

Towards Schenkerian Analysis by Computer: Deriving the Harmonic-Contrapuntal Structure of Music Automatically

Alan Marsden
Lancaster Institute for the Contemporary Arts,
Lancaster University

Schenkerian Analysis

Progressively reduces a score, removing less essential features, to reveal the 'background' structure.

Mozart:



A musical score for a piece by Mozart, likely in G major and 6/8 time. The score is written for piano and consists of four measures. The right hand features a melodic line with various ornaments and grace notes, while the left hand provides a rhythmic accompaniment with chords and single notes. The overall texture is dense and characteristic of the Classical period.

Schenker:



A Schenkerian analysis of the Mozart score, showing the underlying structural framework. The score is written for piano and consists of two measures. The right hand features a single melodic line with a long, sweeping curve, while the left hand provides a simple harmonic accompaniment with a few notes. This analysis reveals the essential structure of the piece, stripping away all ornaments and secondary details.

Alternative Analyses

Forte & Gilbert:

EXAMPLE 139. Mozart, *Sonata in A major*, K. 331, I

The image displays two alternative analyses of a musical passage from Mozart's Sonata in A major, K. 331, I. Analysis (a) shows the original notation with circled numbers 1 and 5. Analysis (b) shows the same passage with fingerings (5, 4, 3, 2, N, 5, 4, 3, 2, 1) and guitar-style fingering (10, 10, 10). Below the bass staff of analysis (b) are chord symbols: I, I, V_s⁶, I, II⁶, V, I, I, V_s⁶, I, II⁶, V, I, I.

Multi-levelled; Tree-like

Lerdahl & Jackendoff:
(Actually a somewhat
different theory from
Schenker, but does
something similar)

The diagram illustrates a multi-levelled, tree-like structure of music. It shows a musical score with five staves, labeled 'a' through 'e' from bottom to top. Above the staves is a tree diagram with nodes labeled 'a', 'b', 'c', 'd', and 'e'. The tree starts at node 'a' at the top, which branches into 'b' and 'c'. Node 'b' branches into two 'c' nodes. The left 'c' node branches into two 'd' nodes, and the right 'c' node branches into 'd', 'e', and 'e' nodes. The musical staves below show the corresponding musical notation for each level: 'a' is a single chord, 'b' is a two-chord phrase, 'c' is a four-chord phrase, 'd' is an eight-chord phrase, and 'e' is the full musical score with various rhythmic and melodic details.

Benefits

- The most influential and widely adopted theory and method of analysis for tonal music since the last quarter of the 20th c.
- Adumbrates many aspects of musical structure (key, harmony, segmentation, metre).
- Some evidence that it corresponds to perception and cognition of music.
- Based on two centuries of previous music theory.

BUT does remain controversial among musicians, and suffers from obscure arguments about detail.

Previous Work

- Kassler (1967, 1975, 1977, 1988)
 - program which successfully analyses three-voice middlegrounds
- Smoliar et al. (1976, 1978, 1980)
 - program capable of verifying an analysis
- Lerdahl & Jackendoff (1983, 2001)
 - rule-based system for quasi-Schenkerian reduction
 - not demonstrably computable
- Mavromatis & Brown (2004)
 - demonstration of theoretical possibility of Schenkerian analysis by context-free grammar
- Hamanaka, Hirata & Tojo (2005-7)
 - implementation of Lerdahl & Jackendoff reduction with adjustment of parameters (now moving towards automatic parameter-setting)
- Gilbert & Conklin (2007)
 - probabilistic grammar for melodic reduction

Formalisation (non contentious)

- 1) Notes are defined by pitch and time (start and duration).
- 2) All notes on the 'surface' of the piece derive by a process of iterative elaboration of a single chord (i.e., several notes all with the same start and duration).
- 3) Only certain kinds of elaboration are possible.
- 4) Elaborations can have an associated key and harmony.
- 5) Simultaneous elaborations (in different parts/voices) must be consistent in key and harmony.

A piece of music is a tree-like structure of elaborations, BUT it has simultaneous trees (for different voices) and these may intertwine (a note can belong to more than one tree).

Elaborations

The image displays two systems of musical notation, each consisting of two staves. The first system is divided into two sections: the first two measures are in 2/4 time, and the last three measures are in 3/4 time. Above the first two measures, the chord is indicated as (G maj.), and above the last three measures, it is (E min.).

The first system includes the following labels in yellow boxes:

- repetition (two instances)
- consonant skip (two instances)
- neighbour note (one instance)

The second system includes the following labels in yellow and orange boxes:

- passing (two instances)
- appoggiatura (one instance)
- suspension (one instance)
- unfolding (one instance)

Labels are connected to the notes on the staves by thin lines, and dotted lines indicate connections between notes in adjacent measures.

Further detail in Marsden, *CHum* (2001) and *JNMR* (2005).

Formalisation (contentious)

- 6) All elaborations produce two 'children'.
- 7) All elaborations have one 'parent' note.
(So trees are binary. Special 'note sequences' are produced in extended passing elaborations. Unfoldings, which should have multiple parents, are represented by multiple elaborations.)
- 8) Elaborations may require a specific preceding or following 'context note'.
(So branches of trees are not independent of each other.)

Restrictions (Temporary?)

In order to allow a less inefficient analysis algorithm:

- 9) Simultaneous branching in trees must produce children with the same durations in each tree.
- 10) Preceding context notes must be present on the surface (e.g., in the case of the preparation of a suspension).
- 11) Voices cannot cross each other.

Plus some arbitrary restrictions to avoid crazy solutions:

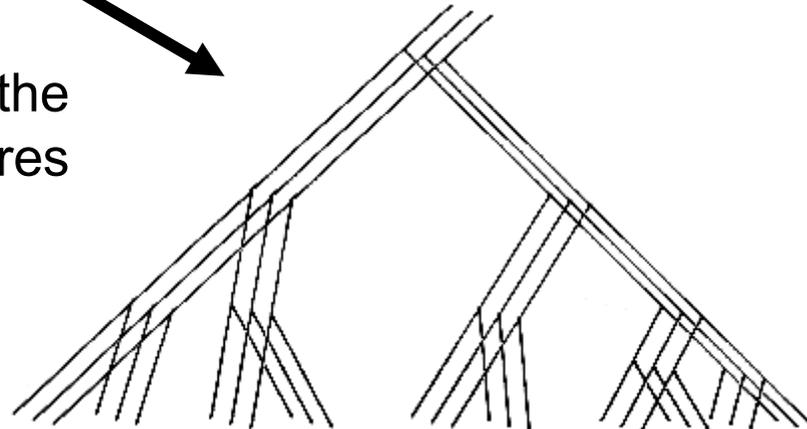
- 12) Chords in reductions must not be larger than a certain small number of notes.
- 13) Pairs of notes reduced must have a moderately simple ratio of durations.

The Problem



From the score ...

... to derive the
tree structures



Local Solution-Finding

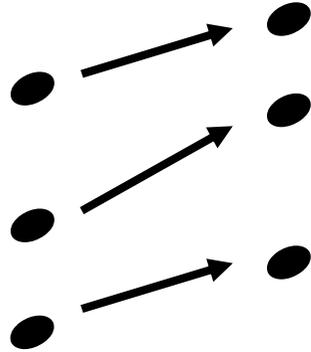
For any pair of notes, given knowledge of the preceding notes (on the surface) and possible and actual following notes (both on the surface and at higher levels), we can determine:

- which elaborations, if any, can produce these notes,
- what the parent note must be for each elaboration,
- what the requirements of key and harmony are for each elaboration.

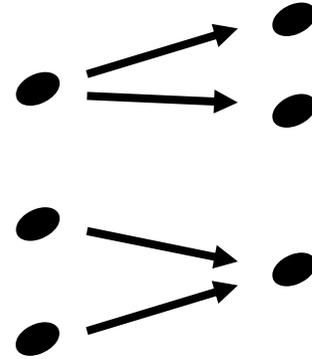
So, given any pair of consecutive chords, knowledge of preceding and following chords, and rules of harmonic and tonal consistency, we can determine the possible parent chords of that sequence.

Combinatorial Problems

1. Voices



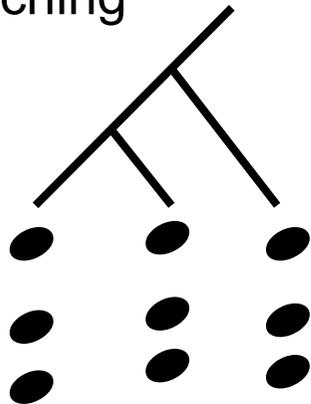
or



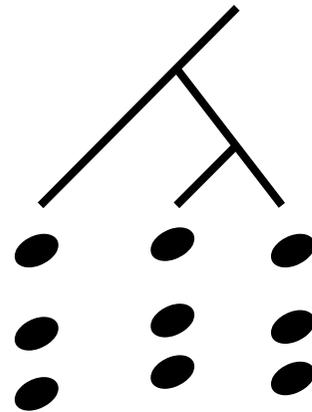
etc. ?

Increases exponentially with the size of a piece

2. Branching



or



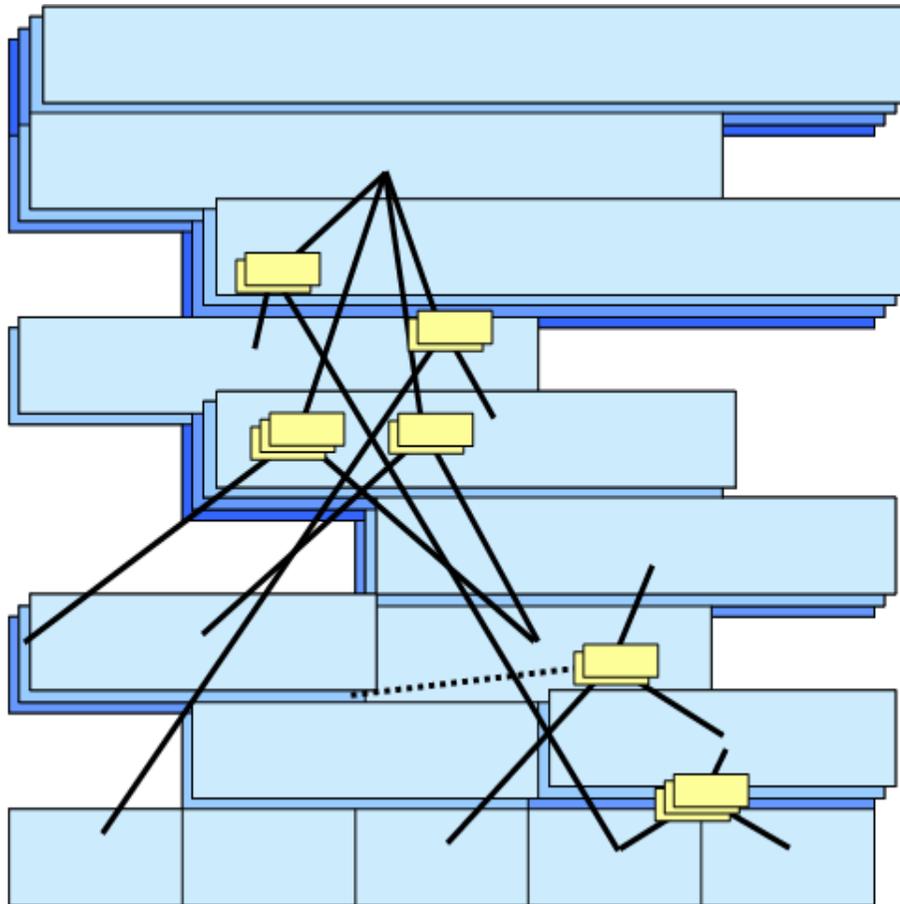
?

Increases factorially with the size of a piece

Attempted Solution

- Inspired by dynamic programming.
- Construct a 3D matrix of valid local solutions.
 - lowest level is all the ‘chords’ of the surface of the piece:
1D, n cells
 - higher levels are all possible chords derived by reduction from all possible pairs of chords below:
2D, $(n - l) * x$ cells
(l level of reduction, x unknown but limited number of possibilities)
- Any valid reduction tree can be derived from the matrix by selecting a top-level cell and then iteratively selecting pairs of possible children.

Illustration



- If we can know metrics of ‘goodness’ for local solutions, the best analysis can be derived by selecting the best children at each point. BUT, there are no accepted metrics.
- In principle, a matrix can be derived in $O(n^3)$ space and $O(n^4)$ time. BUT, 4 bars of Mozart took 170MB of heap space and 1hr 45mins!

Demonstration software

New Segment >>

<< New Segment

Delete Segment

Load Example 1

Load Example 2

Load Example 3

Make Reduction

Clear Reduction

Clear All

Score by: Elaborations per note ▼ Rescore

Show: Best scoring Full All

Pitch-class spelling

C# D# F# G# A#
 Db Eb Gb Ab Bb

row 1 column 0-1

Up
Down
Back
Fwd

Select
Clear

%	Notes
100	F5
67	C5
67	A4

Segment 2 of 3

Previous
Next
Select
Delete

Notes	Cumulative score: 1.0
F5	
A4	

Harmony PCs [F,A]
 Scale PCs [F,A]
 Precontext Pitches []
 Postcontext Pitches []

Row 3	0-3	1-4	2-5	3-6	4-7	5-8			
	58 A5		57 A5	100 _A5	70 G5				
	69 F5		43 G5	33 G5	60 E5				
	54 C5		29 C5	100 C5_	70 C5				
	50 A4		29 Bb4	33 C5	70 Bb4				
	65 F4		86 F4						
Row 2	0-2	1-3	2-4	3-5	4-6	5-7	6-8		
	100 A5_	50 A5		100 _A5	100 G5	100 G5	100 F5		
	50 F5	100 _F5		50 G5	100 C5_	50 E5	67 C5		
	50 C5	100 C5		100 C5	50 Bb4	100 C5	67 A4		
	58 A4	100 F4				100 _Bb4			
	58 F4								
Row 1	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	
	100 F5		100 A5		100 G5	100 G5	67 G5	100 F5	
	67 C5		50 C5		100 Bb4	100 C5_	67 E5	100 _C5	
	67 A4		100 F4			100 _Bb4	100 C5	100 A4	
Row 0	0	1	2	3	4	5	6	7	8
	100 F5_	100 _F5	100 A5_	100 _A5	100 A5	100 G5	100 G5	100 E5	100 F5
	100 A4	100 C5	100 F4	100 C5	100 Bb4_	100 _Bb4	100 C5_	100 _C5	100 A4

Demonstration software (2)

New Segment >>

<< New Segment

Delete Segment

Load Example 1

Load Example 2

Load Example 3

Make Reduction

Clear Reduction

Clear All

Score by: Elaborations per note ▼ Rescore

Show: Best scoring Full All

Pitch-class spelling

C# D# F# G# A#

Db Eb Gb Ab Bb

row column

%	Notes
56	A5
56	F5
59	C5
56	A4
56	F4

Segment of

Notes
F5
C5
A4

Cumulative score: 6.4166665

Harmony PCs [C,F,A]

Scale PCs [C,F,G,A]

Precontext Pitches

Postcontext Pitches

```

Row 8
0-8
56 A5
56 F5
59 C5
56 A4
56 F4
Row 7
0-7      1-8
51 A5      54 A5
35 G5      100 _F5
29 F5      56 F5
37 E5      67 C5
59 C5      56 A4
12 Bb4     54 F4
53 A4
27 F4
Row 6
0-6      1-7      2-8
44 A5      20 A5      62 A5
17 G5      100 _F5     67 F5
46 F5      100 C5      57 C5
100 C5_    20 F4      62 A4
49 C5      62 F4
    
```

Demonstration software (3)

New Segment >>

<< New Segment

Delete Segment

Load Example 1

Load Example 2

Load Example 3

Make Reduction

Clear Reduction

Clear All

Score by: Elaborations per note ▼ Rescore

Show: Best scoring Full All

Pitch-class spelling

C# D# F# G# A#

Db Eb Gb Ab Bb

row column

%	Notes
100	F5
100	C5
100	A4

Segment of

Notes
F5
C5
A4

Cumulative score: 0.6666667

Harmony PCs [C,F,A]

Scale PCs [C,F,G,A]

Precontext Pitches []

Postcontext Pitches []

Row 5	0-5	1-6	2-7	3-8				
	50 G5							
	100 F5							
	100 Bb4							
	50 F4							
Row 4	0-4	1-5	2-6	3-7	4-8			
Row 3	0-3	1-4	2-5	3-6	4-7	5-8		
	100 F5							
	100 C5							
	100 A4							
	100 F4							
Row 2	0-2	1-3	2-4	3-5	4-6	5-7	6-8	
							100 F5	
							100 C5	
							100 A4	
Row 1	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8
	100 F5		100 A5		100 G5		100 G5	
	100 C5		100 C5		100 Bb4		100 E5	
	100 A4		100 F4				100 C5	

Further Work

- Revisions to make reduction procedure more efficient
 - minimising number of segments recorded (separation of constraints and chords)
 - tightening of harmonic constraints (e.g., avoidance of sevenths)
- Testing on 'ground truths' from published analyses
 - Oster archive (Chopin, Beethoven)
 - experimentation with scoring mechanisms based on Plum indices etc.

Further detail at www.lancs.ac.uk/staff/marsdena/research/schenker

Supported by the UK Arts and Humanities Research Council (AHRC):
research-leave award 'Analysing Musical Structure: Harmonic-Contrapuntal
Reduction by Computer'