Price Change: Income and Substitution Effects
THE IMPACT OF A PRICE CHANGE

- Economists often separate the impact of a price change into two components:
  - the substitution effect; and
  - the income effect.
THE IMPACT OF A PRICE CHANGE

- The substitution effect involves the substitution of good $x_1$ for good $x_2$ or vice-versa due to a change in relative prices of the two goods.

- The income effect results from an increase or decrease in the consumer’s real income or purchasing power as a result of the price change.

- The sum of these two effects is called the price effect.
THE IMPACT OF A PRICE CHANGE

- The decomposition of the price effect into the income and substitution effect can be done in several ways.
- There are two main methods:
  (i) The **Hicksian** method; and
  (ii) The **Slutsky** method
THE HICKSIAN METHOD

- Sir John R. Hicks (1904-1989)
- Awarded the Nobel Laureate in Economics (with Kenneth J. Arrow) in 1972 for work on general equilibrium theory and welfare economics.
Optimal bundle is $E_a$, on indifference curve $I_1$. 
THE HICKSIAN METHOD

A fall in the price of $X_1$
The budget line pivots out from $P$
The new optimum is $E_b$ on $I_2$.

The Total Price Effect is $x_a$ to $x_b$.

\[ X_2 \]

\[ X_1 \]
THE HICKSIAN METHOD

- To isolate the substitution effect we ask....
  “what would the consumer’s optimal bundle be if s/he faced the new lower price for $X_1$ but experienced no change in real income?”
- This amounts to returning the consumer to the original indifference curve ($I_1$)
The new optimum is \( E_b \) on \( I_2 \).

The Total Price Effect is \( x_a \) to \( x_b \).
THE HICKSIAN METHOD

Draw a line parallel to the new budget line and tangent to the old indifference curve.
The new optimum on $I_1$ is at Ec. The movement from Ea to Ec (the increase in quantity demanded from Xa to Xc) is solely in response to a change in relative prices.
THE HICKSIAN METHOD

This is the substitution effect.

Substitution Effect
THE HICKSIAN METHOD

- To isolate the income effect …
- Look at the remainder of the total price effect
- This is due to a change in real income.
The remainder of the total effect is due to a change in real income. The increase in real income is evidenced by the movement from $I_1$ to $I_2$. 

The graph shows the income effect ($E_c$) and the direct effect ($E_a$), with $X_c$ and $X_b$ representing different income levels.
THE HICKSIAN METHOD
HICKSIAN ANALYSIS and DEMAND CURVES

A fall in price from $p_1$ to $p_1^*$

Marshallian Demand Curve (A & B)

Hicksian Demand Curve (A & C)

$M_1 = p_1x_1 + p_2x_2$

$M_1 = p_1^*x_1 + p_2x_2$
HICKSIAN ANALYSIS and DEMAND CURVES

Hicksian (compensated) demand curves cannot be upward-sloping (i.e. substitution effect cannot be positive)
THE SLUTSKY METHOD

- Eugene Slutsky (1880-1948)
- Russian economist expelled from the University of Kiev for participating in student revolts.
- In his 1915 paper, “On the theory of the Budget of the Consumer” he introduced “Slutsky Decomposition”.
Optimal bundle is $E_a$, on indifference curve $I_1$. 
THE SLUTSKY METHOD

A fall in the price of $X_1$

The budget line pivots out from $P$

![Graph showing the Slutsky Method with the budget line pivoting out from point P after a fall in the price of $X_1$]
The new optimum is $E_b$ on $I_2$.

The Total Price Effect is $x_a$ to $x_b$.
THE SLUTSKY METHOD

- Slutsky claimed that if, at the new prices,
  - less income is needed to buy the original bundle then “real income” has increased
  - more income is needed to buy the original bundle then “real income” has decreased
- Slutsky isolated the change in demand due only to the change in relative prices by asking “What is the change in demand when the consumer’s income is adjusted so that, at the new prices, s/he can just afford to buy the original bundle?”
THE SLUTSKY METHOD

- To isolate the substitution effect we adjust the consumer’s money income so that s/he change can just afford the original consumption bundle.

- In other words we are holding purchasing power constant.
The new optimum is $E_b$ on $I_2$.

The Total Price Effect is $x_a$ to $x_b$.
THE SLUTSKY METHOD

Draw a line parallel to the new budget line which passes through the point Ea.
THE SLUTSKY METHOD

The new optimum on $I_3$ is at $E_c$. The movement from $E_a$ to $E_c$ is the substitution effect.
The new optimum on $I_3$ is at $Ec$. The movement from $Ea$ to $Ec$ is the substitution effect.
The remainder of the total price effect is the Income Effect.}

The movement from Ec to Eb.
THE SLUTSKY METHOD for NORMAL GOODS

- Most goods are normal (i.e. demand increases with income).
- The substitution and income effects reinforce each other when a normal good’s own price changes.
THE SLUTSKY METHOD for NORMAL GOODS

The income and substitution effects reinforce each other.
THE SLUTSKY METHOD for NORMAL GOODS

- Since both the substitution and income effects increase demand when own-price falls, a normal good’s ordinary demand curve slopes downwards.
- The “Law” of Downward-Sloping Demand therefore always applies to normal goods.
THE SLUTSKY EQUATION

Let

\[ M_1 = p_1 x_1 + p_2 x_2 \]

be the original budget constraint

and let

\[ M_2 = p_1^* x_1 + p_2 x_2 \]

represent the budget constraint after the Slutsky compensating variation in income has been carried out.
THE SLUTSKY EQUATION

Demand for $x_1$ is

$$x_1 = x^d(p_1, p_2, M)$$

$$M_2 = p_1^* x_1 + p_2 x_2$$

$$M_2 < M_1$$

$$M_1 = p_1 x_1 + p_2 x_2$$
**THE SLUTSKY EQUATION**

\[ \Delta M = M_2 - M_1 = \left( p_1^* x_1 + p_2 x_2 \right) - \left( p_1 x_1 + p_2 x_2 \right) \]

\[ \Delta M = M_2 - M_1 = p_1^* x_1 + p_2 x_2 - p_1 x_1 - p_2 x_2 \]

\[ \Delta M = M_2 - M_1 = p_1^* x_1 - p_1 x_1 \]

\[ \Delta M = M_2 - M_1 = x_1 \left( p_1^* - p_1 \right) \]

\[ \Delta M = x_1 \Delta p_1 \quad \text{as} \quad \left( p_1^* - p_1 \right) = \Delta p_1 \]

gives the change in money income needed to consume the original bundle of goods (at \( E_A \))
THE SLUTSKY EQUATION

The demand curve holding M constant is given by

\[ \Delta x_1 = x^d(p^*_1, p_2, M_1) - x^d(p_1, p_2, M_1) \] (1)

which is the change in demand for \( x_1 \) due to the change in its own price, holding M and the price of \( x_2 \) constant
THE SLUTSKY EQUATION

The income effect is given by

\[ \Delta x_m = x^d(p_1^*, p_2, M_1) - x^d(p_1^*, p_2, M_2) \]  \hspace{1cm} (2)

The change in demand due to the Slutsky substitution effect is given by

\[ \Delta x_s = x^d(p_1^*, p_2, M_2) - x^d(p_1, p_2, M_1) \]  \hspace{1cm} (3)
THE SLUTSKY EQUATION

Given

\[ \Delta x_1 = x^d(p^*_1, p_2, M_1) - x^d(p_1, p_2, M_1) \] (1)

\[ \Delta x_m = x^d(p^*_1, p_2, M_1) - x^d(p^*_1, p_2, M_2) \] (2)

\[ \Delta x_s = x^d(p^*_1, p_2, M_2) - x^d(p_1, p_2, M_1) \] (3)

Claim

\[ \Delta x_1 = \Delta x_s + \Delta x_m \] (4)

Show this by substituting equations (1), (2) and (3) into equation (4)
THE SLUTSKY EQUATION

\[ \Delta x_1 = \Delta x_s + \Delta x_m \]

Divide across by \( \Delta p_1 \)

\[ \frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_s}{\Delta p_1} + \frac{\Delta x_m}{\Delta p_1} \]

Recall

\[ \Delta M = x_1 \Delta p_1 \]

so

\[ \Delta p_1 = (-) \frac{\Delta M}{x_1} \]
THE SLUTSKY EQUATION

Substituting

\[ \Delta p_1 = (-) \frac{\Delta M}{x_1} \]

Gives

\[ \frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_s}{\Delta p_1} - \frac{\Delta x_m}{\Delta M} x_1 \]

THE SLUTSKY EQUATION
THE SLUTSKY METHOD: INFERIOR GOODS

- Some goods are (sometimes) inferior (i.e. demand is reduced by higher income).
- The substitution and income effects “oppose” each other when an inferior good’s own price changes.
The substitution effect is as per usual. But, the income effect is in the opposite direction.
GIFFEN GOODS

- In rare cases of extreme inferiority, the income effect may be larger in size than the substitution effect, causing quantity demanded to rise as own price falls.
- Such goods are Giffen goods.
- Giffen goods are very inferior goods.
THE SLUTSKY METHOD for INFERIOR GOODS

In rare cases of extreme income-inferiority, the income effect may be larger in size than the substitution effect, causing quantity demanded to fall as own-price falls.
SLUTSKY’S EFFECT FOR GIFFEN GOODS

- Slutsky’s decomposition of the effect of a price change into a pure substitution effect and an income effect thus explains why the “Law” of Downward-Sloping Demand is violated for very inferior goods.
DECOMPOSITION of TOTAL PRICE EFFECT: PERFECT COMPLEMENTS

A fall in the price of $X_1$

No substitution effect

Original Budget Constraint

New Budget Constraint

$X_2$

$X_1$

$A=C$

$I_1$

$I_2$

$B$

$A=C$
DECOMPOSITION of TOTAL PRICE EFFECT
PERFECT SUBSTITUTES

?