Auctions with Selective Entry and Risk Averse Bidders: Theory and Evidence*

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Abstract

We study auctions with selective entry and risk averse bidders. Our model accounts for risk averse bidders’ endogenous participation decision and thus encompasses the existing entry models. We establish entry and bidding equilibrium in both first price auction and ascending auction mechanisms, and show that bidders’ entry behavior differs between these two mechanisms with different forms of risk aversion. Our approach provides testable implications of risk aversion in terms of entry behavior. We analyze a timber auction data set, and propose a simple test for the form of bidders’ risk aversion based on our model implications.

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1 Introduction

The concept of risk aversion has played a key role in modeling economic agents’ decision with uncertainty since Pratt’s (1964) formulation. Because of uncertainty faced by bidders in auctions, as evidenced from empirical and experimental studies (e.g. Athey and Levin (2001), Cox, Smith and Walker (1988), and Goeree, Holt and Palfrey (2002), among others), bidders can exhibit risk aversion attitude when contemplating their decisions.\(^1\) Therefore, it is important to take into account risk aversion when analyzing bidders’ decision and modeling auctions as games of incomplete information. To this end, auction theory has made much advance in studying risk aversion. For example, Maskin and Riley (1984) formally characterize the bidding equilibrium within the independent private value paradigm with risk aversion and compare revenues generated by different auction mechanisms. A notable result is that the revenue equivalence result derived under the risk neutrality assumption no longer holds for risk averse bidders; in particular, first price auctions generate more revenues than ascending auctions for the seller when bidders are risk averse. Matthews (1987) makes the comparison from a risk averse buyer’s point of view. However, testing for risk aversion and assessing the form and magnitude of bidders’ risk aversion using field data from an econometric perspective has become a challenging problem. For the former, the challenge arises because auction theory does not provide testable implications of risk aversion in terms of bids that can be used to test for risk aversion.\(^2\) For the latter, the difficulty is associated with the non-identification of the first price auction model with risk aversion as shown by Guerre, Perrigne, and Vuong (2009). As a result, a few recent articles using the structural

\(^{1}\text{Bajari and Hortaçsu (2005) also show that a structural model with risk aversion provides the best fit to some experimental data among a set of competing models.}\)

\(^{2}\text{This can be contrasted to the risk neutral case where testable implications can be derived in terms of bids among different information structures such as a private value paradigm versus a common value paradigm, which have been exploited first in Hendricks and Porter (1988) and their subsequent work surveyed in Porter (1995), and later by Haile, Hong and Shum (2003), Hendricks, Pinkse and Porter (2003), among others. Recently this approach has been extended to testing for affiliation using bids from risk neutral bidders by Jun, Pinkse and Wan (2010), and de Castro and Paarsch (2010).}\)
approach have attempted to exploit additional restrictions such as exclusion restrictions (e.g. Guerre, Perrigne and Vuong (2009), Campo, Guerre, Perrigne and Vuong (2011) and Campo (2012)) or combining bidding data from ascending auctions and first price auctions assuming the value distributions in both formats are the same for nonparametric/semiparametric identification of the model with risk aversion as in Lu and Perrigne (2008).

This article attempts to derive testable implications of risk aversion when taking into account bidders’ endogenous entry decision. We build on the affiliated-signal (AS) model as called in Gentry and Li (2014) and extend it to accommodate risk aversion. The AS model is first introduced in Ye (2007) with risk neutral bidders. It assumes that potential bidders draw private signals before entry and the participants’ private values are drawn after entry and are affiliated with their pre-entry private signals. The entry is thus selective because of the affiliation between the pre-entry signal and post-entry private value. The AS model is general as it includes the mixed strategy entry model (with no selection) by Levin and Smith (1994) and the pure strategy entry model (with full selection because potential bidders know their values before entry) by Samuelson (1985). The AS model has recently drawn attention in the empirical/econometrics literature. For example, Gentry and Li (2014) address nonparametric identification of the AS model, Marmer, Shneyrov and Xu (2013) propose a nonparametric test to distinguish among the Levin and Smith model, the Samuelson model, and the AS model, and Roberts and Sweeting (2010, 2012) use a parametric specification of the AS model to study ascending timber auctions.

Specifically, in this article, we extend the AS model with risk neutrality to the case with risk aversion. We characterize entry equilibrium and bidding equilibrium for both first price auctions and ascending auctions, and derive a set of implications for both auction formats. Most importantly, we find that the expected utility between the two formats can be ranked (differently) with different forms of bidders’ absolute risk aversion such as decreasing absolute risk aversion (DARA), or constant absolute risk aversion (CARA), or increasing absolute risk aversion (IARA). As a result, entry threshold and thus entry probability between these two formats can also be ranked differently with DARA, CARA, or IARA. Therefore, these
predictions/implications on the bidders’ entry behavior between the two formats can be exploited to test for one form of risk aversion against another.

It is important to know if bidders are risk averse or risk neutral as bidders’ risk preference can affect their strategic behavior in entry or/and bidding and lead to different policy implications such as revenue comparison between first price and ascending (or second price) auctions. For example, assuming symmetry among bidders and no entry, although the four auction mechanisms, namely, first price auction, second price auction, ascending auction, and descending auction, generate the same expected revenue with risk neutral bidders, the expected revenue is higher in a first price auction than in an ascending (or a second-price) auction with risk averse bidders, see, e.g. Riley and Samuelson (1981). Moreover, as shown in Smith and Levin (1996), although such a ranking is still sustained when bidders adopt mixed strategy entry, exhibit either IARA or CARA, and participants know the number of actual bidders, in the DARA case, such a ranking sometimes can be reversed. This means that identifying the form of risk preferences is important.\(^3\) Also the form of risk preference plays a key role from the mechanism design viewpoint, as the optimal reserve price could be different with different forms of risk preferences. Our result can also be used to test whether bidders are risk averse, as when we confirm from comparison of the two auction formats that bidders are either IARA or DARA, then bidders are risk averse.\(^4\)

On the empirical ground, we study the US Forest Service (henceforth USFS) data on timber auctions. As the data contain both ascending and first price auctions, they allow us to implement an empirical test in view of the model implications we derive with regard to the entry patterns between the two auction formats with different risk aversion forms. The results from our test support a DARA form of bidders’ utility function. It is interesting to note that using a subset of the data we analyze, Lu and Perrigne (2008) find that a CRRA specification better fits the data than a CARA specification. As CRRA is a special case of

\(^3\)We thank James Roberts for raising this point.

\(^4\)Of course if the comparison shows that bidders exhibit CARA, which includes risk neutrality as a special case, then we cannot rule out risk neutrality without further evidence or more information.
DARA, our finding is consistent with that in Lu and Perrigne (2008).5

Our article contributes to the auction literature from both theoretical and empirical perspectives. On one hand, auction models with risk averse bidders have been studied in the literature, most of which has assumed exogenous bidders’ participation. See, e.g., Maskin and Riley (1984) for the theoretical work,6 and Lu and Perrigne (2008), Campo, Guerre, Perrigne and Vuong (2011), Campo (2012), Ackerberg, Hirano and Shahriar (2011) for the econometric/empirical work.7 On the other hand, both the theoretical and empirical literatures on entry in auctions have assumed risk neutrality. See, e.g., Levin and Smith (1994), Samuelson (1985), Ye (2007), among others for entry theory, and Bajari and Hortaçsu (2003), Li (2005), Li and Zheng (2009, 2012), Krasnokutskaya and Seim (2011), Athey, Levin and Seira (2011), Gentry and Li (2014), Marmer, Shneyrov and Xu (2013), and Roberts and Sweeting (2010, 2012) for empirical/econometric analysis of entry. An exception is Fang and Tang (2014), who propose a nonparametric test of risk aversion in ascending auctions based on the idea that bidders’ risk premium required for entry is strictly positive if bidders are risk averse; implementing the test requires observing the distribution of transaction prices, bidders’ entry decisions, and also a measure of entry costs.8

This article is organized as follows. Section 2 presents the entry models with risk averse bidders and with selective entry for both first price auctions and ascending auctions. We establish existence, uniqueness and other properties of the entry equilibrium for both formats. In Section 3 we derive implications on entry patterns for both formats and compare them when risk aversion is of the form of DARA, CARA, or IARA. Section 4 is devoted to an

5Whereas our test does not require bidding data, is simple and easy to implement, a potential caveat of our test is that it requires entry data from both auction formats and that bidders are symmetric.

6An exception is Smith and Levin (1996) who attempt to rank first price auctions and ascending auctions with risk averse bidders where the entry model is characterized in Levin and Smith (1994) with no selection.

7Although the main focus is on the case with exogenous participation, Guerre, Perrigne and Vuong (2009) also discuss the extension to endogenous entry when the exclusion restriction takes the form of instruments not affecting the bidders’ private value distribution.

8Using bidders’ entry behavior to test for model implications in auctions was exploited in Li and Zhang (2010) who propose a test for affiliation using risk neutral bidders’ entry behavior.
empirical analysis of the USFS data and testing for the form of risk aversion based on our theoretical results. Section 5 concludes. The technical derivations and proofs are included in the Appendix.

2 Auction Models

In this article, we study both a first price auction and an ascending auction selling an indivisible single item whereas adopting the framework of the Affiliated-Signal (AS) entry model as in Gentry and Li (2014). Differentiating from their framework, we further generalize the AS entry model by accommodating bidders’ risk aversion.

The AS Entry Model

The Affiliated-Signal entry model is initiated by Ye (2007) in his theoretical analysis on two-stage auction selling an indivisible item, where bidders who observe an imperfect signal about their values bid for entry rights in the first stage, and in the second stage bidders shortlisted further incur an entry cost to discover their values and make their bids for the item auctioned. Gentry and Li (2014) extend Ye (2007) to allow a class of auctions defined by Riley and Samuelson (1981) and address nonparametric identification of the AS entry model.\(^9\)

There are \(N\) potential bidders who are interested in a single item being auctioned. Let \(N = \{1, 2, ..., N\}\) denote the set of all the potential bidders. The game expands in two stages for the AS entry auction model. In stage 1, every bidder \(i \in N\) receives a private signal \(s_i \in [s, \bar{s}]\) about his value, where the signals are independently and identically distributed across \(i\). Without loss of generality, we assume that the marginal distribution of \(s_i\) follows the standard uniform distribution \(U[0, 1]\).\(^{10}\) All \(N\) potential bidders decide simultaneously

\(^9\)The class of auctions defined by Riley and Samuelson (1981) includes all four standard auctions (first price, second price, ascending, and descending), plus other less common types.

\(^{10}\)The normalization comes from the fact that an alternative signal \(\tilde{s}_i = G(s_i)\) can be adopted, which follows a standard uniform distribution. Here \(G(\cdot)\) is the cumulative distribution function of \(s_i\).
whether to pay a fixed entry cost \( (c > 0) \) to enter the second stage. If a bidder chooses to enter, he needs to pay the fixed cost \( c \) regardless of whether he wins or not. To avoid negative wealth after paying the entry cost, we assume bidders are endowed with initial wealth \( w_0 \) such that \( w_0 > c \).

In stage 2, bidders’ true values \( v_i \in [u, v], i \in \mathbf{N} \) are revealed privately to entrant bidders who have opted to incur the entry cost and entered the second stage. All entrant bidders compete in a standard auction at the second stage. In this article, we are particularly interested in a first price auction and an ascending auction with a reserve price \( r \). \((s_i, v_i) \in [0, 1] \times [u, v], i \in \mathbf{N} \) are independently and identically distributed across \( i \) with joint distribution \( F(s_i, v_i), \forall i \). Let \( F(v_i|s_i) \) denote the cumulative distribution function of \( v_i \) conditional on \( s_i \). We assume \( F(v_i|s'_i) < F(v_i|s_i), \forall s'_i > s_i \), i.e. a higher signal \( s_i \) leads to first-order stochastically dominant conditional distribution of \( v_i \). In other words, a high signal \( s_i \) is more likely to be associated with a high value \( v_i \). For example, this is the case when \( s_i \) and \( v_i \) are affiliated in the sense of Milgrom and Weber (1982).\(^{11} \)

The AS model nests two well-studied entry models as polar cases. If bidders’ revealed values at the second stage coincide with their private signals at the first stage, the AS model reduces to the Samuelson (1985) model. If their revealed values at the second stage are independent of their first stage signals, the AS model essentially becomes the Levin and Smith (1994) model. The AS model is more general and realistic in the sense that it accommodates situations where first stage signal \( s_i \) and second stage revealed value \( v_i \) are positively but (possibly) not perfectly interdependent.

**Risk Aversion of Bidders**

Bidders are risk averse and they share an identical concave Bernoulli utility function \( u(w) \) with \( u(0) = 0 \), where \( w \) is the wealth level. For a given Bernoulli utility function \( u(\cdot) \), a well-
established measure of the degree of risk aversion is provided by the *Arrow-Pratt coefficient of absolute risk aversion*, which is defined as $r_A(w; u) = -u''(w)/u'(w) \geq 0$. Depending on whether function $r_A(\cdot; u)$ is increasing, flat or decreasing, bidders’ risk aversion attitude can be completely classified into three categories: increasing, constant or decreasing *absolute risk aversion* (i.e. IARA, CARA or DARA, respectively).

An alternative measure for the degree of risk aversion is provided by the *coefficient of relative risk aversion*, which is defined as $r_R(w; u) = -w \cdot u''(w)/u'(w) \geq 0$. Similarly, depending on whether function $r_R(\cdot; u)$ is increasing, flat or decreasing, bidders’ risk aversion attitude can also be classified into the following three categories: increasing, constant or decreasing *relative risk aversion* (i.e. IRRA, CRRA or DRRA, respectively).\(^{12}\)

**Information Structure**

The number of potential bidders $N$, reserve price $r$, entry cost $c$, initial wealth $w_0$, Bernoulli utility function $u(\cdot)$ and joint distribution $F(\cdot, \cdot)$ are common knowledge to all potential bidders. The realized $s_i$ is privately observed by bidder $i$ at the first stage; the realized $v_i$ is privately observed by bidder $i$ if he enters the second stage auction. It is worth noting that when an entrant $i$ makes his bid at the second stage, he only observes his own $(s_i, v_i)$.

We study in this article the situation where an entrant does not observe the number of other entrants. This means that an entrant has to form his bidding strategy based on his belief about the distribution of the number of other entrants and the distributions of their private values.

### 3 Entry and Bidding Equilibrium

In this section, we will first establish the bidding and entry equilibrium for both first price and ascending auctions with a reserve price $r$. For both auctions, the first stage entry equilibrium

\(^{12}\)It is clear that if $u(\cdot)$ belongs to CRRA or DRRA, it must belong to DARA; if $u(\cdot)$ belongs to IARA or CARA, it must belong to IRRA.
is characterized by an entry threshold $s^e$ where a bidder enters if and only if his signal is higher than $s^e$. On the other hand, whereas it is a dominant strategy for truthful bidding in an ascending auction, in a first price auction, the pure strategy bidding equilibrium $b(v; s^e)$ of the entrants is characterized by a differential equation with appropriate boundary conditions. We then proceed to establish the comparison of the entry equilibria between first price and ascending auctions, when bidders’ risk aversion belongs to IARA, CARA or DARA category, respectively.

**First Price Auction**

We first consider a first price auction with a reserve price $r \in [\underline{v}, \overline{v}]$. We characterize the symmetric Bayesian Nash equilibrium in two steps. When bidders’ entry strategies in stage 1 are fixed, in the second stage we have a first price auction with stochastic entry. Existing studies on first price auctions can be applied to characterize the equilibrium bidding strategy with appropriate modifications to accommodate for entry. After knowing how bidders bid for a given first stage entry strategy, we can further characterize the first stage entry equilibrium.

**Bidding Equilibrium for a Given Entry Threshold**

To look for the pure strategy symmetric entry and bidding equilibrium, we first characterize the symmetric increasing bidding equilibrium $b^F(\cdot; s^e)$ when all bidders adopt the same entry threshold $s^e$. We derive the bidding equilibrium by considering the bidding decision of a representative entrant $i$ who has discovered his value $v_i$. Define

$$
\Psi(v|s_{-i}^{c}) = [s_{-i}^{c} + (1 - s_{-i}^{c})F(v|s \geq s_{-i}^{c})]^{N-1}, \forall v \in [\underline{v}, \overline{v}],
$$

where $F(v|s \geq s_{-i}^{c})$ stands for the probability that the value $v_j$ of a bidder $j$ ($\neq i$) is smaller than $v$ conditional on that he enters, when bidders other than $i$ enter if and only if their signals are above $s_{-i}^{c}$. Here, scalar $s_{-i}^{c}$ represents a uniform entry threshold of all bidders other than $i$. Therefore, $\Psi(v|s_{-i}^{c})$ represents the probability that either no other bidder
enters or the highest value of all entrants other than $i$ is smaller than $v$.\footnote{We treat as if the effective value of a non-entrant is $v$.}

Given an entry threshold $s^c$, we consider a representative bidder $i$’s optimal bidding decision to characterize the condition that a strictly increasing bidding equilibrium $b^F(\cdot; s^c)$ should satisfy. By standard arguments, we must have the boundary condition $b^F(r; s^c) = r$ for a threshold type $r$, and the types lower than $r$ would not bid.\footnote{First, note that it cannot be true that $b^F(v; s^c) = r$ where $v < r$, because type $v$ would gain negative payoff if he wins. Thus type $v$ ($< r$) would not bid. Second, it cannot be true that $b^F(v; s^c) = r$ where $v > r$, because any type $v' \in (r, v)$ also has incentive to bid $r$ and make a positive gain.}

Let $\pi^F_i(v'_i, v_i; s^c)$ denote entrant $i$’s interim expected payoff when his true value is $v_i$ ($\geq r$) but he bids like type $v'_i$ ($\geq r$), whereas other bidders adopt entry threshold $s^c$ and equilibrium bidding $b^F(\cdot; s^c)$. We have

$$\pi^F_i(v'_i, v_i; s^c) = u(v_i - b^F(v'_i; s^c) + w_0 - c)\Psi(v'_i | s^c_i = s^c) + u(w_0 - c)[1 - \Psi(v'_i | s^c_i = s^c)]. \quad (1)$$

To simplify the notation, we define

$$\tilde{u}(w) = u(w + w_0 - c) - u(w_0 - c). \quad (2)$$

It is clear that $\tilde{u}(0) = 0$. It follows immediately that $\tilde{u}(\cdot)$ and $u(\cdot)$ must belong to the same category of absolute risk aversion.\footnote{The proof is omitted to save space.}

**Property 1.** $\tilde{u}(\cdot)$ belongs to IARA, CARA, or DARA category if and only if $u(\cdot)$ belongs to IARA, CARA, or DARA category, respectively.

Equation (1) can then be rewritten as following:

$$\pi^F_i(v'_i, v_i; s^c) = \tilde{u}(v_i - b^F(v'_i; s^c))\Psi(v'_i | s^c_i = s^c) + u(w_0 - c). \quad (3)$$

At the equilibrium, if bidding $b^F(\cdot; s^c)$ is indeed an equilibrium, truth telling is optimal for entrant $i$. This requires that $\pi^F_i(v'_i, v_i; s^c)$ is maximized at $v'_i = v_i$. We thus have the following characterizations for the bidding equilibrium.

**Lemma 1.** Given the first stage entry threshold $s^c$, for the second stage first price auction with a reserve price $r \in [v, \overline{v}]$, the symmetric increasing bidding equilibrium $b^F(\cdot; s^c)$
is uniquely characterized by the following differential equation with the boundary condition
\[ b^F(r; s^c) = r: \]
\[ b^F_v(v; s^c) = \frac{\tilde{u}(v - b^F(v; s^c)) \Psi_v(v|s^c)}{\tilde{u}'(v - b^F(v; s^c)) \Psi(v|s^c)}, \quad \forall v \geq r, \tag{4} \]
where \( b^F_v(v; s^c) \) is the derivative of \( b^F(v; s^c) \) with respect to \( v \), and \( \Psi_v(v|s^c) \) is the derivative with respect to \( v \).

Condition (4) is closely related to the first order condition that characterizes the symmetric bidding equilibrium in a standard first price auction with a reserve price \( r \). \( \Psi(\cdot|\cdot) \) parallels the distribution of the highest value among all rivals in standard auctions without stochastic entry. As the number of entrant bidders is not revealed, every entrant rationally expects to compete with a stochastic pool of entrants, whose value distributions are determined by their entry thresholds. The \( \tilde{u} \) function reflects the impact of risk aversion of bidders on their bidding behavior.

Note that equilibrium bidding strategy depends on the entry threshold. The first stage entry threshold \( s^c \) affects the value distribution of the entrants, which in turn affects their bids.

**Equilibrium Entry**

We now are ready to derive the entry equilibrium in stage 1. For this purpose, we will show that the expected payoff of the marginal type of entrant (i.e. type \( s^c \)) must increase with \( s^c \), given that all entrants adopt the bidding equilibrium \( b^F(\cdot; s^c) \) and entry threshold \( s^c \).

The expected payoff of the marginal type (\( s^c \)) of entrant is
\[ \Pi_i^F(s^c) = u(w_0 - c)F(r|s^c) + \int_r^0 \pi_i^F(v, v; s^c)dF(v|s^c) \]
\[ = \int_r^0 \tilde{u}(v - b^F(v; s^c))\Psi(v|s^c)dF(v|s^c) + u(w_0 - c). \tag{5} \]
Recall \( F(\cdot|s^c) \) denotes the conditional cumulative distribution function of \( v_i \) given \( s_i = s^c \).

To show \( \Pi_i^F(s^c) \) increases with \( s^c \), we consider the following function \( \Pi_i^F(s_i, s^c_{-i}) \):
\[ \Pi_i^F(s_i, s^c_{-i}) = \int_r^0 \tilde{u}(v - b^F(v; s^c_{-i}))\Psi(v|s^c_{-i})dF(v|s_i) + u(w_0 - c). \tag{6} \]
Lemma 2. \( \Pi_i^F(s_i, s_{-i}) \) increases with both \( s_i \) and \( s_{-i} \).

Although the monotonicity of \( \Pi_i^F(s_i, s_{-i}) \) with respect to \( s_i \) is well expected as bidder \( i \)'s value \( v_i \) is affiliated with his signal \( s_i \), its monotonicity with respect to \( s_{-i} \) is rather remarkable. \( s_{-i} \) has two opposite effects on bidder \( i \)'s bidding behavior. First, a higher \( s_{-i} \) means that other bidders would enter with lower probability, which tends to lower bidder \( i \)'s bid when his value is above the reserve price \( r \). Second, a higher \( s_{-i} \) means that other bidders’ value distribution conditional on entry (i.e. \( F(v|s \geq s_{-i}) \)) is first-order improved due to the assumption that \( F(v|s) \) decreases with \( s \). This however tends to increase bidder \( i \)'s bid when his value is above the reserve price \( r \). Lemma 2 establishes that the first negative effect clearly dominates the second positive effect, and thus the overall effect of a higher \( s_{-i} \) on bidder \( i \)'s bids must be negative.

Note \( \Pi_i^F(s^c) = \Pi_i^F(s^c; s^c) \). From Lemma 2, we have the monotonicity of \( \Pi_i^F(s^c) \). In addition, Lemma 2 will be further utilized shortly to establish a bidder’s optimal entry strategy.

Corollary 1. \( \Pi_i^F(s^c) \) increases with \( s^c \).

Let \( s^c = 0 \) if \( \Pi_i^F(0) \geq u(w_0) \); \( s^c = 1 \) if \( \Pi_i^F(1) = E_{v_i|s_i=1}u(\max\{0, v_i - r\} + (w_0 - c)) \leq u(w_0) \);\(^{16} \) otherwise let \( s^c \in (0, 1) \) be the unique solution of

\[
\Pi_i^F(s^c) = u(w_0),
\]

which can be equivalently written as

\[
\int_r^0 \tilde{u}(v - b^F(v; s^c))\Psi(v|s^c)dF(v|s^c) = \tilde{u}(c). \tag{7}
\]

The above uniquely defined \( s^c \) constitutes a unique symmetric equilibrium entry threshold. This is clear for the following reasons. Without loss of generality, we consider the case where \( s^c \in (0, 1) \). Suppose other bidders take entry threshold \( s^c \) and follow bidding strategy \( b^F(\cdot; s^c) \) upon entry. We emphasize that the process of establishing Lemma 1 has shown that regardless

\(^{16}\)When others do not enter for sure, then an entrant would bid the reserve price \( r \) when his value is above \( r \). As pointed out by a referee, if the number of potential bidders is endogenous, then a free-entry condition would guarantee that \( s^c \in (0, 1) \).
of entrant $i$’s signal $s_i$, when his discovered value is $v_i$ upon entry, his optimal bid must be $b^F(v_i; s^e)$. Therefore, if bidder $i$’s type is $s_i$, then his expected payoff upon entry would be

$$\Pi_i^F(s_i, s^e) = \int_r^\beta \bar{u}(v - b^F(v; s^e))\Psi(v|s^e)dF(v|s_i) + u(w_0 - c),$$

which must increase with $s_i$ according to Lemma 2. By definition, we have $\Pi_i^F(s^e, s^e) = u(w_0)$. Therefore, it is clear that bidder $i$ should enter if and only if $s_i \geq s^e$. We thus conclude that the optimal entry threshold $s^e_i$ for bidder $i$ must be $s^e$. The above discussion confirms that bidder $i$’s optimal strategy is to enter if and only if his signal is above $s^e$ and follows the bidding strategy $b^F(\cdot; s^e)$ upon on entry, if this entry and bidding strategy is also adopted by other bidders. A necessary condition for entry equilibrium is the zero expected payoff condition (i.e. $\Pi_i^F(s^e) = u(w_0)$) for the marginal type $s^e$. Due to the monotonicity of $\Pi_i^F(s^e)$ by Corollary 1, we have the uniqueness of $s^e$. Lemma 1 further delivers the uniqueness of the equilibrium bidding strategy upon entry. These results are summarized in the following theorem.

**Theorem 1.** For a first price auction with a reserve price $r \in [v, \bar{v}]$, when it is costly for bidders to enter and they are risk averse, there exists a unique pure strategy symmetric entry and bidding equilibrium. The symmetric interior equilibrium entry threshold $s^e$ is defined as in (7). The equilibrium bidding strategy $b^F(\cdot; s^e)$ is as defined by Lemma 1 for $s^e = s^e$.

We next can establish how the equilibrium entry threshold would depend on the number of potential bidders, $N$.

**Theorem 2.** For a first price auction with a reserve price $r \in [v, \bar{v}]$, the equilibrium entry threshold $s^e$ increases with the number of potential bidders $N$.

Theorem 2 shows that a higher number of potential bidders definitely weakens the incentive of the potential bidders with low signals to proceed to a costly entry. This result is intuitive. Given any entry threshold, a higher $N$ should lead to more aggressive bidding, which lowers the expected payoff of an entrant. For a fix $N$, Corollary 1 means that the expected payoff of the entry threshold type must increase with the threshold type. Combining these two effects leads to that $s^e$ must increase with the number of potential bidders $N$. 

12
Ascending Auction

We now turn to an ascending auction with a reserve price $r \in [v, \bar{v}]$. Bidding truthfully is the dominant strategy for every entrant in the ascending auction even if they are risk averse, thus at the equilibrium $b^A(v; s^c) = v$ for $v \geq r$ regardless of the entry threshold $s^c$. We next establish the symmetric entry equilibrium.

Given the entry threshold $s^c$, the expected payoff of an entrant with value $v \geq r$ is

$$\pi^A_i(v; s^c) = u(v - r + w_0 - c)\Psi(r|s^c) + \int_r^v u(v - t + w_0 - c)d\Psi(t|s^c)$$
$$+u(w_0 - c)(1 - \Psi(v|s^c))$$
$$= \tilde{u}(v - r)\Psi(r|s^c) + \int_r^v \tilde{u}(v - t)d\Psi(t|s^c) + u(w_0 - c), \forall v \geq r. \quad (8)$$

The expected payoff of the marginal type ($s^c$) of entrant is

$$\Pi^A_i(s^c) = u(w_0 - c)F(r|s^c) + \int_r^0 \pi^A_i(v; s^c)dF(v|s^c).$$

The first term corresponds to the event that bidder $i$ draws a value below the reserve price $r$ in the bidding stage. The second term covers the cases where value $v$ is above the reserve price.

To show $\Pi^A_i(s^c)$ increases with $s^c$, we consider the following function $\Pi^A_i(s_i, s^c_{-i})$:

$$\Pi^A_i(s_i, s^c_{-i}) = u(w_0 - c)F(r|s_i) + \int_r^0 \pi^A_i(v; s^c_{-i})dF(v|s_i). \quad (9)$$

Substituting $\pi^A_i(v; s^c_{-i})$ (as defined by equation (8) by replacing $s^c$ by $s^c_{-i}$) and using the equality given by (2), we have

$$\Pi^A_i(s_i, s^c_{-i}) = \int_r^0 \{\tilde{u}(v - r)\Psi(r|s^c_{-i}) + \int_r^v \tilde{u}(v - t)d\Psi(t|s^c_{-i})\}dF(v|s_i) + u(w_0 - c). \quad (10)$$

Similar to Lemma 2, we have the following result.

Lemma 3. $\Pi^A_i(s_i, s^c_{-i})$ increases with both $s_i$ and $s^c_{-i}$.

It is quite intuitive that $\Pi^A_i(s_i, s^c_{-i})$ increases with $s_i$ as a higher $s_i$ means a first-order improved value distribution $F(\cdot|s_i)$ for entrant $i$. The monotonicity of $\Pi^A_i(s_i, s^c_{-i})$ with respect to $s^c_{-i}$ is less obvious. Due to the two opposite effects of a higher $s^c_{-i}$ on different components
of $\Psi(\cdot|s^e_i)$ that have been illustrated in the discussion following Lemma 2, the overall effect of $s^e_i$ can be indeterministic. Lemma 3 however unambiguously establishes that the overall effect of a higher $s^e_i$ on bidder $i$’s expected payoff must be positive.

Note $\Pi_i^A(s^e) = \Pi_i^A(s; s^e)$. From Lemma 3, we have the monotonicity of $\Pi_i^A(s^e)$.

**Corollary 2.** $\Pi_i^A(s^e)$ increases with $s^e$.

Let $s^e = 0$ if $\Pi_i^A(0) \geq u(w_0)$; $s^e = 1$ if $\Pi_i^A(1) = E_{v_i|s_i=1}u(\max\{0, v_i-r\}+(w_0-c)) \leq u(w_0)$; otherwise let $s^e \in (0, 1)$ be the unique solution of

$$\Pi_i^A(s^e) = u(w_0),$$

which can be equivalently written as

$$\int_r^v \{\bar{u}(v-r)\Psi(r|s^e) + \int_r^v \bar{u}(v-t)d\Psi(t|s^e)\}dF(v|s^e) = \bar{u}(c). \tag{11}$$

The above uniquely defined $s^e$ constitutes a unique symmetric equilibrium entry threshold for the same reasons as illustrated for the first price auction case. To save space, we do not repeat here. These results are summarized in the following theorem.

**Theorem 3.** For an ascending auction with a reserve price $r \in [\bar{v}, \bar{v}]$, when it is costly for bidders to enter and they are risk averse, there exists a unique pure strategy symmetric entry and bidding equilibrium. The interior symmetric equilibrium entry threshold $s^e$ is defined as in (11). Every entrant bids truthfully.

As in first price auction, we can establish how the equilibrium entry threshold would depend on the number of potential bidder, $N$.

**Theorem 4.** For an ascending auction with a reserve price $r \in [\bar{v}, \bar{v}]$, the equilibrium entry threshold $s^e$ increases with the number of potential bidders $N$.

As well known, revenue equivalence across standard auctions fails to hold when bidders are risk averse and entry is exogenous (Riley and Samuelson (1981)). On the other hand, Gentry and Li (2012) establish the equivalence among the four standard mechanisms including first price auction and ascending auction in the AS model with risk neutral bidders. In our AS model, because the second stage could be first price auction or ascending auction, we could expect the entry equilibrium to depend on the auction format used in stage 2. In auctions with
risk averse bidders and without entry, the revenue generated in a first price auction is strictly higher than that generated in an ascending auction, because risk aversion of bidders does not change their bidding behavior in ascending auction, whereas in the first price auction risk averse bidders bid higher than what they would bid if they were risk neutral. When entry is considered, in the mixed strategy entry case and assuming that entrants know the number of actual bidders, Smith and Levin (1996) show that such a ranking is preserved in both CARA and IARA cases, but can be reversed sometimes in the DARA case. In our case where we consider selective entry that encompasses the mixed strategy entry model as a polar case, and we assume that entrants do not know the number of actual bidders, a more general and realistic assumption than that in Smith and Levin (1996), it becomes more involved in ranking the revenue across the two auction formats. In fact, we are able to extend the ranking in Smith and Levin (1996) in both CARA and IARA cases to our setting. However, we are unable to find an example in our setting to show that the ranking can sometimes be reversed as in Smith and Levin (1996), though we expect that such an example exists, and leave it for future research. We now proceed to study how auction formats affect bidders’ entry decisions.

**Entry Comparison across First Price and Ascending Auctions**

This section compares the entry equilibria across the two auction formats. We show that given the entry threshold, the comparison of the expected payoffs of the entry threshold type across the two auction formats is determined solely by the category (i.e. IARA, CARA or

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17 This assumption has been used in the recent empirical auction literature; see, e.g., Li and Zheng (2009), Gentry and Li (2014), and Marmer, Shneyerov and Xu (2013), among others. With this general assumption, the entry effect (Li and Zheng (2009)) and the selection effect (Marmer, Shneyerov and Xu (2013)) can be studied to see how bids and seller’s revenue could change as the number of potential bidders changes, in addition to the usual competition effect. Invoking Theorem 2 in Matthews (1987), it can be shown that our results on entry comparison can be readily obtained with the assumption that the participants know the number of participants at the time of bidding.

18 The proofs are available upon request.
DARA) of absolute risk aversion that bidders belong to. This comparison result on expected payoffs further determines a clear ranking of the equilibrium entry thresholds across the two formats of auctions. The entry comparison results are sufficient for testing the form of bidders’ risk aversion.

**Bidders’ Payoff Comparison**

For a given entry threshold $s^c$, we have characterized in Sections 3.1 and 3.2 a representative entrant $i$’s conditional equilibrium expected payoffs in first price and ascending auctions, which are respectively $\pi_i^F(v_i, v_i; s^c)$ and $\pi_i^A(v_i; s^c)$, when his revealed value is $v_i \geq r$.

Recall that when $v_i < r$, $\pi_i^F(v_i, v_i; s^c) = \pi_i^A(v_i; s^c) = u(w_0 - c)$. In addition,

$$\pi_i^F(v_i, v_i; s^c) = \bar{u}(v_i - b^F(v_i; s^c))\Psi(v_i|s^c) + u(w_0 - c), \forall v_i \geq r,$$

and

$$\pi_i^A(v_i; s^c) = \bar{u}(v_i - r)\Psi(r|s^c) + \int_r^v \bar{u}(v_i - t)d\Psi(t|s^c) + u(w_0 - c), \forall v_i \geq r.$$

Define

$$\tilde{\pi}_i^F(v_i; s^c) = \bar{u}(v_i - r)\Psi(r|s^c) + \int_r^v \bar{u}(v_i - t)d\Psi(t|s^c), \forall v_i \geq r,$$

and

$$\tilde{\pi}_i^A(v_i; s^c) = \bar{u}(v_i - r)\Psi(r|s^c) + \int_r^v \bar{u}(v_i - t)d\Psi(t|s^c), \forall v_i \geq r.$$

We next show that $b^F(v_i; s^c)$ can be alternatively interpreted as the equilibrium bidding strategy in a first price auction with reserve price $r$, where $N$ risk averse bidders (with concave Bernoulli utility function $\bar{u}(w)$ with $\bar{u}(0) = 0$) enter independently with probability $1 - s^c$, and their value distribution is $F(\cdot|s \geq s^c)$ and their initial wealth is zero. Payoff (3) can be written as

$$\pi_i^F(v_i', v_i; s^c) = u(w_0 - c) + \left\{ \sum_{n=0}^{N-1} C_{N-1}^n (s^c)^n (1 - s^c)^{(N-1)-n} F(\cdot|s \geq s^c)^{(N-1)-n} \int \bar{u}(v_i' - b^F(v_i'; s^c)) \right\}.$$
\( \nu_i' \geq r \) in a first price auction with a reserve price \( r \), when \( N \) risk averse bidders with the value distribution \( F(\cdot | s \geq s^c) \) and concave Bernoulli utility function \( \tilde{u}(w) \) enter independently with the probability \( 1 - s^c \). Note that for this auction, there is no entry cost involved. Therefore, the equilibrium bidding strategy \( b^F(\cdot ; s^c) \) of Lemma 1 must be the equilibrium bidding strategy in this first price auction with exogenous stochastic entry and no entry cost. It thus follows that \( \tilde{\pi}^F_i(v_i; s^c) \) is the expected payoff of a representative bidder with value \( v_i \geq r \) and zero initial wealth in this auction.

Similarly, \( \tilde{\pi}^A_i(v_i; s^c) \) is the expected payoff of a representative bidder with value \( v_i \geq r \) and zero initial wealth in a second price auction with a reserve price \( r \), when \( N \) risk averse bidders with the value distribution \( F(\cdot | s \geq s^c) \) and concave Bernoulli utility function \( \tilde{u}(w) \) enter independently with exogenous probability \( 1 - s^c \).

For a fixed entry threshold \( s^c \), an entrant with value \( v_i > r \) would bid higher in a first price auction when he is risk averse. On the other hand, he still bids the true value in an ascending auction even when he is risk averse. These results mean that a risk averse bidder would on expectation pay a higher amount in a first price auction if he wins than in an ascending auction. Does this mean that an entrant with \( v_i > r \) necessarily prefers an ascending auction? In a first price auction, a winner with \( v_i > r \) pays his own bid which is a fixed amount, whereas he pays the second highest bid (or the reserve price) which is random. Given that the bidder is risk averse, the uncertainty in the payments in the ascending auction would on the other hand make him be inclined to the first price auction. Based on these discussions, one would expect that the comparison across the two formats of auctions in terms of the bidders’ preference would depend on the characteristics of the risk aversion of bidders.

To address this issue, we are now ready to apply an established result by Matthews (1987). Matthews (1987) compares buyers’ payoffs across first price and ascending auctions with exogenous stochastic entry, when buyers are risk averse and their value distribution is exogenous. Thus we can obtain the following comparison between \( \tilde{\pi}^F_i(v_i; s^c) \) and \( \tilde{\pi}^A_i(v_i; s^c) \) whereas applying Theorem 1 in Matthews (1987).
Lemma 4. For a given $s^c$, i) $\tilde{\pi}_i^F(v_i; s^c) = \tilde{\pi}_i^A(v_i; s^c) = 0, \forall v_i \leq r$; ii) $\tilde{\pi}_i^F(v_i; s^c) < \tilde{\pi}_i^A(v_i; s^c), \forall v_i \in (r, \bar{v}],$ if $\bar{u}(\cdot)$ is DARA; iii) $\tilde{\pi}_i^F(v_i; s^c) = \tilde{\pi}_i^A(v_i; s^c), \forall v_i \in (r, \bar{v}],$ if $\bar{u}(\cdot)$ is CARA; iv) $\tilde{\pi}_i^F(v_i; s^c) > \tilde{\pi}_i^A(v_i; s^c), \forall v_i \in (r, \bar{v}],$ if $\bar{u}(\cdot)$ is IARA.

Based on Property 1 and Lemma 4, we have the comparison of $\pi_i^F(v_i, v_i; s^c)$ and $\pi_i^A(v_i, v_i; s^c)$ for $\forall v_i$.

Corollary 3. For a given $s^c$, i) $\pi_i^F(v_i, v_i; s^c) = \pi_i^A(v_i, v_i; s^c) = u(w_0 - c), \forall v_i \leq r$; ii) $\pi_i^F(v_i, v_i; s^c) < \pi_i^A(v_i, v_i; s^c), \forall v_i \in (r, \bar{v}],$ if $u(\cdot)$ is DARA; iii) $\pi_i^F(v_i, v_i; s^c) = \pi_i^A(v_i, v_i; s^c), \forall v_i \in (r, \bar{v}],$ if $u(\cdot)$ is CARA; iv) $\pi_i^F(v_i, v_i; s^c) > \pi_i^A(v_i, v_i; s^c), \forall v_i \in (r, \bar{v}],$ if $u(\cdot)$ is IARA.

Corollary 3 shows that when the Bernoulli utility function is CARA, the higher payment in a first price auction cancels exactly with the higher uncertainty in payments in an ascending auction. Therefore, the bidder is indifferent between the two formats of auctions. When the Bernoulli utility function is IARA, i.e. the Bernoulli utility function’s concaveness increases with the wealth level, then in the ascending auction the bidder benefits relatively less from the possible low second prices and suffers relatively more from possible higher second price. Thus the bidder favors the first price auction. For the same reason, the bidder favors the ascending auction when the Bernoulli utility function is DARA.

Entry Comparison

For a given entry threshold $s^c$, Corollary 3 provides the comparison of bidders’ expected payoffs across the two formats of auctions for any value $v \in [r, \bar{v}]$. If an entrant draws a value below the reserve price $r$, the payoff is definitely $u(w_0 - c)$ for both formats of auctions. We now look at the ex ante expected payoff for a bidder with a signal $s^c$, which can be computed by integrating bidder’s expected payoff (respectively $\pi_i^F(v_i, v_i; s^c)$ for the first price auction, and $\pi_i^A(v_i, s^c)$ for the ascending auction) over all possible values of $v_i$ drawn from the distribution $F(\cdot|s^c)$. We use $\Pi_i^F(s^c)$ to denote the bidder’s ex ante expected payoff in the first price auction; and $\Pi_i^A(s^c)$ to denote the bidder’s ex ante expected payoff in the ascending auction. Corollary 3 thus leads to following result.

Lemma 5. For a given entry threshold $s^c \in [0, 1)$, i) if $u(\cdot)$ is DARA, $\Pi_i^F(s^c) < \Pi_i^A(s^c)$; ii)
if \( u(\cdot) \) is CARA, \( \Pi^F_i(s^c) = \Pi^A_i(s^c) \); and iii) if \( u(\cdot) \) is IARA, \( \Pi^F_i(s^c) > \Pi^A_i(s^c) \).

Let \( s^e_F \) and \( s^e_A \) denote respectively the equilibrium entry thresholds in first price and ascending auctions. Lemma 5 together with Corollaries 1 and 2 lead to the following comparison between \( s^e_F \) and \( s^e_A \).

**Theorem 5.** i) If \( u(\cdot) \) is DARA, we have \( s^e_F > s^e_A \); ii) if \( u(\cdot) \) is CARA, we have \( s^e_F = s^e_A \); and iii) if \( u(\cdot) \) is IARA, we have \( s^e_F < s^e_A \).

Given an entry threshold, DARA bidders prefer the ascending auction to the first price auction. A lower entry threshold (i.e. higher probability of entry) in the ascending auction may still provide them enough incentive to enter the auction. This leads to a lower entry threshold and a higher participation rate in the ascending auction. Similarly, we can predict the participation rate comparison across the two formats of auctions when bidders exhibit CARA or IARA. Thus Theorem 5 can be translated into the following corollary in terms of the entry probability or the participation rate.

**Corollary 4.** i) If \( u(\cdot) \) is DARA, the ascending auction has a higher participation rate than the first price auction; ii) if \( u(\cdot) \) is CARA, the two formats of auction have the same participation rate; and iii) if \( u(\cdot) \) is IARA, the ascending auction has a lower participation rate than the first price auction.

The entry probability ranking results with different forms of risk aversion established in Corollary 4 are testable implications/predictions from the AS model with selective entry and risk averse bidders, which can be used to test for the form of risk aversion using bidders’ entry behavior.

### 4 Empirical Application

This section analyzes the timber auctions held by the USFS and implements a test for the form of risk aversion based on our theoretical results derived in Section 3.


**Timber Auction Data**

The data we use are from the timber auctions held by the US forest service (USFS). The USFS sells timber from publicly owned forests through both first price auctions and ascending auctions. The timber auctions by the USFS have been frequently analyzed in the empirical auction literature, including the work on collusion (Baldwin, Marshall and Richard (1997), and Athey, Levin and Seira (2011)), resale (Haile (2001)), skewed bidding (Athey and Levin (2001)), among others. The empirical work on auctions with risk aversion also uses the USFS timber data; see e.g. Lu and Perrigne (2008) who assume the underlying value distribution in the first price auction is the same as that in the ascending auction, and combine data on both auctions to estimate the value distribution and the utility function, and Campo, Guerre, Perrigne and Vuong (2011) who impose an exclusion restriction to estimate the first price auction model by assuming the value distribution is independent of the number of bidders. Whereas we use the data from the same year (1979) as in Lu and Perrigne (2008) and Campo, Guerre, Perrigne and Vuong (2011), we use a larger data set as we include all regions to get approximately the same number of observations in the two auction formats whereas these two articles focus on the auctions in the western half (regions 1-6) of the US.

Our data include 1188 first price auctions and 1327 ascending auctions, after dropping the observations that do not contain information on auction types or bidders’ identities. USFS collects information on the date of the auction, auction format, tract characteristics (such as total acres, total volume), appraised value, advertised value, whether the sale is a salvage sale, bids, and bidders’ identities. Although the USFS announces a reserve price before the auction, the reserve prices have been found by previous studies to be too low to be binding (Haile (2001), and Lu and Perrigne (2008)). We also include the number of building permits issued at the county level as a means to control for the market condition.

As the model implications derived in Section 3 are on the entry behavior, to implement an empirical test, we need information on potential bidders. In the timber auctions organized by USFS, however, we cannot directly observe the potential bidders. Therefore, we follow Athey, Levin and Seira (2011) to count the bidders that entered the auctions in both formats...
in the same geographic region in the prior year as potential bidders.\footnote{Li and Zhang (2010, forthcoming) and Li and Zheng (2012) construct the set of potential bidders for a first price auction as consisting of all actual bidders who submit a bid in a first price auction in the same district and during the same quarter.}

Table 1 reports the summary statistics of the data. We note some differences in tract characteristics between the two types of auctions. Although the average acres are similar, the total volume sold in the ascending auction format is almost 2 times larger than that in the first price auction format. As a result, appraisal values in the ascending auction format are almost twice as high as the values in the first price auction format.

As the participation rate is the key quantity of our interest, and Table 1 indicates that the average participation rates in the two formats are about the same in the data, it is necessary to further analyze the bidders’ entry behavior in the two formats in relation to the auction formats and to other auction characteristics. To this end, we present a plot of the participation rate in these two formats. In Figure 1, the horizontal axis represents the number of potential bidders and the vertical axis represents participation rates. Two distinctive features can be seen from the graph: 1) the participation rate decreases substantially when the number of potential bidders increases, and 2) the participation rate in the first price auction is lower than that in the ascending auction format for almost all \( N_s \). In what follows, we implement an inference procedure to investigate the effect of auction format and the number of potential bidders on the bidders’ entry behavior controlling for the auction heterogeneity.

**Inference Strategy**

Corollary 4 gives rankings in terms of participation rates between the two auction formats under different forms of risk aversion. In this section, we propose an inference procedure to evaluate how entry probability is affected by the auction format when other auction characteristics are controlled for.

Suppose \( L \) auctions are observed in the data. In each auction, we have information on
the number of potential bidders \((N)\), the number of entrants \((n)\), auction type \((FPA = 1\) for a first price auction, 0 for an ascending auction), and auction characteristics \((X)\). From the data, we can construct the participation rate \(p = \frac{n}{N}\). Note that \(p\) is a sample analog of the entry probability; in other words, the entry probability is the population mean of \(p\).

To model the entry rate \(p\) which is a fractional response variable, we use the following specification, in which the conditional expectation on participation rate is given by

\[
E(p|X, N, FPA) = G(\alpha + X\beta + \gamma FPA + \delta N),
\]

where \(G(\cdot)\) is a cdf such as a standard normal distribution or a logistic distribution. To estimate the parameters \(\theta = \{\alpha, \beta, \gamma, \delta\}\) in the model above, we use the Quasi-Maximum Likelihood (QML) method proposed by Papke and Wooldridge (1996):

\[
\max_{\theta} \sum_{i=1}^{L} \{p_i \log G(X_i, N_i, FPA_i) + (1 - p_i) \log(1 - G(X_i, N_i, FPA_i))\}.
\]

In essence, the likelihood above uses the Bernoulli likelihood, which is in the linear exponential family. As a result, following Gourieroux, Monfort and Trognon (1984), the resulting QML method is robust as it yields a consistent estimator whenever the conditional mean is correctly specified, and can be efficient for a class of generalized linear models (GLM) as discussed in Papke and Wooldridge (1996), and Wooldridge (2010).

**Results and Robustness Checks**

Our model predicts a positive relationship between the entry threshold and the number of potential bidders, thus a negative relationship between the entry probability and the number of potential bidders. Furthermore, our model provides different predictions of entry pattern for different forms of absolute risk aversion.

The first column in Table 2 reports our main estimation results, where we use the standard normal distribution as the functional form for \(G(\cdot)\) and control for number of potential bidders, log acres, log appraisal value, log density (defined as the ratio of total volume and acres), salvage dummy, and log number of building permits. Higher \(N\) decreases the entry
probability, which is consistent with our prediction that higher number of potential bidders deters entry. The coefficient for the FPA dummy is negative and significant, showing a lower participation rate and higher entry threshold in the first price auction when other auction characteristics are controlled for. This confirms the pattern depicted in Figure 1 where the auction heterogeneity is not controlled for.

The QML estimates are consistent when the conditional mean for participation rate is correctly specified, and the conditional mean is specified as the standard normal distribution. To check how robust the results are with different specifications on the conditional mean, we conduct QML estimation with the conditional mean specified as a logistic distribution, and also a linear regression. The second and the third columns in Table 2 report the results from these two specifications, respectively. As is clear, the estimates from these two specifications have the same signs as those in the first specification. Moreover, the results from the standard normal distribution as the conditional mean function and from the logistic distribution as the conditional mean function are qualitatively similar.\footnote{Hereafter, the standard errors are reported in the parenthesis and are calculated using the usual sandwich estimator for the asymptotic variance of the QMLE estimator, unless otherwise mentioned.}

The results in Tables 2 are obtained by using the measure of potential bidders as in Athey, Levin and Seira (2011) to count the number of potential bidders as those distinctive actual bidders in both formats in the same geographic region in the prior year. It is clear that this measure can be best considered as a proxy for the number of potential bidders, and could suffer from measurement error problems as discussed in Athey, Levin and Seira (2011). Alternatively there are other ways to construct a proxy for the number of potential bidders, all of which could also suffer from a degree of measurement error. To check how sensitive our results are to different ways of constructing the measure of potential competition, we have conducted QMLE with the number of potential bidders in one format counted as those participants in the same geographic region in the same format in the prior year, and also with the number of potential bidders counted as those participants in the same geographic region in both formats in the same quarter, among others. We have found that the results
are qualitatively similar across different measures of the number of potential bidders. To save space and for illustration purposes, in Table 3 we report the QMLE results with the conditional mean standard normal specification and also logistic distribution as well as OLS estimates from a linear specification, with the number of potential bidders counted as those participants in the same geographic region in the same format in the prior year.

It is evident in our data that the average volume in ascending auctions is almost three times of the average volume in first price auctions; we also conduct estimation by excluding the ascending auctions whose volumes are in the top 95 percentile. The results are reported in the first column of Table 4. Also there are about 5.1% first price auctions where the participation rate is one if the number of potential bidders is counted as the participants in both formats in the same geographic region in the prior year whereas there are 4% ascending auctions where the participation rate is one, thus we conduct estimation by excluding all these auctions where all potential bidders entered, and report the results in the second column of Table 4. Once again the results, in particular, the estimates on the coefficient of FPA dummy, are qualitatively similar to the results reported in Table 2 and Table 3 with the full data, confirming the robustness of our results.

Another important issue to address in testing whether the participation rate is lower on average in first price auctions than in ascending auctions is the possibility of endogeneity of the FPA dummy.\textsuperscript{21} Athey, Levin and Seira (2011) use the auctions in the Northern forests for the years between 1982 and 1990 because there is some evidence shown by Schuster and Niccolucci (1994) that it is possible the sale method was adopted in this region in a random manner. Focusing on the Northern region in our data set, however, would reduce the data size significantly to only about 200 auctions.\textsuperscript{22} To take into account possible endogeneity

\textsuperscript{21}As pointed out by a referee, one possible source of endogeneity of the FPA dummy is unobserved auction heterogeneity. For example, it could be possible that the auctioneer may use the ascending format to sell the tracts with high levels of the unobservable.

\textsuperscript{22}Nevertheless, estimation using this subsample on the Northern region yields an insignificant FPA dummy, supporting a CARA hypothesis. However, the insignificant FPA dummy could probably be due to the relatively small sample size. The results using this subsample are available upon request from the authors.
of the auction format, we follow Wooldridge (2010) to conduct a QMLE using the bivariate probit log-likelihood for the fractional response model with a possibly endogenous binary explanatory variable; see detailed discussion on pages 754 and 755 in Wooldridge (2010).\textsuperscript{23} Table 5 reports the results for both measures of the number of potential bidders. As can be seen from Table 5, the FPA dummy is no longer statistically significant with both measures of the number of potential bidders. Moreover and more importantly, the insignificant correlation (denoted as Rho in Table 5) between the error term in the outcome equation and the error term in the selection equation means that the hypothesis that the choice of auction format is exogenous cannot be rejected. Therefore the results in Table 2 and Table 3 can be viewed as providing a convincing support of DARA because the FPA dummy is significantly negative, which is consistent with our model prediction on entry probabilities between the two formats.

Lastly, we divide the data set into four subsamples based on seasons of the year to study how the type of risk aversion could differ because of seasonal effect.\textsuperscript{24} Table 6 reports the QMLE results with the conditional mean function specified as a standard normal distribution. Once again the coefficient of the auction format dummy FPA is negative across all the four subsamples. Interestingly, although the FPA dummy is highly significant in the subsamples for the Winter, Spring, and Summer, as the corresponding p-values are all zero, the FPA dummy is barely significant with a p-value 0.101 for the Fall sample. Therefore the empirical evidence indicates that whereas bidders could exhibit CARA risk attitude in the Fall, they exhibit DARA risk attitude in the other three seasons.

\textsuperscript{23}The specification of the reduced form equation for the auction format dummy (FPA) is a probit model. In our data, we cannot find a variable that can be excluded from the main outcome (entry probability) equation but can affect the auction format. As a result, identification of the (rescaled) coefficients is attained through nonlinearity of the model. As most of the estimates reported in Table 5 have small standard errors and are significant, the model is well identified through nonlinearity alone.

\textsuperscript{24}We thank a referee for making this suggestion.
5 Conclusion

In this article, we study auction models with selective entry and risk averse bidders. Our model is general as it encompasses the existing entry models by allowing for risk averse bidders’ endogenous participation decision. Our novel approach lies in the idea that bidders’ risk aversion can affect bidders’ entry decision; in particular, bidders’ entry behavior can differ across different auction formats such as first price auction and ascending auction with different risk aversion forms. Specifically we establish entry and bidding equilibrium in both first price auction and ascending auction mechanisms, and show that with DARA, the entry probability is lower in a first price auction than in an ascending auction; with CARA, the entry probability is the same between the two formats, and with IARA, the entry probability is higher in a first price auction than in an ascending auction. In contrast to the previous theoretical work on auctions with risk averse bidders that focuses on studying bidding equilibrium and thus does not provide testable implications of risk aversion in terms of bids, our approach provides implications of risk aversion in terms of entry behavior, which can be tested in empirical applications.25

We analyze the timber auctions held by the USFS, and propose a simple test for our model implications based on the bidders’ entry behavior. We find that the data support a DARA utility. A further structural analysis of the data based on a DARA utility taking into account of selective entry within the general framework we consider in the article would be a natural direction of future research. For example, one can specify a parametric form of the utility function imposing DARA; with the private value distribution unspecified, the estimation problem becomes semiparametric. Such a problem could be dealt with by extending Campo, Guerre, Perrigne and Vuong (2011) who consider semiparametric estimation of first price auctions with risk aversion and Gentry and Li (2014) who address nonparametric (partial)

25Although our approach does not require the same set of potential bidders in both auction formats, the symmetry assumption is needed in the sense that both sets of potential bidders in both formats have identical entry cost, draw private values from the same distribution, and have the same utility function, and initial wealth.
identification of the AS model with risk neutral bidders. A more general question would be nonparametric identification and inference of the AS model with risk averse bidders in first price auctions, which raises some challenging issues. These problems are studied in Gentry, Li and Lu (2015).
Appendix

Proof of Lemma 1: In the first price auction, suppose bidder \( i \)'s true value is \( v_i \geq r \) but is reported as \( v'_i \geq r \), his interim expected utility is given by equation (3):

\[
\pi_i^F(v'_i, v_i; s^c) = \tilde{u}(v_i - b^F(v'_i; s^c))\Psi(v'_i|s^c) + u(w_0 - c).
\]

Take the first order derivative with respect to \( v'_i \),

\[
\frac{\partial \pi_i^F(v'_i, v_i; s^c)}{\partial v'_i} = -\tilde{u}'(v_i - b^F(v'_i; s^c))b^F(v'_i|s^c)\Psi(v'_i|s^c) + \tilde{u}(v_i - b^F(v'_i; s^c))\Psi_v(v'_i|s^c).
\]

In the equilibrium, revealing the true type is optimal if \( b^F(\cdot; s^c) \) is the bidding function. This requires \( \frac{\partial \pi_i^F(v'_i, v_i; s^c)}{\partial v'_i} = 0 \) when \( v'_i = v_i \), i.e.:

\[
\frac{\partial \pi_i^F(v_i, v_i; s^c)}{\partial v'_i} = -\tilde{u}'(v_i - b^F(v_i; s^c))b^F(v_i|s^c)\Psi(v_i|s^c) + \tilde{u}(v_i - b^F(v_i; s^c))\Psi_v(v_i|s^c) = 0.
\]

Equivalently,

\[
b^F(v; s^c) = \frac{\tilde{u}(v - b^F(v; s^c)) \Psi_v(v|s^c)}{\tilde{u}'(v - b^F(v; s^c)) \Psi(v|s^c)}, v \geq r.
\]

Proof of Lemma 2: By (6), the expression for \( \Pi_i^F(s_i, s_{-i}^c) \) is

\[
\Pi_i^F(s_i, s_{-i}^c) = \int_r^0 \tilde{u}(v - b^F(v; s_{-i}^c))\Psi(v|s_{-i}^c)dF(v|s_i) + u(w_0 - c).
\]

We first prove \( \Pi_i^F \) is increasing in \( s_i \).

\[
\frac{\partial \tilde{u}(v - b^F(v; s_{-i}^c))\Psi(v|s_{-i}^c)}{\partial v}
= \tilde{u}'(v; s_{-i}^c)(1 - b^F(v; s_{-i}^c))\Psi(v|s_{-i}^c) + \tilde{u}(v - b^F(v; s_{-i}^c))\Psi_v(v|s_{-i}^c)
= \tilde{u}'(v; s_{-i}^c)\Psi(v|s_{-i}^c) > 0, v \geq r.
\]
The second equality follows the expression of \( b^F_v(v; s^c_{-i}) \) established in Lemma 1. Note that 
\[
\frac{\partial \tilde{u}(v-b^F(v; s^c_{-i}))}{\partial v} \Psi(v|s^c_{-i}) = 0, \forall v < r.
\]
Note that \( F(v_i|s'_i) < F(v_i|s_i), \forall s'_i > s_i \). Take any function
\( \lambda(v) \) such that \( \lambda'(v) \geq 0 \) and \( \lambda(r) = 0 \), where \( r \in [v, \pi] \). We claim that 
\[
\int^0_r \lambda(v)dF(v|s'_i), \forall s'_i > s_i.
\]
This is true because 
\[
\int^0_r \lambda(v)d[F(v|s'_i) - F(v|s_i)] = \lambda(v)[F(v|s'_i) - F(v|s_i)]\lambda(v)dv \geq 0.
\]
We have shown that 
\[
\frac{\partial \tilde{u}(v-b^F(v; s^c_{-i}))}{\partial v} \Psi(v|s^c_{-i}) \geq 0, \forall v \geq r.
\]
Note \( \tilde{u}(r-b^F(r; s^c_{-i}))\Psi(r|s^c_{-i}) = \tilde{u}(0)\Psi(r|s^c_{-i}) = 0 \).

Thus, 
\[
\Pi^F_i(s_i, s^c_{-i}) = \int^0_r \tilde{u}(v-b^F(v; s^c_{-i}))\Psi(v|s^c_{-i})dF(v|s_i) + u(w_0 - c) \text{ must increase with } s_i.
\]

We now show \( \Pi^F_i \) is also increasing in \( s^c_{-i} \). To prove \( \Pi^F_i(s_i, s^c_{-i}) \) is increasing in \( s^c_{-i} \), we only need to show the integrand \( \tilde{u}(v-b^F(v; s^c_{-i}))\Psi(v|s^c_{-i}) \) is increasing in \( s^c_{-i} \). We prove this result by two steps: we firstly show \( \Psi(v|s^c_{-i}) \) is an increasing function of \( s^c_{-i} \), and then prove \( \tilde{u}(v-b^F(v; s^c_{-i})) \) is also increasing in \( s^c_{-i} \).

We now show \( \Psi(v|s^c_{-i}) \) increases with \( s^c_{-i} \).
\[
\Psi(v|s^c_{-i}) = [s^c_{-i} + (1 - s^c_{-i})F(v|s \geq s^c_{-i})]^{N-1}
\]
\[
= [s^c_{-i} + (1 - s^c_{-i})\int^1_{s^c_{-i}} F(v|s) \frac{1}{1-s^c_{-i}} ds]^{N-1}
\]
\[
= [s^c_{-i} + \int^1_{s^c_{-i}} F(v|s) ds]^{N-1}.
\]
Therefore,
\[
\frac{\partial \Psi(v|s^c_{-i})}{\partial s^c_{-i}} = (N-1)[s^c_{-i} + \int^1_{s^c_{-i}} F(v|s) ds]^{N-2}(1 - F(v|s^c_{-i})) > 0,
\]
which means \( \Psi(v|s^c_{-i}) \) is increasing in \( s^c_{-i} \).

We now show \( \tilde{u}(v-b^F(v; s^c_{-i})) \) increases with \( s^c_{-i} \), which is equivalent to showing \( b^F(v; s^c_{-i}) \) decreases with \( s^c_{-i} \). Suppose \( s'^i > s^c_{-i} \). As \( b^F(r; s^c_{-i}) = b^F(r; s'^i) = r \), we only need to show 
\[
b^F_v(v; s^c_{-i}) > b^F_v(v; s'^i) \]
whenever \( b^F(v; s^c_{-i}) = b^F(v; s'^i) \) for \( v \geq r \).

As in (4),
\[
b^F_v(v; s^c_{-i}) = \frac{\tilde{u}(v-b^F(v; s^c_{-i})) \Psi_v(v|s^c_{-i})}{\tilde{u}(v-b^F(v; s^c_{-i})) \Psi(v|s^c_{-i})}.
\]
When \( b^F(v; s^c_{-i}) = b^F(v; s'^i) \), \( \frac{\tilde{u}(v-b^F(v; s))}{\tilde{u}(v-b^F(v; s))} \) is the same for \( s = s^c_{-i} \) and \( s = s'^i \). We need to show 
\( \frac{\Psi_v(v|s^c_{-i})}{\Psi(v|s^c_{-i})} \) decreases with \( s^c_{-i} \). Using the definition of \( \Psi(v|s^c_{-i}) \), 
\( \frac{\Psi_v(v|s^c_{-i})}{\Psi(v|s^c_{-i})} \) can be written
as:

\[
\frac{\Psi_v(v|s_{-i}^c)}{\Psi(v|s_{-i}^c)} = (N - 1) \int_{s_{-i}^c}^{1} f(v|s) ds = (N - 1) \int_{s_{-i}^c}^{1} f(v|s) ds \frac{\Psi(v|s_{-i}^c)^{1/N-1}}{\Psi(v|s_{-i}^c)}
\]

where \( f(v|s) = F_v(v|s) \). The denominator is an increasing function of \( s_{-i}^c \), as shown previously; the numerator is clearly a decreasing function of \( s_{-i}^c \). As a result, \( \frac{\Psi_v(v|s_{-i}^c)}{\Psi(v|s_{-i}^c)} \) must strictly decrease with \( s_{-i}^c \), which leads to that \( b_v^F(v; s_{-i}^c) \) must decrease with \( s_{-i}^c \) for \( v \) such that \( b_v^F(v; s_{-i}^c) = b_v^F(v; s_{-i}') \). Moreover, \( b_v^F(v; s_{-i}^c) \) must strictly decrease with \( s_{-i}^c \) when \( v > r \), because \( b_v^F(v; s_{-i}^c) < v \) if \( v > r \).\(^{26}\) When \( v = r \), we have \( \bar{u}(v - b_v^F(v; s_{-i}^c)) = 0 \). Therefore \( b_v^F(r; s_{-i}^c) = b_v^F(r; s_{-i}') = 0 \). For concave \( \bar{u}(\cdot) \), \( \frac{\bar{u}(v)}{v} \) is an increasing function. Therefore, for \( s_{-i}' > s_{-i}^c \), we have \( b_v^F(v; s_{-i}'') > b_v^F(v; s_{-i}^c) \) whenever \( b_v^F(v; s_{-i}'') \leq b_v^F(v; s_{-i}) \) for \( v > r \). We call this Property A.

We now are ready to prove \( b_v^F(v; s_{-i}^c) > b_v^F(v; s_{-i}') \) for \( v > r \), \( s_{-i}' > s_{-i}^c \) by contradiction. Suppose \( b_v^F(v^*; s_{-i}^c) > b_v^F(v^*; s_{-i}') \) for some \( v^* > r \). By Property A and the fact that \( b_v^F(r; s_{-i}^c) = b_v^F(r; s_{-i}') = r \), it must be the case that \( b_v^F(v; s_{-i}^c) < b_v^F(v; s_{-i}') \) for all \( v \in [r, v^*] \). (Otherwise, denote \( v^{**} \) as the highest \( v \in [r, v^*] \) such that \( b_v^F(v; s_{-i}^c) = b_v^F(v; s_{-i}') \). By above Property A, \( b_v^F(v^{**}; s_{-i}^c) > b_v^F(v^{**}; s_{-i}') \). It cannot be the case that \( b_v^F(v^*; s_{-i}^c) < b_v^F(v^*; s_{-i}') \). However, if \( b_v^F(v; s_{-i}^c) < b_v^F(v; s_{-i}') \) for all \( v \in (r, v^*) \), using above property A again, we must have \( b_v^F(v; s_{-i}^c) > b_v^F(v; s_{-i}') \) for all \( v \in (r, v^*) \). Because \( b_v^F(r; s_{-i}^c) = b_v^F(r; s_{-i}') \), \( b_v^F(r; s_{-i}^c) = b_v^F(r; s_{-i}^c) \) and \( b_v^F(v; s_{-i}^c) > b_v^F(v; s_{-i}') \) for all \( v \in (r, v^*) \), we must have that \( b_v^F(v^*; s_{-i}^c) > b_v^F(v^*; s_{-i}') \). This contradicts with our initial assumption that \( b_v^F(v^*; s_{-i}^c) < b_v^F(v^*; s_{-i}') \).

Aggregating the above arguments leads to that \( \Pi_v^F(s_i, s_{-i}^c) \) is an increasing function of \( s_{-i}^c \).

**Proof of Theorem 2:** Without loss of generality, we focus on the case where the equilibrium

\(^{26}\)Note that for first price auction, bidders never bid higher than their values. If \( v > r \), by bidding \( v \), the bidder’s payoff is definitely zero. By placing a bid in between \( r \) and \( v \), he has some chance to win and make a positive payoff.
entry thresholds \( s^e(N) \) are in \((0, 1)\). Recall entry equilibrium condition (7):

\[
\int_r^\theta \tilde{u}(v - b^F(v; s^e))\Psi(v|s^e)dF(v|s^e) = \tilde{u}(c).
\]

Note that we have established that \( \int_r^\theta \tilde{u}(v - b^F(v; s^e))\Psi(v|s^e)dF(v|s^e) \) increases with \( s^e \), which is implied by Corollary 1. To establish a positive relationship between \( N \) and \( s^e \), we only need to show that \( \int_r^\theta \tilde{u}(v - b^F(v; s^e))\Psi(v|s^e)dF(v|s^e) \) decreases with \( N \) for fixed \( s^e \). A sufficient condition is to show that both \( \tilde{u}(v - b^F(v; s^e)) \) and \( \Psi(v; s^e) \) decrease with \( N \), for fixed \( s^e \). We use notation \( b^F(v; s^e, N) \) to explicitly incorporate the impact of \( N \) on the symmetric bidding strategy when entry threshold is \( s^e \).

To prove \( \tilde{u}(v - b^F(v; s^e, N)) \) is decreasing in \( N \), we only need to show \( b^F(v; s^e, N) \) is increasing in \( N \), which is a very intuitive result because bidders bid more aggressively if they face more competitors. Note that at the reserve price, \( b^F(r; s^e, N) = r \) for every \( N \). By Lemma 1, we have

\[
b^F(v; s^e, N) = \frac{\tilde{u}(v - b^F(v; s^e, N)) \Psi_v(v|s^e)}{\tilde{u}(v - b^F(v; s^e, N)) \Psi(v|s^e)}, \ v \geq r,
\]

where

\[
\frac{\Psi_v(v|s^e, N)}{\Psi(v|s^e, N)} = \frac{(N - 1) \int_{s^c}^1 f(v|s)ds}{s^c + \int_{s^c}^1 F(v|s)ds}
\]

is increasing in \( N \). Here \( f(v|s) = F_v(v|s) \). Consider \( N_2 > N_1 \). The above expressions mean that we have \( b^F_v(v; s^e, N_2) > b^F_v(v; s^e, N_1) \) at any \( v > r \) whenever \( b^F_v(v; s^e, N_2) = b^F(v; s^e, N_1) \). Based on same arguments elaborated in the proof of Lemma 2 for the monotonicity of \( b^F_v(v; s^e_{-i}) \) with respect to \( s^e_{-i} \), we have that \( b^F(v; s^e, N_2) > b^F(v; s^e, N_1) \) for \( v > r \), i.e. \( b^F(v; s^e, N) \) is an increasing function of \( N \).

Note by definition \( \Psi(v|s^e) = [s^e + (1 - s^e)F(v|s \geq s^e)]^{N-1} \). Because \( s^e + (1 - s^e)F(v|s \geq s^e) \) lies in the interval \([0, 1]\), \( \Psi(v|s^e_{-i}) \) is also a decreasing function of \( N \). Therefore, we showed that both \( \tilde{u}(v - b^F(v; s^e)) \) and \( \Psi(v; s^e) \) decrease with \( N \), for fixed \( s^e \). It thus follows that \( \int_r^\theta \tilde{u}(v - b^F(v; s^e)) \cdot \Psi(v|s^e)dF(v|s^e) \) decreases with \( N \) for fixed \( s^e \). As a result, the entry equilibrium \( s^e(N) \) increases with \( N \).
Proof of Lemma 3: The proof for Lemma 3 is very similar to the proof for Lemma 2. Recall equation (10),

\[ \Pi_i^A(s_i, s_{c-i}^e) = \int_r^0 \{ \tilde{u}(v-r)\Psi(r|s_{c-i}^e) + \int_r^v \tilde{u}(v-t)d\Psi(t|s_{c-i}^e) \}dF(v|s_i) + u(w_0 - c). \]

We first show \( \Pi_i^A(s_i, s_{c-i}^e) \) increases with \( s_i \). Note that we have established that \( \Psi(.|s_{c-i}^e) \) is an increasing function of \( s_{c-i}^e \). This result suffices for showing \( \tilde{u}(v-r)\Psi(r|s_{c-i}^e) + \int_r^v \tilde{u}(v-t)d\Psi(t|s_{c-i}^e) \) is increasing in \( s_{c-i}^e \). Take \( s_{c-i}^e > s_{c-i}^e \), we have

\[
\begin{align*}
[\tilde{u}(v-r)\Psi(r|s_{c-i}^e) + \int_r^v \tilde{u}(v-t)d\Psi(t|s_{c-i}^e)] - [\tilde{u}(v-r)\Psi(r|s_{c-i}^e) + \int_r^v \tilde{u}(v-t)d\Psi(t|s_{c-i}^e)] \\
= \tilde{u}(v-r)[\Psi(r|s_{c-i}^e) - \Psi(r|s_{c-i}^e)] + \int_r^v \tilde{u}(v-t)[\Psi(t|s_{c-i}^e) - \Psi(t|s_{c-i}^e)]dF(v|s_i) \\
= \tilde{u}(v-r)[\Psi(r|s_{c-i}^e) - \Psi(r|s_{c-i}^e)] + \tilde{u}(v-t)[\Psi(t|s_{c-i}^e) - \Psi(t|s_{c-i}^e)]dF(v|s_i) \\
+ \int_r^v [\Psi(t|s_{c-i}^e) - \Psi(t|s_{c-i}^e)]\tilde{u}'(v-t)dt \\
= \int_r^v [\Psi(t|s_{c-i}^e) - \Psi(t|s_{c-i}^e)]\tilde{u}'(v-t)dt \geq 0.
\end{align*}
\]

We thus have \( \Pi_i^A(s_i, s_{c-i}^e) \) increases with \( s_{c-i}^e \).

Proof of Theorem 4: Without loss of generality, we focus on the case where the equilibrium entry thresholds \( s^e(N) \) are in \((0, 1)\). Recall entry equilibrium condition (11): 

\[ \int_r^0 \{ \tilde{u}(v-r)\Psi(r|s^e) + \int_r^v \tilde{u}(v-t)d\Psi(t|s^e) \}dF(v|s^e) = \tilde{u}(c). \]

Note that we have established that \( \int_r^0 \{ \tilde{u}(v-r)\Psi(r|s^e) + \int_r^v \tilde{u}(v-t)d\Psi(t|s^e) \}dF(v|s^e) \) increases with \( s^e \), which is implied by Corollary 2. To establish a positive relationship between \( N \) and \( s^e \), we only need to show that \( \int_r^0 \{ \tilde{u}(v-r)\Psi(r|s^e) + \int_r^v \tilde{u}(v-t)d\Psi(t|s^e) \}dF(v|s^e) \) decreases with \( N \) for fixed \( s^e \). We have shown in the proof of Theorem 2 that \( \Psi(v; s^e) \) is a decreasing function of \( N \). Note that \( \tilde{u}(v-t) \) decreases with \( t \) for \( \forall t \in (r, v], \forall v \in [r, \tilde{v}] \). We thus have \( \int_r^0 \{ \tilde{u}(v-r)\Psi(r|s^e) + \int_r^v \tilde{u}(v-t)d\Psi(t|s^e) \}dF(v|s^e) \) is decreasing in \( N \).
References


35


Figure 1: Participation Rates in First Price Auction andAscending Auction
Table 1: Summary Statistics

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<th>First Price Auction (L=1188)</th>
<th>Ascending (L=1327)</th>
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<tr>
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<td>Mean</td>
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Table 2: QMLE and Linear Regression Using N as 78-both-year

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Table 3: QMLE and Linear Regression Using N as 78-sep-year

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<tr>
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<td>-0.087</td>
<td>-0.017</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>(log) Acres</td>
<td>0.107</td>
<td>0.175</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.015)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>(log) Density</td>
<td>0.122</td>
<td>0.204</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.021)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>(log) Building permits</td>
<td>0.056</td>
<td>0.091</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.013)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>(log) Appraisal value</td>
<td>0.058</td>
<td>0.099</td>
<td>0.019</td>
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<tr>
<td></td>
<td>(0.013)</td>
<td>(0.021)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Salvage</td>
<td>-0.032</td>
<td>-0.040</td>
<td>-0.013</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.054)</td>
<td>(0.011)</td>
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Table 4: Results from Excluding Auctions with Extreme High Volume or Full Participation

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<tr>
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<th>78-both-year</th>
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<th>78-sep-year</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vol &lt;95%</td>
<td>p ≠ 1</td>
<td>Vol &lt;95%</td>
<td>p ≠ 1</td>
</tr>
<tr>
<td># Obs</td>
<td>2374</td>
<td>2402</td>
<td>2263</td>
<td>2234</td>
</tr>
<tr>
<td>FPA</td>
<td>-0.213</td>
<td>-0.156</td>
<td>-0.272</td>
<td>-0.188</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.023)</td>
<td>(0.031)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>N</td>
<td>-0.040</td>
<td>-0.030</td>
<td>-0.052</td>
<td>-0.037</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>(log) Acres</td>
<td>0.090</td>
<td>0.091</td>
<td>0.109</td>
<td>0.104</td>
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<tr>
<td></td>
<td>(0.009)</td>
<td>(0.007)</td>
<td>(0.010)</td>
<td>(0.008)</td>
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<tr>
<td>(log) Density</td>
<td>0.110</td>
<td>0.112</td>
<td>0.123</td>
<td>0.124</td>
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<tr>
<td></td>
<td>(0.012)</td>
<td>(0.010)</td>
<td>(0.013)</td>
<td>(0.010)</td>
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<tr>
<td>(log) Building permits</td>
<td>0.051</td>
<td>0.036</td>
<td>0.057</td>
<td>0.042</td>
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<td>(0.008)</td>
<td>(0.006)</td>
<td>(0.008)</td>
<td>(0.007)</td>
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<td>(log) Appraisal value</td>
<td>0.016</td>
<td>0.054</td>
<td>0.054</td>
<td>0.081</td>
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<tr>
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<td>(0.012)</td>
<td>(0.009)</td>
<td>(0.013)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Salvage</td>
<td>-0.039</td>
<td>-0.036</td>
<td>-0.028</td>
<td>-0.023</td>
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<tr>
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<td>(0.030)</td>
<td>(0.026)</td>
<td>(0.033)</td>
<td>(0.028)</td>
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Table 5: QMLE from the Fractional Response Model with a Dummy Endogenous Variable

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<tbody>
<tr>
<td># Obs</td>
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<td>2390</td>
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<td>(0.005)</td>
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<tr>
<td>(log) Acres</td>
<td>0.110</td>
<td>0.112</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>(log) Density</td>
<td>0.133</td>
<td>0.128</td>
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<td>(0.036)</td>
<td>(0.036)</td>
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<td>(log) Building permits</td>
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<td>0.056</td>
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<td>(log) Appraisal value</td>
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<td>0.063</td>
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<tr>
<td>Salvage</td>
<td>-0.037</td>
<td>-0.031</td>
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<td>(0.076)</td>
<td>(0.076)</td>
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<tr>
<td>Rho</td>
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<td>(0.161)</td>
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Table 6: Seasonality

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<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
<th>Winter</th>
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<td>739</td>
<td>739</td>
<td>395</td>
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<td>-0.280</td>
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<td>(0.053)</td>
<td>(0.053)</td>
<td>(0.070)</td>
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<td>-0.036</td>
<td>-0.038</td>
<td>-0.043</td>
<td>-0.040</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>(log) Acres</td>
<td>0.120</td>
<td>0.086</td>
<td>0.078</td>
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<tr>
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<td>(0.018)</td>
<td>(0.015)</td>
<td>(0.018)</td>
<td>(0.021)</td>
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<td>0.102</td>
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<td>(0.024)</td>
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<td>(0.021)</td>
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<td>0.060</td>
<td>0.061</td>
<td>0.001</td>
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<tr>
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<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.020)</td>
</tr>
<tr>
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<td>0.048</td>
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<td>(0.023)</td>
<td>(0.022)</td>
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<td>(0.034)</td>
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<tr>
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<td>-0.045</td>
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<td>(0.064)</td>
<td>(0.056)</td>
<td>(0.052)</td>
<td>(0.081)</td>
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