)) - \varphi(b_D(v^{c*})-D^*)(b_D(v^{c*})-D^*)\Big] = 0.
\end{aligned} \tag{69}
$$

Given that $\frac{\partial b_D(v_i)}{\partial v_i} = \frac{1-\Phi(v_i)}{1-\Phi(b_D(v_i)-D)}$ and $b_D(v^{c*}) - D^* = r^* = \underline{v}$, simplifying (69) gives us

$$\frac{(1-F(v^{c*}))(1-\Phi(v^{c*}))}{f(v^{c*})} = \frac{\underline{v} + D^*}{1-\varphi(\underline{v})\underline{v}} \tag{70}$$

and then D^* is determined by

$$\int_{\underline{v}}^{v^{c*}} [1-\Phi(p)]dp = D^*. \tag{71}$$

Clearly we see that equations (70) and (71) give the exactly same v^{c*} and D^* as in the exclusive deposit setting. Thus, in equilibrium both settings generate the same expected revenue to the seller. Note that although v^* and D^* are exactly the same, optimal reserve prices are different across these two settings. In the exclusive deposit setting, optimal $r^* = b_c(v^{c*}) = \underline{v}$, while in the inclusive deposit setting, the optimal $r^* = b_D(v^{c*}) = \underline{v} + D^*$. □

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