Model-based Geostatistics: geospatial statistical methods for public health applications

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Overview: spatial epidemiology and model-based geostatistics

Epidemiological data

- **incidence:** number of new cases per unit time per unit population
- **prevalence:** number of existing cases per unit population
- **risk:** probability that a person will contract the disease (per unit time or per life-time)

General objective is to understand spatial variation in disease incidence and/or prevalence and/or risk according to context

Relevant books include

Elliott et al (2000); Gelfand et al (2010); Rothman (1986); Waller and Gotway (2004); Woodward (1999);

Epidemic vs endemic patterns of incidence

- **Foot-and-mouth in Cumbria (the 2001 epidemic)**

  Diggle (2006)

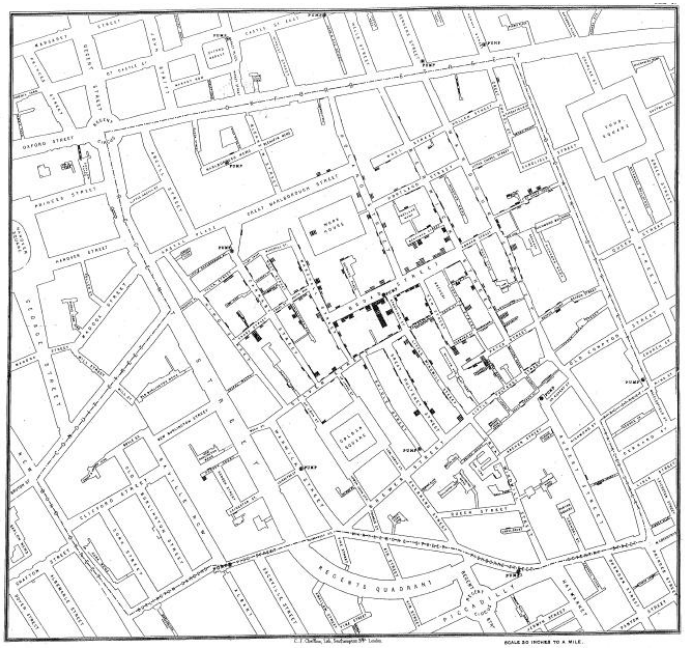
- **Gastro-enteric disease in Hampshire (AEGISS)**

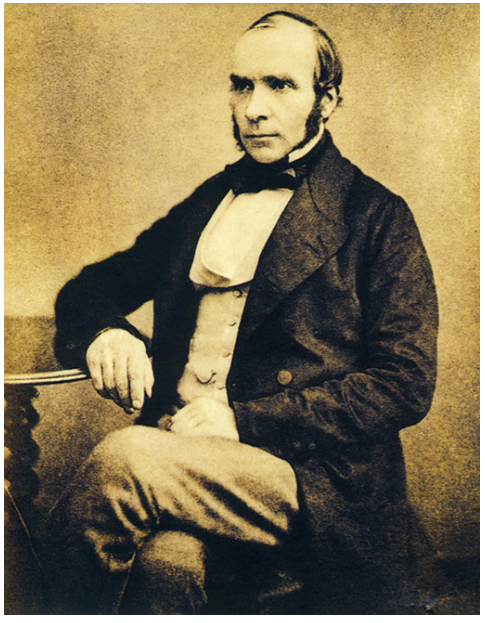
  Diggle, Rowlingson and Su (2005)

Animations at: `http://www.lancaster.ac.uk/staff/diggle/`

What are the similarities and differences between the two phenomena?

In the beginning: Cholera in Victorian London, 1854





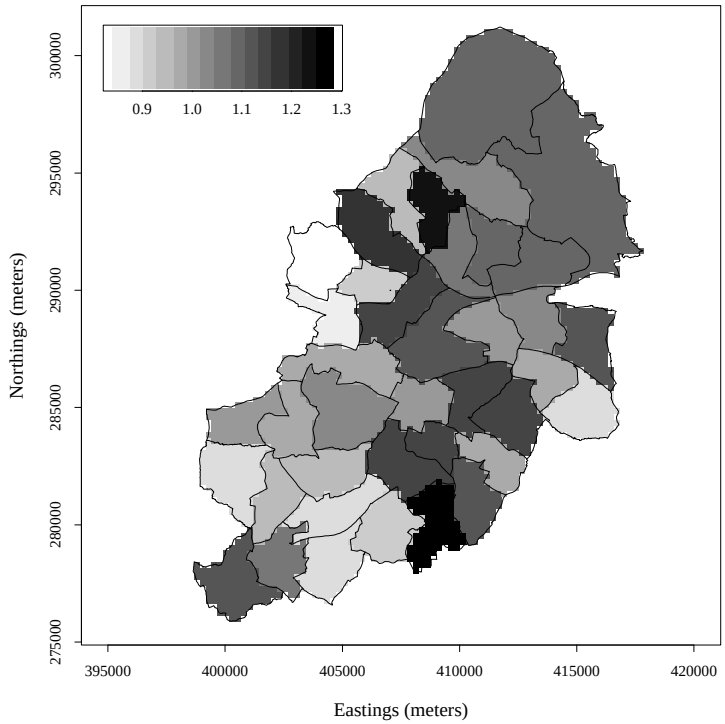
The physician John Snow famously removed the handle of the Broad Street water-pump, having concluded (correctly) that infected water was the source of the disease contrary to conventional wisdom at the time.

`https://en.wikipedia.org/wiki/1854_Broad_Street_cholera_outbreak`

Study-designs

- **Registry**
  - case-counts in sub-regions to partition study-region (numerators)
  - population size in each sub-region (denominators)
  - collateral information from national census (covariates)
- **Case-control**
  - **cases:** all known cases within study region
  - **controls:** probability sample of non-cases within study-region
- **Survey**
  - sample of locations within study-region
  - collect data from each location
  - commonly used in developing country settings

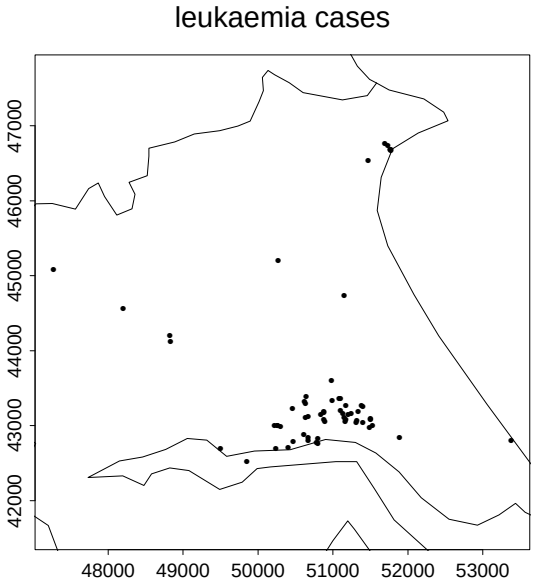
Registry example. Colorectal cancer in Birmingham

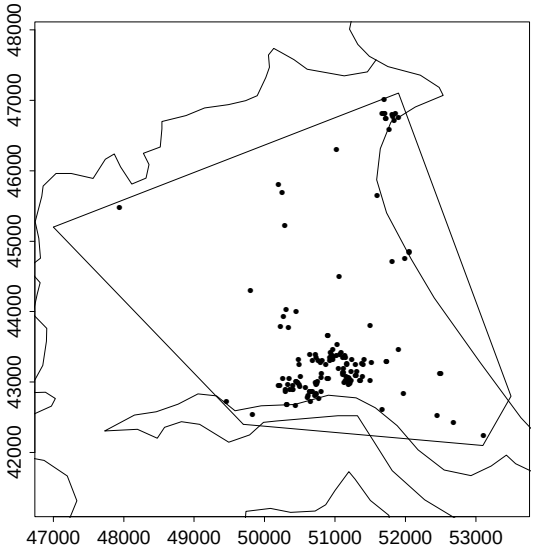


Smoothed estimates of relative risk in 36 electoral wards.

Kelsall and Wakefield (2002).

Case-control example Childhood leukaemia in Humberside





- **residential locations of all known cases of childhood leukaemia in Humberside, England, over the period 1974-82;**
- **residential locations of a random sample of births**

Cuzick and Edwards (1990); Diggle and Chetwynd (1991).

Survey example Loa loa prevalence in Cameroon

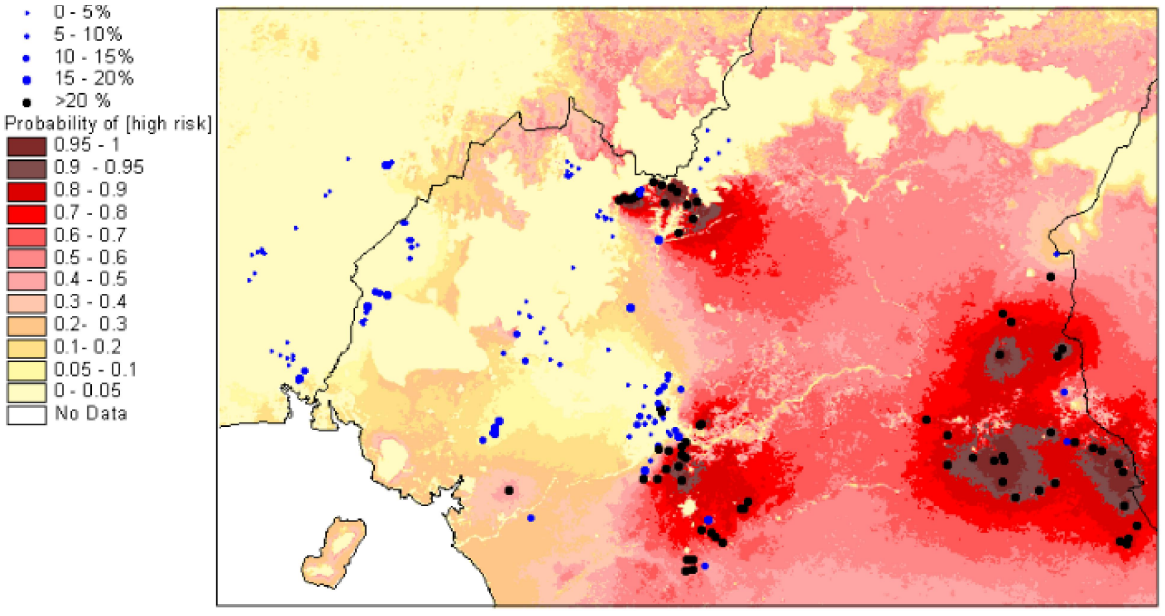


Figure 6: PCM for [high risk] in Cameroon based on ERMr with ground truth data.

Data are empirical prevalences in surveyed villages

Map shows predictive probabilities of exceeding 20% prevalence threshold

Diggle et al (2007)

What is the public health question?

1. **Colorectal cancer in Birmingham**
   - **Does the risk of contracting the disease vary spatially?**
   - **And if so, why?**

2. **Childhood leukaemia in Humberside**
   - **Do cases show a surprising tendency to cluster together?**

3. **Loa loa in Cameroon**
   - **What environmental characteristics affect the risk of disease?**
   - **Can we predict where the prevalence of the disease exceeds a policy-based intervention threshold?**

Spatial stochastic processes

1. **A stochastic process is a collection of random variables**

2. **A spatial stochastic process is a stochastic process in which each random variable is associated with a position in space**

3. **Three important types of spatial stochastic process:**

   - **discrete spatial variation: the random variables associate a real value with a particular, pre-specified, set of points in space, hence $\{(S_i, x_i) : i = 1, \ldots, n\}$**
   - **point processes: the random variables are the locations themselves, $\{x_i : i = 1, \ldots, n\}$**
   - **continuous spatial variation: the random variables associate a real value with every point in the space, hence $\{S(x) : x \in \mathbb{R}^2\}$**

This course covers continuous spatial variation, with a focus on its application to prevalence mapping

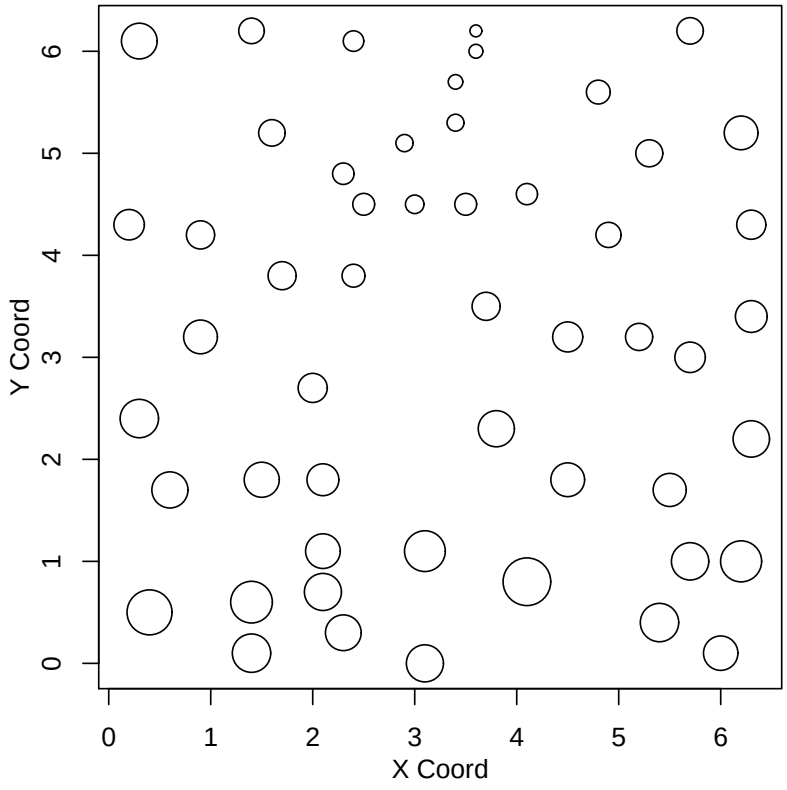
Geostatistics

- **traditionally, a self-contained methodology for spatial prediction:**
  - **origins in the South African mining industry**
  - **subsequently developed at École des Mines, Fontainebleau, France**
- **nowadays, that part of spatial statistics which is concerned with data obtained by spatially discrete sampling of a spatially continuous process**

Model-based geostatistics: the application of general principles of statistical modelling and inference to geostatistical problems

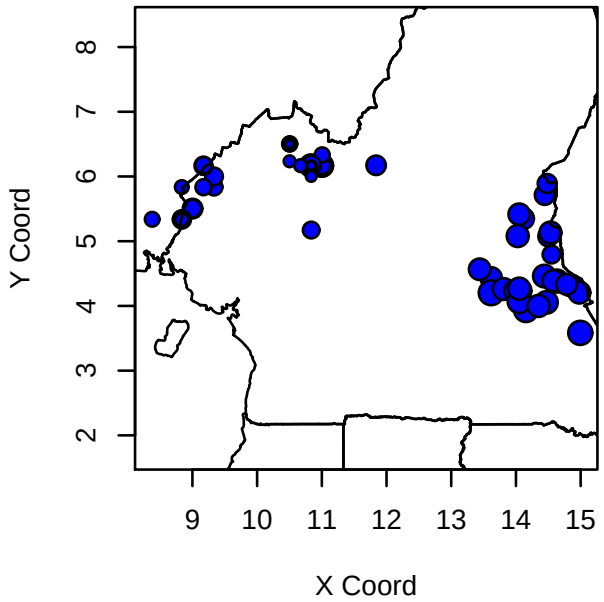
Diggle, Moyeed and Tawn (1998)

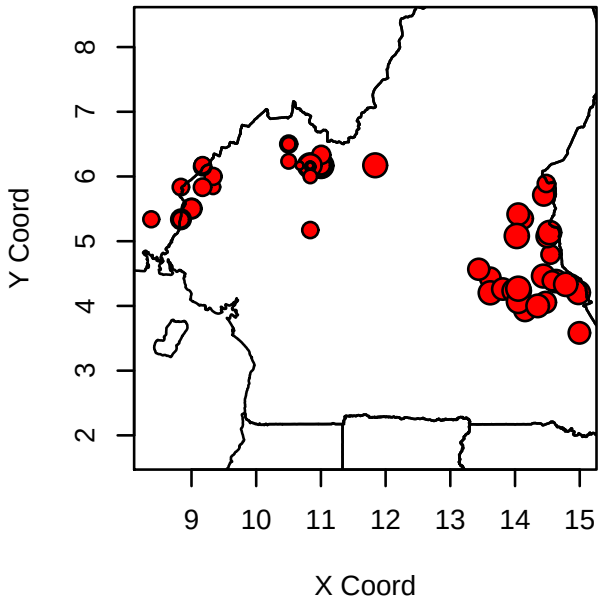
Measured surface elevations

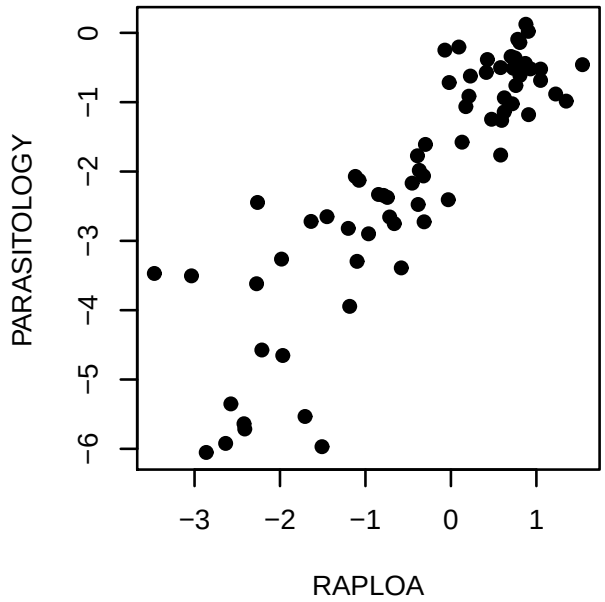


- **simplest form of geostatistical data**
- **locations and associated measurements**
- **no covariates**

Loa loa prevalence surveys



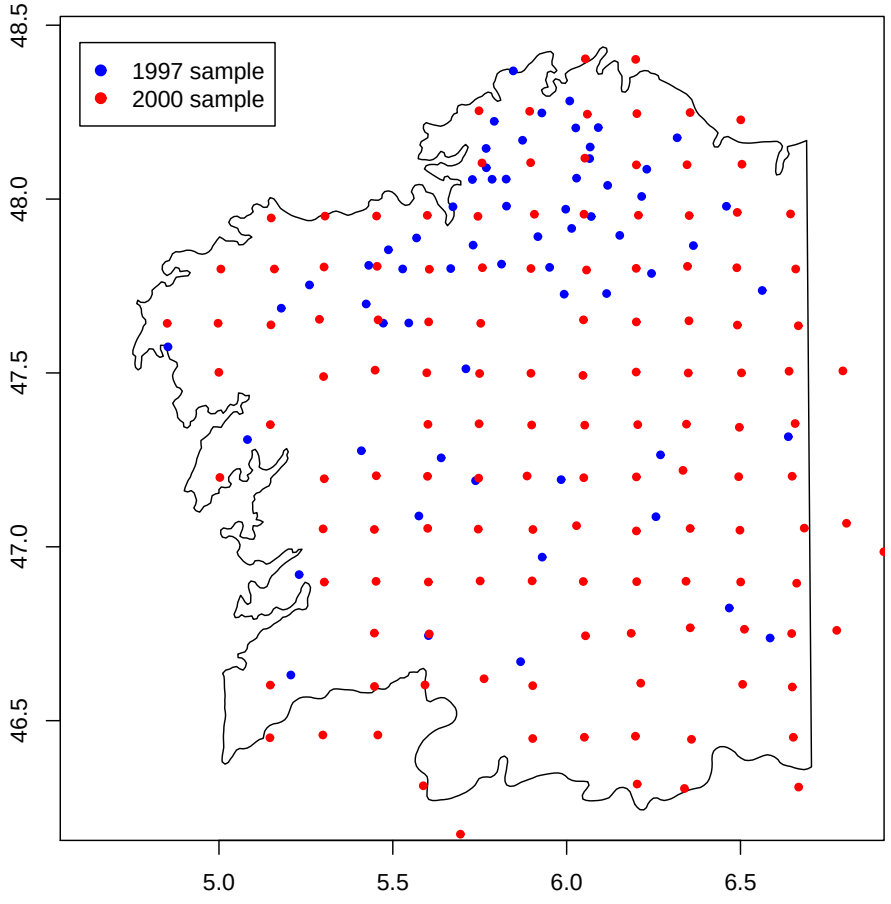




Maps show prevalence as estimated by each of two methods:

- **parasitology (blood sample, red)**
- **simple questionnaire (RAPLOA, blue)**
- **covariates include elevation and NDVI (green-ness of vegetation) at 1km resolution**

Environmental monitoring in Galicia, north-west Spain



- **spatially irregular sample in 1997**
- **potential for selection bias?**

Geostatistical problems

- **Design:** how to choose locations x_i at which to collect outcome data?
- **Estimation:** how to investigate relationship between outcome and covariates when data may be spatially correlated?
- **Prediction:** how to map (expected value of) outcome throughout the study-region?

Practical point:

- **Estimation** only requires covariate information at locations x_i
- **Prediction** requires covariate information throughout the study-region.

A non-spatial model for prevalence survey data

Design

- **Sample communities $i = 1, \ldots, n$.**
- **In community i, sample m_i individuals of whom Y_i test positive for disease of interest.**
- **Associated covariates w_i**

Model

- **ρ_i = probability that a randomly sampled individual in community i will test positive**
- **$\log\{\rho_i/(1-\rho_i)\} = \alpha + w_i'\beta$**
- **$Y_i \sim \text{Binomial}(m_i, \rho_i)$, mutually independent**

A spatial model for prevalence survey data

Design

- **Sample communities $i = 1, \ldots, n$ at locations x_i**
- **In community i, sample m_i individuals of whom Y_i test positive for disease of interest.**
- **Associated covariates $w_i = w(x_i)$**

Model

- ρ_i **= probability that a randomly sampled individual in community i will test positive**
- $\log\{\rho_i/(1-\rho_i)\} = \alpha + w(x_i)'\beta + S(x_i)$
- $Y_i \sim \mathrm{Binomial}(m_i, \rho_i)$**, conditionally independent given** $S(\cdot)$

A spatial model for prevalence survey data (continued)

Two kinds of covariates

- **$w(x_i)$ an intrinsic property of the location x_i**
- **$w(x_i)$ a property of the people who live at location x_i**

Practical implication: when mapping prevalence we need to be able to assign a value w(x) to every location in the study-region.

What is S(x)?

- **an unobserved spatially varying stochastic process**
- **a proxy for unmeasured, spatially structured covariates**

Practical implication: in any application where S(x) turns out to be important, it is worth asking what the missing covariate(s) might be.

Person or place?

Extend spatial model to

$$\log\{\rho_i/(1-\rho_i)\} = \alpha + \{w(x_i)'\beta + S(x_i)\} + \{d_i'\gamma + U_i\}$$

- **w(x) : measured properties of location x**
- **S(x) : stochastic process, proxy for unmeasured properties of x**
- **d_i : measured properties of ith community**
- **U_i : independent random variables, proxy for unmeasured properties of ith community**

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