Prevalence mapping

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Overview

1. Prevalence mapping using linear models and binomial models.
2. Plug-in and Bayesian predictions.
3. Predictive summaries: prevalence, odds and exceedance probabilities.
4. Combining data from multiple surveys (continue).

Geostatistical prediction

- $\mathcal{S} = \{S(x) : x \in \mathbb{A} \subset \mathbb{R}^2\}$
- $X = \{x_1, \ldots, x_n\}$ (sampled locations).
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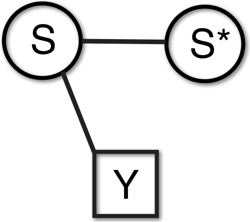

$$[S, S^*, Y] = [Y][S|Y][S^*|S, Y] = [Y][S|Y][S^*|S]$$

- **Linear model based on logit transformed prevalence.**

$$\begin{aligned} Z_i &= \log\{(Y_i + 0.5)/(m_i - Y_i + 0.5)\} \\ &= d(x_i)^\top \beta + S(x_i) + U_i \end{aligned}$$

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Prevalence mapping: a general framework

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- $T^*|T \sim$ MVN(μ^*, Σ^*).

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How to estimate prevalence?

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$$[T^*|T] = \int [T^*|T, \theta][\theta]\, d\theta.$$

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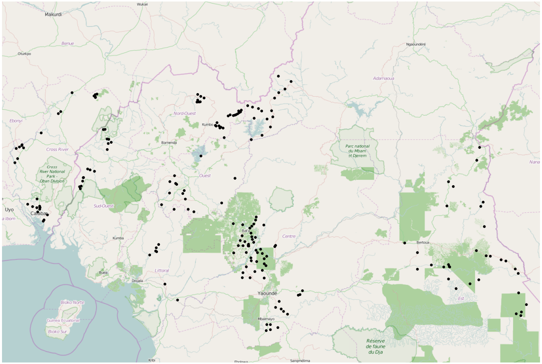
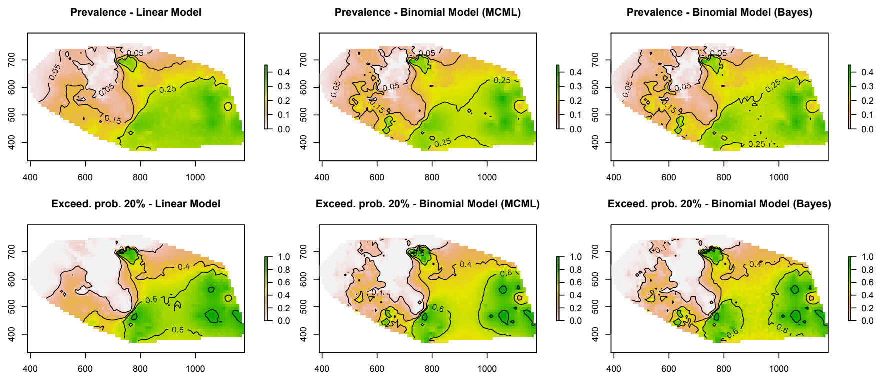
Loa loa prevalence mapping (1)



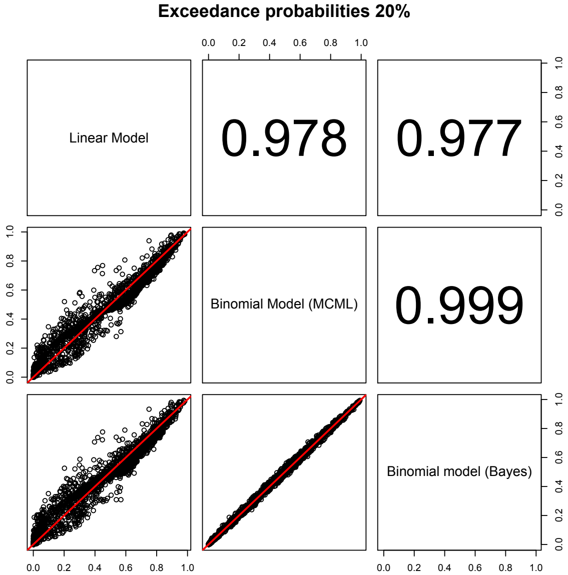
Epidemiological and geostistical questions

- What are the main risk factors of Loa loa?
- How do we identify Loa-loa hotspots?

Loa loa prevalence mapping (2)



Loa loa prevalence mapping (3)



Effects decomposition

- $d(x)^\top \beta = \beta_0 + \beta_1 \mathsf{elev}(x) + \beta_2 \mathsf{elev}^2(x)$

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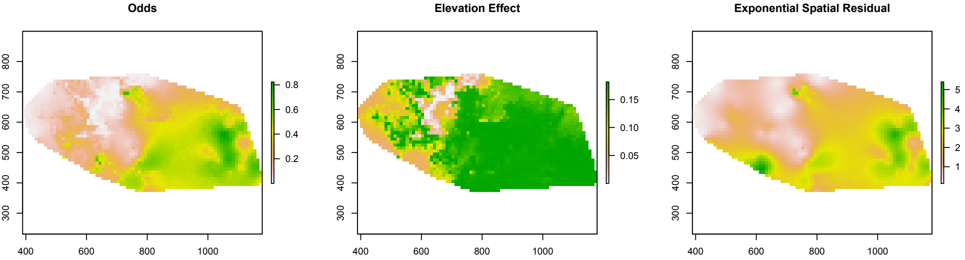
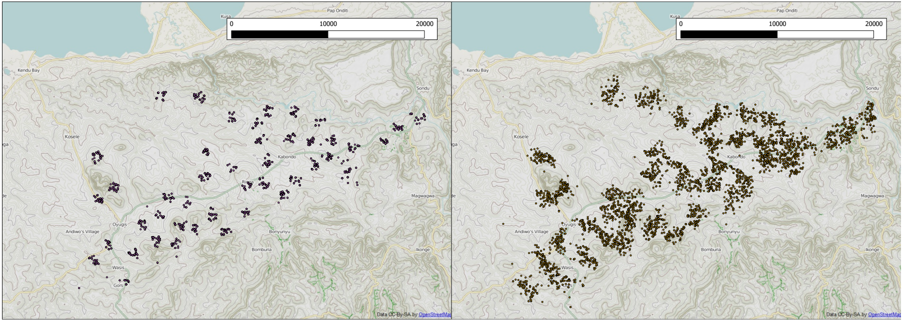


Combining information from multiple surveys



- **Community survey:** 1430 individuals; 740 compounds.
- **School survey:** 4852 pupils (46 schools); 3791 compounds.

Model formulation

- **Community survey.**

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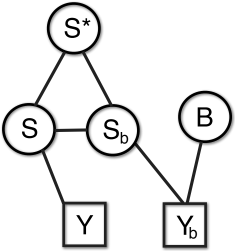
- **School survey.**

$$\log\{p_{ij}/(1-p_{ij})\} = d_{ij}^\top (\beta + \delta) + S(x_i) + B(x_i) + Z_i.$$



What is the gain in accuracy in estimating S^*?

Prevalence mapping using convenience samples

$$[S, S_b, S^*, B|Y, Y_b] = [B|Y_b][S|Y][S_b|Y_b][S^*|S, S_b]$$

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- **Odds within the convenience samples.**

$$O_j(x) \quad = \quad \exp\{d_j^\top \beta + S(x)\} \exp\{d_j^\top \delta\} \exp\{B(x)\}$$

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(b) $1 - P[0.9 < \exp\{B(x)\} < 1.1|Y, Y_b]$

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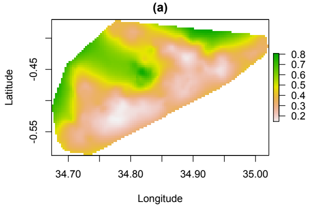

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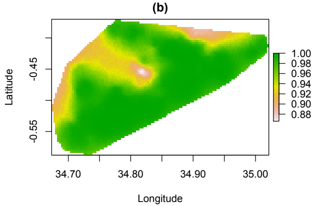
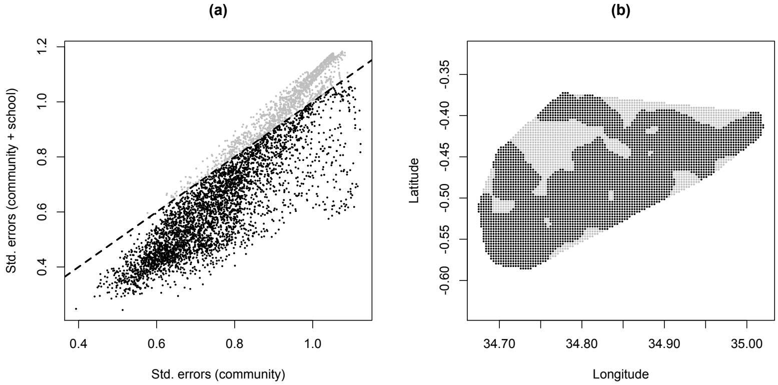


Gain in efficiency



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