LECTURE 6: MULTI-INPUT PRODUCTION FUNCTIONS

QUESTIONS AND PROBLEMS

True/False Questions

____ The returns to scale of a multi-input production function depend on the composition of its inputs.

____ If the isoquants of the production function $f(K, L)$ touch the L-axis, then this means that input K is essential for production, i.e., that you need at least some of input K to produce a positive output.

____ If the isoquants of the production function $f(K, L)$ touch the K-axis, then this means that input K is essential for production, i.e., that you need at least some of input K to produce a positive output.

____ The *MRTS* gives the trade off between two inputs holding output constant, i.e., the amount of one input that would necessary to make up for the reduction of another input so that output remains constant.

____ The concept of the *MRTS* is defined only for production functions of two inputs, because the concept involves two inputs.

____ The concept of *MRTS* is defined even for production functions of more than two inputs.

____ The *MRTS* between two inputs is independent of the usage level of any additional inputs, because the *MRTS* is equal to the ratio of the marginal products of these two inputs.

____ If the isoquants of the production function $f(K, L)$ touch the L-axis, then this means that one can produce a given level of output using only labor inputs.

Short Questions

1. Consider the production function

$$q = 3\,L^{0.7}\,K^{0.1} + 2\,L^{0.3}\,E$$

where L is the level of labor inputs, K the level of capital inputs, and E the level of energy inputs.

a. What is the marginal product of labor?

b. Does the marginal product of labor depend on the level of capital employed by the firm?

c. Does more capital make labor more productive? (in other words, does the marginal product of labor increase as capital increases?)

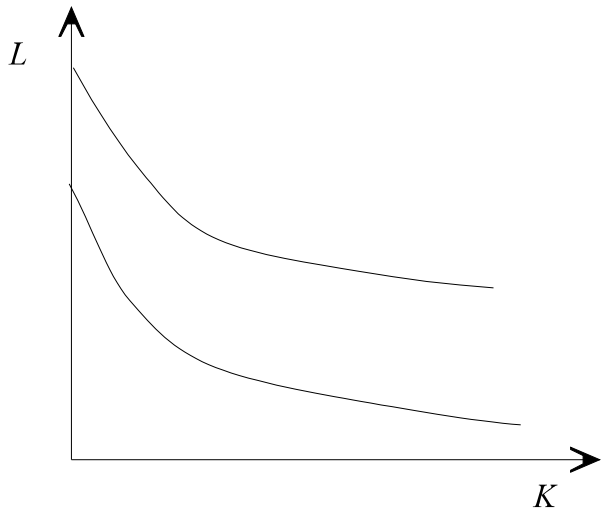
2. Consider the following production function

$$f(x,y,z) = x^2 + y^2 + x\,z$$

A. Graph the isoquant of y and z for this production function for output equal to 20 and $x = 2$. Put z on the vertical axis and y in the horizontal axis. Label any intercepts. Show your work.

B. Does this isoquant exhibit decreasing MRTS? Why or why not?

3. Consider the following two isoquants of a production function:



A. Does this production function exhibit diminishing MRTS? Why or why not?

B. Is capital (K) an essential input for this production process? Explain your answer.

4. Consider the production function

$$f(K, L) = \alpha\ (2\ K^2 + \beta\ L^2)$$

where K is a measure of capital inputs, L is a measure of labor inputs, the parameter α reflects overall productivity and the parameter β reflects productivity specific to labor (e.g., increased ability or education).

What is the $MRTS_{K,L}$? Does it depend on the overall productivity parameter α? Does it depend on the labor specific productivity parameter β?

5. Consider the production function

$$Q = L + \log(1+K)$$

a. Write the equation for the isoquant of this production function that corresponds to an output level of Q=10.

b. Show that this production function is characterized by diminishing *MRTS*. [You can do this using the above isoquant, if you like, or using the expression for the production function given above.]

Problems

1. The production function for trucks in a single plant is given by

$$Q = \alpha K^{0.3} L^{0.6}$$

where K is the amount of capital in the production process, L the amount of labor employed (in thousands), and α a parameter that measures the efficiency of production. It is immediately apparent that the greater the capital and labor inputs, the greater the output. Also, an increase in production efficiency increases output proportionately.

Suppose that in the US the value of α is equal to 2, while in China it is equal to 1, that is, US plants are twice as efficient as Chinese plants. Also suppose that Chinese plants have only a fifth as much capital per worker as US plants (simply because capital is relatively scarce in China). In particular, a US plant that employs 2,000 workers has 5 units of capital, while its Chinese counterpart (which also employees 2,000 workers) only has a single unit of capital.

a. What is the marginal product of labor the US and in Chinese truck plants described above?

b. Suppose that the plant efficiency in the Chinese plant increases (through technological diffusion) to the level of the US plant. What is the new marginal product of labor in the Chinese plant?

c. Suppose instead of the increase in plant efficiency, investment in the Chinese plant increases (through capital inflows from abroad) so that the capital in the Chinese plant reaches the level of its US counterpart. What is the new marginal produce of labor in the Chinese plant?

d. Which of the two changes described above is the most beneficial in terms of raising labor productivity?

2. Consider the production function of a manufacturing plant given by

$$Q = (K^{0.5} + 4 L^{0.5} + E^{0.5})^2$$

The plant is currently having a capital stock equal to 1, and employees 4 units of labor. Its energy use is 2 units.

a. What is the marginal product of labor for this plant, and for this level of input use?

b. Suppose an energy crisis forces the plant to cut back its energy usage to only 1 unit. What is the effect of this reduction in energy inputs on the marginal product of labor? Has labor

become more or less productive?

c. Suppose to compensate for this loss of energy inputs the plant decides to increase its capital stock. What would the increase in the capital stock have to be for the marginal product of labor to go up to what it was in part (a)?