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Externalities

Externalities

- An **externality** is a cost or a benefit imposed upon someone by actions taken by others. The cost or benefit is thus generated externally to that somebody.
- An externally imposed benefit is a **positive externality**.
- An externally imposed cost is a **negative externality**.

Examples of Negative Externalities

- Air pollution.
- Water pollution.
- Loud parties next door.
- Traffic congestion.
- Second-hand cigarette smoke.
- Increased insurance premiums due to alcohol or tobacco consumption.

Examples of Positive Externalities

- A well-maintained property next door that raises the market value of your property.
- A pleasant cologne or scent worn by the person seated next to you.
- Improved driving habits that reduce accident risks.
- A scientific advance.

Externalities and Efficiency

- Crucially, an externality impacts a **third party**; i.e. somebody who is not a participant in the activity that produces the external cost or benefit.

Externalities and Efficiency

- Externalities cause Pareto inefficiency; typically
 - too much scarce resource is allocated to an activity which causes a negative externality
 - too little resource is allocated to an activity which causes a positive externality.

Externalities and Property Rights

- An externality will be viewed as a **purely public commodity**.
- A commodity is purely public if
 - it is consumed by everyone (**nonexcludability**),
and
 - everybody consumes the entire amount of the commodity (**nonrivalry in consumption**).
- E.g. a broadcast television program.

Inefficiency & Negative Externalities

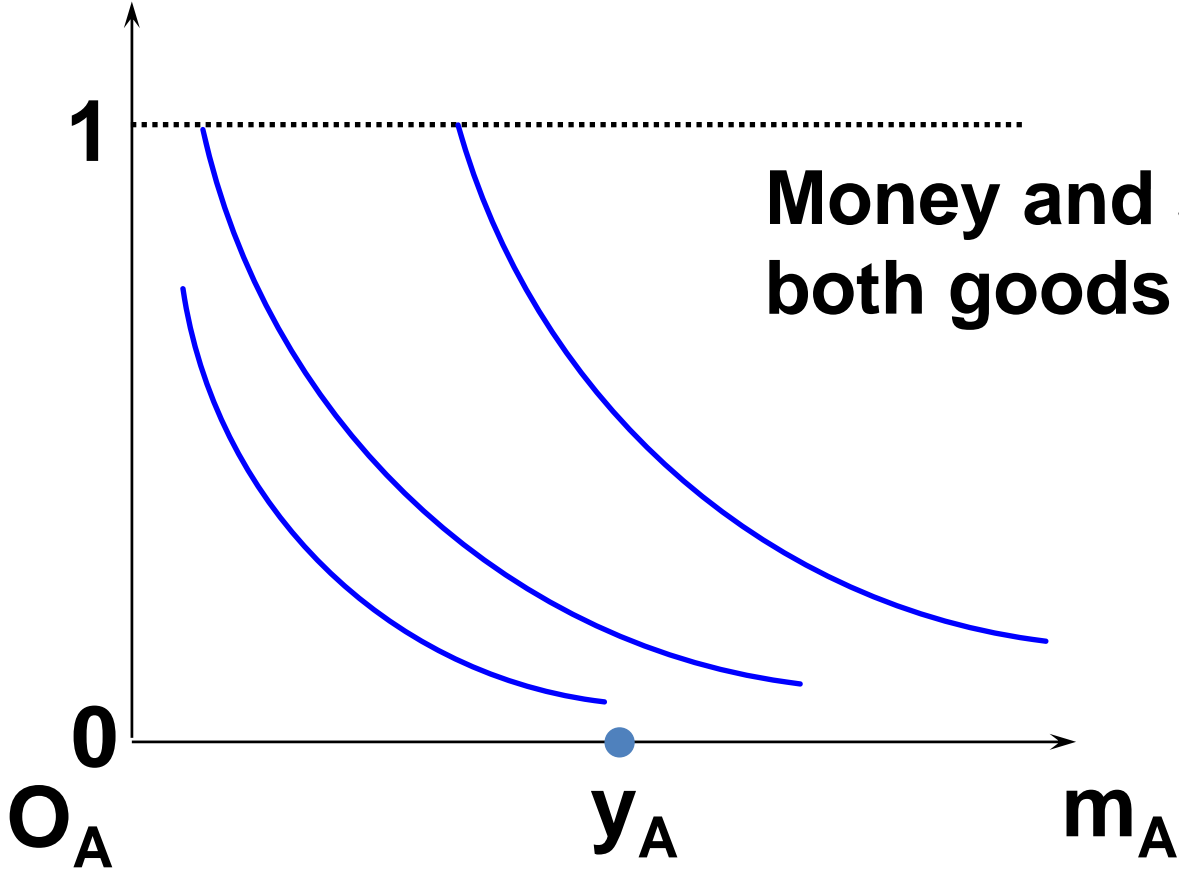
- Consider two agents, A and B, and two commodities, money and smoke.
- Both smoke and money are goods for Agent A.
- Money is a good and smoke is a bad for Agent B.
- Smoke is a purely public commodity.

Inefficiency & Negative Externalities

- Agent A is endowed with y_A .
- Agent B is endowed with y_B .
- Smoke intensity is measured on a scale from 0 (no smoke) to 1 (maximum concentration).

Inefficiency & Negative Externalities

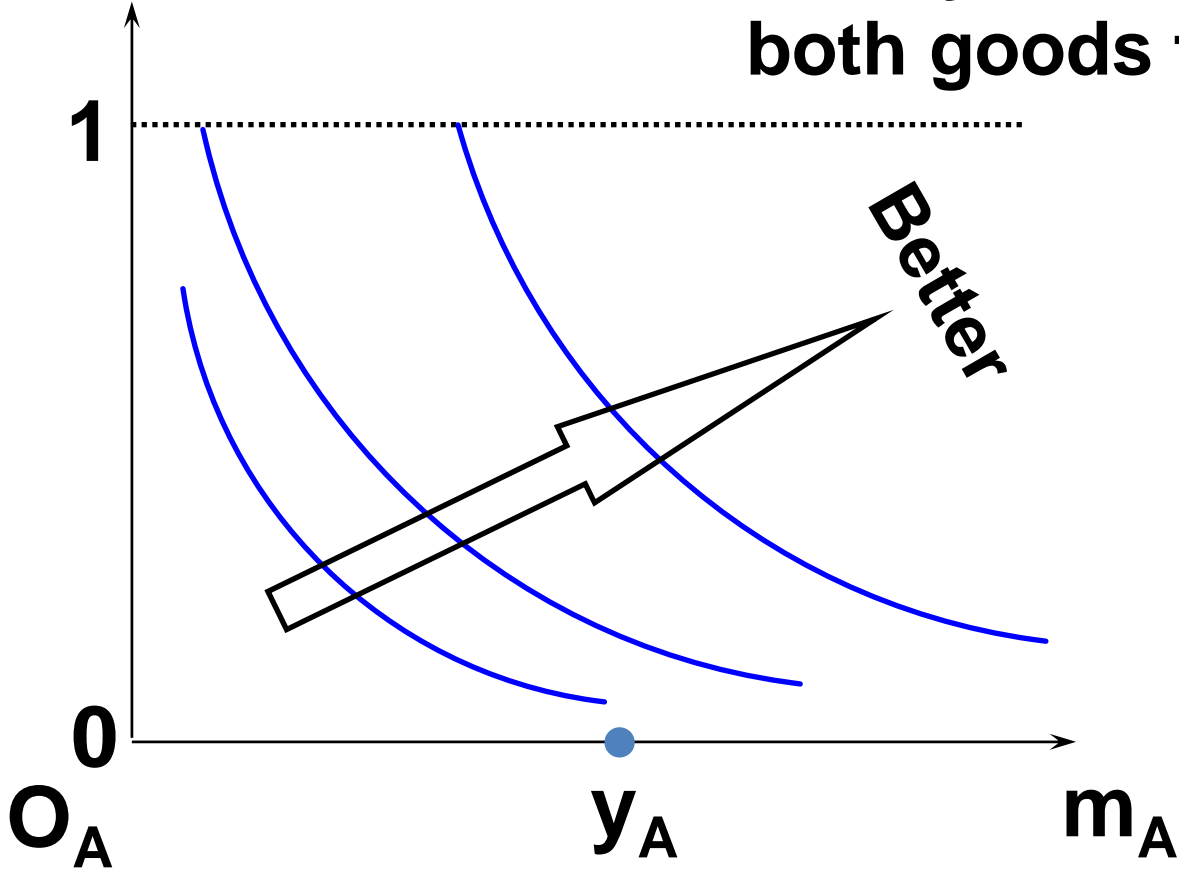
Smoke



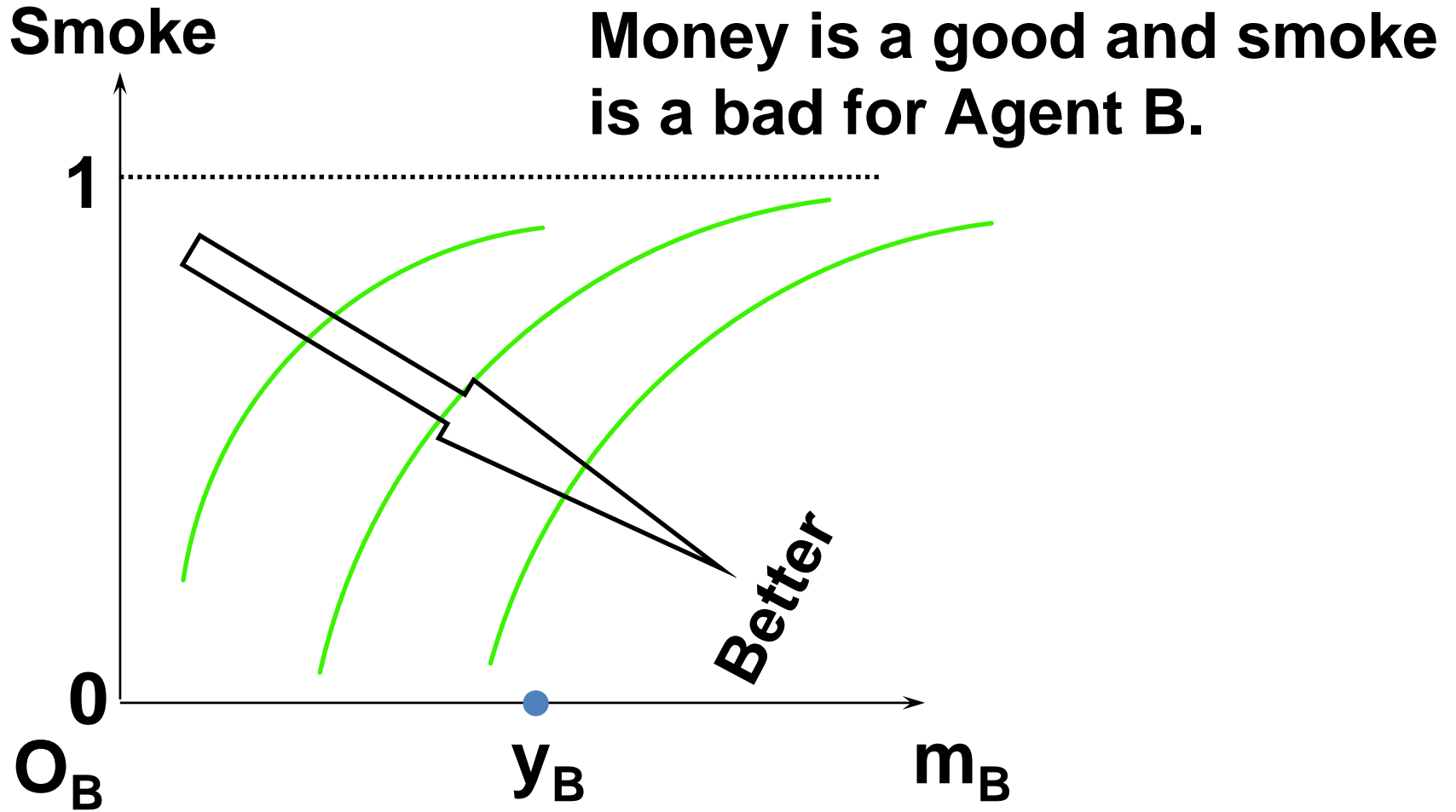
Inefficiency & Negative Externalities

Smoke

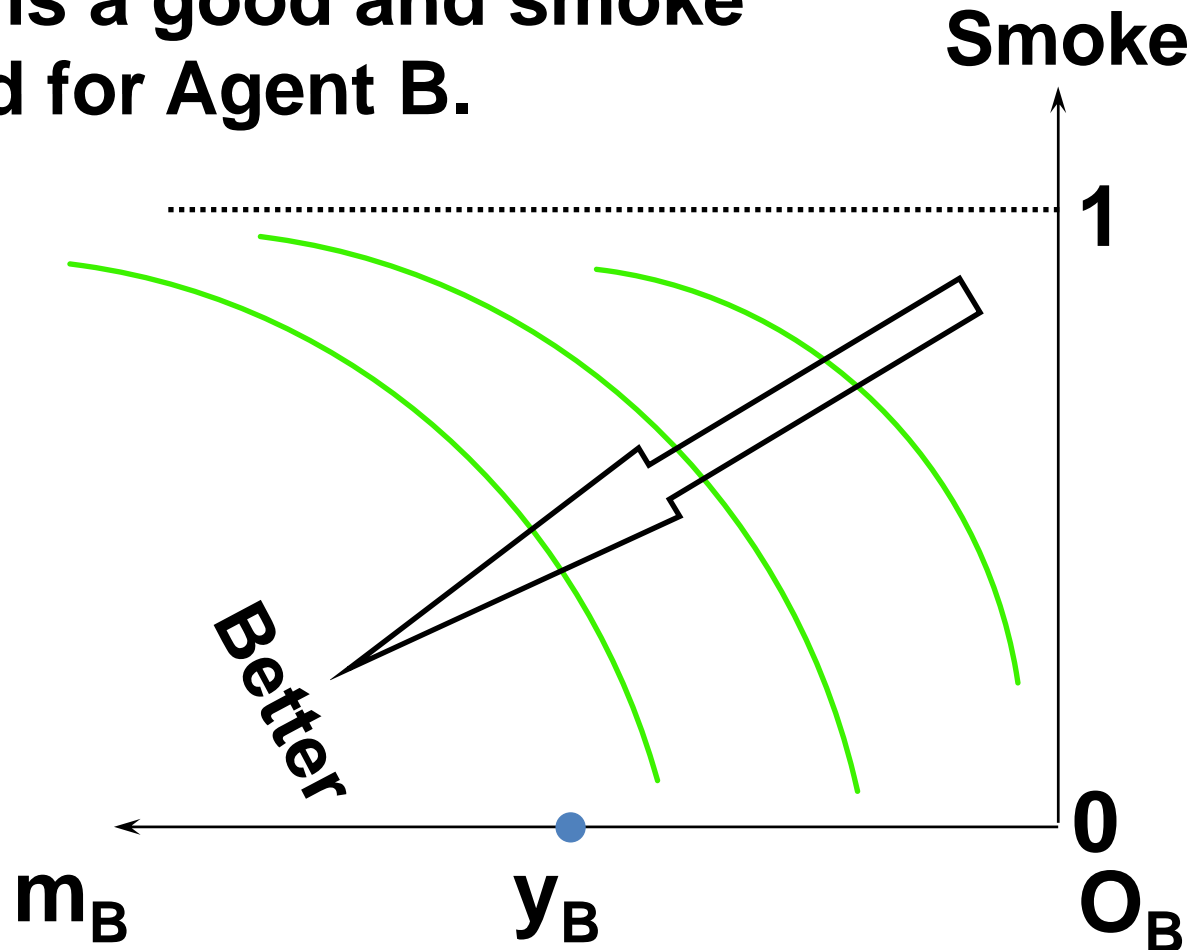
Money and smoke are both goods for Agent A.



Inefficiency & Negative Externalities



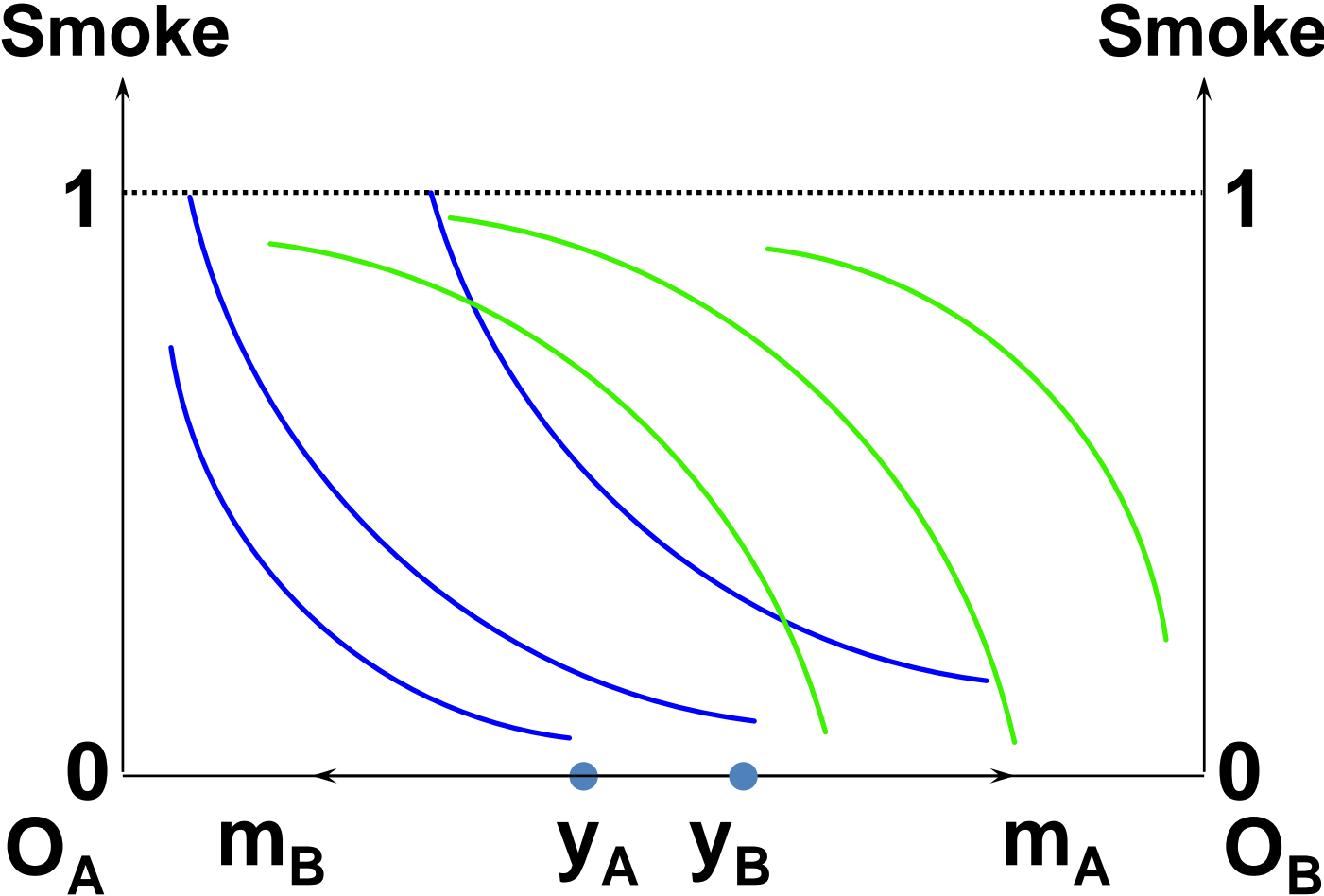
Inefficiency & Negative Externalities
Money is a good and smoke
is a bad for Agent B.



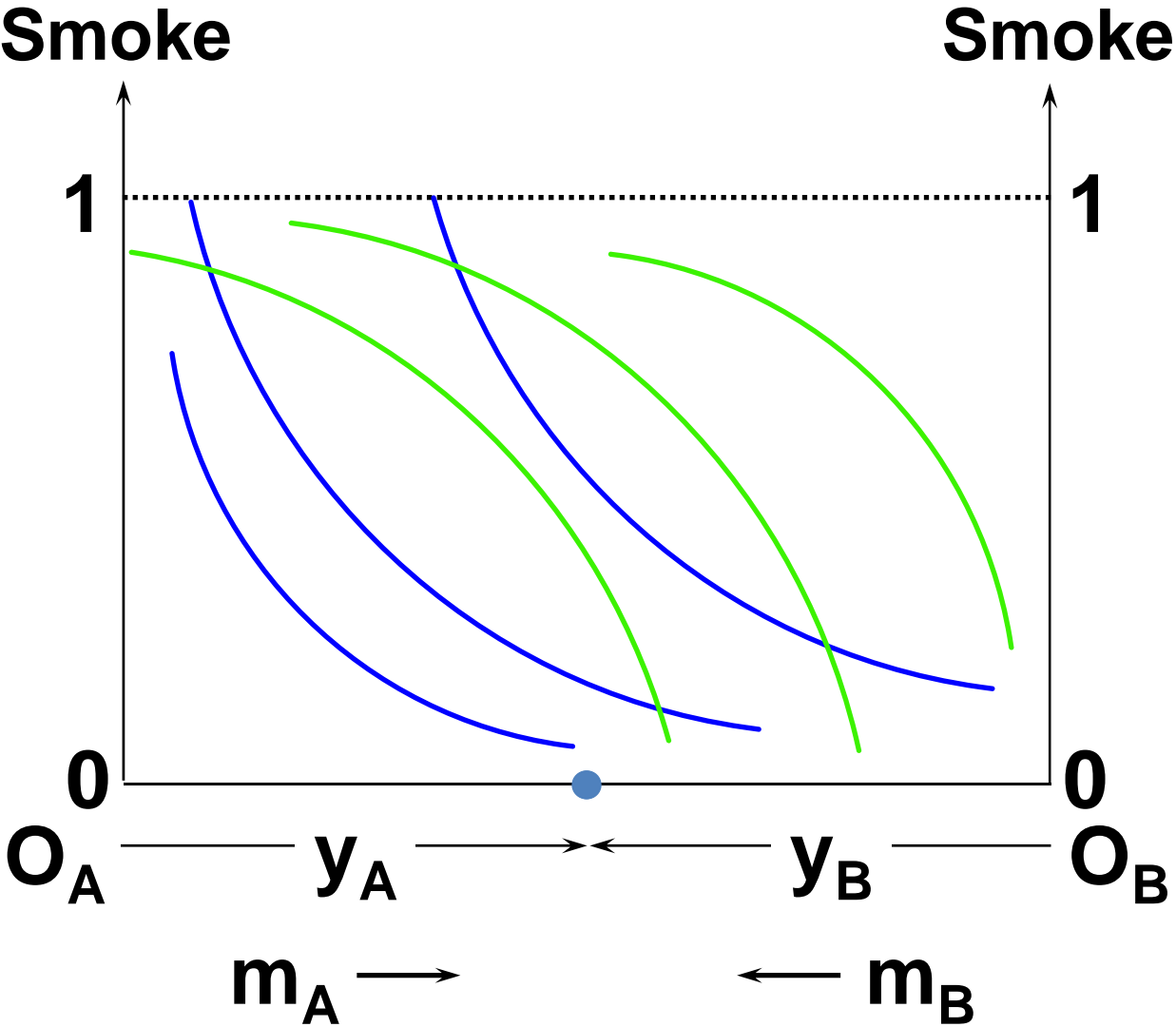
Inefficiency & Negative Externalities

- What are the efficient allocations of smoke and money?

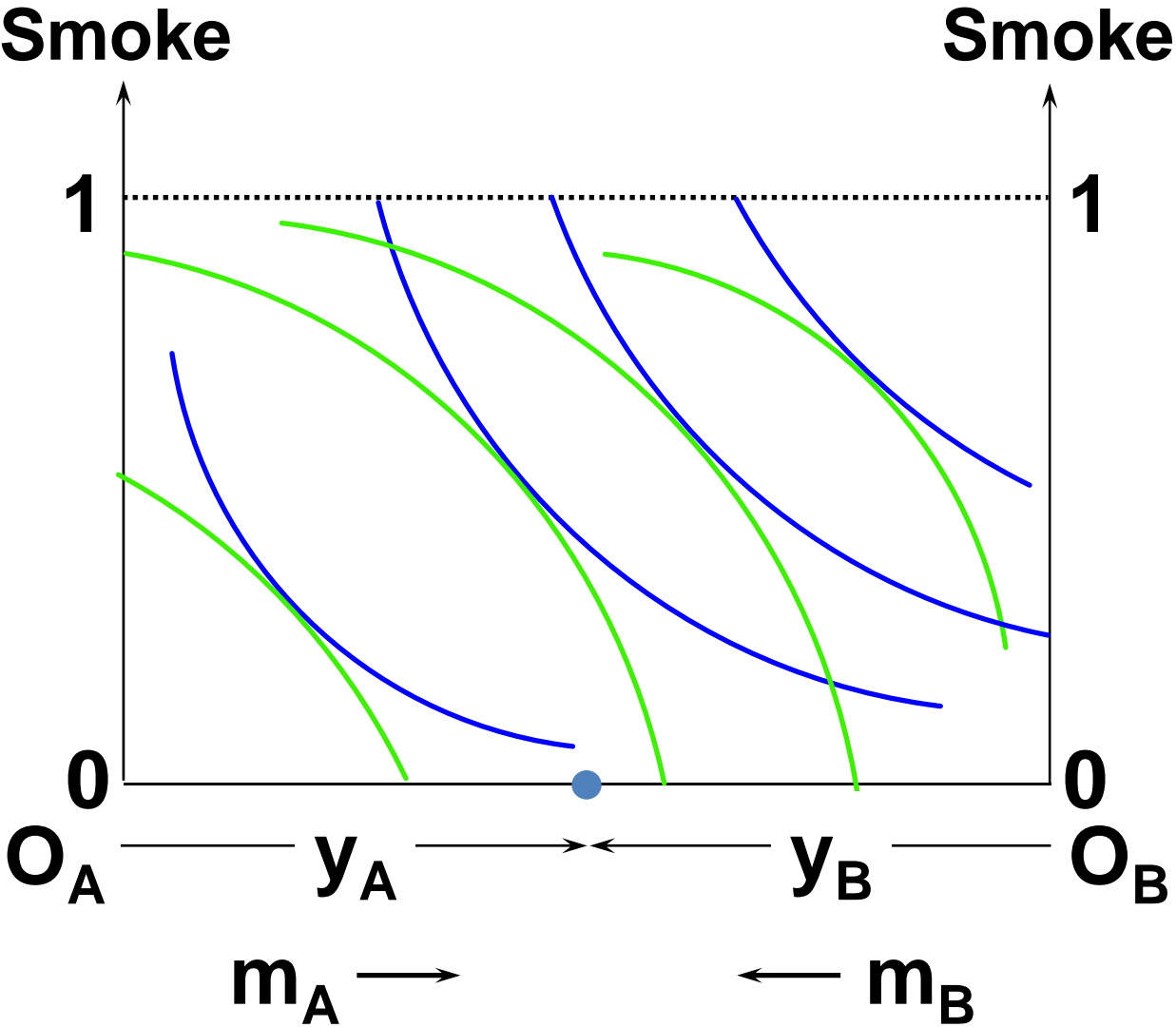
Inefficiency & Negative Externalities



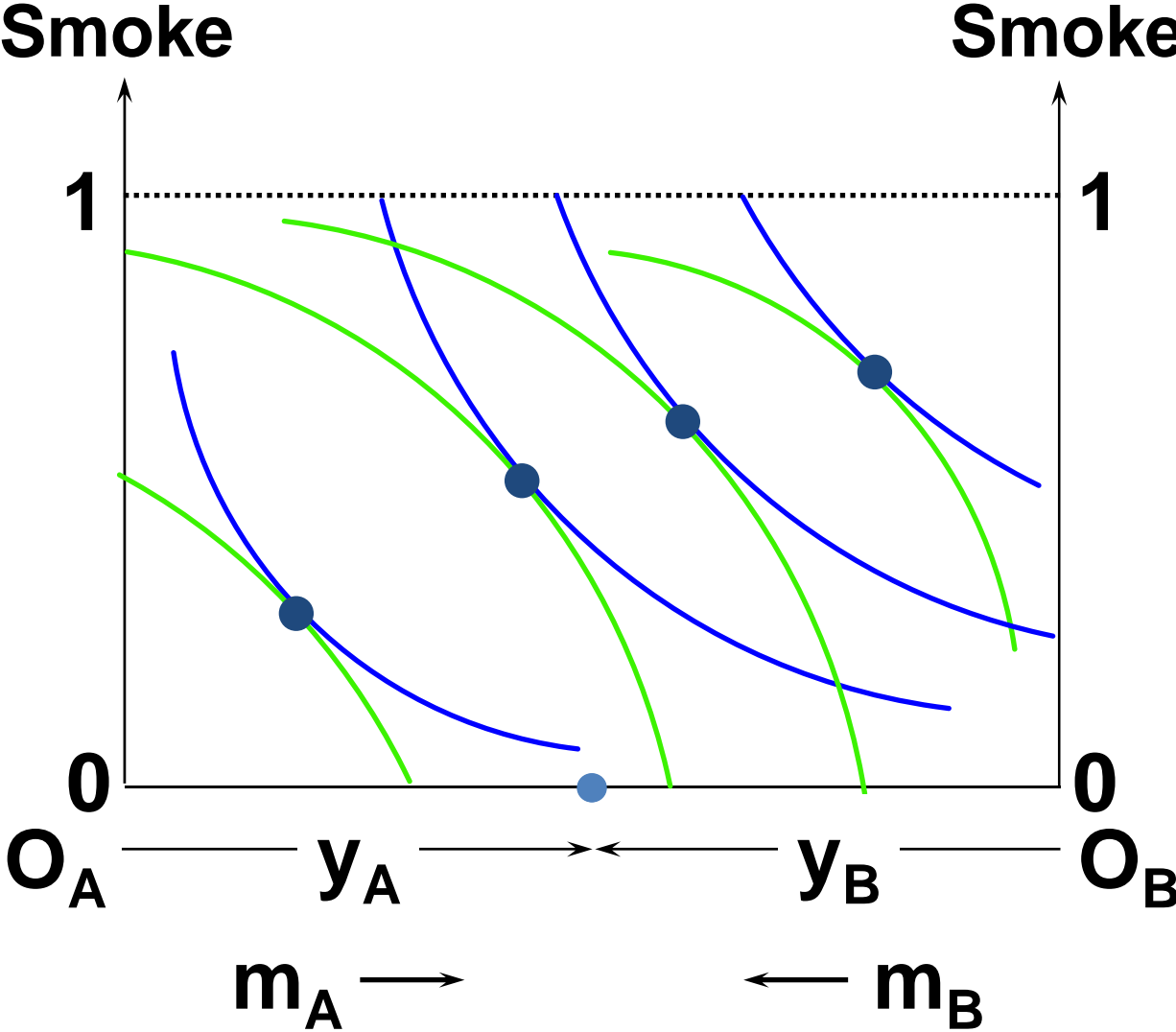
Inefficiency & Negative Externalities



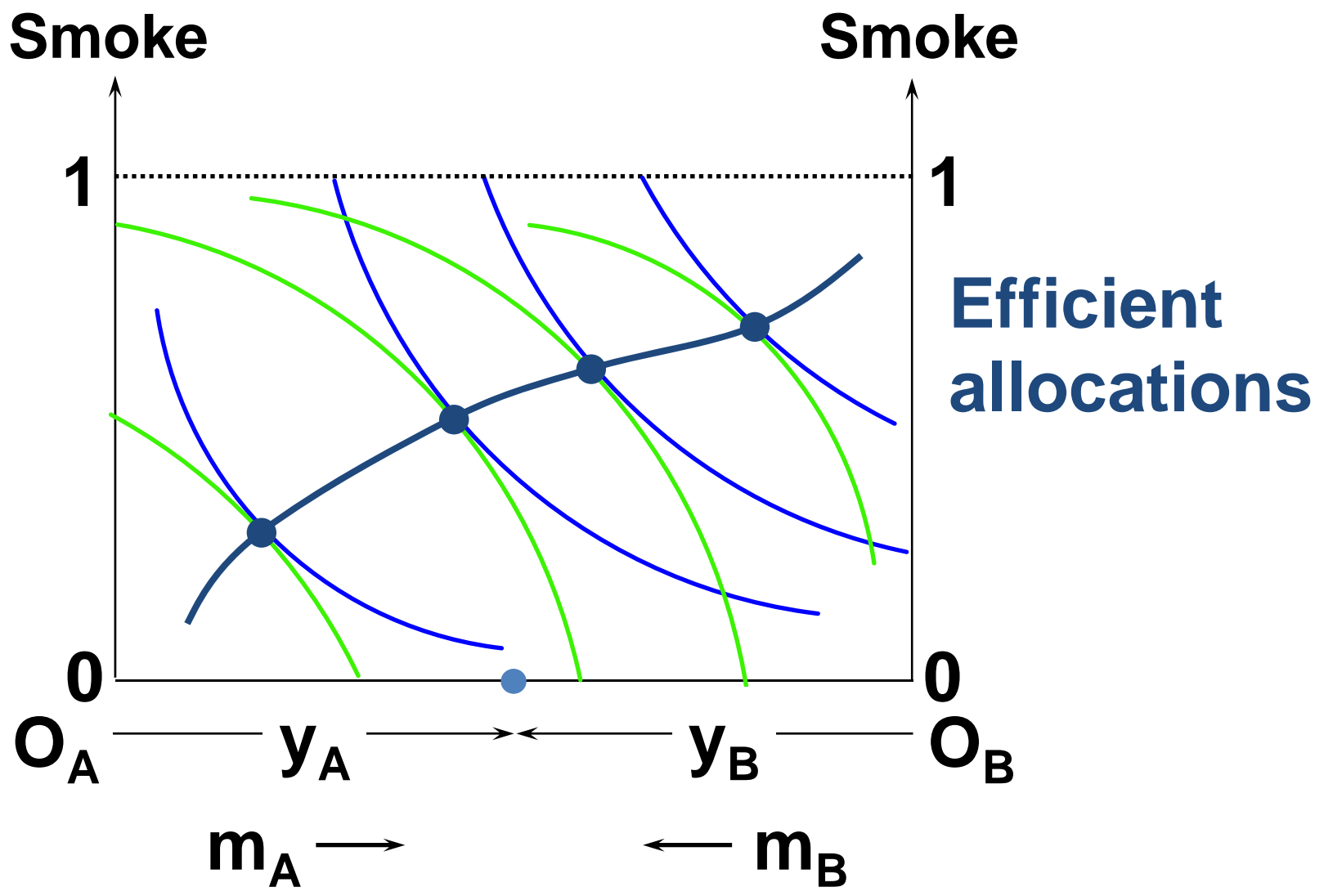
Inefficiency & Negative Externalities



Inefficiency & Negative Externalities



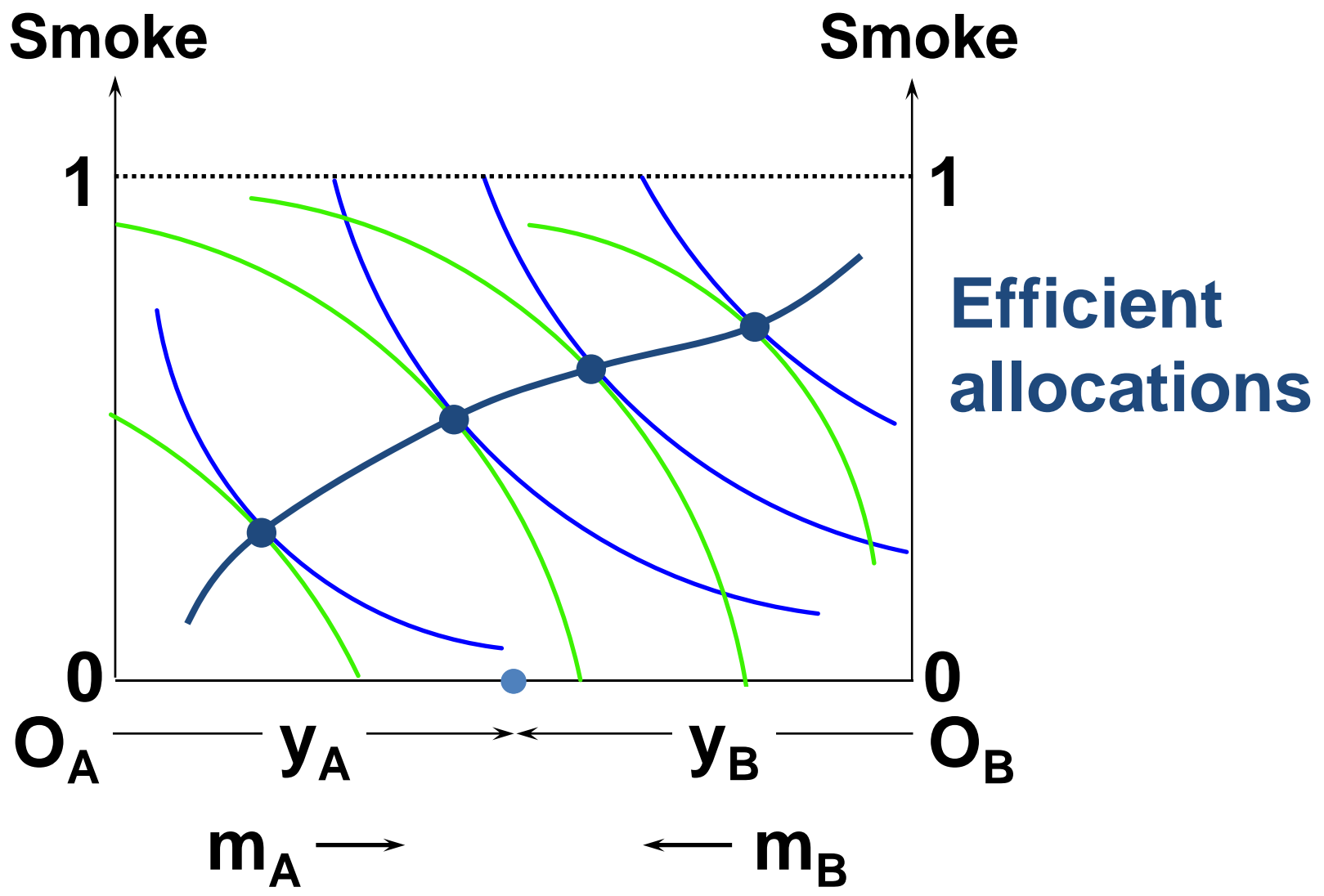
Inefficiency & Negative Externalities



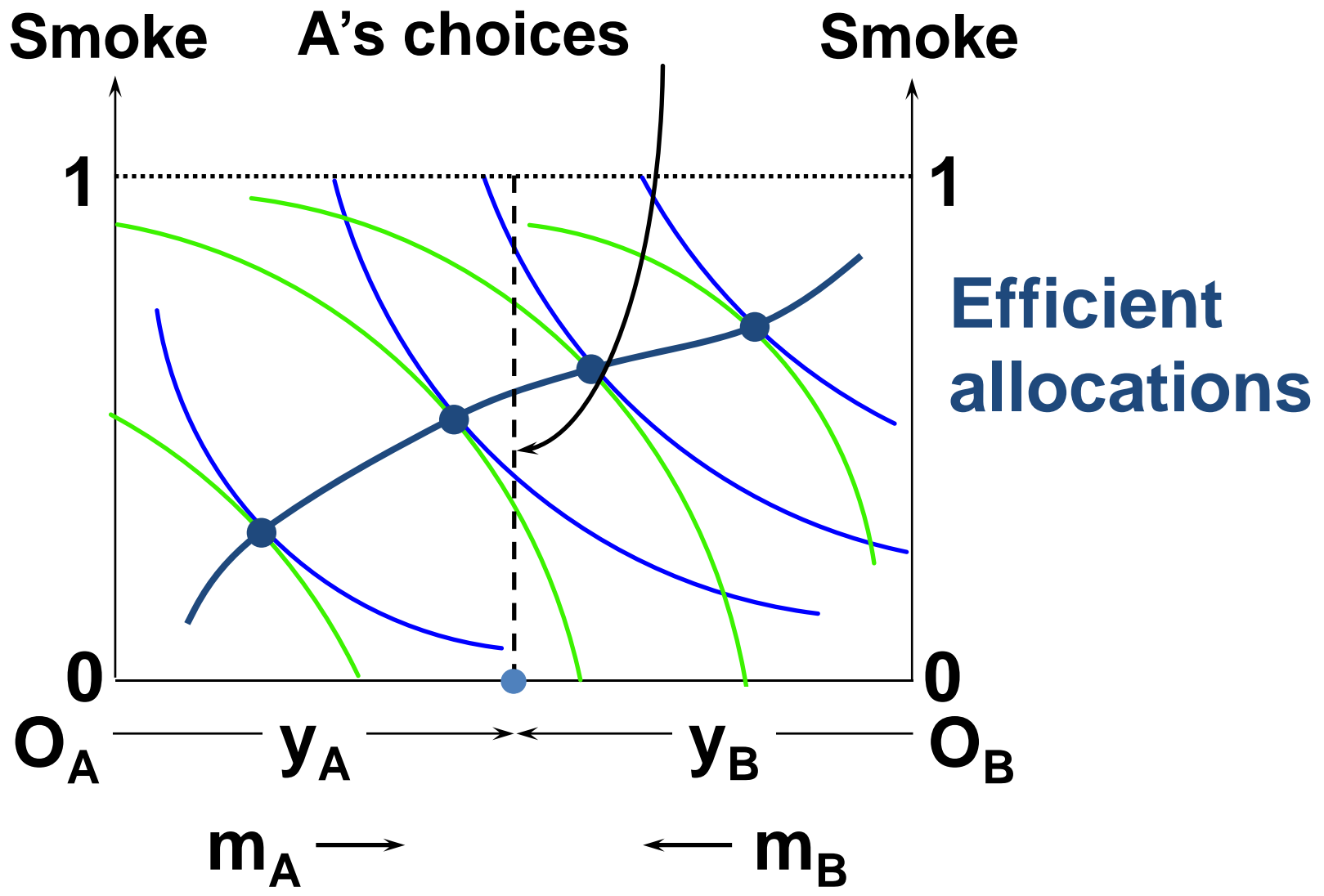
Inefficiency & Negative Externalities

- Suppose there is no means by which money can be exchanged for changes in smoke level.
- What then is Agent A's most preferred allocation?
- Is this allocation efficient?

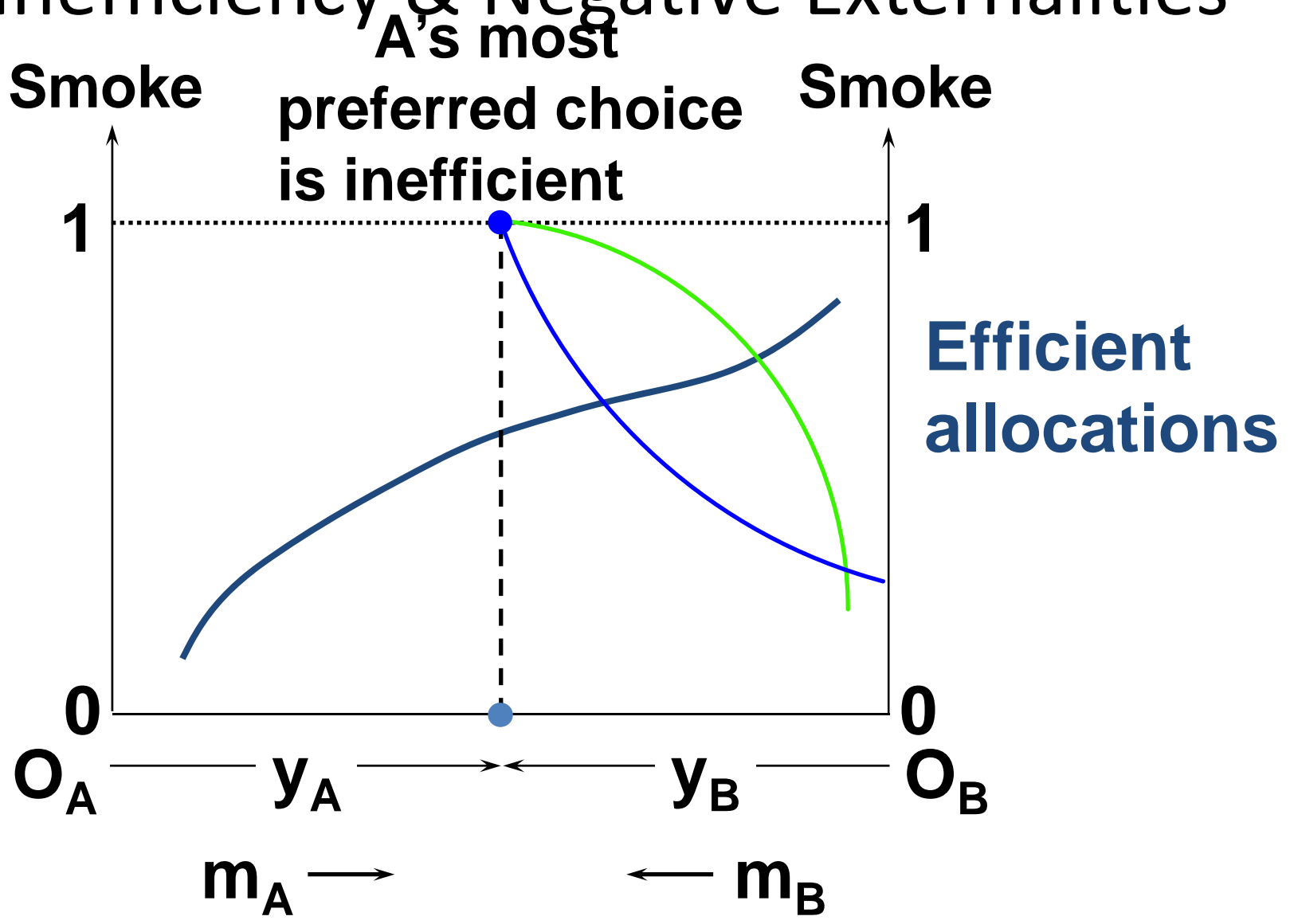
Inefficiency & Negative Externalities



Inefficiency & Negative Externalities



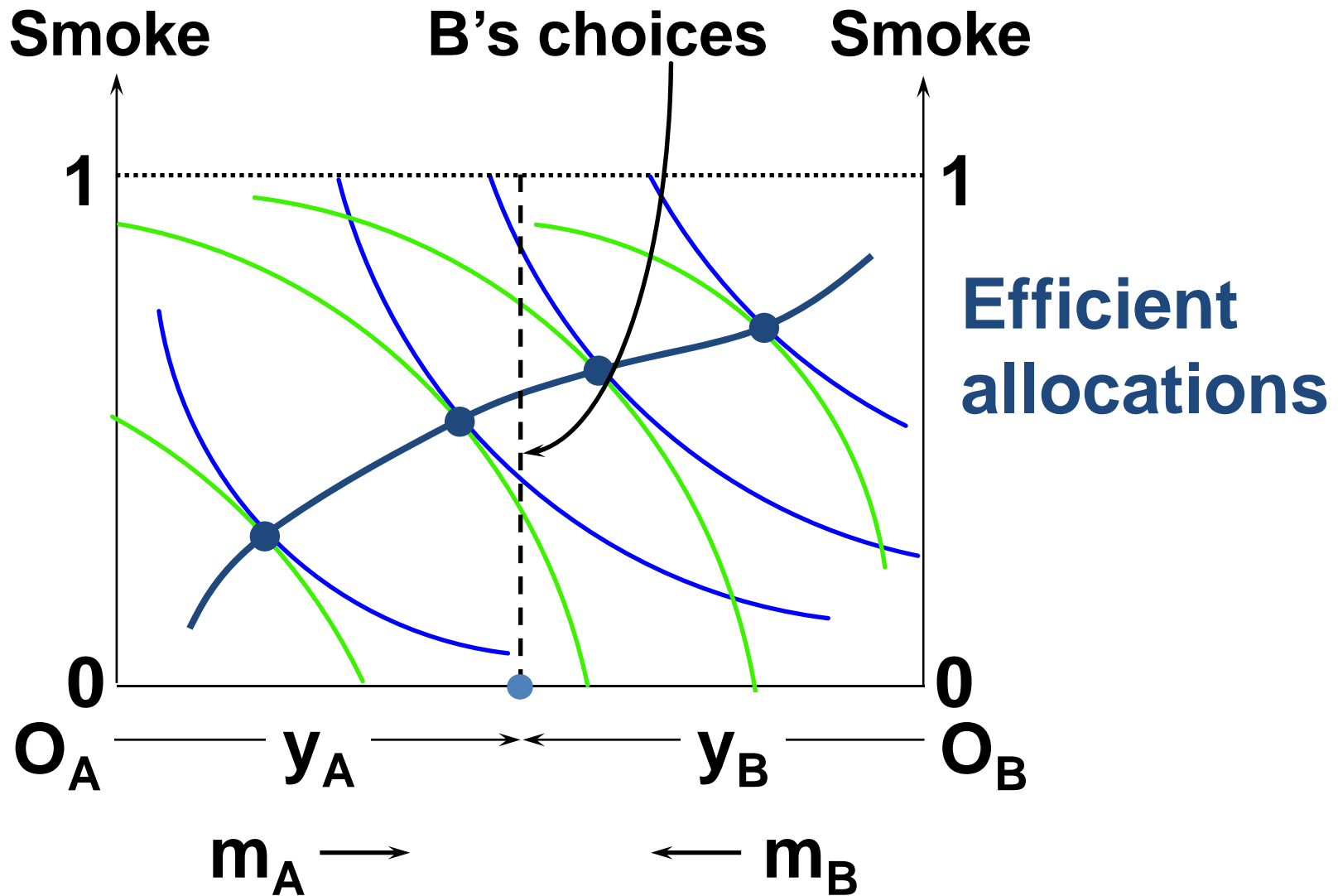
Inefficiency & Negative Externalities



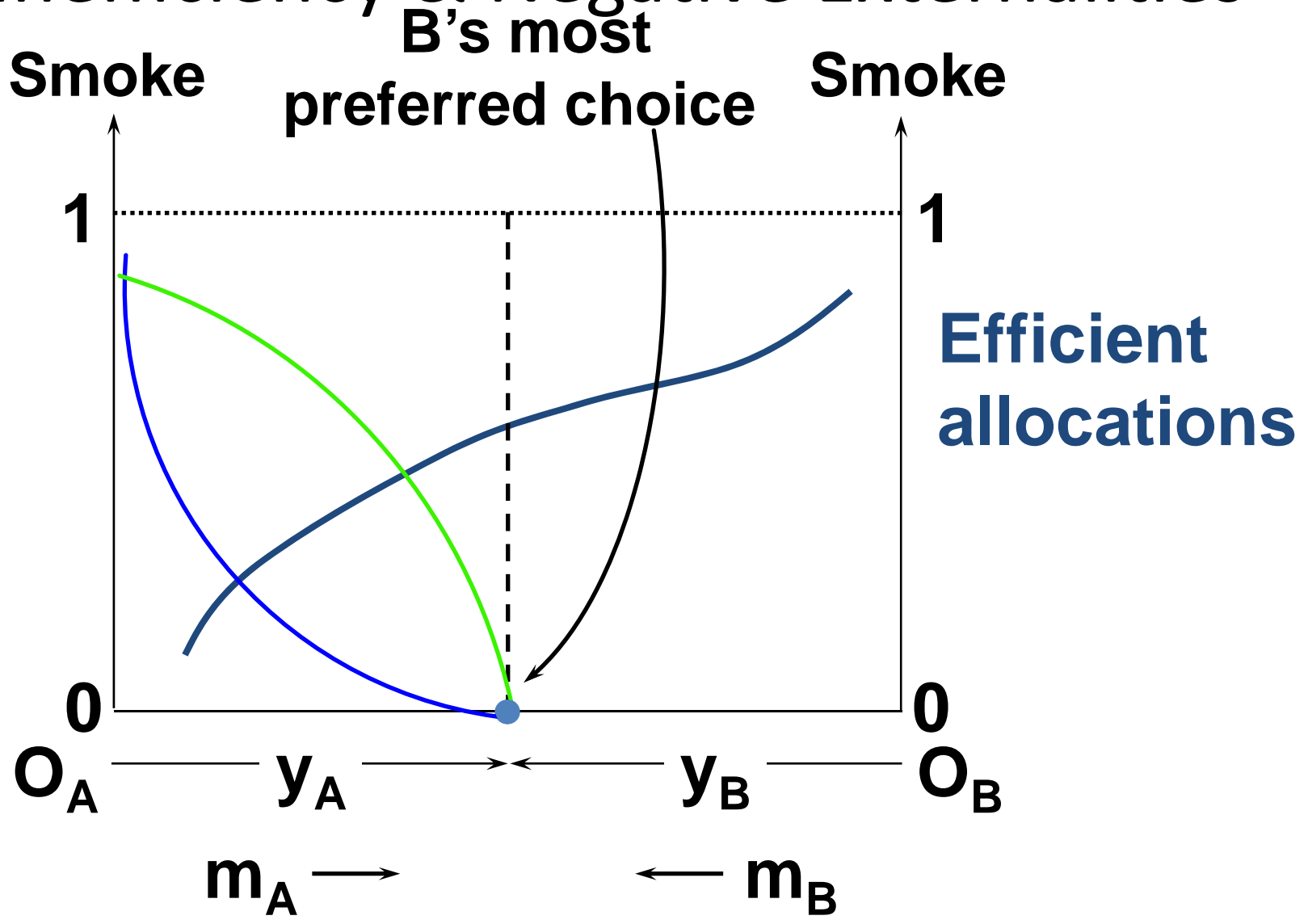
Inefficiency & Negative Externalities

- Continue to suppose there is no means by which money can be exchanged for changes in smoke level.
- What is Agent B's most preferred allocation?
- Is this allocation efficient?

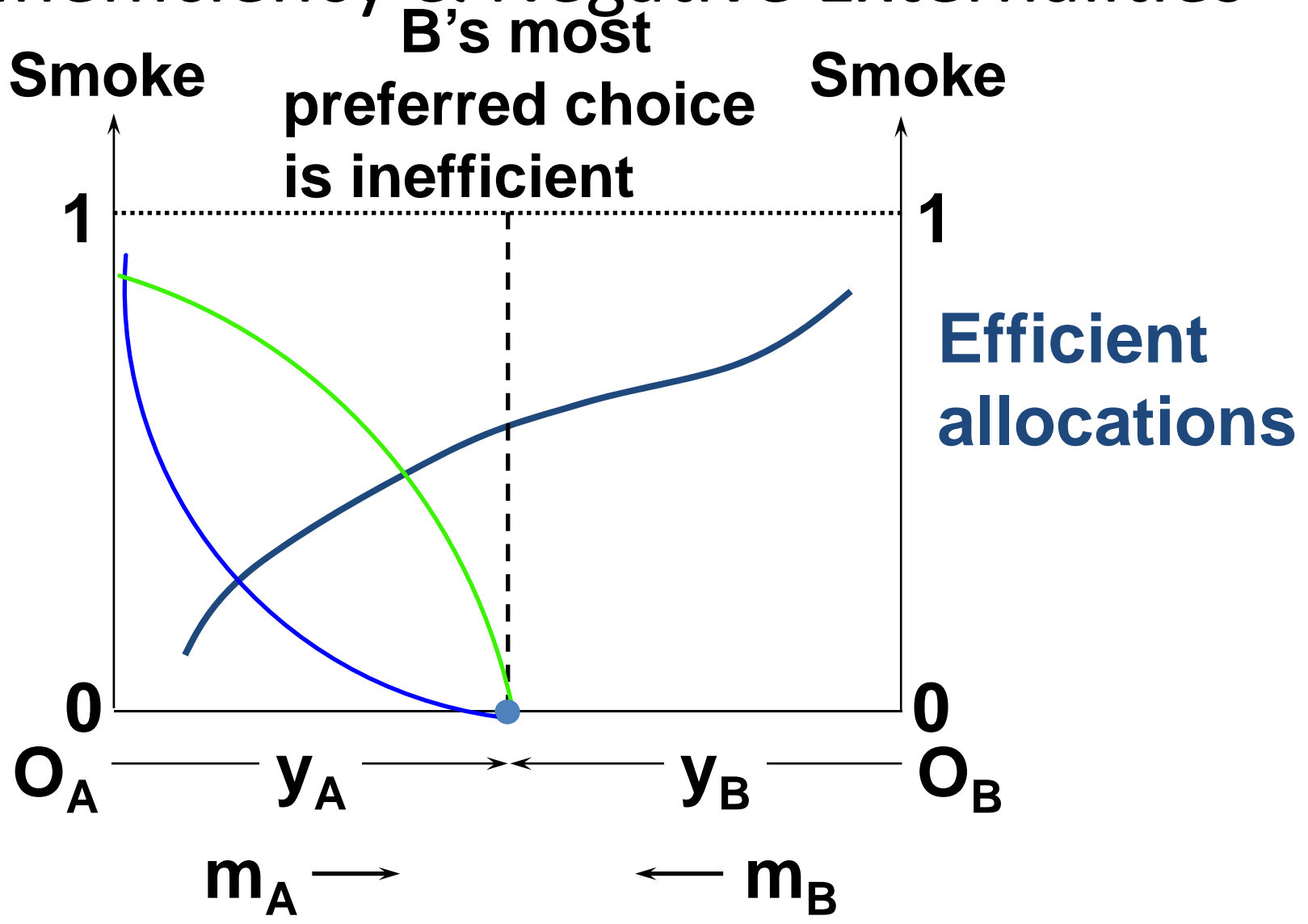
Inefficiency & Negative Externalities



Inefficiency & Negative Externalities



Inefficiency & Negative Externalities



Inefficiency & Negative Externalities

- So if A and B cannot trade money for changes in smoke intensity, then the outcome is inefficient.
- Either there is too much smoke (A's most preferred choice) or there is too little smoke (B's choice).

Externalities and Property Rights

- Ronald Coase's insight is that most externality problems are due to an inadequate specification of property rights and, consequently, an absence of markets in which trade can be used to internalize external costs or benefits.

Externalities and Property Rights

- Causing a producer of an externality to bear the full external cost or to enjoy the full external benefit is called **internalizing the externality**.

Externalities and Property Rights

- Neither Agent A nor Agent B owns the air in their room.
- What happens if this property right is created and is assigned to one of them?

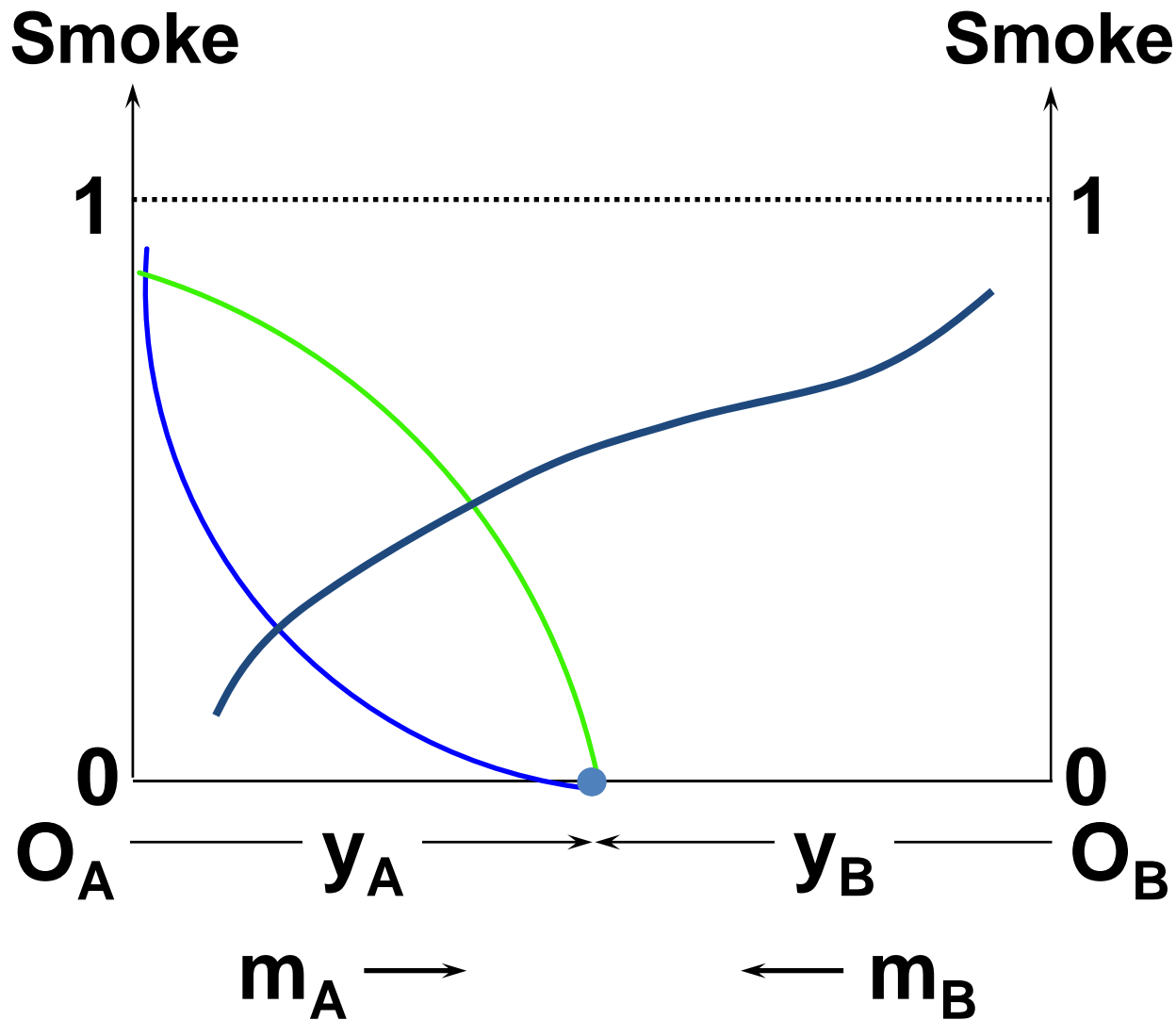
Externalities and Property Rights

- Suppose Agent B is assigned ownership of the air in the room.
- Agent B can now sell “rights to smoke”.
- Will there be any smoking?
- If so, how much smoking and what will be the price for this amount of smoke?

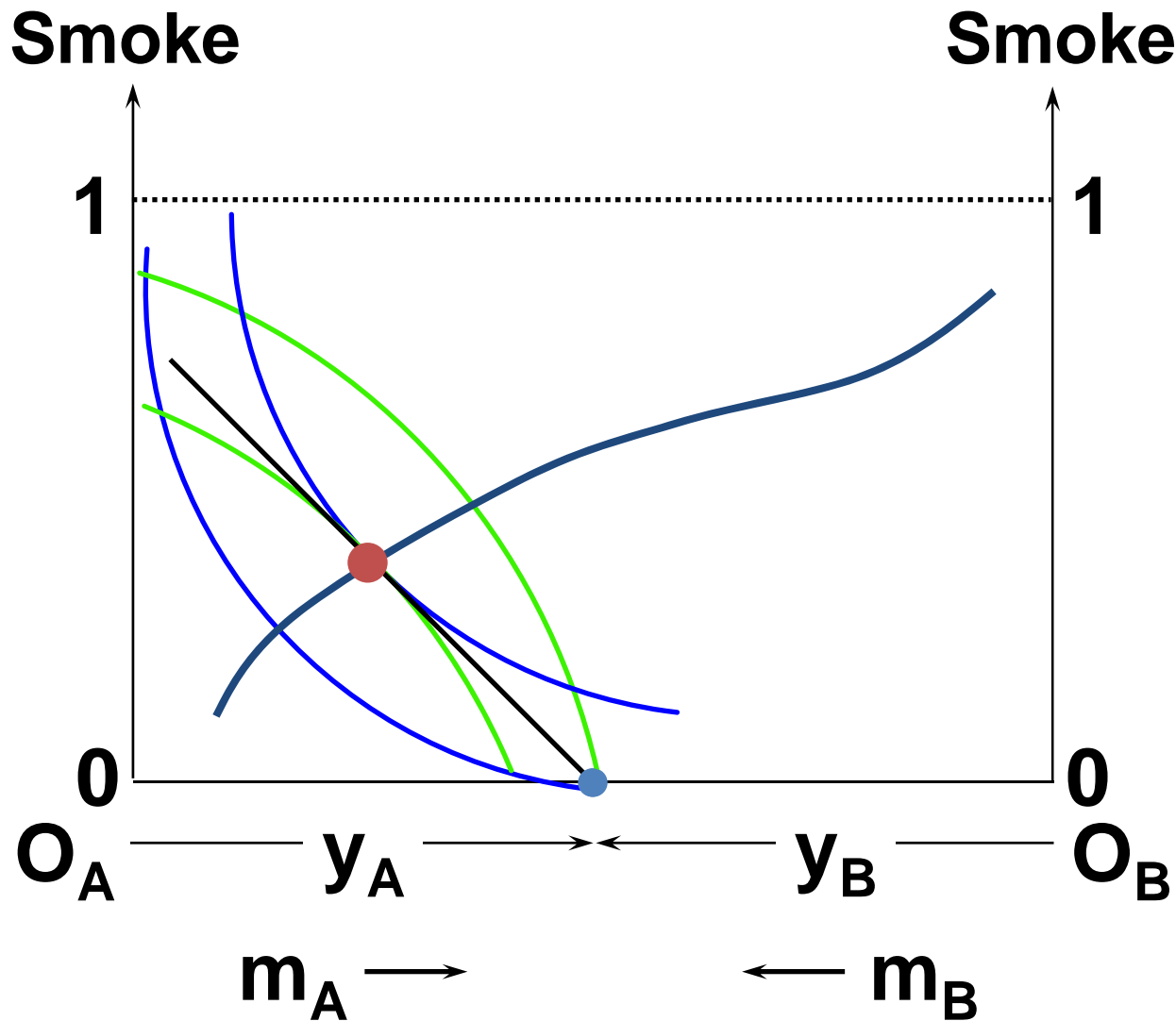
Externalities and Property Rights

- Let $p(s_A)$ be the price paid by Agent A to Agent B in order to create a smoke intensity of s_A .

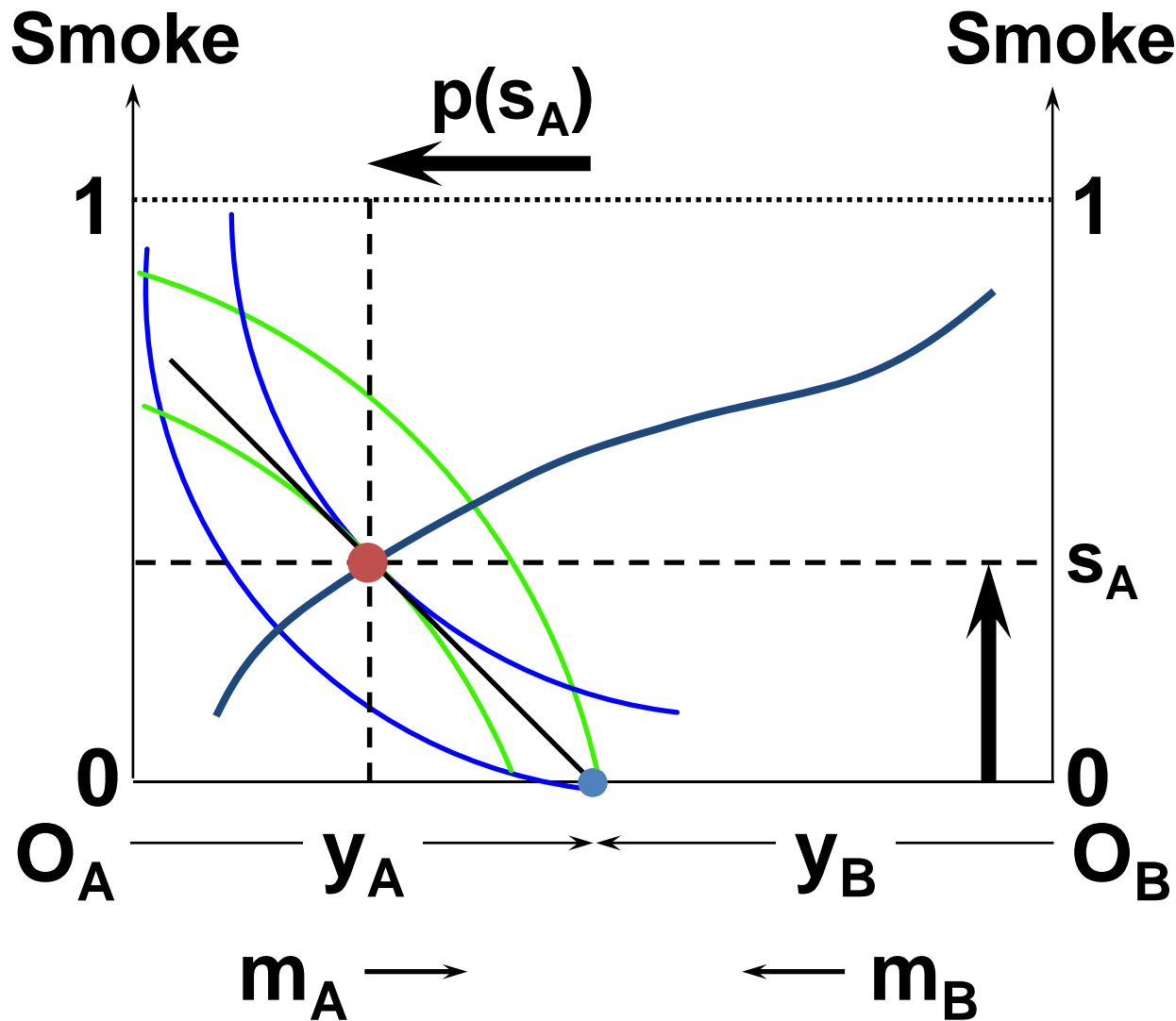
Externalities and Property Rights



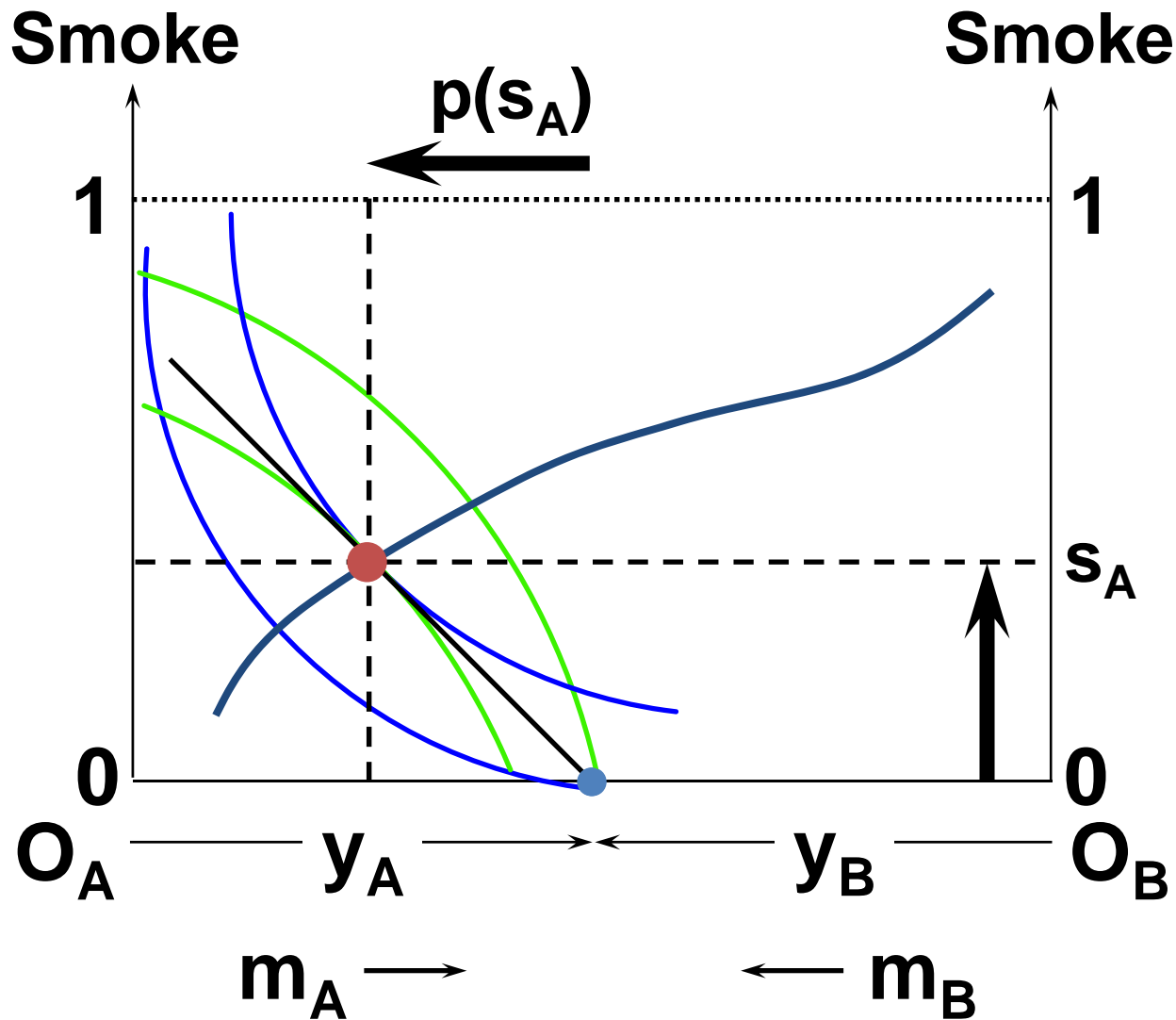
Externalities and Property Rights



Externalities and Property Rights

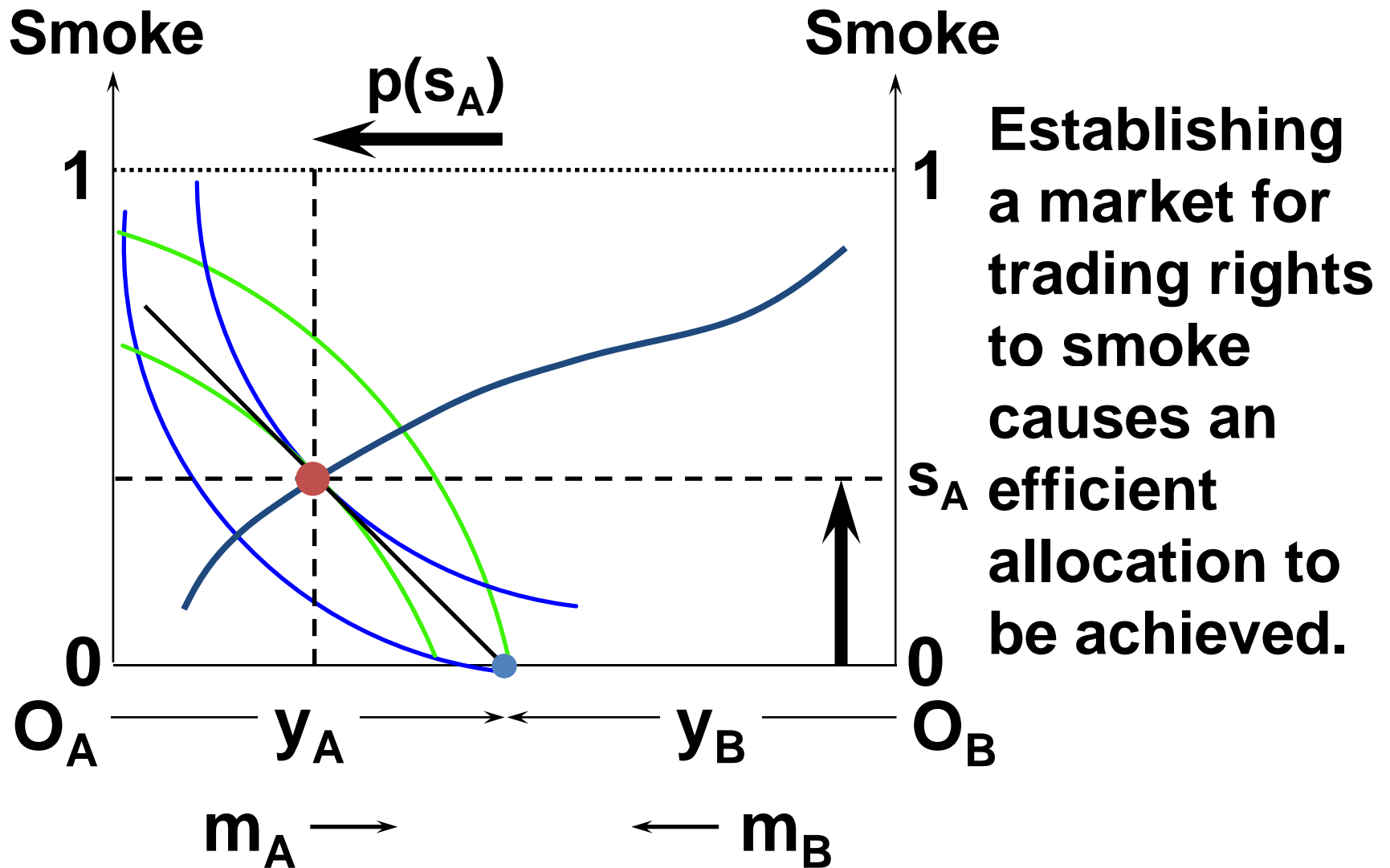


Externalities and Property Rights



**Both agents
gain and
there is a
positive
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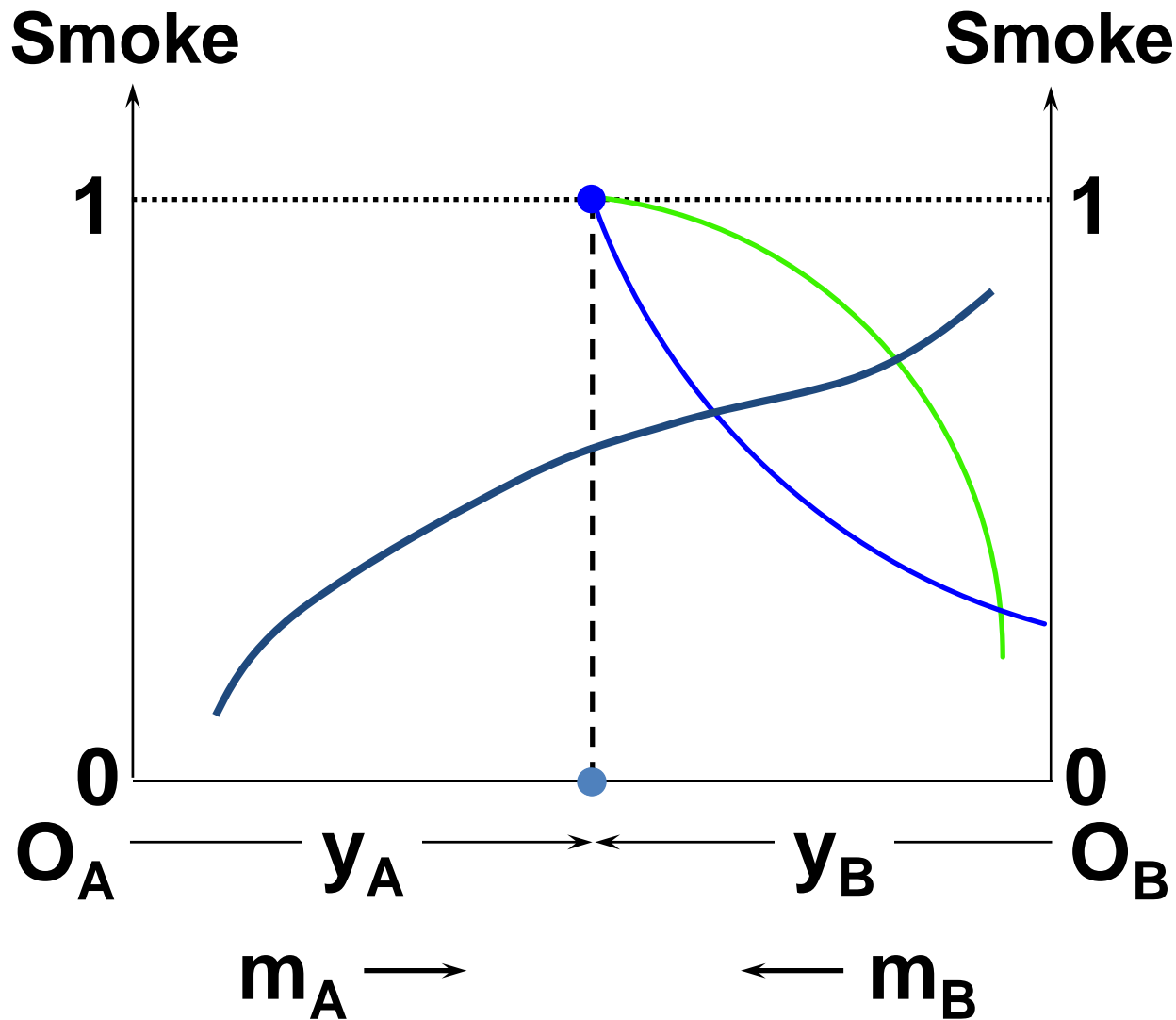
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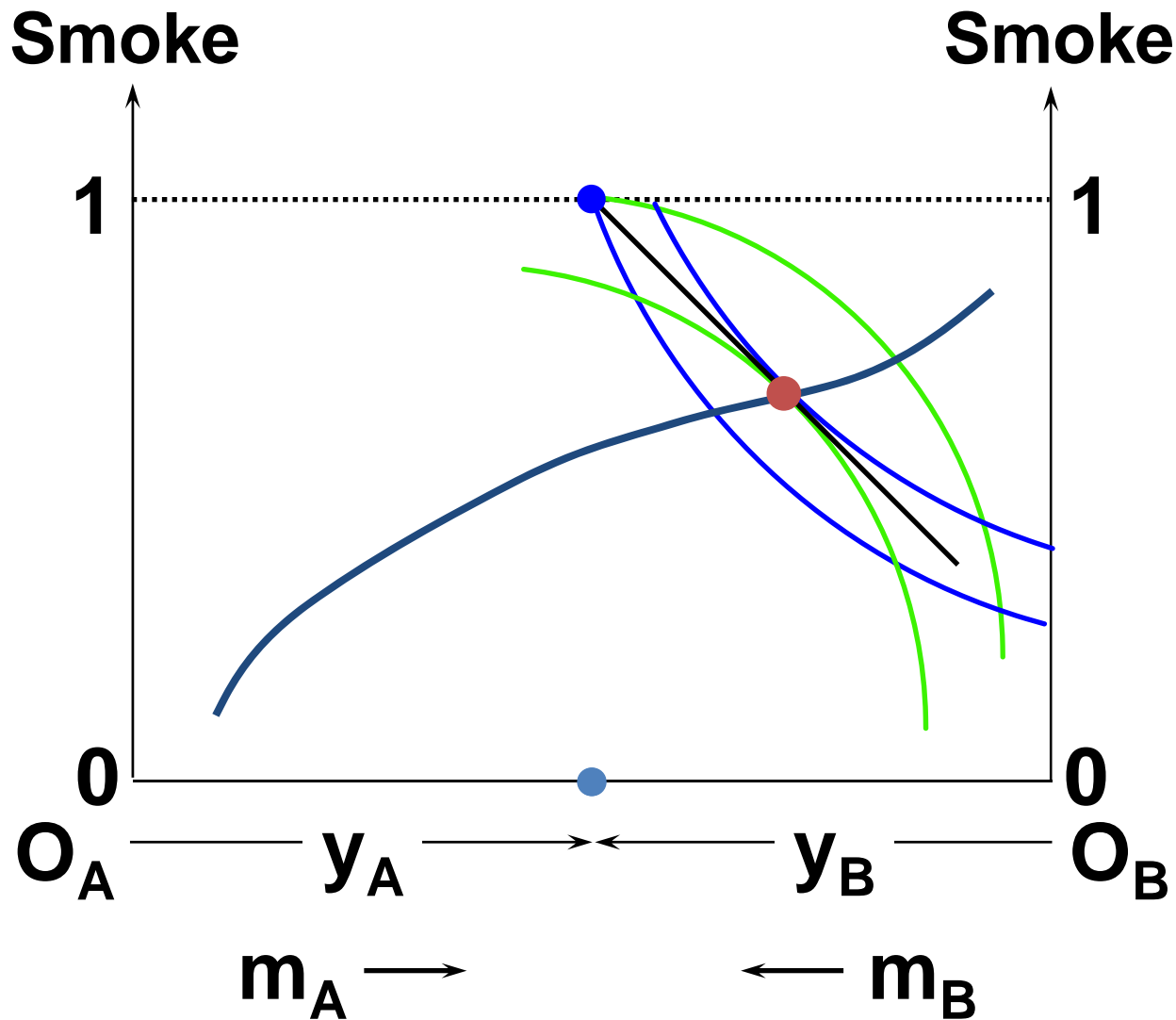
Externalities and Property Rights

- Suppose instead that Agent A is assigned the ownership of the air in the room.
- Agent B can now pay Agent A to reduce the smoke intensity.
- How much smoking will there be?
- How much money will Agent B pay to Agent A?

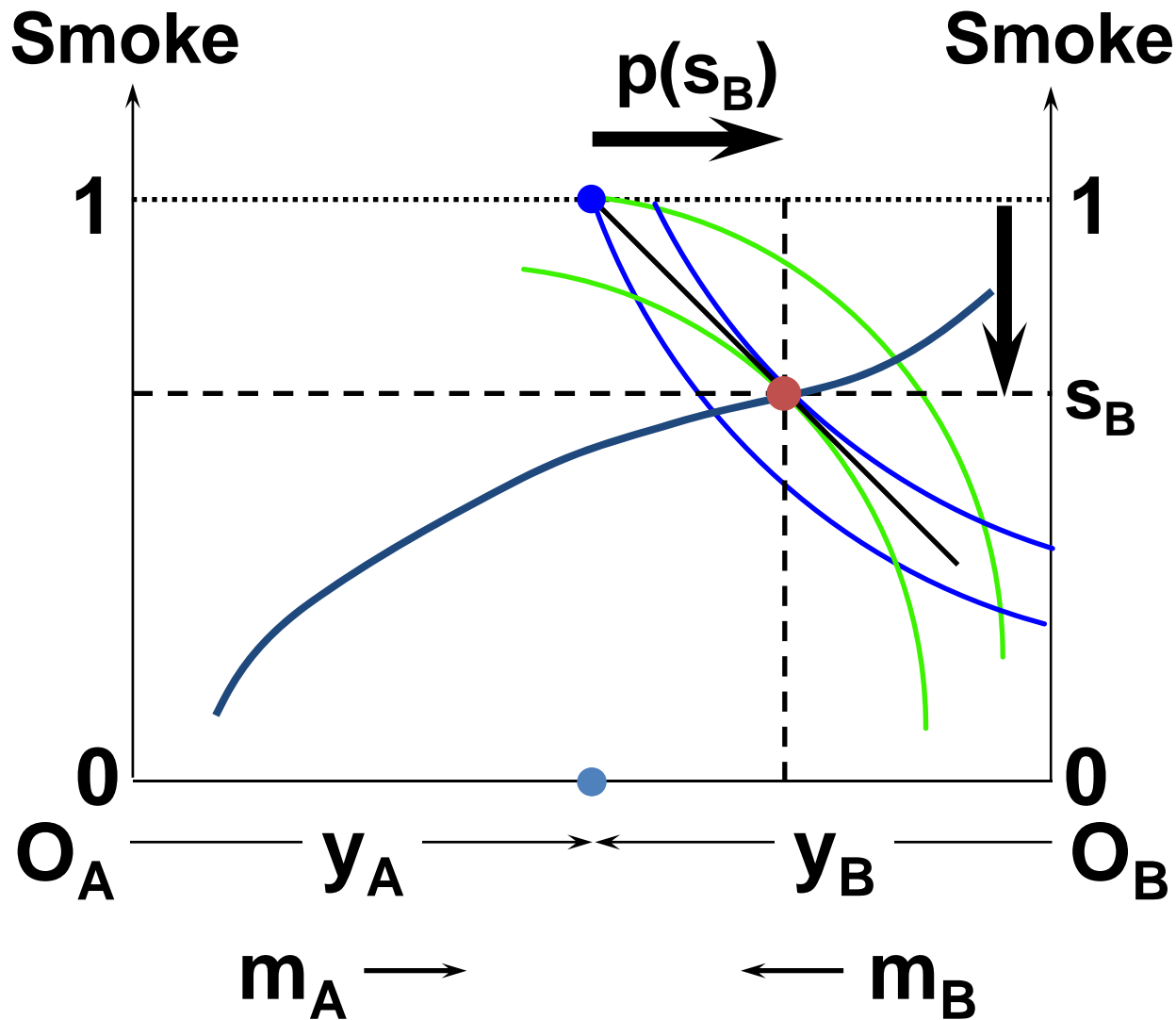
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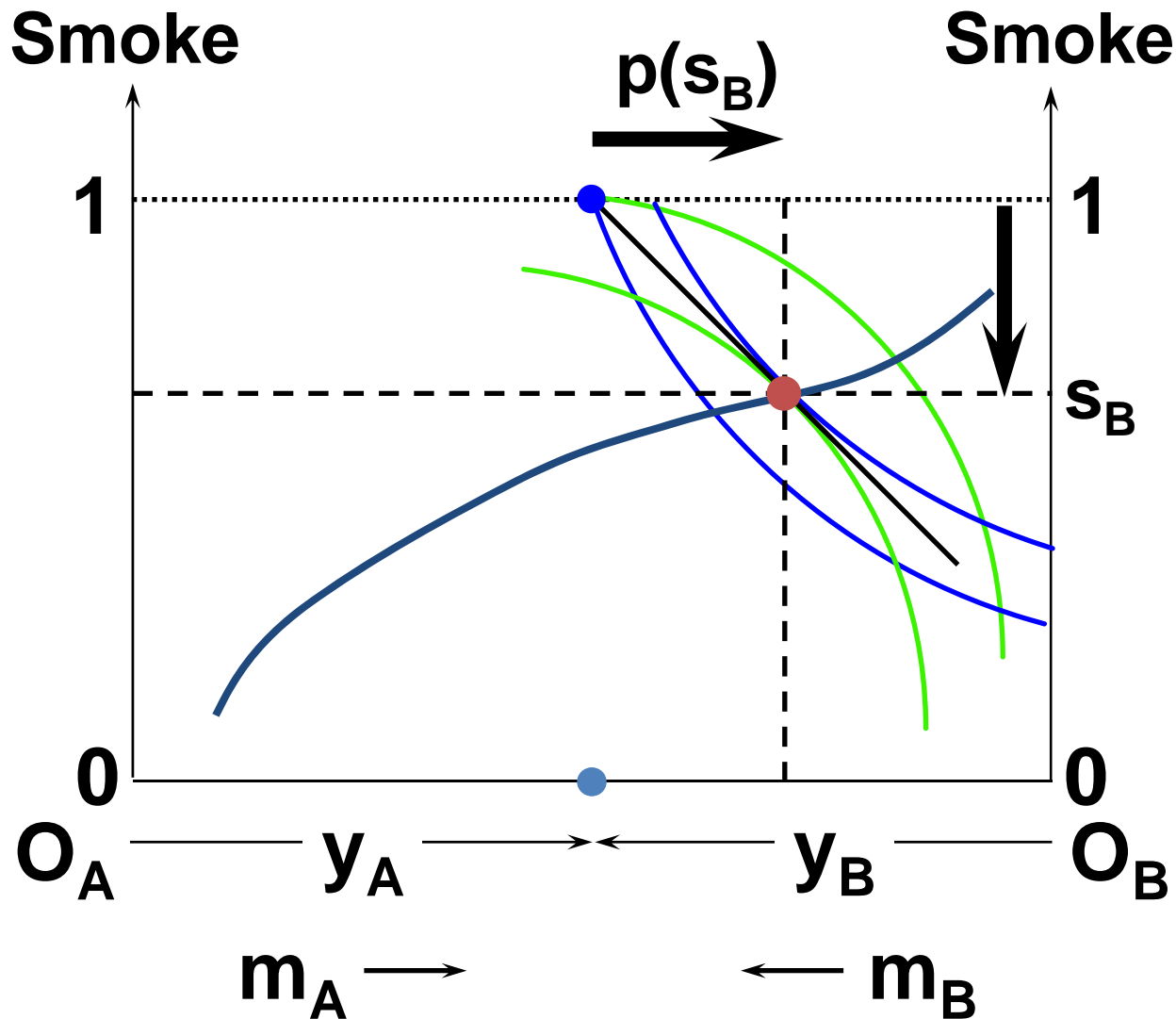
Externalities and Property Rights



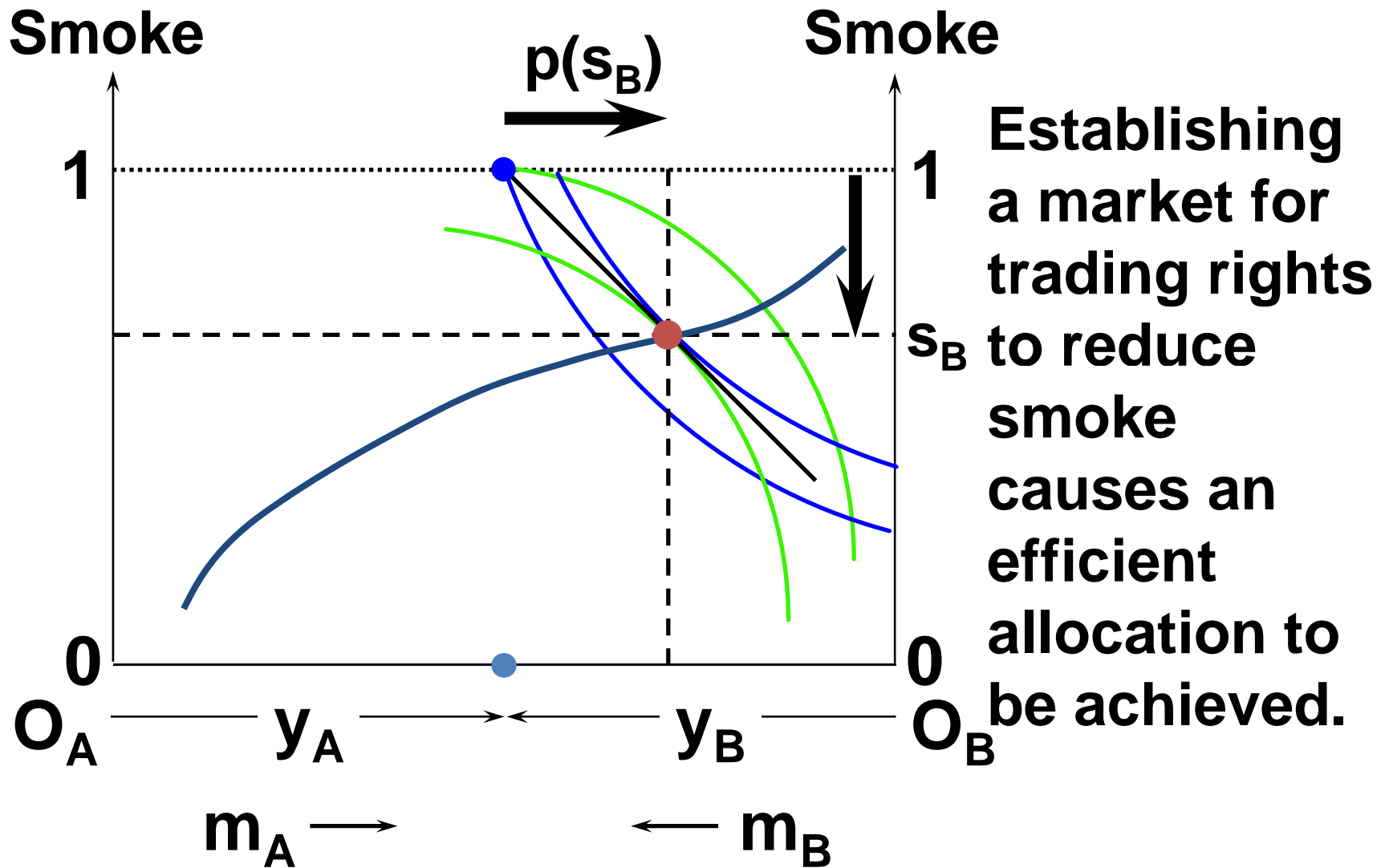
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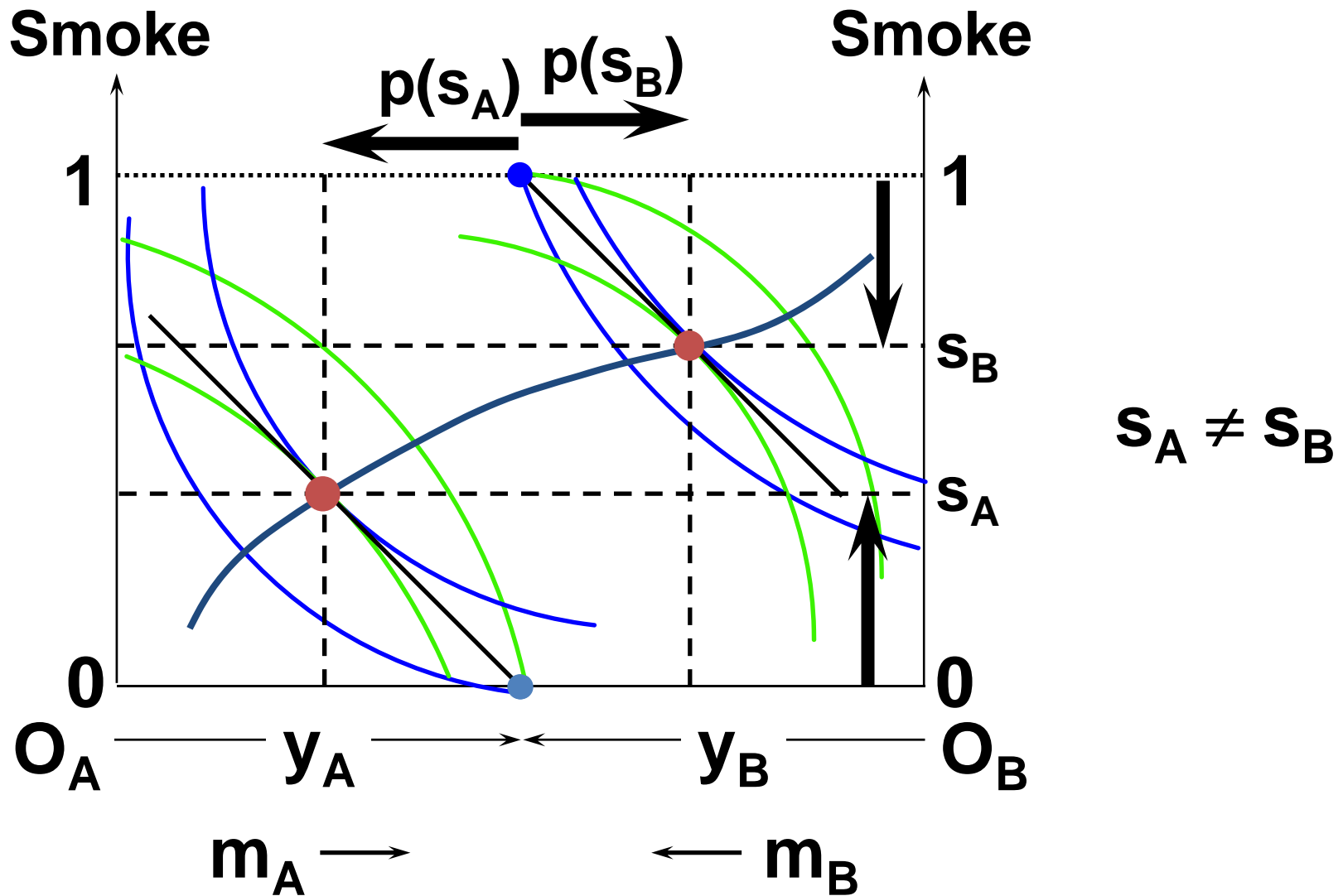
Externalities and Property Rights



Externalities and Property Rights

- Notice that the
 - agent given the property right (asset) is better off than at her own most preferred allocation in the absence of the property right.
 - amount of smoking that occurs in equilibrium depends upon which agent is assigned the property right.

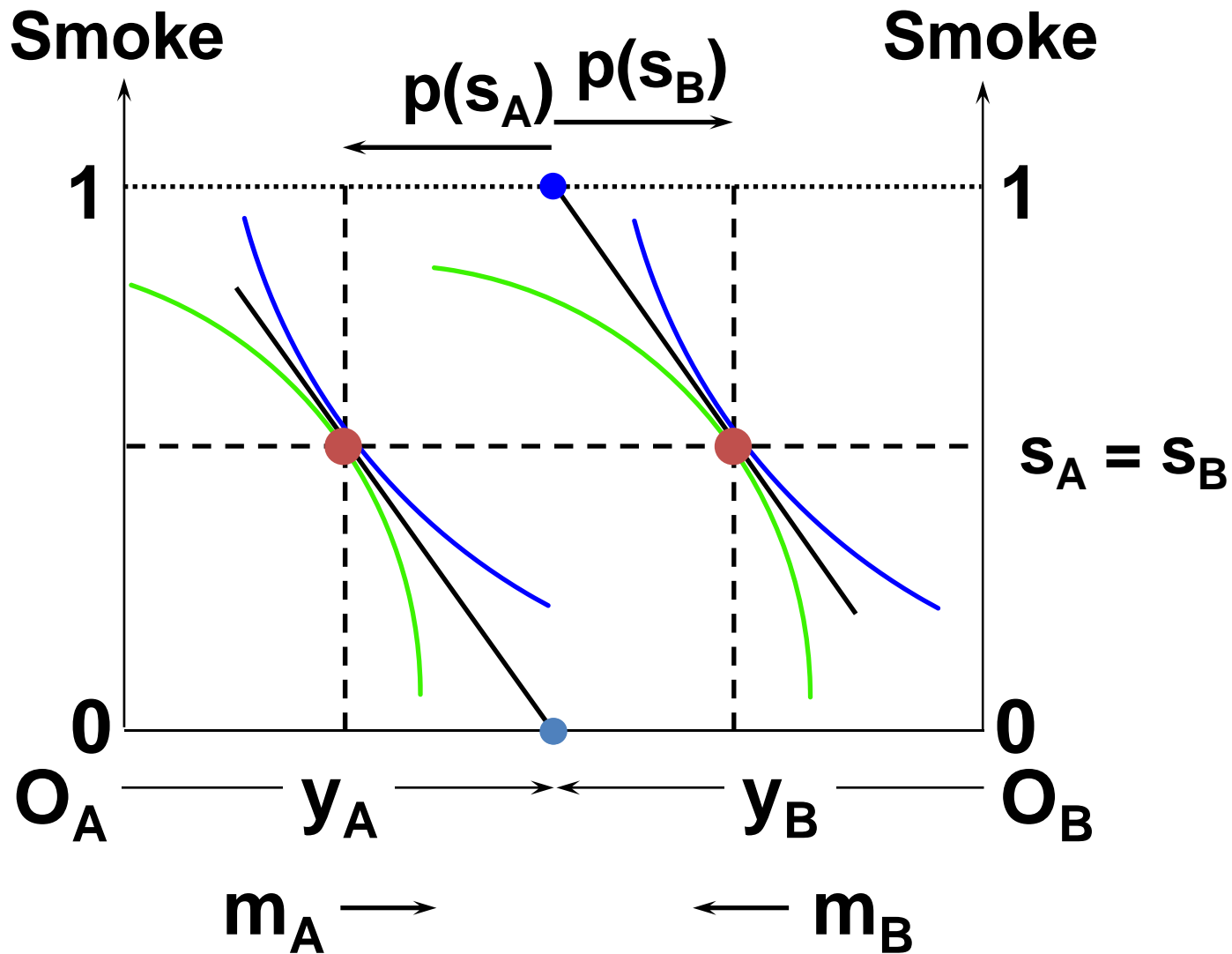
Externalities and Property Rights



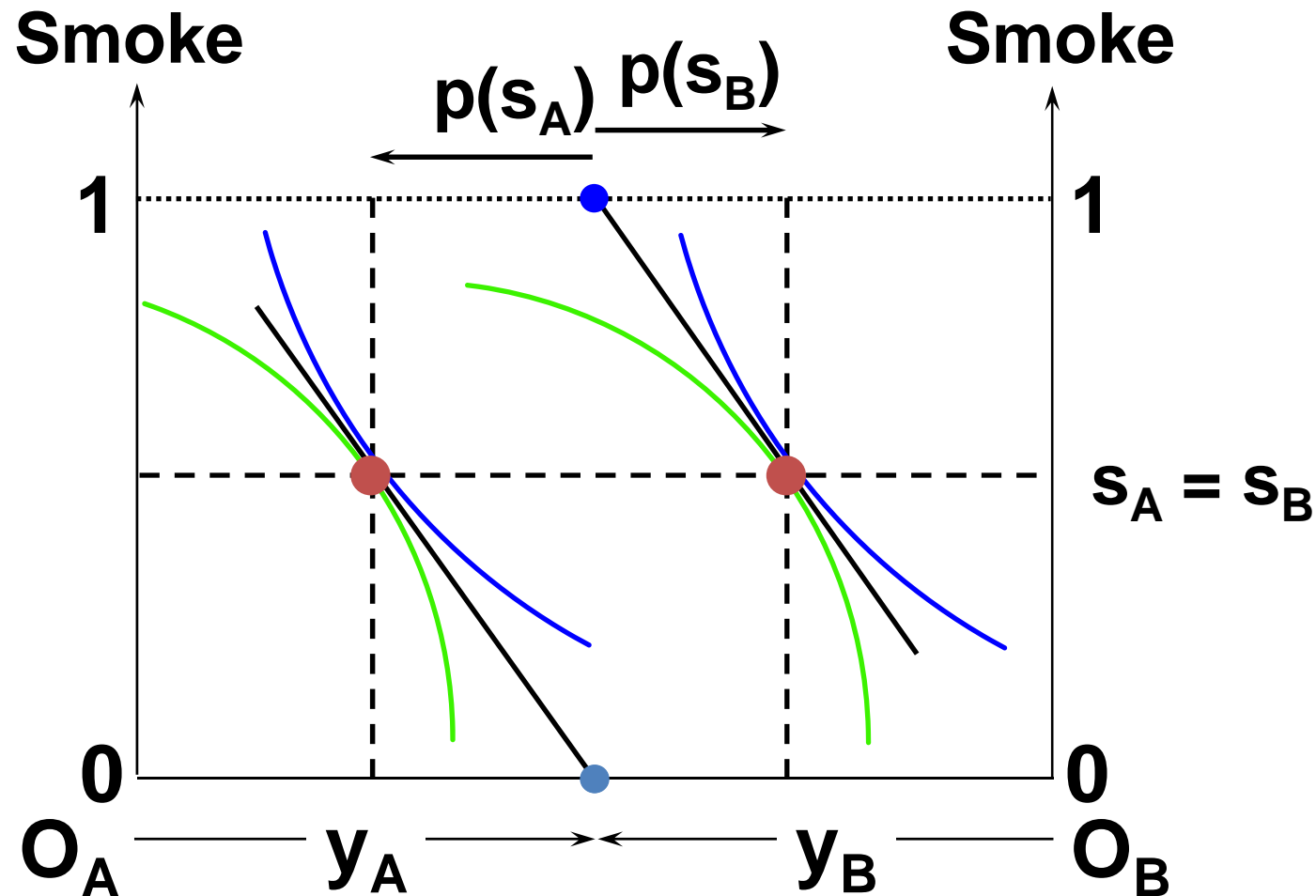
Externalities and Property Rights

- Is there a case in which the same amount of smoking occurs in equilibrium no matter which agent is assigned ownership of the air in the room?

Externalities and Property Rights

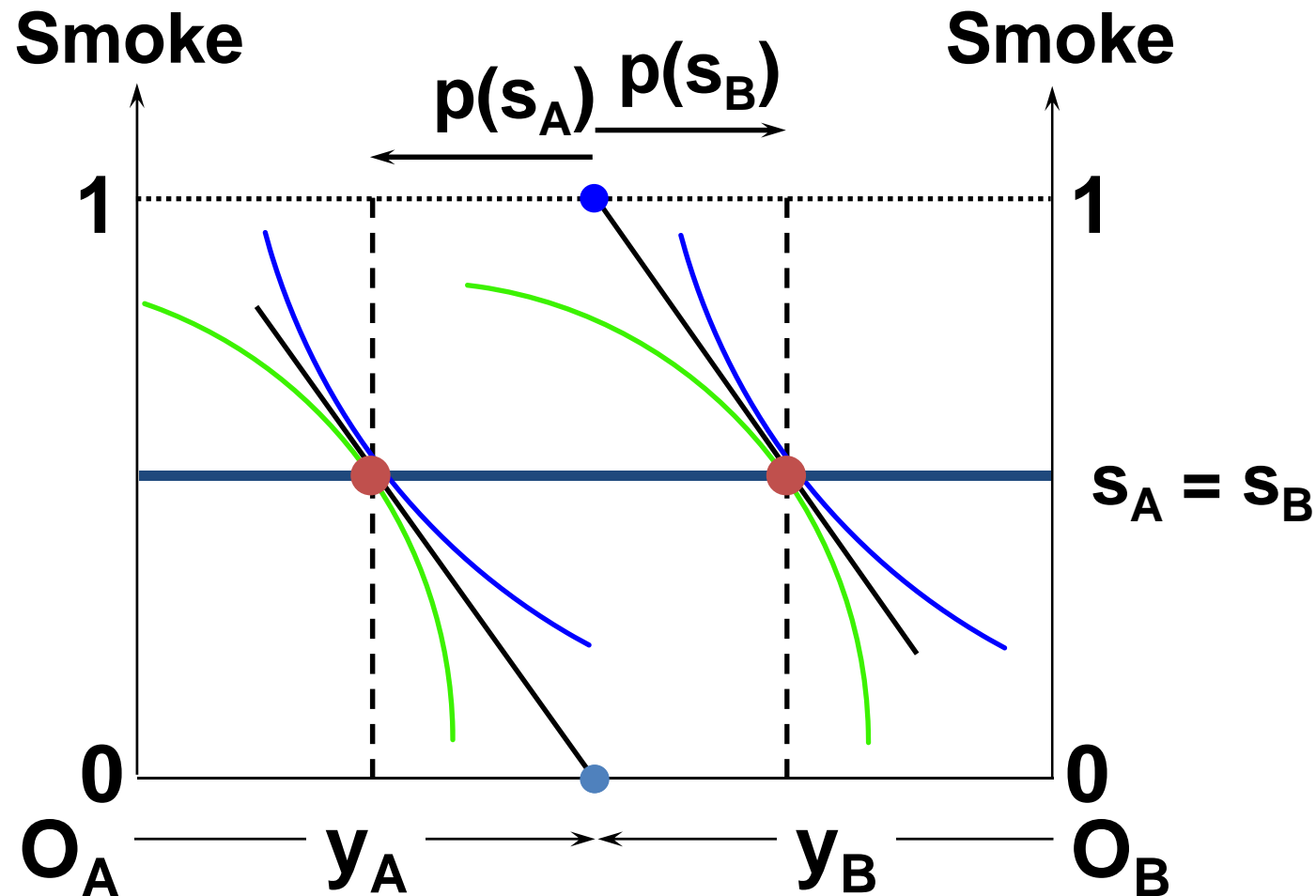


Externalities and Property Rights



For both agents, the MRS is constant as money changes, for given smoke intensity.

Externalities and Property Rights



So, for both agents, preferences must be quasilinear in money; $U(m,s) = m + f(s)$.

Coase's Theorem

- Coase's Theorem is: If all agents' preferences are quasilinear in money, then the efficient level of the externality generating commodity is produced no matter which agent is assigned the property right.

Production Externalities

- A steel mill produces **jointly** steel and pollution.
- The pollution adversely affects a nearby fishery.
- Both firms are price-takers.
- p_S is the market price of steel.
- p_F is the market price of fish.

Production Externalities

- $c_s(s,x)$ is the steel firm's cost of producing s units of steel jointly with x units of pollution.
- If the steel firm does not face any of the external costs of its pollution production then its profit function is

$$\Pi_s(s, x) = p_s s - c_s(s, x)$$

and the firm's problem is to

Production Externalities

$$\max_{\mathbf{s}, \mathbf{x}} \Pi_s(\mathbf{s}, \mathbf{x}) = \mathbf{p}_s \mathbf{s} - \mathbf{c}_s(\mathbf{s}, \mathbf{x}).$$

The first-order profit-maximization conditions are

Production Externalities

$$\max_{s, \mathbf{x}} \Pi_s(s, \mathbf{x}) = p_s s - c_s(s, \mathbf{x}).$$

The first-order profit-maximization conditions are

$$p_s = \frac{\partial c_s(s, \mathbf{x})}{\partial s} \quad \text{and} \quad \mathbf{0} = \frac{\partial c_s(s, \mathbf{x})}{\partial \mathbf{x}}.$$

Production Externalities

$p_s = \frac{\partial c_s(s, \mathbf{x})}{\partial s}$ states that the steel firm

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Production Externalities

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should produce the output level of steel for which price = marginal production cost.

$\frac{\partial c_s(s, \mathbf{x})}{\partial \mathbf{x}}$ is the rate at which the firm's

internal production cost goes down as the pollution level rises, so

$-\frac{\partial c_s(s, \mathbf{x})}{\partial \mathbf{x}}$ is the marginal cost to the firm of pollution **reduction**.

Production Externalities

$-\frac{\partial c_s(s, \mathbf{x})}{\partial \mathbf{x}}$ is the marginal cost to the firm of pollution reduction.

What is the marginal benefit to the steel firm from reducing pollution?

Production Externalities

$-\frac{\partial c_s(s, \mathbf{x})}{\partial \mathbf{x}}$ is the marginal cost to the firm of pollution reduction.

What is the marginal benefit to the steel firm from reducing pollution?

Zero, since the firm does not face its external cost.

Hence the steel firm chooses the pollution level for which $-\frac{\partial c_s(s, \mathbf{x})}{\partial \mathbf{x}} = 0$.

Production Externalities

E.g. suppose $c_s(s,x) = s^2 + (x - 4)^2$ and $p_s = 12$. Then

$$\Pi_s(s,x) = 12s - s^2 - (x - 4)^2$$

and the first-order profit-maximization conditions are

$$12 = 2s \quad \text{and} \quad 0 = -2(x - 4).$$

Production Externalities

$p_s = 12 = 2s$, determines the profit-max.
output level of steel; $s^* = 6$.

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$-2(x - 4)$ is the marginal cost to the firm
from pollution reduction. Since it gets
no benefit from this it sets $x^* = 4$.

The steel firm's maximum profit level is

$$\begin{aligned}\text{thus } \Pi_s(s^*, x^*) &= 12s^* - s^{*2} - (x^* - 4)^2 \\ &= 12 \times 6 - 6^2 - (4 - 4)^2 \\ &= \$36.\end{aligned}$$

Production Externalities

- The cost to the fishery of catching f units of fish when the steel mill emits x units of pollution is $c_F(f,x)$. Given f , $c_F(f,x)$ increases with x ; i.e. the steel firm inflicts a negative externality on the fishery.

Production Externalities

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- The fishery's profit function is

$$\Pi_F(\mathbf{f};\mathbf{x}) = \mathbf{p}_F\mathbf{f} - \mathbf{c}_F(\mathbf{f};\mathbf{x})$$

so the fishery's problem is to

Production Externalities

$$\max_{\mathbf{f}} \Pi_{\mathbf{F}}(\mathbf{f}; \mathbf{x}) = \mathbf{p}_{\mathbf{F}} \mathbf{f} - \mathbf{c}_{\mathbf{F}}(\mathbf{f}; \mathbf{x}).$$

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Production Externalities

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Production Externalities

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The first-order profit-maximization condition is

$$\mathbf{p}_{\mathbf{F}} = \frac{\partial \mathbf{c}_{\mathbf{F}}(\mathbf{f}; \mathbf{x})}{\partial \mathbf{f}}.$$

Higher pollution raises the fishery's marginal production cost and lowers both its output level and its profit. This is the external cost of the pollution.

Production Externalities

E.g. suppose $c_F(f;x) = f^2 + xf$ and $p_F = 10$.

The external cost inflicted on the fishery by the steel firm is xf . Since the fishery has no control over x it must take the steel firm's choice of x as a given. The fishery's profit function is thus

$$\Pi_F(f;x) = 10f - f^2 - xf$$

Production Externalities

$$\Pi_F(f; \mathbf{x}) = 10f - f^2 - \mathbf{x}f$$

Given \mathbf{x} , the first-order profit-maximization condition is $10 = 2f + \mathbf{x}$.

Production Externalities

$$\Pi_F(f; x) = 10f - f^2 - xf$$

Given x , the first-order profit-maximization condition is $10 = 2f + x$.

So, given a pollution level x inflicted upon it, the fishery's profit-maximizing output level is $f^* = 5 - \frac{x}{2}$.

Production Externalities

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So, given a pollution level x inflicted upon it, the fishery's profit-maximizing output level is $f^* = 5 - \frac{x}{2}$.

Notice that the fishery produces less, and earns less profit, as the steel firm's pollution level increases.

Production Externalities

$f^* = 5 - \frac{x}{2}$. The steel firm, ignoring its external cost inflicted upon the fishery, chooses $x^* = 4$, so the fishery's profit-maximizing output level given the steel firm's choice of pollution level is $f^* = 3$, giving the fishery a maximum profit level of

$$\begin{aligned}\Pi_F(f^*; x) &= 10f^* - f^{*2} - xf^* \\ &= 10 \times 3 - 3^2 - 4 \times 3 = \$9.\end{aligned}$$

Notice that the external cost is \$12.

Production Externalities

- Are these choices by the two firms efficient?
- When the steel firm ignores the external costs of its choices, the sum of the two firm's profits is $\$36 + \$9 = \$45$.
- Is $\$45$ the largest possible total profit that can be achieved?

Merger and Internalization

- Suppose the two firms merge to become one. What is the highest profit this new firm can achieve?

Merger and Internalization

- Suppose the two firms merge to become one. What is the highest profit this new firm can achieve?

$$\Pi^m(s, f, x) = 12s + 10f - s^2 - (x - 4)^2 - f^2 - xf.$$

- What choices of s , f and x maximize the new firm's profit?

Merger and Internalization

$$\Pi^m(s, f, x) = 12s + 10f - s^2 - (x - 4)^2 - f^2 - xf.$$

The first-order profit-maximization conditions are

$$\frac{\partial \Pi^m}{\partial s} = 12 - 2s = 0$$

$$\frac{\partial \Pi^m}{\partial f} = 10 - 2f - x = 0.$$

$$\frac{\partial \Pi^m}{\partial x} = -2(x - 4) - f = 0.$$

The solution is

$$s^m = 6$$

$$f^m = 4$$

$$x^m = 2.$$

Merger and Internalization
And the merged firm's maximum profit level is

$$\begin{aligned}\Pi^m(s^m, f^m, x^m) &= 12s^m + 10f^m - s^{m2} - (x^m - 4)^2 - f^{m2} - x^m f^m \\ &= 12 \times 6 + 10 \times 4 - 6^2 - (2 - 4)^2 - 4^2 - 2 \times 4 \\ &= \$48.\end{aligned}$$

This exceeds \$45, the sum of the non-merged firms.

Merger and Internalization

- Merger has improved efficiency.
- On its own, the steel firm produced $x^* = 4$ units of pollution.
- Within the merged firm, pollution production is only $x^m = 2$ units.
- So merger has caused both an improvement in efficiency and less pollution production.
Why?

Merger and Internalization

The steel firm's profit function is

$$\Pi_s(s, x) = 12s - s^2 - (x - 4)^2$$

so the marginal cost of producing x units of pollution is $MC_s(x) = 2(x - 4)$

When it does not have to face the external costs of its pollution, the steel firm increases pollution until this marginal cost is zero; hence $x^* = 4$.

Merger and Internalization

In the merged firm the profit function is

$$\Pi^m(s, f, x) = 12s + 10f - s^2 - (x - 4)^2 - f^2 - xf.$$

The marginal cost of pollution is thus

$$MC^m(x) = 2(x - 4) + f$$

Merger and Internalization

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The marginal cost of pollution is

$$MC^m(x) = 2(x - 4) + f > 2(x - 4) = MC_s(x).$$

Merger and Internalization

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The marginal cost of pollution is

$$MC^m(x) = 2(x - 4) + f > 2(x - 4) = MC_s(x).$$

The merged firm's marginal pollution cost is larger because it faces the full cost of its own pollution through increased costs of production in the fishery, so less pollution is produced by the merged firm.

Merger and Internalization

- But why is the merged firm's pollution level of $x^m = 2$ efficient?

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- The steel firm's cost of **reducing** pollution is $-MC^m(x) = 2(x - 4)$.

Merger and Internalization

- But why is the merged firm's pollution level of $x^m = 2$ efficient?
- The external cost inflicted on the fishery is xf , so the marginal external pollution cost is

$$MC_x^E = f.$$

- The steel firm's cost of **reducing** pollution is

$$-MC^m(x) = 2(x - 4).$$

- Efficiency requires

$$MC_x^E = -MC^m(x) \Rightarrow f = 2(x - 4).$$

Merger and Internalization

- Merger therefore internalizes an externality and induces economic efficiency.
- How else might internalization be caused so that efficiency can be achieved?

Coase and Production Externalities

- Coase argues that the externality exists because neither the steel firm nor the fishery owns the water being polluted.
- Suppose the property right to the water is created and assigned to one of the firms. Does this induce efficiency?

Coase and Production Externalities

- Suppose the fishery owns the water.
- Then it can sell pollution rights, in a competitive market, at $\$p_x$ each.
- The fishery's profit function becomes

$$\Pi_F(\mathbf{f}, \mathbf{x}) = p_f \mathbf{f} - \mathbf{f}^2 - \mathbf{x}\mathbf{f} + p_x \mathbf{x}.$$

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$$\Pi_F(\mathbf{f}, \mathbf{x}) = p_f \mathbf{f} - \mathbf{f}^2 - \mathbf{x}\mathbf{f} + p_x \mathbf{x}.$$

- Given p_f and p_x , how many fish and how many rights does the fishery wish to produce? (Notice that x is now a choice variable for the fishery.)

Coase and Production Externalities

$$\Pi_{\mathbf{F}}(\mathbf{f}, \mathbf{x}) = \mathbf{p}_{\mathbf{f}}\mathbf{f} - \mathbf{f}^2 - \mathbf{x}\mathbf{f} + \mathbf{p}_{\mathbf{x}}\mathbf{x}.$$

The profit-maximum conditions are

$$\frac{\partial \Pi_{\mathbf{F}}}{\partial \mathbf{f}} = \mathbf{p}_{\mathbf{f}} - 2\mathbf{f} - \mathbf{x} = \mathbf{0}$$

$$\frac{\partial \Pi_{\mathbf{F}}}{\partial \mathbf{x}} = -\mathbf{f} + \mathbf{p}_{\mathbf{x}} = \mathbf{0}$$

Coase and Production Externalities

$$\Pi_F(f, x) = p_f f - f^2 - xf + p_x x.$$

The profit-maximum conditions are

$$\frac{\partial \Pi_F}{\partial f} = p_f - 2f - x = 0$$

$$\frac{\partial \Pi_F}{\partial x} = -f + p_x = 0$$

and these give $f^* = p_x$ (fish supply)
 $x_S^* = p_f - 2p_x$ (pollution right supply)

Coase and Production Externalities

- The steel firm must buy one right for every unit of pollution it emits so its profit function becomes

$$\Pi_S(s, x) = p_s s - s^2 - (x - 4)^2 - p_x x.$$

- Given p_f and p_x , how much steel does the steel firm want to produce and how many rights does it wish to buy?

Coase and Production Externalities

$$\Pi_S(s, \mathbf{x}) = p_s s - s^2 - (\mathbf{x} - 4)^2 - p_x \mathbf{x}.$$

The profit-maximum conditions are

$$\frac{\partial \Pi_S}{\partial s} = p_s - 2s = 0$$

$$\frac{\partial \Pi_S}{\partial \mathbf{x}} = -2(\mathbf{x} - 4) - p_x = 0$$

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and these give $s^* = \frac{p_s}{2}$ (steel supply)

$\mathbf{x}_D^* = 4 - \frac{p_x}{2}$ (pollution right demand)

Coase and Production Externalities

In a competitive market for pollution rights the price p_x must adjust to clear the market so, at equilibrium,

$$x_D^* = 4 - \frac{p_x}{2} = p_f - 2p_x = x_S^*.$$

Coase and Production Externalities

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The market-clearing price for pollution rights is thus $p_x = \frac{2p_f - 8}{3}$

and the equilibrium quantity of rights traded is $x_D^* = x_S^* = \frac{16 - p_f}{3}$.

Coase and Production Externalities

$$s^* = \frac{p_s}{2}; \quad f^* = p_x; \quad x_D^* = x_S^* = \frac{16 - p_f}{3};$$

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Coase and Production Externalities

$$s^* = \frac{p_s}{2}; \quad f^* = p_x; \quad x_D^* = x_S^* = \frac{16 - p_f}{3};$$
$$p_x = \frac{2p_f - 8}{3}.$$

So if $p_s = 12$ and $p_f = 10$ then

$$s^* = 6; \quad f^* = 4; \quad x_D^* = x_S^* = 2; \quad p_x = 4.$$

This is the efficient outcome.

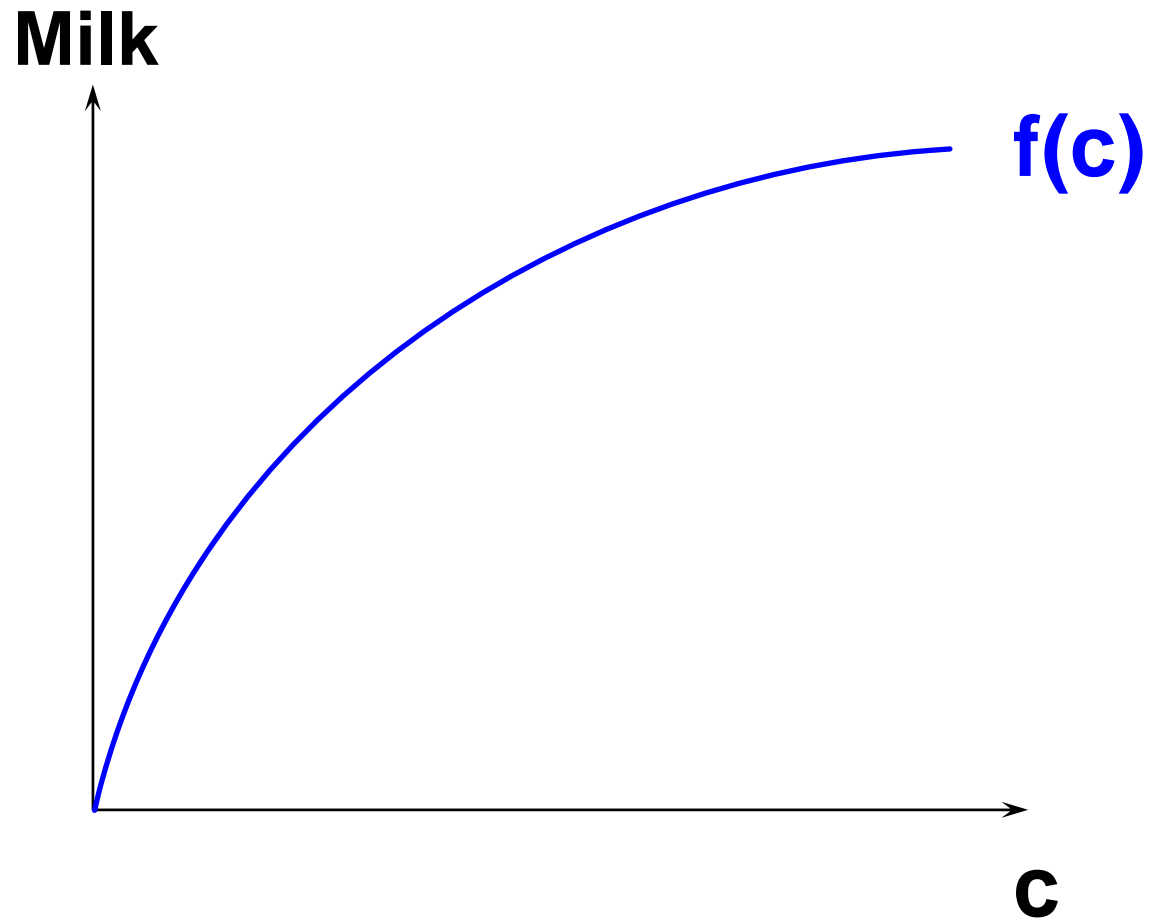
Coase and Production Externalities

- Q: Would it matter if the property right to the water had instead been assigned to the steel firm?
- A: No. Profit is linear, and therefore quasi-linear, in money so Coase's Theorem states that the same efficient allocation is achieved whichever of the firms was assigned the property right. (And the asset owner gets richer.)

The Tragedy of the Commons

- Consider a grazing area owned “in common” by all members of a village.
- Villagers graze cows on the common.
- When c cows are grazed, total milk production is $f(c)$, where $f' > 0$ and $f'' < 0$.
- How should the villagers graze their cows so as to maximize their overall income?

The Tragedy of the Commons



The Tragedy of the Commons

- Make the price of milk \$1 and let the relative cost of grazing a cow be p_c . Then the profit function for the entire village is

$$\Pi(\mathbf{c}) = \mathbf{f}(\mathbf{c}) - p_c \mathbf{c}$$

and the village's problem is to

$$\max_{\mathbf{c} \geq 0} \Pi(\mathbf{c}) = \mathbf{f}(\mathbf{c}) - p_c \mathbf{c}.$$

The Tragedy of the Commons

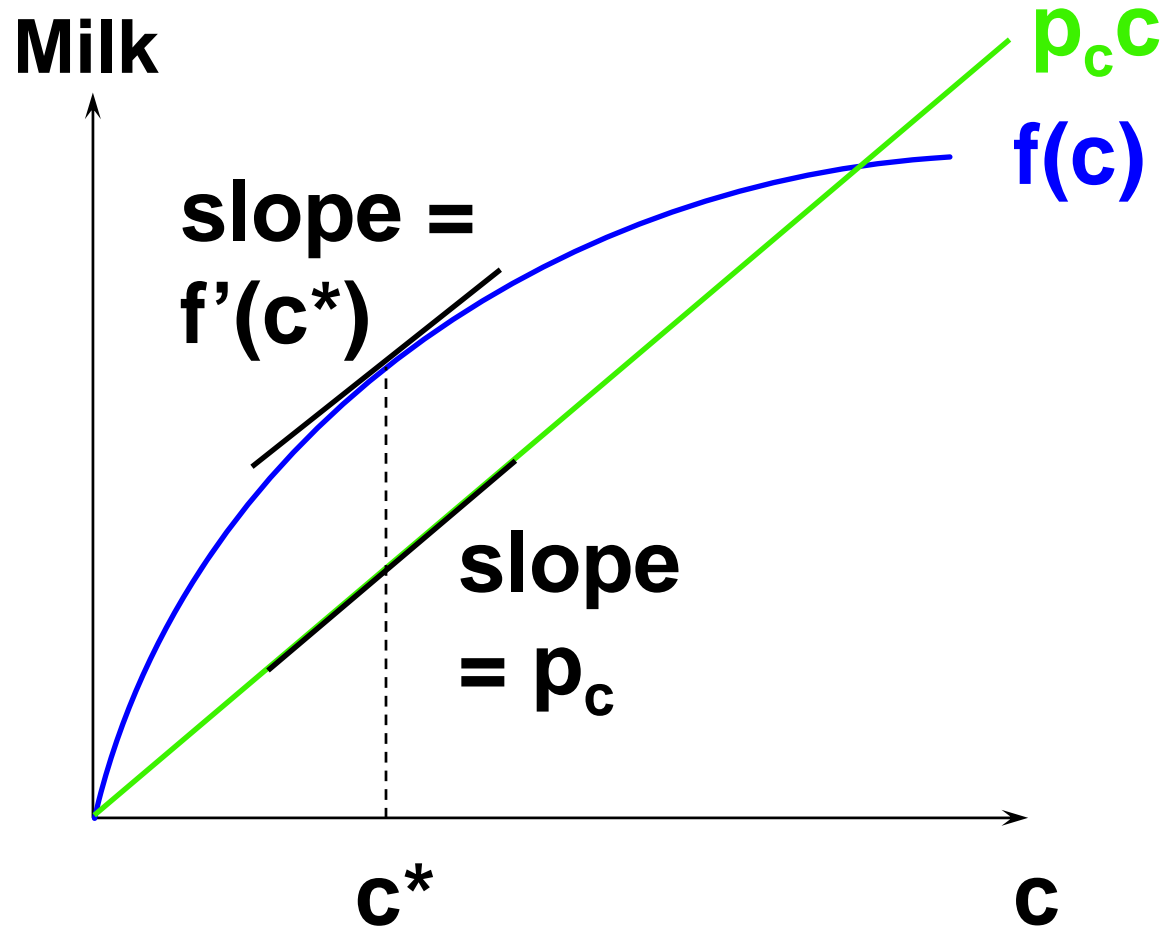
$$\max_{c \geq 0} \Pi(c) = f(c) - p_c c.$$

The income-maximizing number of cows to graze, c^* , satisfies

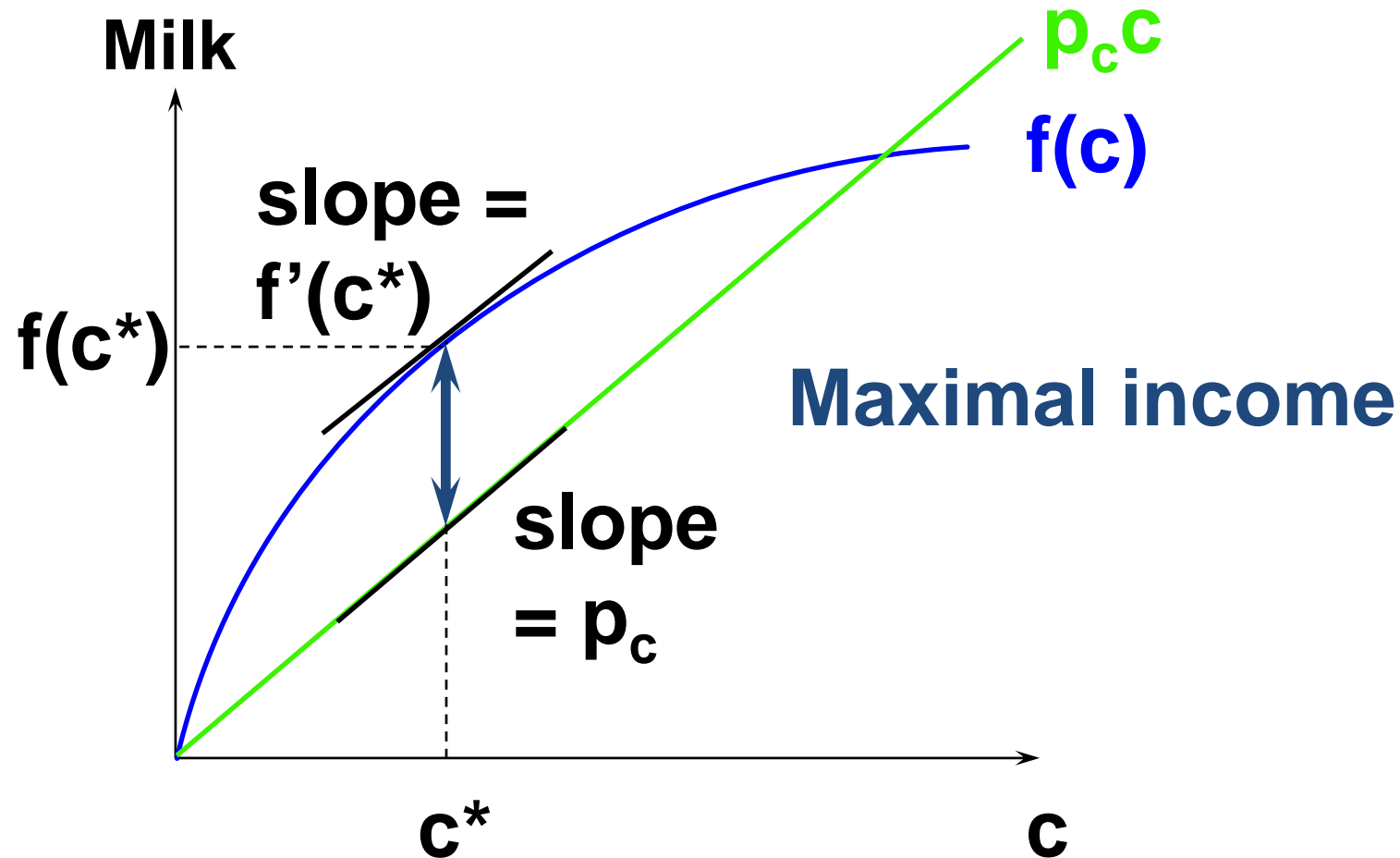
$$f'(c) = p_c$$

i.e. the **marginal** income gain from the last cow grazed must equal the **marginal cost** of grazing it.

The Tragedy of the Commons



The Tragedy of the Commons



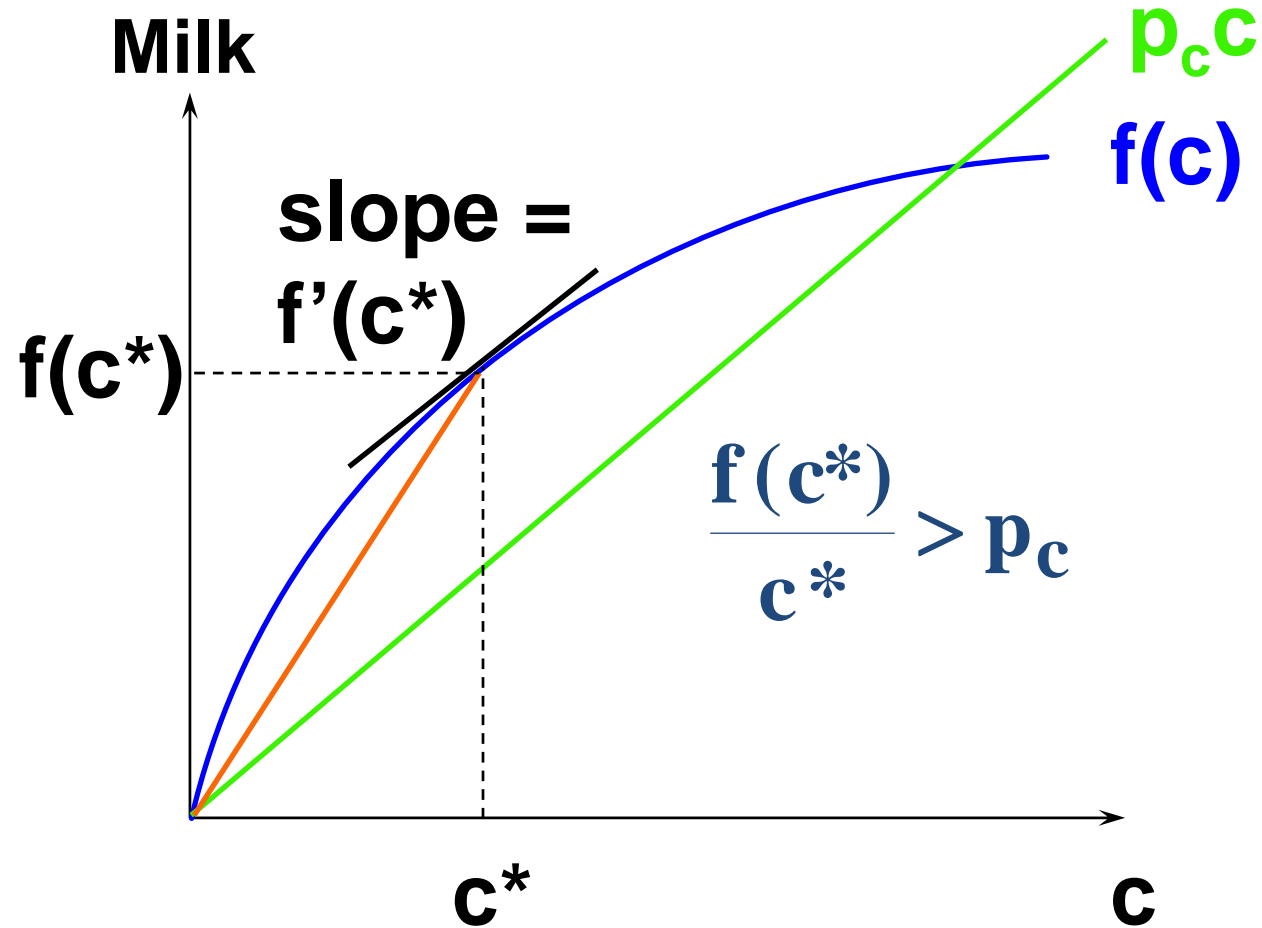
The Tragedy of the Commons

- For $c = c^*$, the average gain per cow grazed is

$$\frac{\Pi(c^*)}{c^*} = \frac{f(c^*) - p_c c^*}{c^*} = \frac{f(c^*)}{c^*} - p_c > 0$$

because $f' > 0$ and $f'' < 0$.

The Tragedy of the Commons



The Tragedy of the Commons

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because $f' > 0$ and $f'' < 0$. So the economic profit from introducing one more cow is positive.

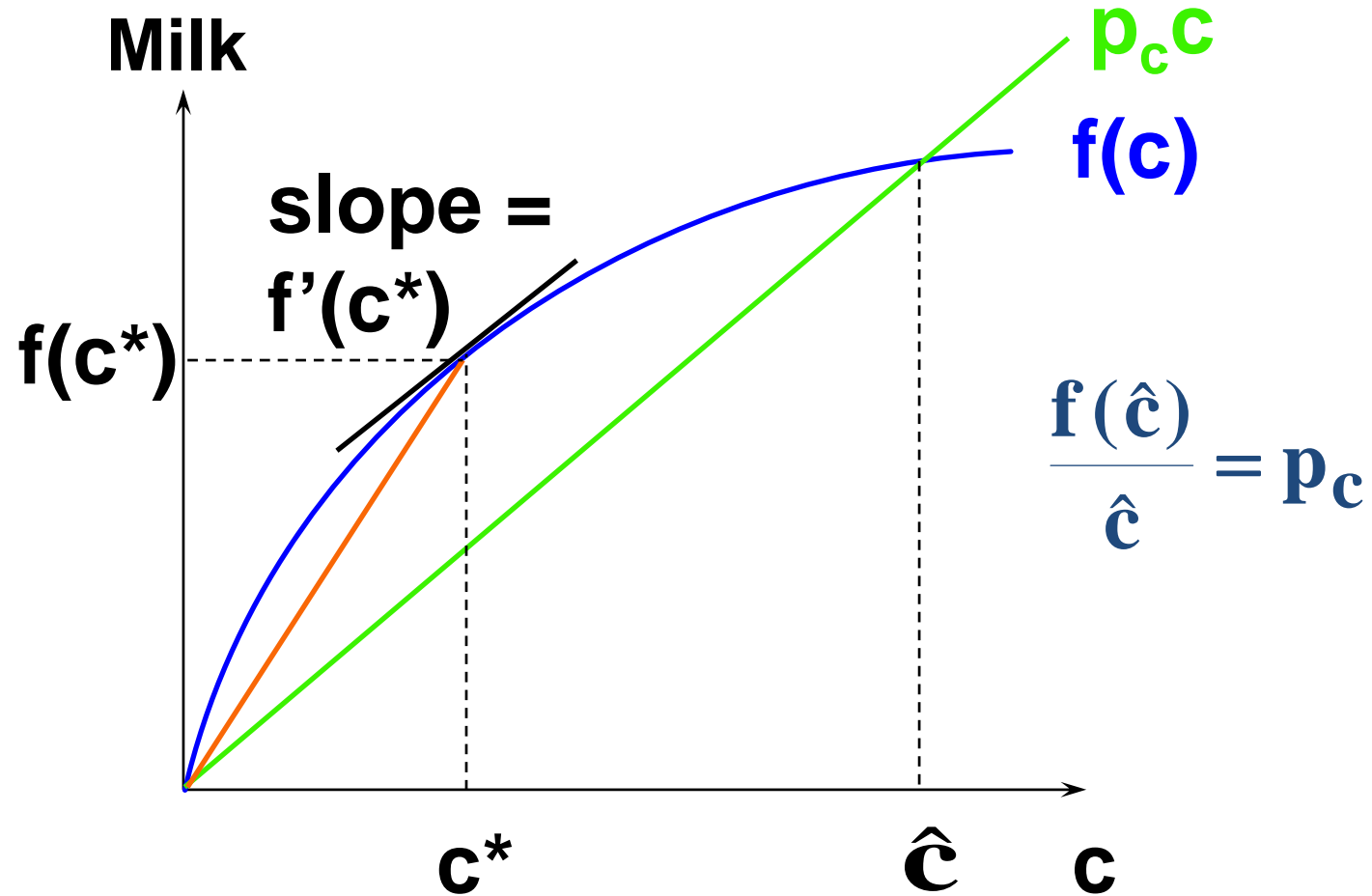
- Since nobody owns the common, entry is not restricted.

The Tragedy of the Commons

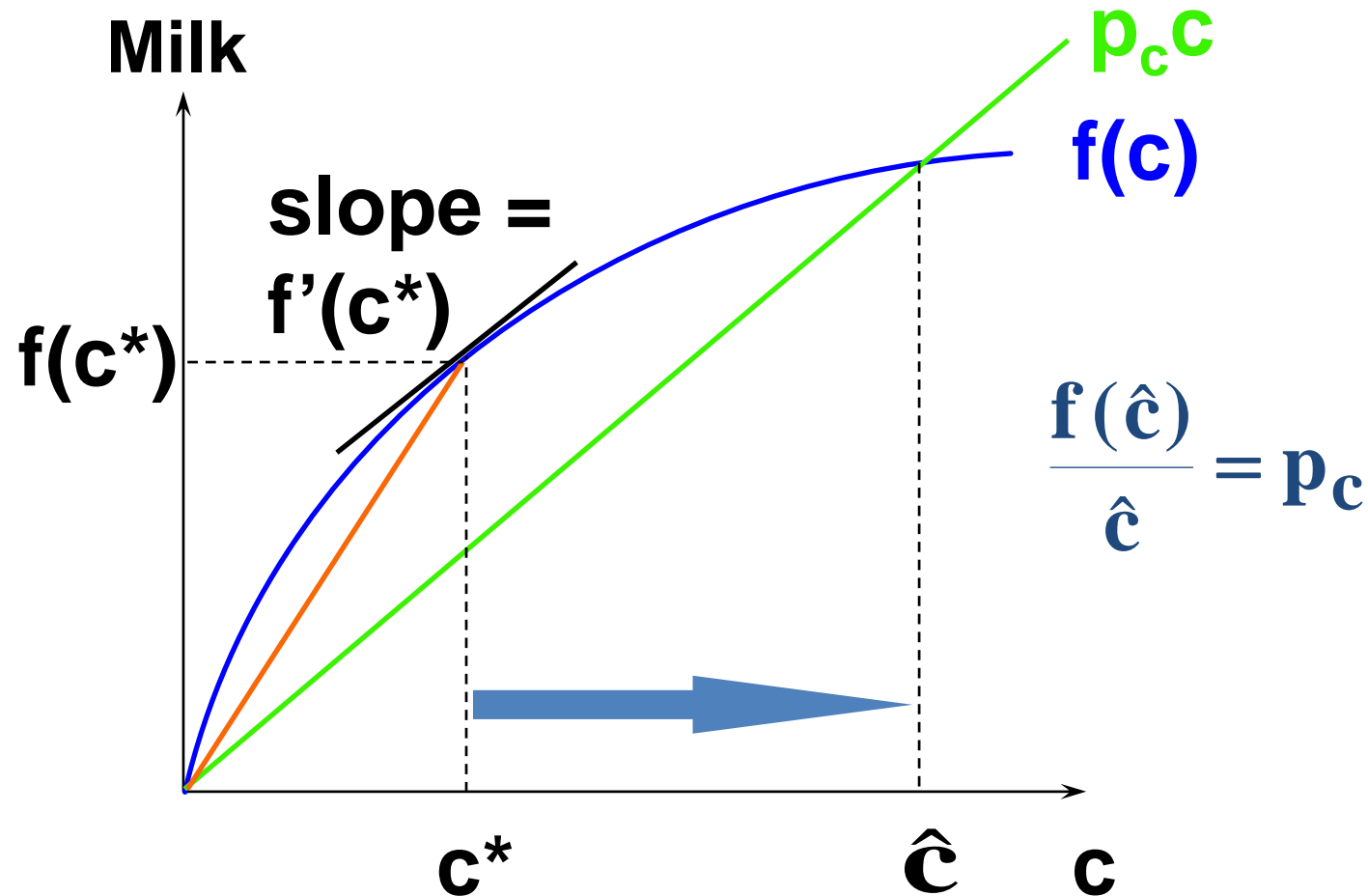
- Entry continues until the economic profit of grazing another cow is zero; that is, until

$$\frac{\Pi(c)}{c} = \frac{f(c) - p_c c}{c} = \frac{f(c)}{c} - p_c = 0.$$

The Tragedy of the Commons



The Tragedy of the Commons



The commons are over-grazed, tragically.

The Tragedy of the Commons

- The reason for the tragedy is that when a villager adds one more cow his income rises (by $f(c)/c - p_c$) but every other villager's income falls.
- The villager who adds the extra cow takes no account of the cost inflicted upon the rest of the village.

The Tragedy of the Commons

- Modern-day “tragedies of the commons” include
 - over-fishing the high seas
 - over-logging forests on public lands
 - over-intensive use of public parks; e.g. Yellowstone.
 - urban traffic congestion.