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General Routing Problem

GRP

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MSC2000: 90B20

Article Outline

Keywords

Routing

The *general routing problem* (GRP) is a routing problem defined on a graph or network where a minimum cost tour is to be found and where the route must include visiting certain required vertices and traversing certain required edges. More formally, given a connected, undirected graph G with vertex set V and (undirected) edge set E, a cost c_e for traversing each edge e

$\in E$, a set $V_R \subseteq V$ of *required vertices* and a set $E_R \subseteq E$ of *required edges*, the GRP is the problem of finding a minimum cost vehicle route, starting and finishing at the same vertex, passing through each $v \in V_R$ and each $e \in E_R$ at least once ([13]).

The GRP contains a number of other routing problems as special cases. When $E_R = \emptyset$, the GRP reduces to the *Steiner graphical traveling salesman problem* (SGTSP) ([4]), also called the *road traveling salesman problem* in [7]. On the other hand, when $V_R = \emptyset$, the GRP reduces to the *rural postman problem* (RPP) ([13]). When $V_R = V$, the SGTSP in turn reduces to the *graphical traveling salesman problem* or GTSP ([4]). Similarly, when $E_R = E$, the RPP reduces to the *Chinese postman problem* or CPP ([5,8]).

The CPP can be solved optimally in polynomial time by reduction to a matching problem ([6]), but the RPP, GTSP, SGTSP and GRP are all NP-hard. This means that the computational effort to solve such a problem increases exponentially with the size of the problem. Therefore exact algorithms are only practical for a GRP if it is not too large, otherwise a heuristic algorithm is appropriate. The GRP was proved to be NP-hard in [10].

In [3], an integer programming formulation of the GRP is given, along with several classes of valid inequalities which induce facets of the associated polyhedra under mild conditions. Another class of valid inequalities for the GRP is introduced in [11] and in [12] it is shown how to convert facets of the GTSP polyhedron into valid inequalities for the GRP polyhedron. These valid inequalities form the basis for a promising branch and cut style of algorithm described in [2] which can solve GRPs of moderate size to optimality.

In [9], a heuristic algorithm for the GRP is described. The author adapts Christofides' heuristic for the TSP to show that when the triangle inequality holds in the graph, the heuristic has a worst-case ratio of heuristic solution value to optimum value of 1.5.

There are many vehicle routing applications of the GRP. In these cases, the edges of the graph are used to represent streets or roads and the vertices represent road junctions or particular locations on a map. In any practical application there are likely to be many additional constraints which must also be taken into account such as the capacity of the vehicles, time-window constraints for when the service may be carried out, the existence of one-way streets and prohibited turns etc.

Many applications are for the special cases when either $E_R = \emptyset$ or $V_R = \emptyset$. However, there are some types of vehicle routing applications where the problem is most naturally modeled as a GRP with both required edges and required vertices. For example, in designing routes for solid waste collection services, collecting waste from all houses along a street could be modeled as a required edge and collecting waste from the foot of a multistory apartment block could be modeled as a required vertex. Other examples include postal delivery services where some customers with heavy demand might be modeled as required vertices, while other customers with homes in the same street might be modeled together as a required edge. School bus services are other examples of GRPs where a pick-up in a remote village could be modeled as a required vertex, but if the school bus must pick-up at some point along a street (and is not allowed to perform a U-turn in the street) then that may best be modeled as a required edge.

Further details about solution methods and applications for various network routing problems can be found in [1].

See also

- ▶ Stochastic Vehicle Routing Problems
- ▶ Vehicle Routing
- ▶ Vehicle Scheduling

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Genetic Algorithms

GA

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Article Outline

Keywords

Optimization; Genetic algorithms; Evolution; Stochastic global optimization; Population; Fitness; Crossover; Mutation; Binary encoding; Individual; Chromosome; Generation; Elitism; Premature convergence; Gray code; Random walk search; Roulette wheel procedure; Population size; Schema theorem; Schema; Local minimum; Selection; Evolution strategy

Genetic algorithms (GAs) comprise a class of *stochasticglobal optimization* methods based on several strategies from biological evolution. The basic genetic algorithm was developed by J.H. Holland and his students ([5,6,7,8]), and was based on the observation that selection (either natural or artificial) can produce highly optimized individuals in a relatively short number of generations. This is true despite the fact that the space of all gene mutations through which a population must sort is astronomical. For instancethe genome of the yeast *Saccharomyces cerevisiae*, which is the simplest eukaryote, contains just over 6000 genes, each of which can occur in several mutant forms. Despite this, *S. cerevisiae* can reoptimize itself to survive and flourish in many new environments in a relatively short number of generations. This is equivalent to having a computer search for a near-optimal solution to a 6000-dimensional problem where each of the 6000 variables can take on any one of a large number of values.

The most important notion from natural systems that the GA employs is the use of a *population* of individuals which go through a *selection* step to produce offspring and pass on their genetic material.Optimality or *fitness* is measured by how many offspring an individual produces. A second notion is the use of *crossover* in which individuals share genetic information and pass the shared information onto their offspring. A third borrowing from nature is the idea of *mutation*, the consequence of which is that the transfer of genetic informationis prone to random errors. This helps maintain the level of genetic diversity in a population.

The implementation of a simple GA (SGA) which uses these ideas is straightforward. The description that follows uses a *binary encoding*, but all of the ideas follow identically for integer or even real number encodings. The most important idea is that one works with a population of *individuals* which will interact through genetic operators to carry out an optimization process. An individual is specified by a *chromosome* C which is a bit string of length N_c that can be decoded to give a set of N parameters x_i which are the natural parameters for the optimization application. Each parameter x_i is encoded by n_i bits so that $\sum_i^N n_i = N_c$. In what follows, chromosome and bit string are synonymous. A fitness function $f(x_1, \dots, x_N)$, which is the function to be optimized, is used to rank the individual chromosomes. An initial population of N_{pop} individuals is formed by choosing N_{pop} bit strings at random, and evaluating each individual's fitness. (Decode $C \to (x_1, \dots, x_N)$, calculate $f(x_1, \dots, x_N)$.)Subsequent *generations* are formed as follows. All parents (members of the current generation) are ranked by fitness and the highest fitness individual is placed directly into the next generation with