Covariate effects in oceanographic applications of marginal, conditional and spatial extremes

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Lancaster STORi extremes workshop
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Covariates in extremes
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Motivation

- Rational and consistent design and assessment of marine structures
  - Reduce bias and uncertainty in estimation of structural integrity
  - Quantify uncertainty as well as possible
- Non-stationary marginal, conditional and spatial extremes
  - Multiple locations, multiple variables, time-series
  - Multidimensional covariates
- Improved understanding and communication of risk
  - Incorporation within established engineering design practices
  - Knock-on effects of improved inference
- Other current applications in Shell
  - Earthquake hazards
  - Corrosion and fouling
Motivation

- Environmental extremes vary smoothly with multidimensional covariates
  - Model parameters are functions of covariates

- Uncertainty quantification for whole inference
  - Data acquisition (simulator or measurement)
  - Data pre-processing (storm peak identification)
  - Extreme value threshold
  - Model form (marginal measurement scale effect, spatial extremal dependence)

- Statistical and computational efficiency
  - Slick algorithms
  - Parallel computation
  - Bayesian inference
Motivation: storm model

$H_S \approx 4 \times$ standard deviation of ocean surface time-series at specific location corresponding to a specified period (typically three hours)
Covariate effects in:

- Marginal models
  - Simple introductory example (directional model)
  - Storm peak $H_S$ with 2D, 3D and 4D covariates
- Conditional extremes models
  - Associated values of other wave field parameters given extreme storm peak $H_S$
- Spatial extremes models
  - Directional dependence in max-stable process parameters for storm peak $H_S$

North Sea example used as “connecting theme”; other examples to embellish
Outline: North Sea application

Storm peak $H_S$ from gridded NEXTRA winter storm hindcast for North Sea locations; directional variability in storm severity; “strips” of locations with different orientations; central location for directional model.
Marginal: simple gamma-GP model

- Sample of peaks over threshold $y$, with covariates $\theta$
  - $\theta$ is 1D in motivating example: directional
  - $\theta$ is $nD$ later: e.g. 4D spatio-directional-seasonal
- Below threshold $\psi$
  - $y$ follows truncated gamma with shape $\alpha$, scale $1/\beta$
  - Hessian for gamma better behaved than Weibull
- Above $\psi$
  - $y$ follows generalised Pareto with shape $\xi$, scale $\sigma$
  - $\xi, \sigma, \alpha, \beta, \psi$ all functions of $\theta$
  - $\psi$ for pre-specified threshold probability $\tau$
    - Generalise later to estimation of $\tau$

- Frigessi et al. [2002], Behrens et al. [2004], MacDonald et al. [2011]
- Randell et al. [2016]
Marginal: simple gamma-GP model

- Density is $f(y|\xi, \sigma, \alpha, \beta, \psi, \tau)$

$$= \begin{cases} \tau \times f_{TG}(y|\alpha, \beta, \psi) & \text{for } y \leq \psi \\ (1 - \tau) \times f_{GP}(y|\xi, \sigma, \psi) & \text{for } y > \psi \end{cases}$$

- Likelihood is $L(\xi, \sigma, \alpha, \beta, \psi, \tau|\{y_i\}_{i=1}^n)$

$$= \prod_{i:y_i \leq \psi} f_{TG}(y_i|\alpha, \beta, \psi) \prod_{i:y_i > \psi} f_{GP}(y_i|\xi, \sigma, \psi) \times \tau^{n_B}(1 - \tau)^{(1-n_B)} \text{ where } n_B = \sum_{i:y_i \leq \psi} 1.$$  

Estimate all parameters as functions of $\theta$
Marginal: count rate $c$

- Whole-sample rate of occurrence $\rho$ modelled as Poisson process given counts $c$ of numbers of occurrences per covariate bin

- Chavez-Demoulin and Davison [2005]
Physical considerations suggest $\alpha, \beta, \rho, \xi, \sigma, \psi$ and $\tau$ vary smoothly with covariates $\theta$.

Values of $\eta \in \{\alpha, \beta, \rho, \xi, \sigma, \psi, \tau\}$ on some index set of covariates take the form $\eta = B\beta_\eta$.

For $nD$ covariates, $B$ takes the form of tensor product $B_{\theta_n} \otimes \ldots \otimes B_{\theta_\kappa} \otimes \ldots \otimes B_{\theta_2} \otimes B_{\theta_1}$.

Spline roughness with respect to each covariate dimension $\kappa$ given by quadratic form $\lambda_{\eta \kappa} \beta'_{\eta \kappa} P_{\eta \kappa} \beta_{\eta \kappa}$.

$P_{\eta \kappa}$ is a function of stochastic roughness penalties $\delta_{\eta \kappa}$.

Brezger and Lang [2006]
Marginal: priors and conditional structure

Priors

\[ \text{density of } \beta_{\eta \kappa} \propto \exp \left( -\frac{1}{2} \lambda_{\eta \kappa} \beta'_{\eta \kappa} P_{\eta \kappa} \beta_{\eta \kappa} \right) \]

\[ \lambda_{\eta \kappa} \sim \text{gamma} \]

( and \( \tau \sim \text{beta, when } \tau \text{ estimated} \) )

Conditional structure

\[ f(\tau|y, \Omega \setminus \tau) \propto f(y|\tau, \Omega \setminus \tau) \times f(\tau) \]

\[ f(\beta_{\eta}|y, \Omega \setminus \beta_{\eta}) \propto f(y|\beta_{\eta}, \Omega \setminus \beta_{\eta}) \times f(\beta_{\eta}|\delta_{\eta}, \lambda_{\eta}) \]

\[ f(\lambda_{\eta}|y, \Omega \setminus \lambda_{\eta}) \propto f(\beta_{\eta}|\delta_{\eta}, \lambda_{\eta}) \times f(\lambda_{\eta}) \]

\[ \Omega = \{\alpha, \beta, \rho, \xi, \sigma, \psi, \tau\} \]
Marginal: inference

- Elements of $\beta_\eta$ highly interdependent, correlated proposals essential for good mixing
- “Stochastic analogues” of IRLS and back-fitting algorithms for maximum likelihood optimisation used previously
- Estimation of different penalty coefficients for each covariate dimension

- Gibbs sampling when full conditionals available
- Otherwise Metropolis-Hastings (MH) within Gibbs, using suitable proposal mechanisms
  - mMALA where possible

- Roberts and Stramer [2002], Girolami and Calderhead [2011], Xifara et al. [2014]
Marginal: posterior parameter

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Marginal: posterior roughness penalty

Different scales so must be careful: rate is roughest, GP shape is smoothest

Smoothness Parameters: $\lambda$

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Marginal: validation

Compare sample with simulated values on partitioned covariate domain

Covariates in extremes
Marginal: extension to 2D

Directional-seasonal model for location in northern North Sea; $\tau$ estimated; land-shadow effect of Norway obvious; Randell et al. [2016]
Marginal: extension to 2D

Summary statistics for return value distributions

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Spatio-directional-seasonal model for location in South China Sea; ML/CV/BS estimation; bootstrap median estimate after integration over season; clear spatial and directional effects; Raghupathi et al. [2016]
Marginal: extension to 4D

Bootstrap median estimate after integration over direction; clear spatial and seasonal effects
Conditional: summary

- Heffernan and Tawn [2004] and derivatives
- Evidence for covariate effects in conditional extremes of sea-state and storm peak variables
  - Marginal non-stationary extreme value model
  - Marginal transformation to standard scale removing marginal covariate dependence
  - Conditional dependence structure showing covariate effects
- Examples
  - Wave peak period | Significant wave height
  - Ocean current at one depth | Current at another depth
  - Significant wave height | Wind speed
  - Weather-vaning
Conditional: $T_P | H_S$ example

On Gumbel scale, extend with covariates $\theta$

$$(Y_2 | Y_1 = y, \theta) = \alpha_\theta y + y^{\beta_\theta} (\mu_\theta + \sigma_\theta Z) \text{ for } y > \psi_\theta(\tau)$$

where

- $\psi_\theta(\tau)$ is a high non-stationary quantile of $Y_1$ on Gumbel scale, for non-exceedance probability $\tau$, above which the model fits well
- $\alpha_\theta \in [0, 1]$, $\beta_\theta \in (-\infty, 1]$, $\sigma_\theta \in [0, \infty)$
- $Z$ is a random variable with unknown distribution $G$, assumed Normal for estimation

Application

- Estimate spectral peak wave period $T_P$ for storm sea states with extreme severity (energy) $H_S$
- In $T_P, H_S$ case, $\psi = \theta_j = \theta_k$
- Jonathan et al. [2014]
Conditional: $T_P|H_S$ example

ML/CV/BS inference; uncertainty bands capture uncertainty from marginal and dependence estimation; in conditional model, only $\alpha$ shows directional effect; reduction in conditional return value
Spatial: outline

Why do spatial extremes?

- Improved inference at one location using data from spatial neighbourhood
- Insurance risk of damage to multiple structures from single “event”

Evidence for covariate effects in spatial extremes of storm peak significant wave height

- Neighbourhood of spatial locations
- Storm peak events corresponding to storm events observed at all spatial locations
- Marginal transformation per location to standard scale removing marginal covariate dependence
- Extremal spatial dependence structure showing anisotropy and location effect
Spatial: North Sea application

Storm peak $H_5$ from gridded NEXTRA winter storm hindcast for North Sea locations; directional variability in storm severity; “strips” of locations with different orientations on bicycle wheel; multiple strips with same orientation
Spatial: storm on physical and Frechet scales

Storm peak $H_S$ on physical and Frechet scales; marginal effects important
Estimates for (a) $\eta$ and (b) $\chi(x)$ against estimates for Spearman's $\rho$ for sample size $n = 10^6$, from the Smith (magenta), Schlather (red), Brown-Resnick (blue), extremal-t (green) and Gaussian (black) processes, and the inverted logistic distribution (cyan). Estimation methods use model (19) for $\eta$ with $q = 0.99$, and the empirical estimate for $\chi(x)$ with $x = 100$. Solid lines are median estimates from 1000 sample replications, dashed lines give 2.5% and 97.5% quantiles.

- $\eta = 1$ AD, $\chi = 0$ AI
- AD: Smith (magenta), Schlather (red), Brown-Resnick (blue)
- AI: extremal-t (green), Gaussian (black)
- Kereszturi et al. [2016]
Spatial: Diagnosing dependence - all sea states

Estimates of $\eta$ with (a) $q = 0.90$ and (c) $q = 0.99$, and $\chi(x)$ with (b) $x = 10$ and (d) $x = 100$, plotted against Spearman’s $\rho$ for sea-state $H_S$ sample of size $n = 58585$. Coloured points identify estimates from corresponding strip. Lines identify estimates using simulated samples of same size from Smith (black) and Gaussian (red) processes, and from the inverted logistic distribution (green); Kereszturi et al. [2016]

- $\eta$ for $q = 0.9$, $\chi(x)$ for $x = 10$; $n = 58585$ individual sea states
- AD: Smith (black)
- AI: Gaussian (red), inverted logistic (green)
Spatial: Diagnosing dependence - all sea states

Estimates of $\eta$ with (a) $q = 0.90$ and (c) $q = 0.99$, and $\chi(x)$ with (b) $x = 10$ and (d) $x = 100$, plotted against Spearman’s $\rho$ for sea-state $H_S$ sample of size $n = 58585$. Coloured points identify estimates from corresponding strip. Lines identify estimates using simulated samples of same size from Smith (black) and Gaussian (red) processes, and from the inverted logistic distribution (green); Kereszturi et al. [2016]

- $\eta$ for $q = 0.99$, $\chi(x)$ for $x = 100$; $n = 58585$ sea states
- AD: Smith (black)
- AI: Gaussian (red), inverted logistic (green)
Spatial: Diagnosing dependence - storm peaks

 Estimates of (a) $\eta$ with $q = 0.90$ and (b) $\chi(x)$ with $x = 10$, plotted against Spearman’s $\rho$ for storm-peak $H_S$ sample of size $n = 916$. Points and lines as described in previous slide; Kereszturi et al. [2016]

- $\eta$ for $q = 0.9$, $\chi(x)$ for $x = 10$; $n = 916$ storm peak events
- AD: Smith (black)
- AL: Gaussian (red), inverted logistic (green)
Spatial: models and estimation

Marginal model
- Estimate non-stationary model
- Propagate uncertainty; this will be large in general

Max-stable process
- Smith, Schlather, Brown-Resnick, ...
- All AD; conservative estimates
- Wadsworth and Tawn [2012], Davison et al. [2012]

Composite likelihood
- Full likelihood unavailable; approximated by pairwise
- Padoan et al. [2010]

Censored likelihood
- Dependence structure required for peaks over threshold margins not block maxima
- Threshold selection required; confirmed choice not affecting main inferences; need to propagate uncertainty
- Huser and Davison [2014]
Spatial: Smith

Construction
- Max-stable process $Z(x) \sim \max_{i \geq 1} \xi_i f(x, U_i)$
- $f(x, U_i) \sim N(U_i, \Sigma)$
  - storm profile
- $\xi_i =$ storm intensity
  - from point process
- $U_i =$ storm centre
  - uniform RV

Estimation
- (Censored composite) likelihood available but messy
- 1D (“strip”): Estimate $\Sigma = \sigma^2$
- 2D (neighbourhood): Estimate $\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$
Spatial: Smith dependence anisotropy

Box plots: 1D parameter estimates; black lines: 2D parameter estimates; ML estimates with bootstrap 95% uncertainty bands accounting for uncertainty from marginal and dependence estimation; spatial dependence is higher WE than NS, consistent with large spatial events sweeping down from north.
Spatial: Schlather & Brown-Resnick

Construction

- Max-stable process \( Z(x) \sim \max_{i \geq 1} \xi_i Y_i(x) \)
- Schlather
  - \( Y_i(x) = \) standard normal Gaussian field, correlation \( \rho(h) \)
  - \( \rho(h) = \exp(-0.5h'\Sigma^{-1}h) \) here
- Brown-Resnick

\[
Y_i(x) = \frac{\exp(W_i(x) - \gamma(x - U_i))}{n^{-1} \sum_{l=1}^{n} \exp(W_i(x_l) - \gamma(x_l - U_i))}
\]

- \( W_i(x) = \) fractional Brownian motion, Hurst parameter \( H \in [0, 1] \), variogram \( 2\gamma(h) = (h'\Sigma^{-1}h)^H \)
- Dieker and Mikosch [2014]

Estimation

- 1D: Estimate \( \Sigma = \sigma^2 \) (for range of \( H \) with BR)
- 2D: Estimate \( \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} \)
Spatial: Dependence anisotropy

Smith, Schlather and Brown-Resnick consistent; confirmed that censored likelihood threshold not affecting relative size of dependence with direction
Spatial: Dependence location effect?

Box plots: 1D parameter estimates; black lines: 2D parameter estimates; ML estimates with bootstrap 95% uncertainty bands accounting for uncertainty from marginal and dependence estimation; spatial dependence is higher WE than NS, consistent with large spatial events sweeping down from north.
Spatial: Dependence location effect?

Box plots: 1D parameter estimates; black lines: 2D parameter estimates; ML estimates with bootstrap 95% uncertainty bands accounting for uncertainty from marginal and dependence estimation; spatial dependence is higher WE than NS, consistent with large spatial events sweeping down from north.
Summary

- Evidence for covariate effects in marginal, conditional and spatial extremes of ocean storms
  - Modelling non-stationarity essential for understanding extreme ocean storms, and estimating marine risk well
  - Non-parametric P-spline flexible basis for covariate description
  - Essential that non-stationary models are used for marginal, conditional and spatial extremes inference of ocean environment
  - Cradle-to-grave uncertainty quantification

- Further investigation of covariate effects in spatial ocean extremes needed
  - Anisotropy in North Sea hindcast, maybe absolute location (or fetch) effect?
  - Currently examining satellite altimeter measurements
  - Asymptotic independence?

- Goal: Bayesian inference for whole-basin spatial models with 4D covariates


