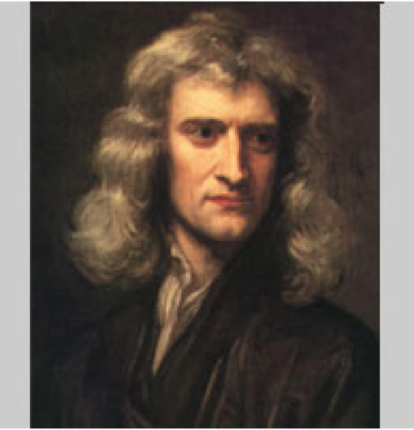
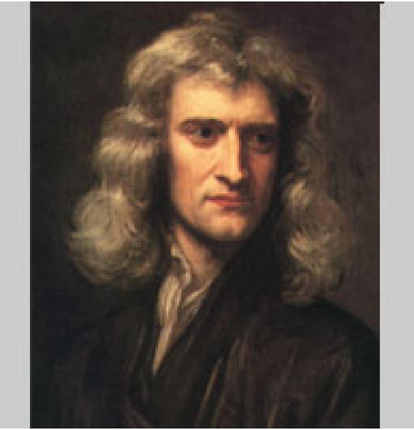
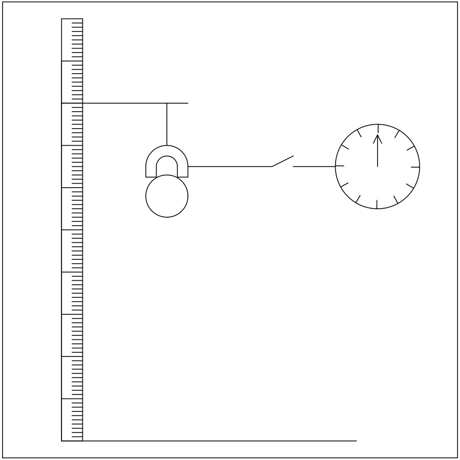




Isaac Newton (1643–1727)

An experiment to illustrate Newton's Law



Experimental results

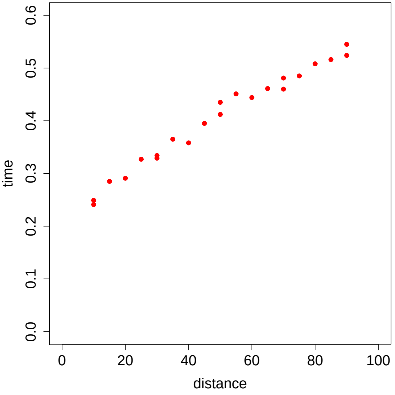
The data reproduced below are the results obtained by one student from 22 experimental runs.

t(sec)	d(cm)	t(sec)	d(cm)	t(sec)	d(cm)
0.241	10	0.358	40	0.460	70
0.249	10	0.395	45	0.485	75
0.285	15	0.435	50	0.508	80
0.291	20	0.412	50	0.516	85
0.327	25	0.451	55	0.524	90
0.329	30	0.444	60	0.545	90
0.334	30	0.461	65		
0.365	35	0.481	70		

Points for discussion

- choice of design points, d?
- inputs and outputs?
- sources of systematic variation in results?
- sources of random variation in results?

Experimental results in graphical form



- **How would you describe the relationship between distance and time?**
- **Why did I put time on the y-axis of the graph, rather than on the x-axis?**

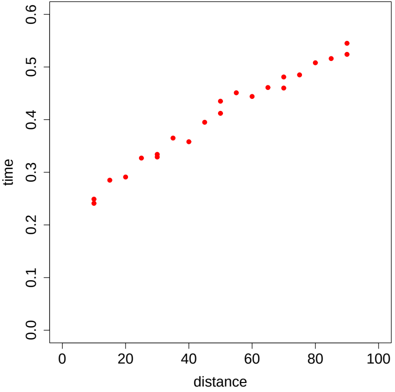
Newtonian mechanics

Newton's law states that the vertical distance d travelled in time t by a body initially at rest and falling under the influence of gravity is given by the formula

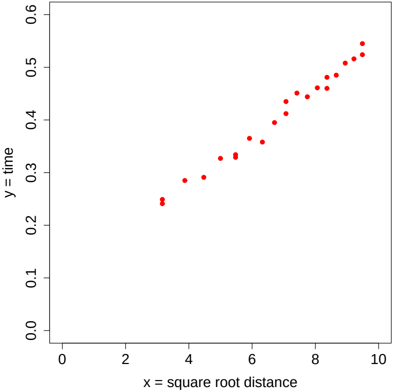
$$\mathbf{d = \frac{1}{2}gt^2}$$

where g is a constant.

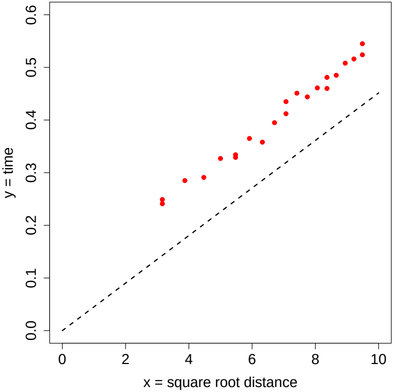
Newtonian mechanics and experimental data



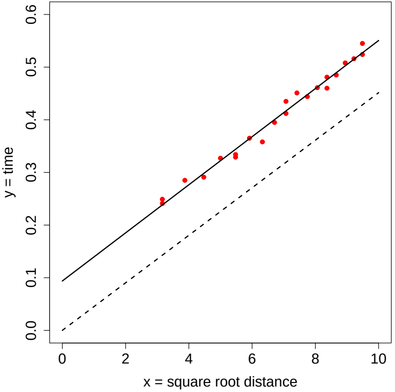
- **The experimentally observed relationship is non-linear, as predicted by Newton's law.**



- **Transformation of the data, from distance d to the new variable $x = \sqrt{d}$ makes the relationship linear, also as predicted by Newton's law.**

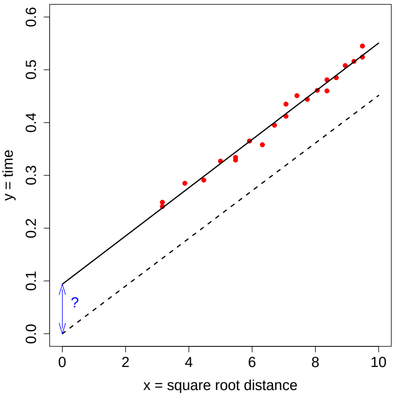


- **But Newton's law does not fit the data!**



- **We need to add an** **intercept** **to the straight line**

.



- **What does the intercept represent?**
- **What does the variation of the data around the fitted line represent?**

A statistical model for the experiment

Start with Newtonian mechanics,

$$\mathbf{d} = \frac{1}{2}\mathbf{g} \times \mathbf{t}^2$$

Now transform to $\mathbf{x} = \sqrt{\mathbf{d}}$ and $\mathbf{Y} = \mathbf{t}$, and put $\mathbf{Y}$ on the left-hand side of the equation,

$$\mathbf{x} = \sqrt{\mathbf{g}/2} \times \mathbf{Y} \qquad \mathbf{Y} = \beta \times \mathbf{x} \quad (\beta = \sqrt{2/\mathbf{g}})$$

Now incorporate the effects of the experimenter's reaction-time,

$$\mathbf{Y} = \alpha + \beta\mathbf{x} + \mathbf{Z}$$

Interpreting the model

$$\mathbf{Y} = \alpha + \beta \mathbf{x} + \mathbf{Z}$$

- α **represents mean reaction time**
- $\beta = \sqrt{2/g}$ **is the quantity of scientific interest**
- $\mathbf{Z}$ **is a random error, which varies independently between different runs of the experiment**

What have we learnt so far?

1. **Graphical presentation of data is almost always useful.**
2. **Statistical models should:**
   - **respect the data;**
   - **respect the underlying science.**
3. **Transforming the data can help to achieve both goals.**
4. **The results of a statistical analysis should always be interpreted in relation to the original scientific question.**