Progress Towards Schenkerian Analysis by Computer

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LANCASTER INSTITUTE FOR
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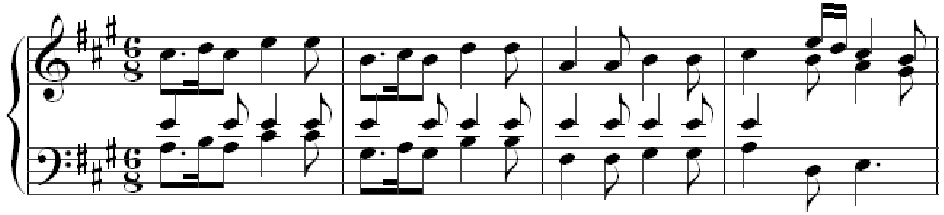
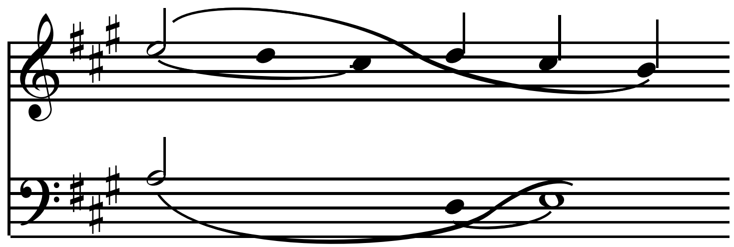
Schenkerian Analysis

Progressively reduces a score, removing less essential features, to reveal the 'background' structure.

Mozart:



Schenker:



Lerdahl & Jackendoff GTTM

F. Lerdahl & R. Jackendoff,
A Generative Theory of Tonal Music
(1983), MIT Press

Benefits

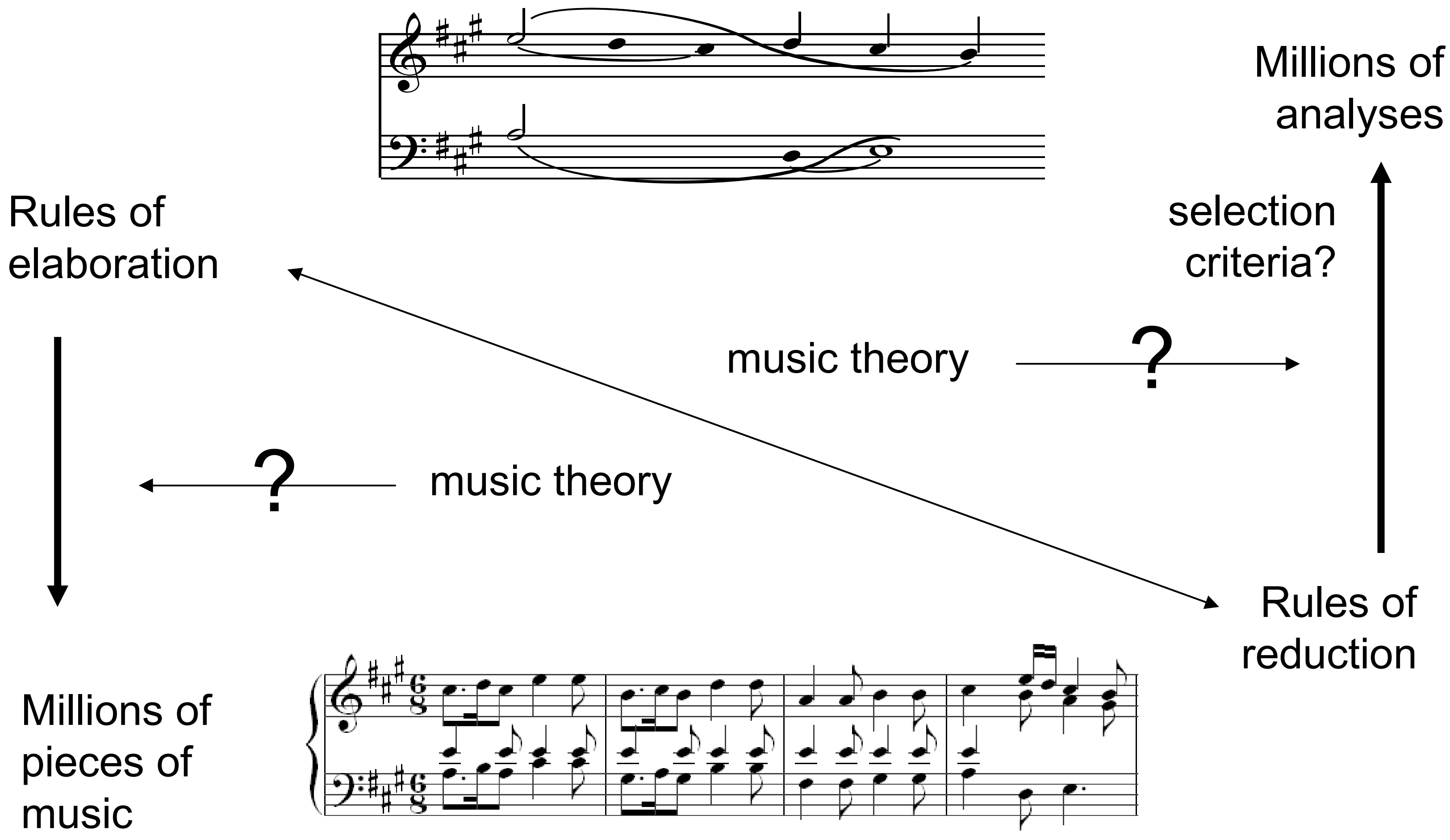
- The most influential and widely adopted theory and method of analysis for tonal music since the last quarter of the 20th c.
- Adumbrates many aspects of musical structure (key, harmony, segmentation, metre).
- Some evidence that it corresponds to perception and cognition of music.
- Based on two centuries of previous music theory.

BUT does remain controversial among musicians, and suffers from obscure arguments about detail.

Previous Work

- Kassler (1967, 1975, 1977, 1988)
  - program which successfully analyses three-voice middlegrounds
- Smoliar et al. (1976, 1978, 1980)
  - program capable of verifying an analysis
- Mavromatis & Brown (2004)
  - demonstration of theoretical possibility of Schenkerian analysis by context-free grammar
- Hamanaka, Hirata & Tojo (2005-7)
  - implementation of Lerdahl & Jackendoff reduction with adjustment of parameters (now moving towards automatic parameter-setting)
- Gilbert & Conklin (2007)
  - probabilistic grammar for melodic reduction

The Research Problem



A Framework for Empirical Research

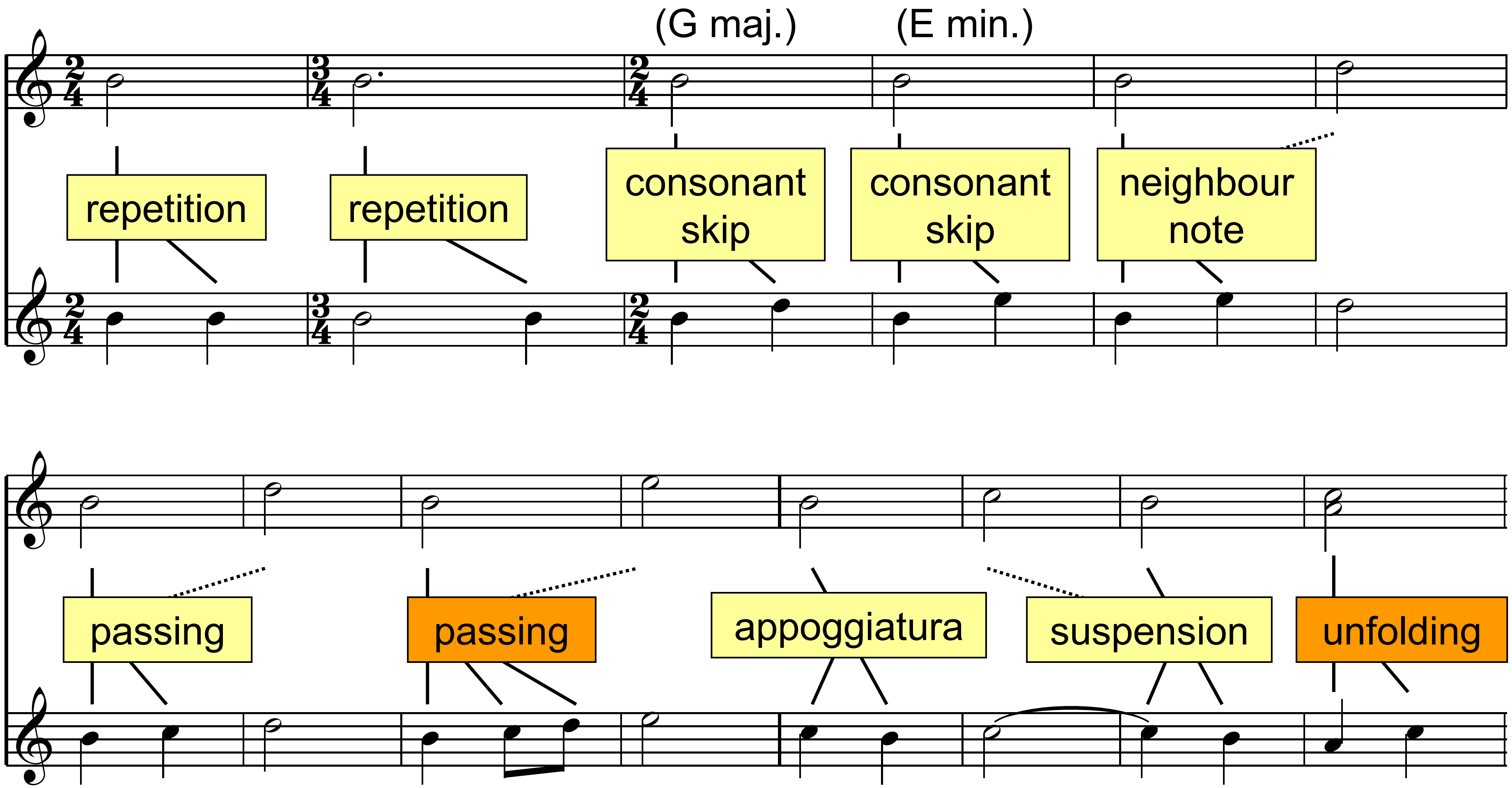
1. Formalise rules of reduction.
2. Derive all possible reductions of a fragment of music.
3. Measure certain characteristics of a sample.
4. Measure the same characteristics in ‘correct’ analyses of the same fragments.
5. Compare the distribution of values from the sample to the values from the analyses.
6. Possible selection criteria can be derived from characteristics where the distribution of values in ‘correct’ analyses differ consistently from those in the sample.

1. Formalisation of Reduction

- See Alan Marsden, ‘Generative Structural Representation of Tonal Music’, *Journal of New Music Research*, 34 (2005), 409-428

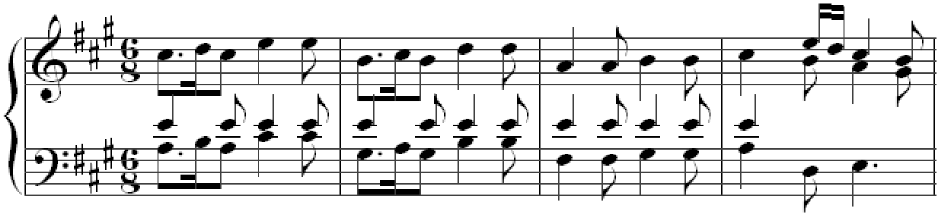
1. All elaborations are binary.
   - elaborations producing more than one new note accommodated by special intermediate ‘notes’
   - analysis is a set of binary trees, each corresponding roughly to a voice of the structure
   - trees can share nodes (one note can be elaborated in more than one way; a note can arise from more than one elaboration)
2. Elaborations generate new notes within the same time-span (cf. Lerdahl & Jackendoff, Komar).
3. Only certain kinds of elaborations are possible.
4. Elaborations have harmonic constraints.
5. Some elaborations require specific preceding or following context notes.

Elaborations



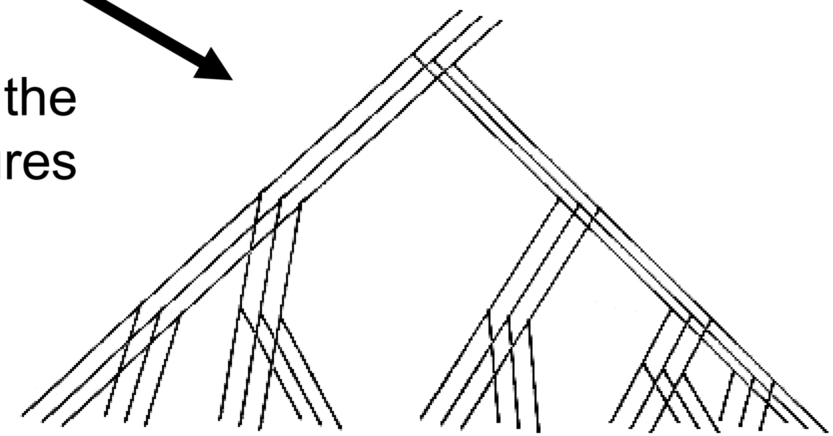
Further detail in Marsden, *CHum* (2001) and *JNMR* (2005).

2. Derivation of Possible Reductions



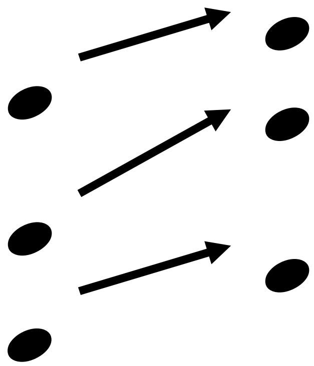
From the score …

… to derive the
tree structures

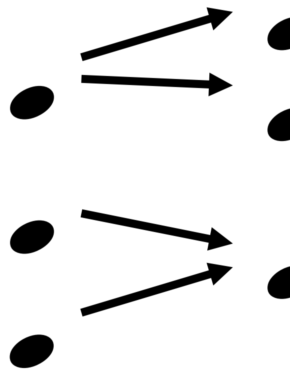


Combinatorial Problems

1. Voices



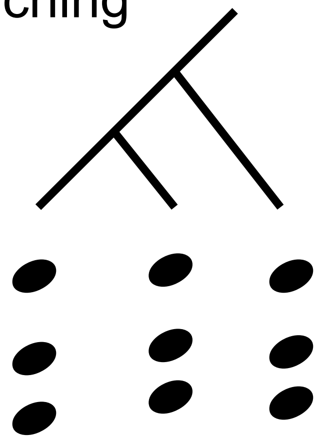
or



etc. ?

Increases exponentially with the size of a piece

2. Branching



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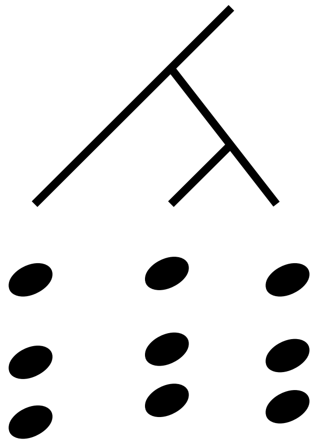
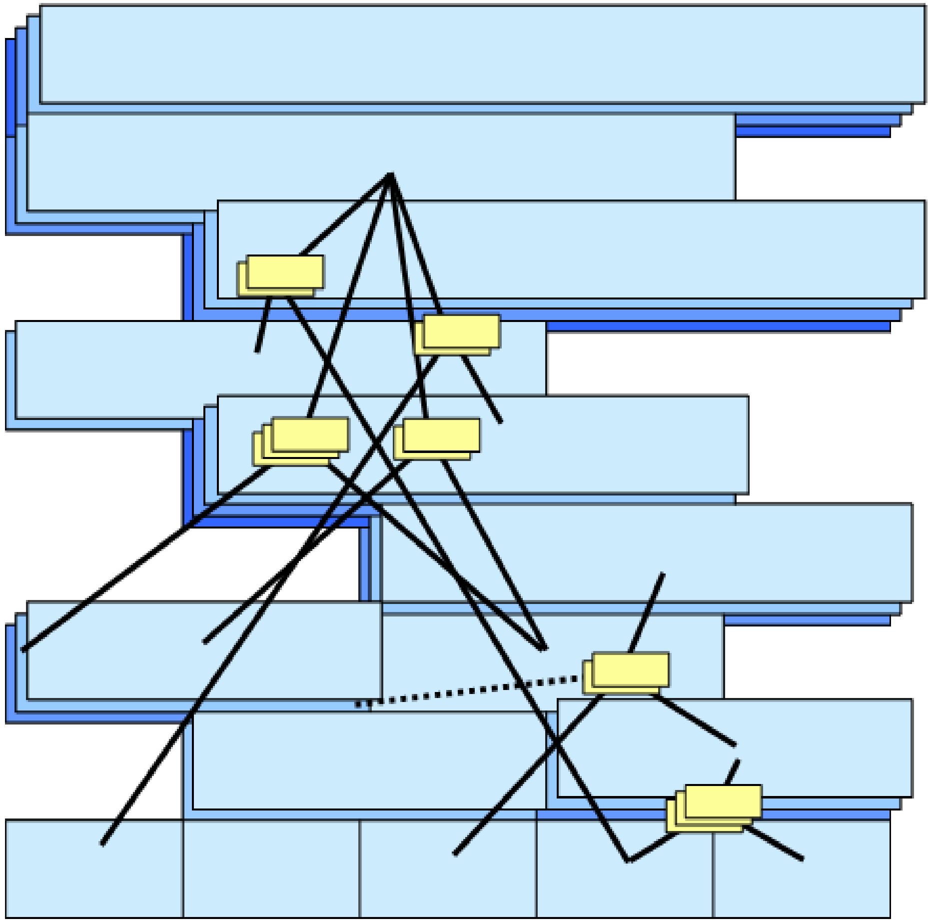


?

Increases factorially with the size of a piece

‘Reduction Matrix’

- A ‘matrix’ of local solutions, from which all possible reductions may be derived
- Complexity related to n^3



‘Chart-Parser’ Derivation (CYK Algorithm)

- Similar to dynamic programming
- Construct a 3D matrix of valid local solutions.
  - lowest level is all the ‘chords’ of the surface of the piece:
    1D, *n* cells
  - higher levels are all possible chords derived by reduction from all possible pairs of chords below:
    2D, (*n* – *l*) * *x* cells
    (*l* = level of reduction, *x* = unknown but limited number of possible local solutions)
- Any valid reduction tree can be derived from the matrix by selecting a top-level cell and then iteratively selecting pairs of possible children.

Example of Reduction Matrix

```
Row 5
0-5 16
 67 E5
 67 C5
 75 C4
 50 A3
 25 G3
Row 4
0-4 8        1-5 14
 63 E5        67 _E5
 38 D5        67 C5
 25 C4        75 C4
 50 B3        50 A3
 25 A3        25 G3
 38 G3
Row 3
0-3 7        1-4 6        2-5 12
 67 E5        33 _E5      100 C5
 33 D5        33 D5        75 C4
 33 C4        67 B3        50 A3
 33 B3        22 A3        25 G3
 50 A3        44 G3
Row 2
0-2 6        1-3 5        2-4 4        3-5 10
100 E5        50 _E5       43 D5       100 C5
 50 C4        30 D5        57 B3       100 C4
 25 B3        40 pB3-G3    14 A3        50 G3
 50 A3        40 B3        57 G3
              40 A3
Row 1
0-1 4        1-2 4        2-3 3        3-4 2        4-5 9
100 E5        67 _E5       50 D5       100 D5       100 C5
 33 pC4-A3    50 pB3-G3    50 B3        67 B3       100 C4
 33 C4        17 B3        50 A3        67 G3        50 G3
 33 B3        67 A3
Row 0
0 2          1 2          2 2          3 1          4 1          5 8
100 E5       100 _E5      100 A3       100 D5       100 _D5      100 C5
100 C4       100 B3                    100 B3       100 G3       100 C4
```

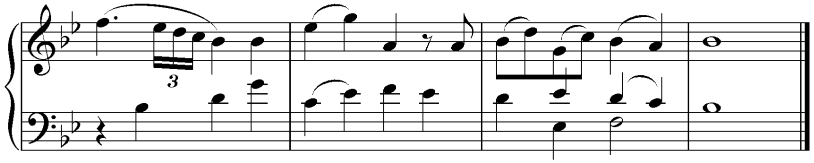
Example of Selection

Row 5					
0-5 16					
100 E5					
100 C4					
Row 4					
0-4 8	1-5 14				
100 E5					
100 C4					
Row 3					
0-3 7	1-4 6	2-5 12			
Row 2					
0-2 6	1-3 5	2-4 4	3-5 10		
100 E5					
100 C4					
Row 1					
0-1 4	1-2 4	2-3 3	3-4 2	4-5 9	
	100 _E5		100 D5		
	100 pB3-G3		100 G3		
Row 0					
0 2	1 2	2 2	3 1	4 1	5 8
100 E5	100 _E5	100 A3	100 D5	100 _D5	100 C5
100 C4	100 B3		100 B3	100 G3	100 C4

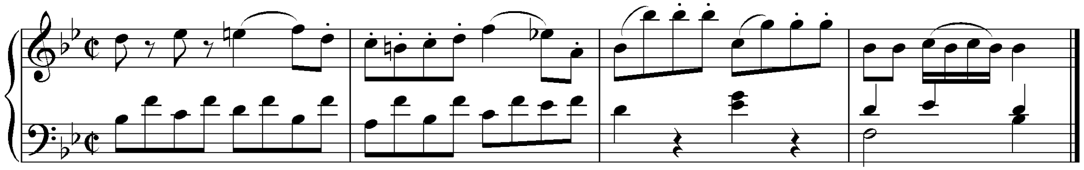
Current Research Materials

Rondo themes from Mozart piano sonatas

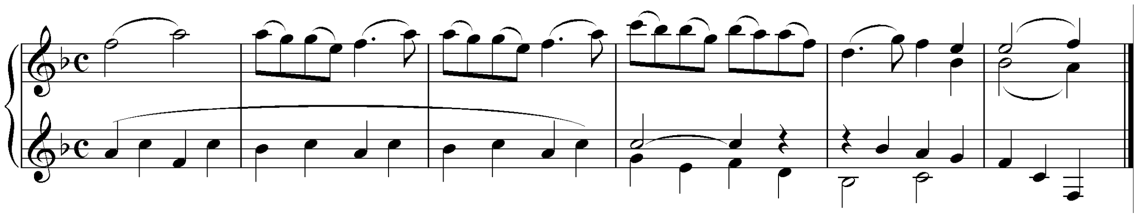
$5 * 10^{8}$ solutions, not including the ‘correct’ one



$7 * 10^{10}$ solutions, including the ‘correct’ one



$2 * 10^{20}$ solutions, including the ‘correct’ one



$7 * 10^{23}$ solutions, including the ‘correct’ one

3. Selection and Measurement of a Sample

- Selecting a random sample is not trivial
  - selecting an option at one point in the matrix affects options at other points
  - currently selects top-down giving equal likelihood to each remaining option at each point
- 400 samples from each example which included the ‘correct’ analysis
  - aiming at 1000 samples per example
- ‘Correct’ analyses derived from teaching materials
  - original analyses less detailed than computationally derived reductions
  - selection of a close match from the possibilities in the reduction matrix

4. Measurement of Characteristics (1)

1. **duration ratio** of children
2. **short-long:** number of reductions with shorter first child
3. **syncopations:** reductions which cover a beat stronger than the beat at their start
4. **harmonic simplicity:** harmonies which are I or $V^{(7)}$
5. **root position** harmonies
6. **second-inversion** harmonies
7. **harmonic support:** proportion of the ‘surface’ covered by a reduction which is consonant with the reduction
8. **pitch support:** proportion of the ‘surface’ covered by a reduction which contains the pitches of the reduction

4. Measurement of Characteristics (2)

9. **interval** between children
10. **voice split/join:** reductions which share a child
11. **delay:** number of reductions with a rest as first child
12. **shortening:** number of reductions with a rest as second child
13. **post-context from parent:** number of levels between lowest common ancestor and required context
14. **post-context from surface:** number of levels between surface and required context

5. Comparison of Measures: Rhythm

Measure	Example	Average	Std. dev.	'Correct'	Deviation
duration ratio	Mozart2	1.95	0.230	1.39	-2.41
	Mozart3	2.17	0.258	1.40	-2.98
	Mozart5	2.17	0.348	1.69	-1.38
short-long	Mozart2	0.262	0.0750	0.303	-3.09
	Mozart3	0.270	0.0746	0	-3.62
	Mozart5	0.188	0.0795	0.111	-0.967
syncopation	Mozart2	0.0935	0.0293	0	-3.19
	Mozart3	0.147	0.0414	0	-3.54
	Mozart5	0.0683	0.0357	0.0270	-1.16

Comparison of Measures: Harmony

Measure	Example	Average	Std. dev.	‘Correct’	Deviation
harmonic simplicity (I/V)	Mozart2	0.909	0.0606	0.939	0.500
	Mozart3	0.846	0.0313	0.900	1.71
	Mozart5	0.747	0.102	0.833	0.840
root position	Mozart2	0.530	0.149	0.545	0.105
	Mozart3	0.544	0.120	0.600	0.471
	Mozart5	0.510	0.156	0.909	2.56
second inversion	Mozart2	0.250	0.135	0.182	-0.503
	Mozart3	0.243	0.106	0.200	-0.411
	Mozart5	0.257	0.134	0	-1.92

Comparison of Measures: Support & Interval

Measure	Example	Average	Std. dev.	'Correct'	Deviation
harmonic support	Mozart2	0.725	0.0328	0.737	0.361
	Mozart3	0.676	0.0335	0.712	1.08
	Mozart5	0.683	0.0463	0.753	1.52
pitch support	Mozart2	0.818	0.0123	0.821	0.250
	Mozart3	0.558	0.0104	0.559	0.138
	Mozart5	0.337	0.0216	0.313	-1.10
interval	Mozart2	0.545	0.172	0.263	-1.65
	Mozart3	1.45	0.447	1.07	-0.86
	Mozart5	0.734	0.318	0.250	-1.52

Comparison of Measures: Voices

Measure	Example	Average	Std. dev.	'Correct'	Deviation
voice split/join	Mozart2	0.0121	0.0185	0	-0.657
	Mozart3	0.00717	0.0510	0	-0.476
	Mozart5	0.0148	0.0289	0	-0.514
delay	Mozart2	0.0538	0.0412	0	-1.31
	Mozart3	0.121	0.0517	0	-2.34
	Mozart5	0.172	0.0837	0.111	-0.725
shortening	Mozart2	0.0774	0.0440	0.0303	-1.07
	Mozart3	0.208	0.0643	0.1	-1.68
	Mozart5	0.0942	0.0670	0.0556	-0.577

Comparison of Measures: Post-Context

Measure	Example	Average	Std. dev.	'Correct'	Deviation
post-context from parent	Mozart2	0.240	0.211	0	-1.13
	Mozart3	0.312	0.228	0.083	-1.00
	Mozart5	0.350	0.297	0.200	-0.505
post-context from surface	Mozart2	0.420	0.214	1	2.70
	Mozart3	0.213	0.183	0.167	-0.251
	Mozart5	0.470	0.242	0.2	-1.11

6. Possible Criteria

- Prefer reductions with
  - few syncopations
  - few short-long reductions
  - equal durations
  - small intervals
  - no voice splitting/joining
  - few ‘delay’ and ‘shortening’ reductions
  - post-contexts close to lowest common parent

Further Work

- Incorporation of the most obvious selection criteria to prune derivation
- Experimentation on search procedures (with Geraint Wiggins)
- Testing for derivation of published analyses
  - Oster archive (Chopin, Beethoven)
  - *Das Meisterwerk in der Musik*

Further detail at www.lancs.ac.uk/staff/marsdena/research/schenker

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