Geostatistical modelling of the relationship between microfilariae and antigenaemia prevalence of lymphatic filariasis infection

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Overview

- Lymphatic filariasis: what is it? What diagnostics?
- Bivariate geostatistical modelling of prevalence from two different diagnostics.
  1. A semi-mechanistic model for lymphatic filariasis microfilariae and antigenaemia prevalence.
  2. An empirical model for prevalence from any two diagnostics.
- Application to lymphatic filariasis prevalence data from West Africa.
- Discussion.

Lymphatic filariasis: the disease

Figure 1: Microfilaria of *Wuchereria*.

Figure 2: Microfilaria of *Brugia malayi*.

Figure 3: Patient with lymphedema.

Lymphatic filariasis: the disease

Figure 4: Endemic areas for LF in red.

Lymphatic filariasis: the vector

Figure 5: *Anopheles*.

Figure 6: *Culex*.

Figure 7: *Aedes*.

Lymphatic filariasis: the life cycle

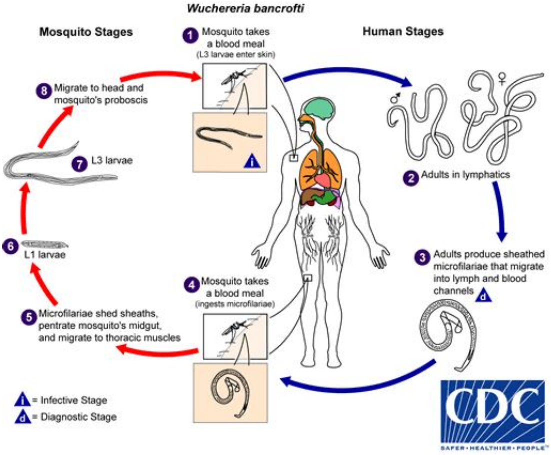

Figure 8: Life Cycle of *Wuchereria bancrofti*.

Lymphatic filariasis: diagnosis

Figure 9: Counting microfilariae at night.

Figure 10: ICT card for LF antigens detection.

Research question

- The data

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Objective

How should we build a bivariate geostatistical model for $p_1(x)$ and $p_2(x)$?

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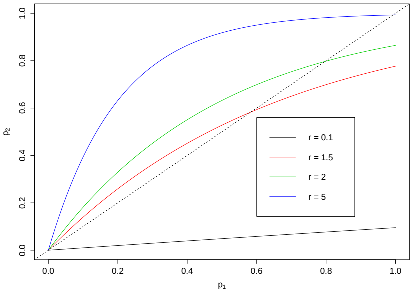

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- If $\gamma = 1$ and $f(p_1(x)) = \log\{p_1(x)/[1-p_1(x)]\} - d_1(x)^\top\beta_1 - Z_1(x)$, we recover Giorgi, Sesay, Terlouw and Diggle (2015).

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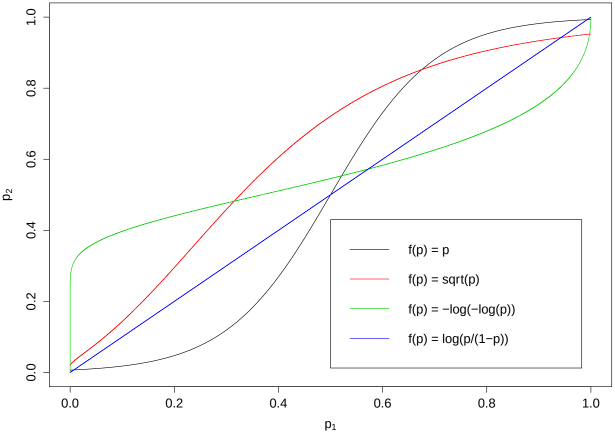

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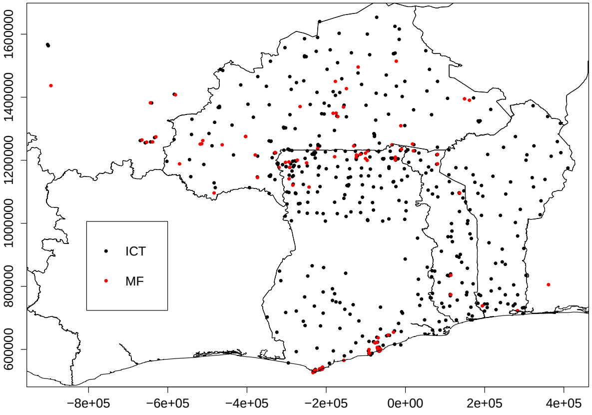
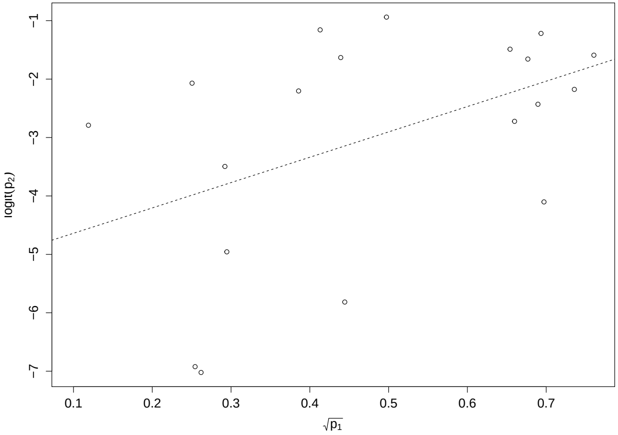
- 479 ICT surveys; on average 61 individuals sampled per village.
- 90 MF surveys; on average 245 individuals sampled per village.



Empirical relationship



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Term	Estimate	95% CI
μ	-2.562	(-3.991, -1.132)
α	0.722	(0.636, 0.820)
γ	0.106	(0.010, 0.203)
σ^2	2.974	(1.392, 6.355)
ϕ	275.995	(117.374, 648.980)
ν^2	0.201	(0.091, 0.445)

Estimation (2)

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Term	Estimate	95% CI
μ_1	-2.244	(-4.201, -0.287)
μ_2	-3.055	(-3.763, -2.348)
γ	0.476	(0.200, 0.752)
σ_1^2	4.205	(1.667, 10.608)
ϕ_1	354.055	(126.608, 990.106)
ν_1^2	0.172	(0.066, 0.449)
σ_2^2	1.796	(0.909, 3.550)
ϕ_2	82.555	(34.412, 198.052)
ν_2^2	0.228	(0.069, 0.760)

Model diagnostic (1)

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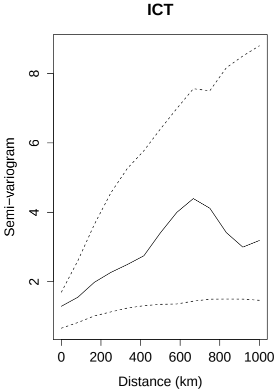
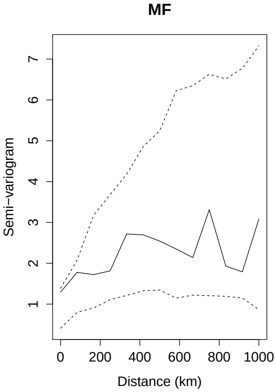
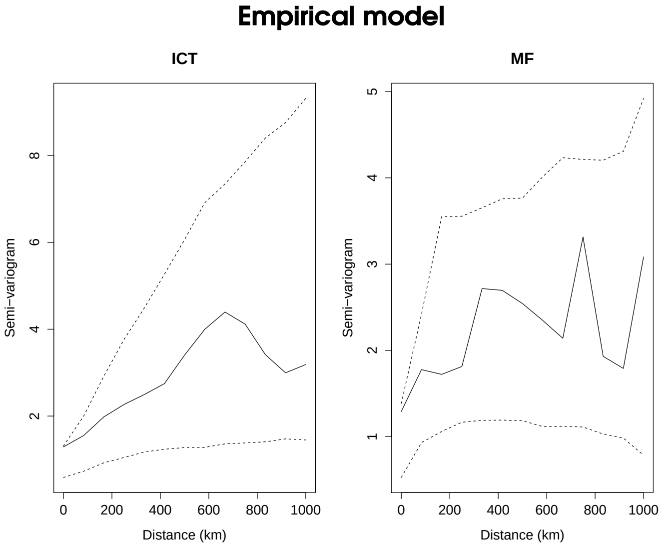




Model diagnostic (2)



Exceeding 1% MF prevalence

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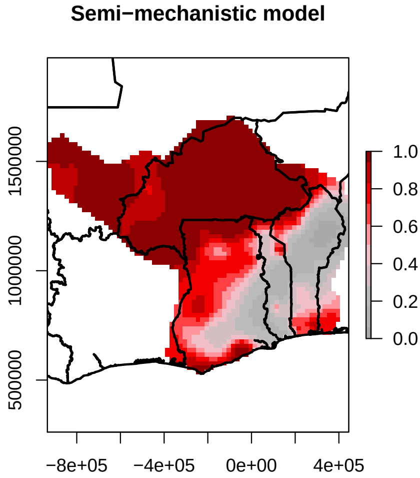

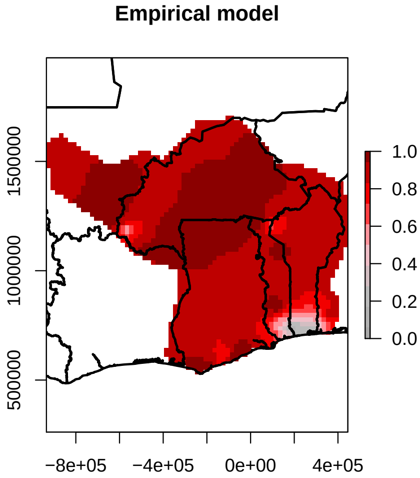

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Thank you for your attention!

Bibliography

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