Towards Schenkerian Analysis by Computer: Deriving the Harmonic-Contrapuntal Structure of Music Automatically

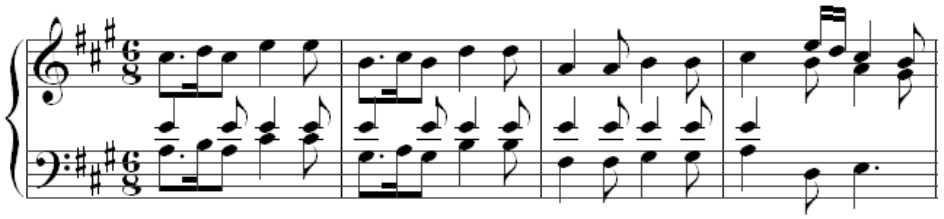
Alan Marsden
Lancaster Institute for the Contemporary Arts,
Lancaster University

City University, 14 Nov. 2007

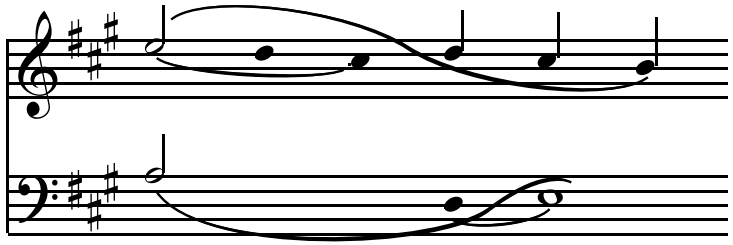
Schenkerian Analysis

Progressively reduces a score, removing less essential features, to reveal the ‘background’ structure.

Mozart:

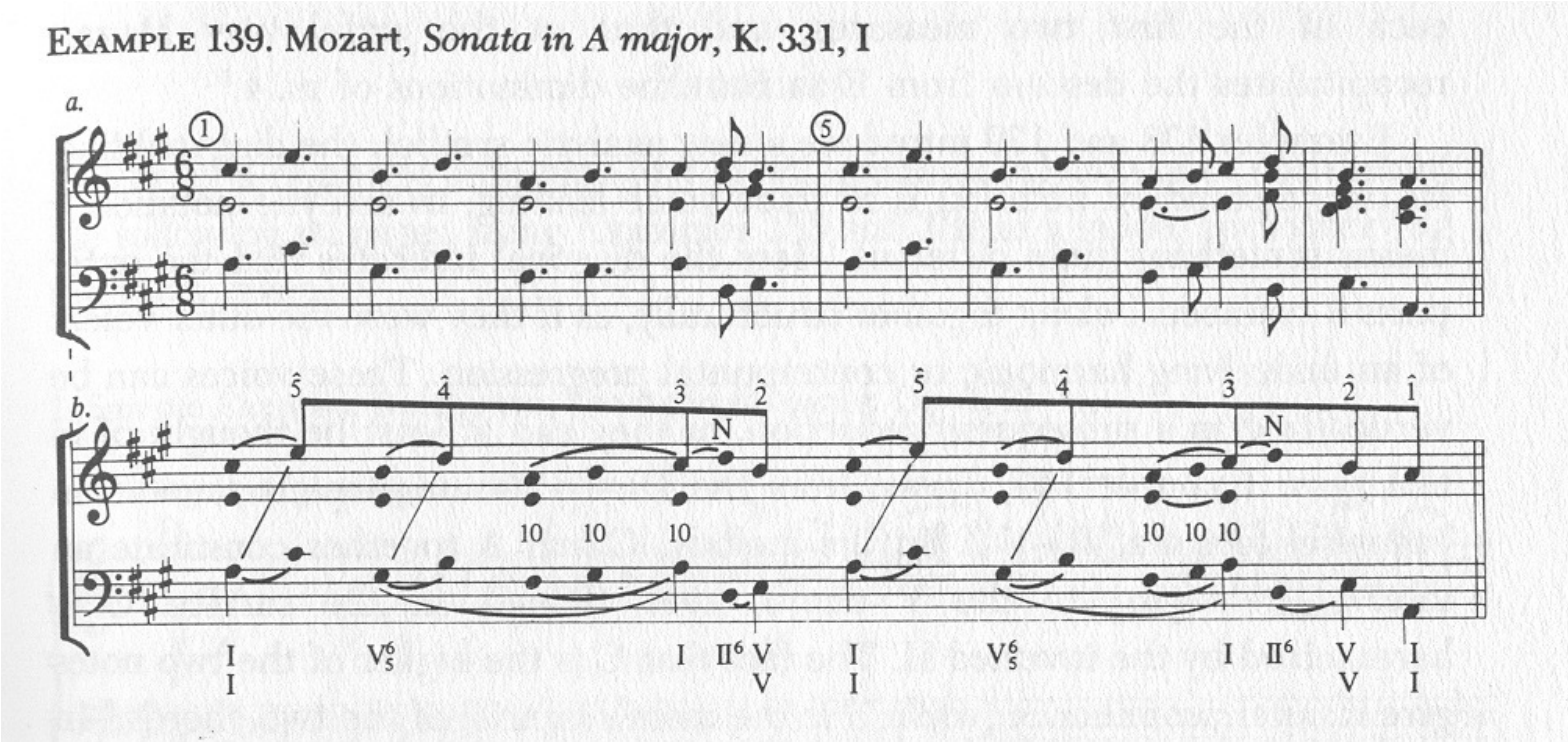


Schenker:



Alternative Analyses

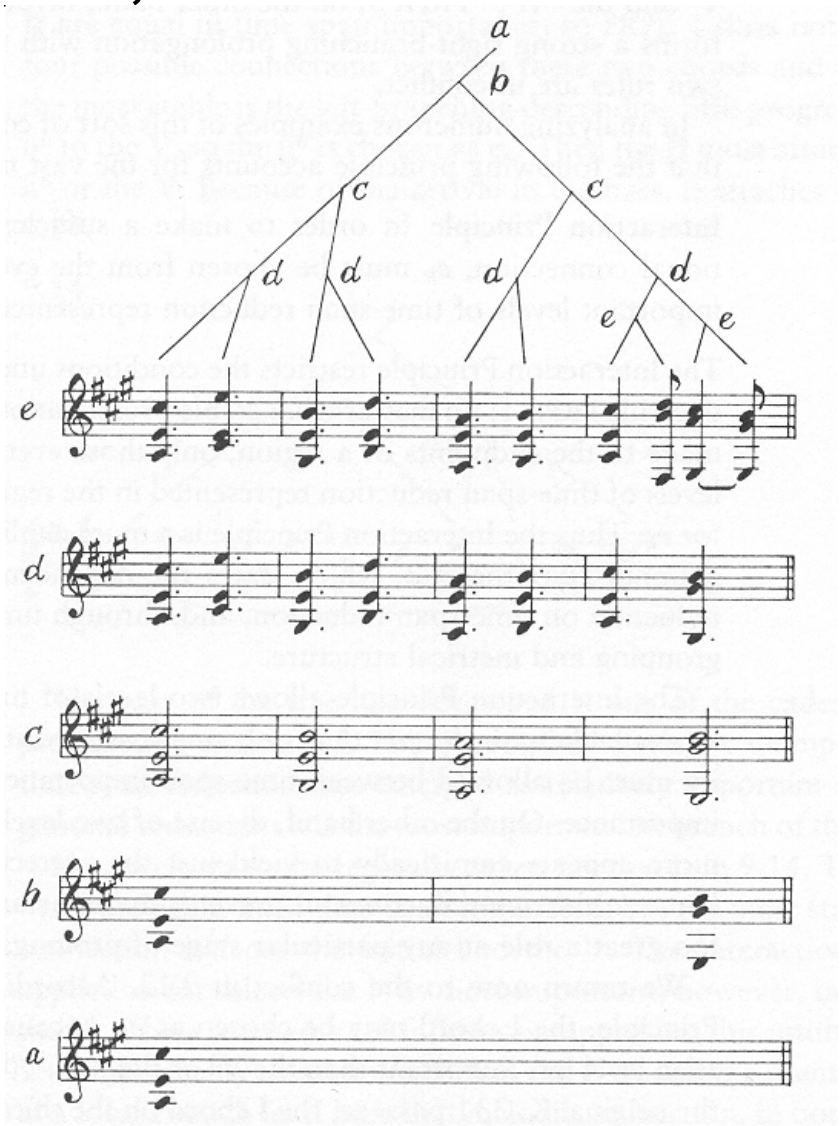
Forte & Gilbert:



Multi-levelled; Tree-like

Lerdahl & Jackendoff:

(Actually a somewhat different theory from Schenker, but does something similar)



Benefits

- The most influential and widely adopted theory and method of analysis for tonal music since the last quarter of the 20th c.
- Adumbrates many aspects of musical structure (key, harmony, segmentation, metre).
- Some evidence that it corresponds to perception and cognition of music.
- Based on two centuries of previous music theory.

BUT does remain controversial among musicians, and suffers from obscure arguments about detail.

Previous Work

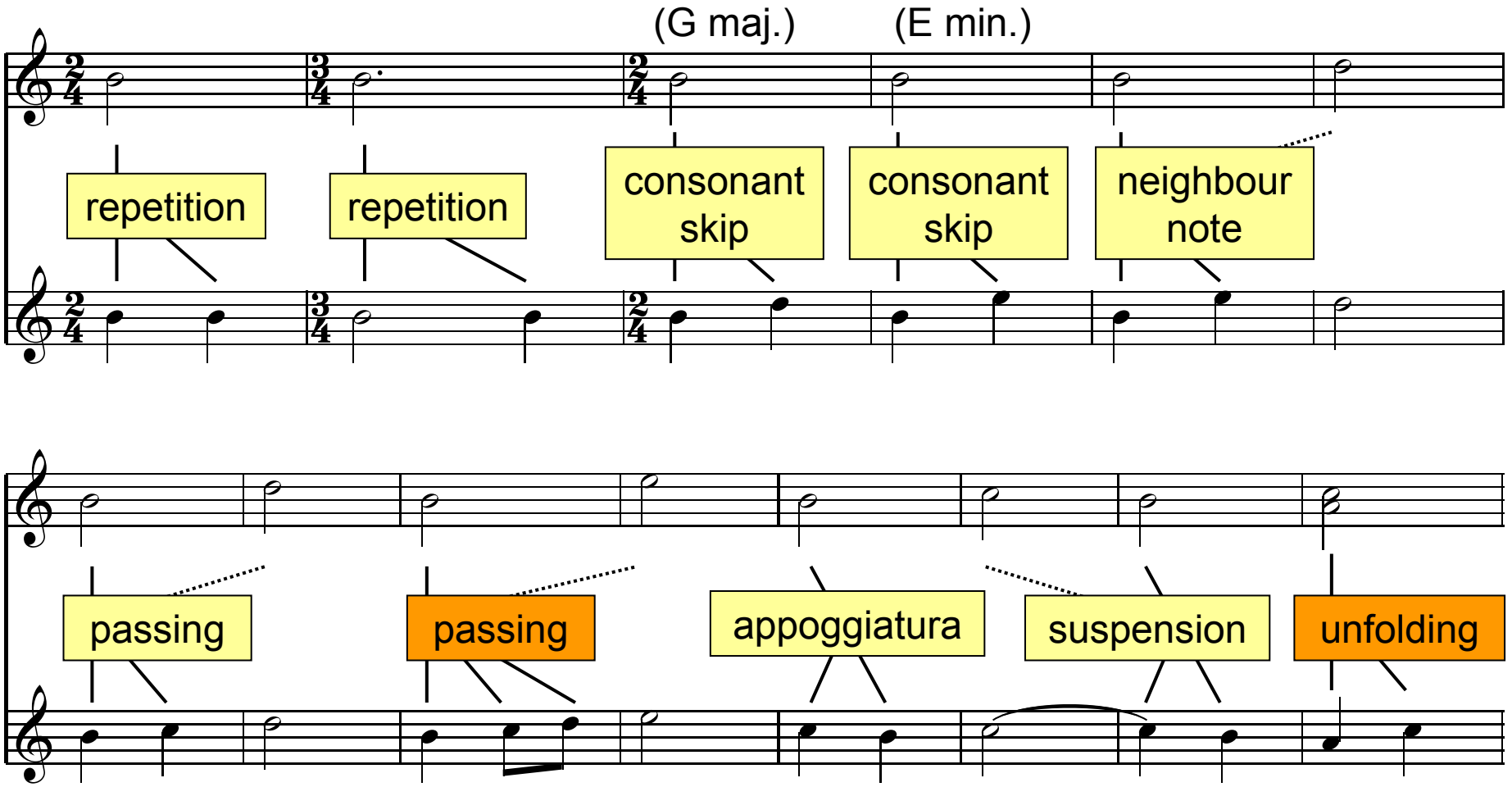
- Kassler (1967, 1975, 1977, 1988)
  - program which successfully analyses three-voice middlegrounds
- Smoliar et al. (1976, 1978, 1980)
  - program capable of verifying an analysis
- Lerdahl & Jackendoff (1983, 2001)
  - rule-based system for quasi-Schenkerian reduction
  - not demonstrably computable
- Mavromatis & Brown (2004)
  - demonstration of theoretical possibility of Schenkerian analysis by context-free grammar
- Hamanaka, Hirata & Tojo (2005-7)
  - implementation of Lerdahl & Jackendoff reduction with adjustment of parameters (now moving towards automatic parameter-setting)
- Gilbert & Conklin (2007)
  - probabilistic grammar for melodic reduction

Formalisation (non contentious)

1) Notes are defined by pitch and time (start and duration).
2) All notes on the ‘surface’ of the piece derive by a process of iterative elaboration of a single chord (i.e., several notes all with the same start and duration).
3) Only certain kinds of elaboration are possible.
4) Elaborations can have an associated key and harmony.
5) Simultaneous elaborations (in different parts/voices) must be consistent in key and harmony.

A piece of music is a tree-like structure of elaborations, BUT it has simultaneous trees (for different voices) and these may intertwine (a note can belong to more than one tree).

Elaborations



Further detail in Marsden, *CHum* (2001) and *JNMR* (2005).

Formalisation (contentious)

6) All elaborations produce two ‘children’.
7) All elaborations have one ‘parent’ note.

   (So trees are binary. Special ‘note sequences’ are produced in extended passing elaborations. Unfoldings, which should have multiple parents, are represented by multiple elaborations.)
8) Elaborations may require a specific preceding or following ‘context note’.

   (So branches of trees are not independent of each other.)

Restrictions (Temporary?)

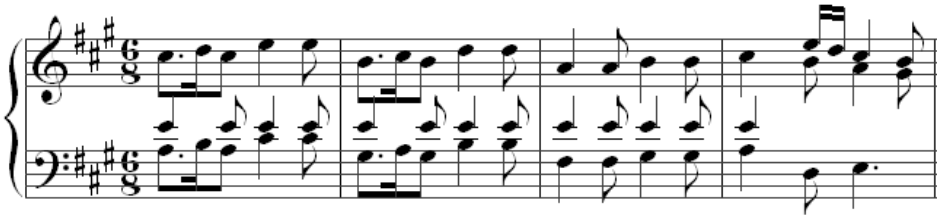
In order to allow a less inefficient analysis algorithm:

9) Simultaneous branching in trees must produce children with the same durations in each tree.
10) Preceding context notes must be present on the surface (e.g., in the case of the preparation of a suspension).
11) Voices cannot cross each other.

Plus some arbitrary restrictions to avoid crazy solutions:

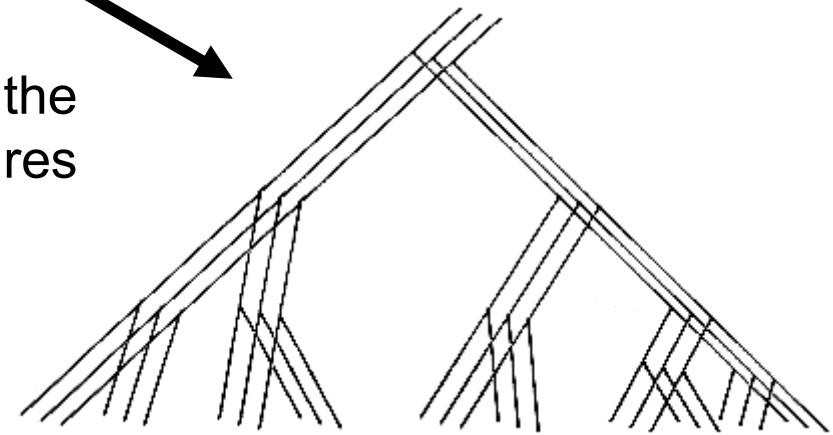
12) Chords in reductions must not be larger than a certain small number of notes.
13) Pairs of notes reduced must have a moderately simple ratio of durations.

The Problem



From the score …

… to derive the
tree structures



Local Solution-Finding

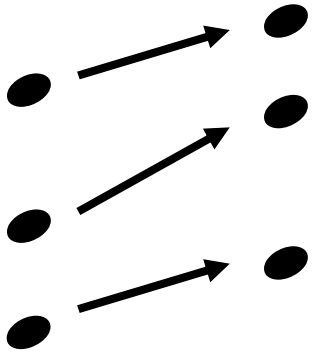
For any pair of notes, given knowledge of the preceding notes (on the surface) and possible and actual following notes (both on the surface and at higher levels), we can determine:

- which elaborations, if any, can produce these notes,
- what the parent note must be for each elaboration,
- what the requirements of key and harmony are for each elaboration.

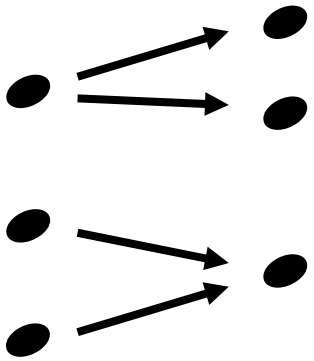
So, given any pair of consecutive chords, knowledge of preceding and following chords, and rules of harmonic and tonal consistency, we can determine the possible parent chords of that sequence.

Combinatorial Problems

1. Voices



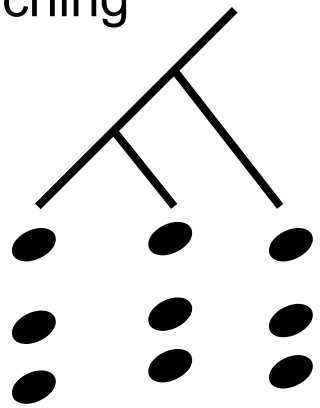
or



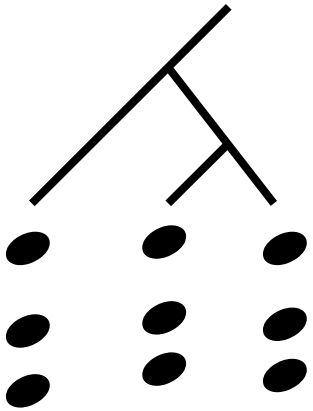
etc. ?

Increases exponentially with the size of a piece

2. Branching



or



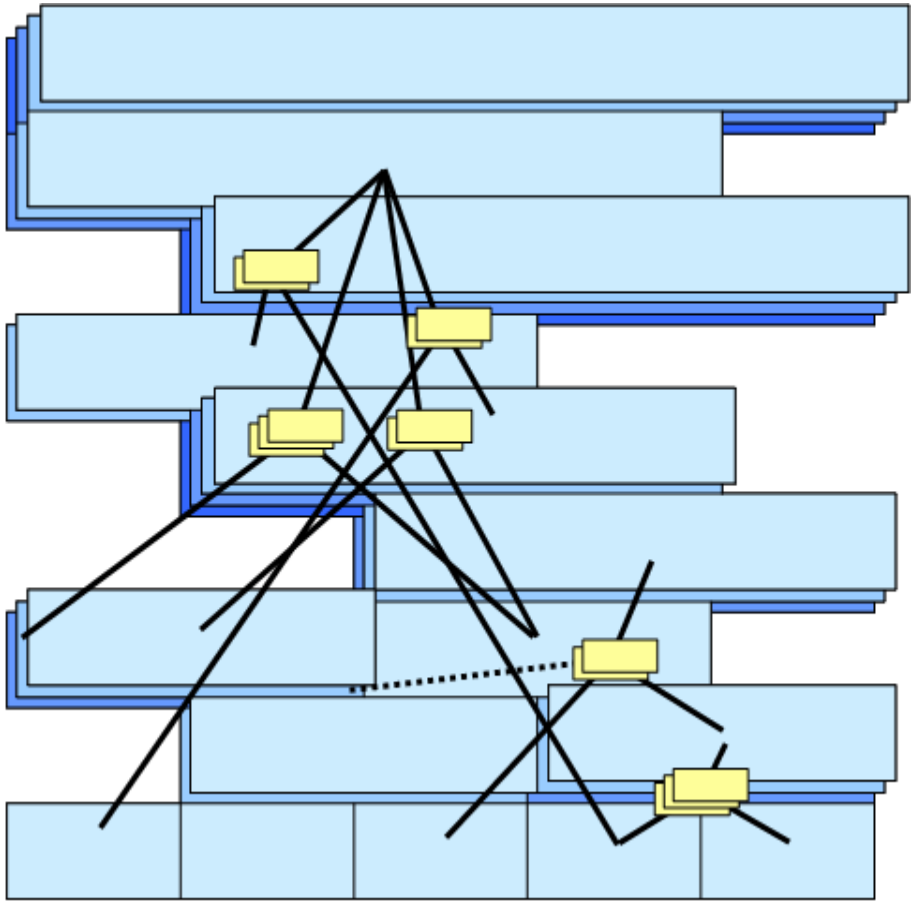
?

Increases factorially with the size of a piece

Attempted Solution

- Inspired by dynamic programming.
- Construct a 3D matrix of valid local solutions.
  - lowest level is all the ‘chords’ of the surface of the piece:
    1D, *n* cells
  - higher levels are all possible chords derived by reduction from all possible pairs of chords below:
    2D, (*n* – *l*) * *x* cells
    (l level of reduction, *x* unknown but limited number of possibilities)
- Any valid reduction tree can be derived from the matrix by selecting a top-level cell and then iteratively selecting pairs of possible children.

Illustration



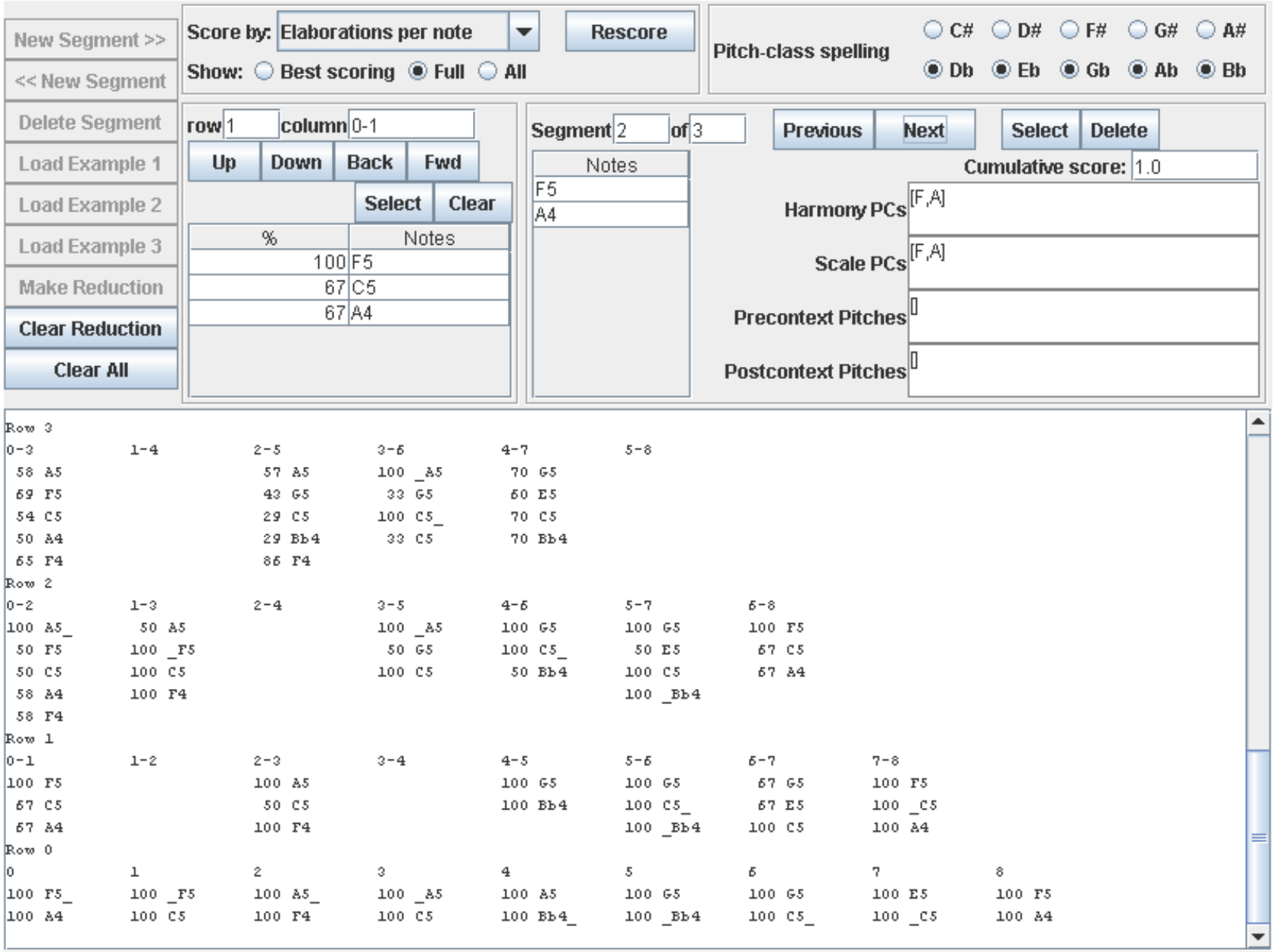
- If we can know metrics of ‘goodness’ for local solutions, the best analysis can be derived by selecting the best children at each point.

  BUT, there are no accepted metrics.

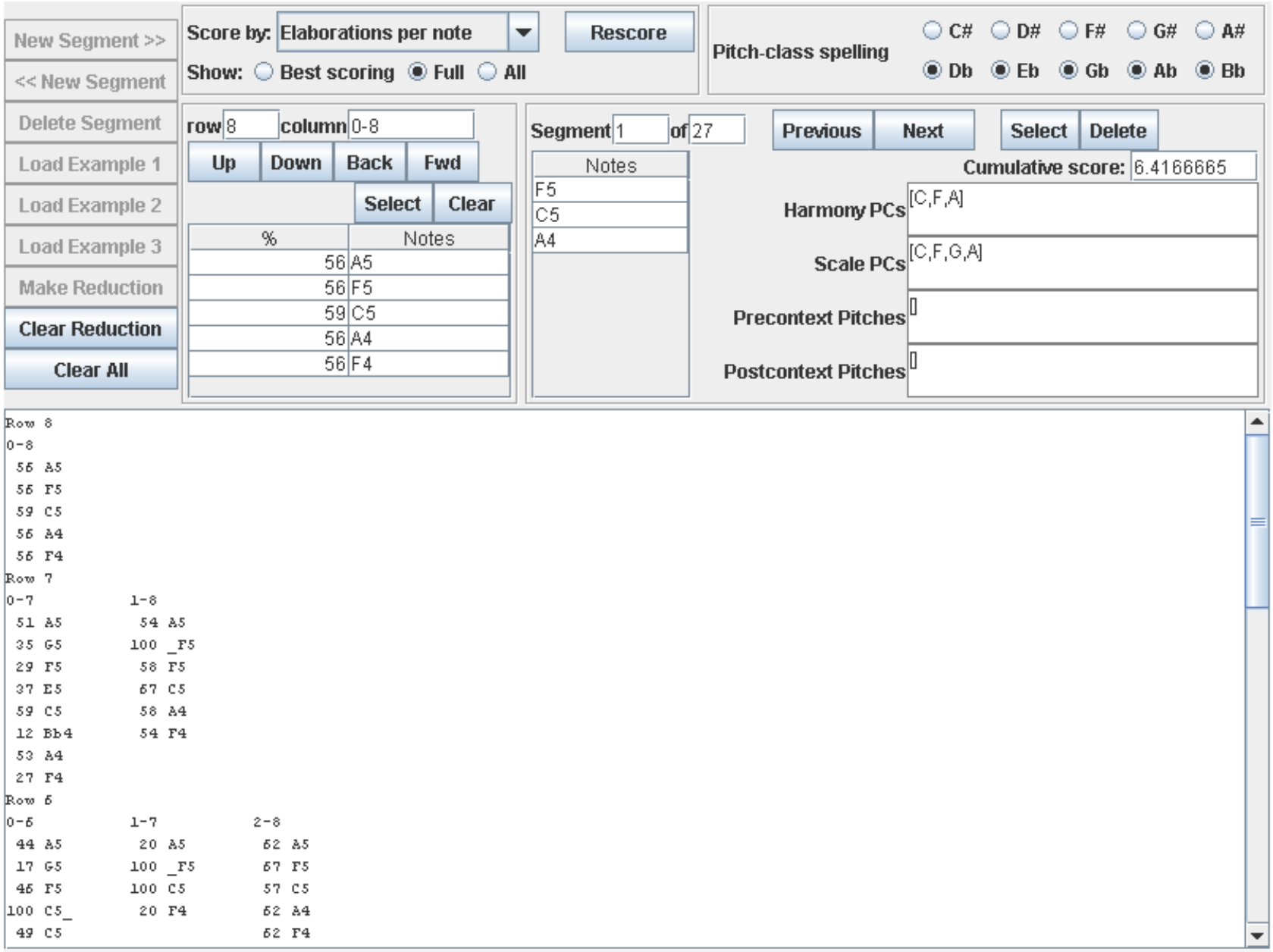
- In principle, a matrix can be derived in $O(n^3)$ space and $O(n^4)$ time.

  BUT, 4 bars of Mozart took 170MB of heap space and 1hr 45mins!

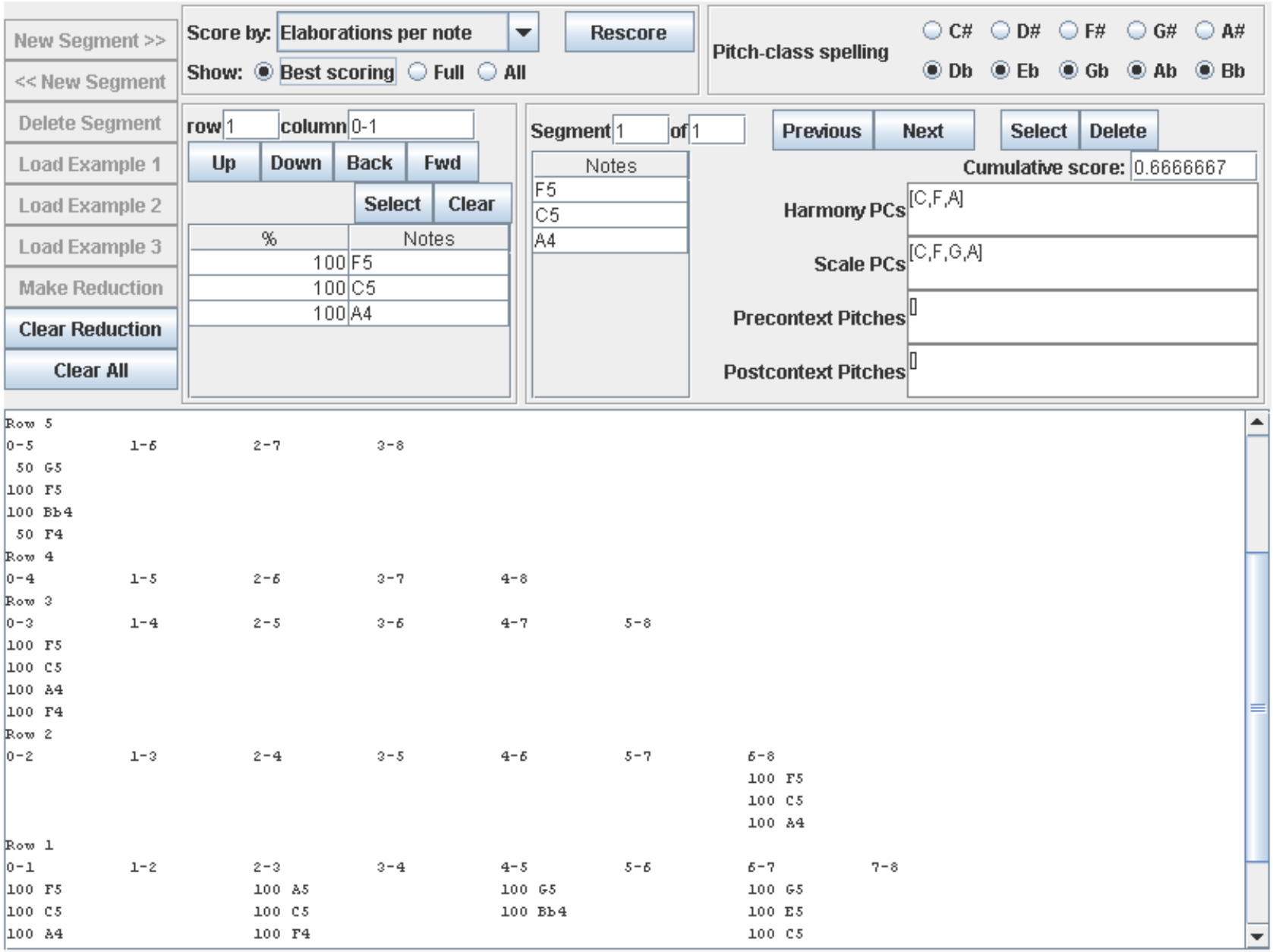
Demonstration software



Demonstration software (2)



Demonstration software (3)



Further Work

- Revisions to make reduction procedure more efficient
  - minimising number of segments recorded (separation of constraints and chords)
  - tightening of harmonic constraints (e.g., avoidance of sevenths)
- Testing on ‘ground truths’ from published analyses
  - Oster archive (Chopin, Beethoven)
  - experimentation with scoring mechanisms based on Plum indices etc.

Further detail at www.lancs.ac.uk/staff/marsdena/research/schenker

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