7

Fiscal Policy and the government budget constraint

Up to now we have discussed fiscal policy as an element of demand management, separate from monetary policy and useable both as an automatic stabiliser and for feedback policy. However, the main focus of our discussion of stabilization policy has been on monetary policy, which in practice has been almost exclusively the instrument governments have used for this purpose. In this chapter we consider the role of and contraints on fiscal policy in more detail. We start with accounting, the implications of the government budget constraint; then go on to various models of how fiscal policy affects the economy; finally we turn to policy implications.

GOVERNMENT SOLVENCY AND THE GOVERNMENT BUDGET CONSTRAINT

The first question we must ask about government tax, any resources it raises by printing money with consequent inflation (its 'seigniorage'), its spending and its borrowing is: what are the limits on a government's actions? These limits are implied by its budget constraint, which relates these elements. It can conveniently be thought of as the borrowing required to finance the budget deficit, the gap between spending, inclusive of debt interest, and tax plus seigniorage. Sometimes it is useful to refer to the 'primary' deficit which is spending *excluding* debt interest, less tax. To keep the exposition as simple as possible we will assume that all debt is one-period indexed bonds, the number of which we denote by b_t: such a bond is a promise to pay (next period, $t+1$) £1 plus the rate of inflation and a real rate of interest, r_t; hence its face value today is always £1. Later we will discuss complexities introduced by nominal bonds.

Let us define the budget constraint then as:

$$b_{t+1} - b_t = g_{t+1} - \tau_{t+1} + r_t b_t - \frac{H_{t+1} - H_t}{P_{t+1}} = d_{t+1} + r_t b_t \qquad (1)$$

where d_t is the primary deficit (in real terms); $g_t,\ \tau_t$ are respectively real government spending excluding debt interest and real taxation; and H_t is the monetary base (government's own monetary liabilities) and so the real value of its increase is the seigniorage. This equation plainly tells us how debt will accumulate as the government runs primary deficits . We can project it into the future and track out the implications of such policies for debt.

However, matters do not end there since plainly debt must eventually be paid off. 'Eventually' here could in practice mean at a future date which is constantly postponed — in other words, the government does not actually pay it off by any particular date because it continually persuades its creditors to take more debt ('rolling it over'). Thus we can let the date of repayment be infinitely far in the future; nevertheless we assume that that day is some time in the future. We can put this another way: if a government is seen to be in a position where it will never pay its debts and so will have to default, then it will be violating the terms of its debt and so of its budget constraint: it will be insolvent. We will assume that the government does not behave in this way. (We will deal later with what happens if it does and a default is expected.)

On this assumption the government's outstanding debt defines the limits on its future deficits. It can only issue debt that is 'backed' by future primary surpluses exactly sufficient to pay it off with its interest. If its debt is so backed, then it is said to be solvent. Solvency is guaranteed if the government budget constraint is met at all times.

This can be seen by repeated forward substitution for b_{t+i} in (1); we will assume a constant real interest rate, r. So

$$b_t = \frac{s_{t+1} + b_{t+1}}{1+r} \qquad (2)$$

where s_t is the primary surplus. Continuing to substitute for b_{t+i} we obtain:

$$b_t = \sum_{i=1}^{N} \frac{s_{t+i}}{(1+r)^i} + \frac{b_{t+N}}{(1+r)^N} \qquad (3)$$

If we now let N go to infinity and impose the condition that the debt will eventually be paid off (which we interpret as above: that the government may roll over its debts but is expected not to default, that is to be in a position to pay them off some time), then the last term goes to zero and we obtain:

$$b_t = \sum_{i=1}^{\infty} \frac{s_{t+i}}{(1+r)^i} \tag{4}$$

which is the solvency condition.

In fact, we note from (3) that the condition for the remainder term to go to zero is weaker than that debt should be totally paid off at some (indefinite) date: it is merely that real debt should grow less rapidly than the real rate of interest. This shows a peculiarity of infinite time: that the government not only need never pay off its debts but may even raise its debt, provided the growth rate does not exceed the rate of interest. Buiter (1999) has called this the great 'puzzle' of fiscal arithmetic. Plainly if the economy has a finite life then when it ends debts must be paid off: everyone will wish to use up their wealth in consumption (or transfer it to another economy) before the end. The same applies if the economy has a finite life whose end is constantly postponed.

In fact there is some difficulty in assuming the economy comes to an end; for behaviour being forward-looking, those of our theories that have people acting in the interests partly of their descendants would fail were the economy to stop — the meaning attached by people to life's purposes would be drained away, as would the incentive to maintain any of society's institutions or infrastructure. Our assumption is that people act as if they only care about themselves, in which case they would simply consume all they have before the economy's and their end; implying that the government must also pay off its debt, since people will call it in and consume it. However, law and order (which rely on social institutions being respected) might also break down. The end of the economy is effectively the end of its inhabitants' world; imagining how people would behave if the world were to end is hard, if not meaningless (an amusing attempt is in Douglas Adams, 1978, 'The restaurant at the end of the universe' where the whole point is that people cannot really imagine the end).

This seems to force us to consider time which is truly infinite — that is, where the economy never stops. In this situation how can we interpret the constraint that the government must simply have its debt grow at less than the rate of interest? Clearly it may be asked to pay it off (unlike money, where it knows it will never be asked to redeem its fiat currency). Anyone redeeming state debt will force the government to sell the debt to someone else: solvency then requires that when resold it has a present value equal to its face value (at least equal — implicitly we assume that if its present value was greater, then taxes would be cut or spending raised, in effect redistributing the excess back to the citizens.) On reflection, this is no more troublesome as a way of avluing government paper than the way we value say private equity (the present value of future dividends).

Government debt in other words is sold at a price that reflects its 'value' that is, its discounted stream of primary surpluses, its cash flow available for paying interest and repayment on the debt. The transversality condition on b_{t+N} forces exactly this condition on that future debt.

It may be useful to illustrate this point with an example. Consider three cases.

First, let debt be increasing faster than the rate of interest. By definition:

$$\Delta b_t = rb_{t-1} - s_t \tag{5}$$

Because $\frac{\Delta b_t}{b_{t-1}} > r$, s_t must be negative. This is the case of Ponzi finance, where debt is run up to pay off interest and pay for extra net spending.

Suppose this situation prevails from time $t + N + 1$, where the remainder term of (7.3) cuts in, and let s be constant and negative. Then the present value of b_{t+N} at time $t + N$ can be written by discounting future surpluses as:

$$\text{p.v.of } b_{t+N} = s[\frac{1}{1+r} + ... + \frac{1}{(1+r)^{N+1}} + ...] = \frac{s}{r} \tag{6}$$

Plainly, s being negative the debt will be valueless.

More generally, let debt be growing at the constant rate $g(> r)$, so that $\frac{\Delta b_t}{b_{t-1}} = r - \frac{s_t}{b_{t-1}} = g$ and thus $\frac{s_t}{b_{t-1}} = -(g-r)$ is constant, implying that primary surpluses are both negative and also growing at the same rate as debt, g.

Then the present value of b_{t+N} becomes:

$$\begin{aligned}\text{p.v.of } b_{t+N} &= \frac{s_{t+N+1}}{b_{t+N}}\frac{b_{t+N}}{1+r} + \frac{s_{t+N+2}}{b_{t+N+1}}\frac{b_{t+N+1}}{(1+r)^2} + ... \\ &= b_{t+N}[-(g-r)][\frac{1}{1+r} + \\ &\quad \frac{1+g}{(1+r)^2} + \frac{(1+g)^2}{(1+r)^3} + ...] \\ &= -\frac{(g-r)}{1+g} b_{t+N} \sum_{i=1}^{\infty} \frac{(1+g)^i}{(1+r)^i} = -\infty \end{aligned} \tag{7}$$

The present value of the government's future cash flow in (6) is negative and in (7) is infinitely negative; in both cases its debt at $t + N$,

b_{t+N}, is valueless. In other words it will be impossible to issue debt at $t+N+1$, b_{t+N+1}, to pay off b_{t+N}; it follows that b_{t+N} itself could not have been issued, given this was known. And so on, back down the chain to the present time, t. In period t the government will be unable to issue any more debt, b_{t+1}, and will therefore default on its current debt, b_t.

Secondly, consider the case where $g = r$. Then (7) equals 0. There are neither primary deficits or surpluses from $t + N$.

The accumulated debt, b_{t+N}, has no value, as there is no future cash flow to service it; hence there is also insufficient cash flow from t onwards to pay off the current debt, b_t. The government is not therefore solvent; it will again be unable to issue new debt and will default on b_t.

Thirdly and finally, consider the case where $g < r$. Then (7) becomes finite and equal to b_{t+N}, in other words its face value. Hence the present value at t of future surpluses from $t + N + 1$ onwards in this case is equal to that of the debt which will have been accumulated up to that point by previous deficits, b_{t+N}. The government's outstanding debt is therefore 'backed': the government could pay off its debt at any time (by just surrendering its cash flow to creditors). The essential point is that with $g < r$, there is a primary surplus from $t + N + 1$ which starts equal to $(r - g)b_{t+N}$ and grows at the same rate, g, as debt (hence in effect remaining equal to this growing value).

The government's solvency or transversality condition can therefore be seen to be the commonsense condition that at all times the debt must be backed by primary surpluses with a present value equal to the debt's face value.

If this were not to be satisfied then the market would react by marking the debt's price below face value. Call this price D_t. At all points of time this price will move to make the market value of government debt equal to its present value. Hence we can write the price equation as:

$$D_t = \frac{\sum_{i=1}^{\infty} \frac{s_{t+i}}{(1+r)^i}}{b_t} \tag{8}$$

What this means is that a government can issue debt which is not fully backed by its future policies but only at a price that reflects this. We have been dealing with these policies as if they are fully known in advance. In this case when future policies change from those that are solvent to those that offer inadequate present value, there will be an immediate and sudden fall in the price of outstanding debt below its face value. In other words, if the government announces policies which mean it cannot pay its debts, it is as if it is 'writing down' the obligations (coupons etc.) on those debts; which of course implies that their present value, D_t (their price in the market-place) is correspondingly lowered.

In practice, when future surpluses are uncertain, we can think of r as the pure risk-free real interest rate and D_t as the risk discount factor (in present value) on expected future surpluses when valued at the risk-free rate (their 'face value' as we have put it). Thus equivalently we could value these expected future surpluses at the risk-free rate *plus* the risk-free premium implied by D_t, that is, at the market rate of interest on government bonds. This would be the bonds' market value. This is the usual situation in the bond market where a government is following an official policy of servicing its debt fully (no default). When a government defaults, that is announces it will no longer pay its interest and repayment obligations on old bonds in full, then D_t is marked in the market as an explicit discount (e.g. 20 pence in the pound) on the old bonds when valued at the rate of interest applicable to any fresh debt the government might raise.

THE 'FISCAL THEORY OF THE PRICE LEVEL'

In the past few years a surprising literature has grown up claiming that the (general consumer) price level, P_t, can be determined by fiscal policy — for example, Sims (1994) and Woodford (1995) and see Buiter (1999) for a critique on which this section draws. The claim can be seen quite simply by considering equation (4) above, the solvency condition. In that equation assume that instead of issuing indexed bonds the government has outstanding nominal bonds, so that $b_t = \frac{B_t}{P_t}$; let B_t be the market value of these bonds in money terms, computed by the normal discounting method given current interest rates (thus for example if bonds were perpetuities paying £1 each period their present value would be the number of these bonds divided by R_t the long-term rate of interest). Now suppose the government makes plans for future primary surpluses (on the RHS of the equation) whose present value is lower than the real value of the debt — this policy these authors call 'non-Ricardian' (if the government plans an RHS equal to debt's real value, a 'Ricardian' policy, then none of the following is relevant; the price level is then not claimed to be affected by fiscal policy — this use of 'Ricardian' has nothing to do with our later discussion of 'Ricardian Equivalence') . The Fiscal Theory states that prices will adjust to reduce the real value of the debt to equality with this RHS in order to produce solvency at the new real value; in effect this adjustment is a devaluation of the debt in response to fiscal policies that do not give it sufficient 'backing'.

One oddity of this theory is that it implies the price level will be overdetermined under the usual assumption that the government sets the level of the money supply as well as its rate of growth (seigniorage). To avoid this the theory assumes that the government sets the interest rate and not the level of the money supply; it does set seigniorage however (hence we can think of the interest rate path being set by a rule that targets the growth of the money supply or inflation and possibly real variables also). In a flexible-price economy this leaves the price level indeterminate because the choice of interest rate only sets real money balances demanded and cannot fix the split of these between nominal money supply and prices. Thus the Fiscal Theory claims that in such a situation the fiscal plans will set prices via (4) as above.

We can immediately note that this argument does not work in a model where prices are sticky (as in overlapping wage contracts) and there is some Phillips Curve mechanism translating excess demand into current inflation; for in this case an interest rate rule will fix the price level because it will fix excess demand, so inflation and so given past prices the current price level. So in this case, too, as in the flexible-price case where money supply is set, there would be overdetermination of the price level.

Overdetermination of a variable is fatal to a theory because it implies an internal inconsistency. Thus we can say at once that for many possible economies the Fiscal Theory of the price level will produce an incoherent theory overall. Furthermore, we can see from our discussion in the above section that the Fiscal Theory leaves out the way in which insolvency is dealt with by bond markets, viz. an adjustment of the bond price itself (D_t), not of the (goods) price level over which bond markets have no control.

Can we accept the Fiscal Theory as a possible mechanism for determining the price level where it is left indeterminate by an inadequately-specified interest rate rule? Not really, because the bond market, facing an equation (4) that was not satisfied, would have two degrees of freedom — either adjusting D_t or P_t. Hence either the price level or the bond price would be indeterminate. The model under these policies would thus be inadequate to explain a world in which both are seen to be determinate. Implictly Fiscal Theorists impose $D_t = 1$ to resolve this problem: but this is not merely arbitrary, it is at variance with the facts of debt downgrade (that is, a rise in the risk-premium added to the risk-free rate).

The Fiscal Theory therefore does not solve the problem of price level indeterminacy due to an inadequately-specified interest rate rule; such a world cannot therefore apply since we observe determinate prices. Under other assumptions it produces overdetermination of the price level — and thus must also be rejected. Buiter has called this theory a 'fallacy' — a correct assessment in our view if we interpret it in the bald way we have set it out above.

There is however a constructive way to think of the theory: as defining a particular combination of fiscal and monetary policies. By such a combination we mean a set of government plans that, given existing debts and projected real interest rates,

(a) set primary surpluses through spending and tax plans

(b) set the money supply (either directly or indirectly via plans for interest rates and prices).

We can distinguish four types of plan:

1. 'Classical' monetary policy leadership: the money supply is set implying a price level sequence (or alternatively an interest rate rule targeting inflation or the money growth rate from a given initial price level: this implies a determinate money supply sequence). Fiscal policy sets surpluses required to validate the real debts given this price sequence.

2. 'Sargent-Wallace fiscal leadership' (discussed further below under 'selfish overlapping generations'): here the *current* money supply is set to fix current prices but fiscal policy implies deficits which require a further inflation (and implied price level) sequence to validate the debt, so defining the future money supply sequence (or equivalent interest rate rule alternative as in type 1 policy).
   Both policy types 1 and 2 are 'Ricardian' in terms of the fiscal theory.

3. 'Non-Ricardian' fiscal leadership: fiscal policy implies deficits and an inflation path is set via *future* money supply growth (or implicit future interest rate policy); to validate the debt a *current* price level rise is required, implying a current money supply jump (or equivalent current interest rate policy). Hence type 3 is to be understood as fiscal leadership like type 2, only whereas 2 fixes current prices via monetary policy, 3 fixes future inflation via monetary policy; thus in 2 future monetary policy must reflect fiscal requirements whereas in 3 current monetary policy must do so via a jump in the levels of both money and prices.
   In none of the above cases does the government plan a default which leaves:

4. 'Planned default': here fiscal policy implies deficits while monetary policy fixes a price path that cannot validate the debt. The consequence is that the debt is automatically devalued by a fall in its

own price, D_t. This is the case of deliberate outright default without resort to 'indirect default' via price-devaluation (3) or inflation tax (2).

(1)–(4) is thus a classification of possible fiscal-monetary strategies. The 'fiscal theory' can then be thought of as drawing attention to 3. The essential point is that behind monetary policy lies a fiscal constraint: the government in the end has to plan how this will be met and the monetary (and hence price) consequences, present and future, are one part of this.

MODELS OF FISCAL EFFECT

The Ricardian equivalence model

In considering how fiscal policy might affect the economy, we begin with the model of 'Ricardian equivalence', named after Ricardo, who suggested the model without really believing it. If we assume that households have an infinite life, that all taxes are lump-sum (so creating no distortions), and that markets are complete (no unrealised opportunities for insurance or lending exist), then household consumption will depend, through intertemporal optimization, on the real rate of interest and permanent income. It follows that government spending plans will require a certain permanent level of taxation-plus-seigniorage in line with our previous discussion. Let us hold seigniorage constant as it is to do with monetary policy. If the government varies the pattern of taxation between periods, it will make no difference to this required permanent level. Cutting taxes in the present by TC_t for example will simply imply more taxes later, by an amount equal in present value to the tax cuts: for the extra debt issued will be rolled up with interest until future extra taxes are raised to pay it off, say in $t + N$. When they are, the accumulated extra debt will be equal to $TC_t(1 + r)^{t+N}$ and its present value will therefore be this discounted by the same interest rate, thus simply TC_t. This implies that households will treat the extra bonds they hold as not providing them with any extra wealth; thus government bonds are said to have no 'wealth effect'.

This is not to be confused with the idea that government spending changes have no effect: they will in principle. A higher permanent level of spending, the increase spread equally across periods, will raise permanent taxation and so lower permanent income by an equal amount. However, even Though they must reduce their permanent consumption by this amount, households may not reduce their spending in the current period by it; for example if interest rates are currently very high so that current consumption is low, they may reduce consumption proportionately in all periods given the pattern of interest rates, which in turn implies only a small absolute cut in the current period. Hence there would be a 'balanced budget multiplier' effect here, putting interest rates up still further.

If government spending is increased temporarily, with permanent spending held constant, then households will not change their consumption at given interest rates, so implying a rise in aggregate demand which will put upward pressure on interest rates. So the pattern of government spending over time will affect demand and the economy and the permanent level of government spending may also do so. The Ricardian Equivalence point applies to the pattern of taxation.

Its implications are that fiscal policy, interpreted as bond-financed tax changes, has no effect on the economy. Since there are also no distortions to be affected as taxes are speeded up or deferred through borrowing changes, it is a matter of supreme indifference in this model whether the government runs deficits or not in financing its spending plans. The only element in fiscal policy that matters is the level of those plans and their pattern over time. However it is usual to define fiscal policy to exclude this (considering it as allocative or strategic policy) and to refer to the extent of deficits: on this definition fiscal policy is irrelevant to aggregate demand.

Plainly too there is no relation in this model between deficits and monetary policy; a higher deficit requires no extra money supply growth to reduce the strain of financing it by bonds as there is no strain because private agents spend the same and so provide extra savings exactly equal to the required extra borrowing. Nor does a higher deficit increase interest rates, tightening monetary conditions (as in the IS-LM models we have used up to now). Monetary policy can be chosen entirely independently of fiscal policy (the only relation would be with the *permanent* level of taxes, which being lump-sum impose no distortions whereas seigniorage creates distortionary costs through money demand).The Ricardian assumptions can of course be relaxed, to dilute this harsh implication of a zero wealth effect. First, if taxes are not lump-sum, then a change in their intertemporal pattern will cause distortions, which will have real effects. We will discuss below (model (c)) how this alters the situation.

Secondly, we may appeal to various failures in the 'complete markets' assumption. These are not likely to have a large effect quantitatively; but one example is given in Box 7.1.

Box: 7.1

AN EXAMPLE OF MARKET INCOMPLETENESS: INCOMPLETE INSURANCE.

Suppose each individual faces considerable uncertainty about how much tax he and his heirs will pay; he does not know future income, future family size, the possibility of emigration, etc. Let us ignore emigration, in which case the change in tax prospects will have no impact on him; this no doubt affects only a minority. Let us assume that the tax system, as is typically the case in Western economies, is progressive; also assume that people are identical and risk averse. Suppose everyone receives the same transfer and holds it in the form of bonds. Then the individual will perceive his and his heirs' potential net income after tax as follows: at the one extreme they will be poor, receive the bond interest, but pay little tax, from there progressing with higher income towards the other extreme where they will be rich, receive the bond interest, and pay a lot of tax. If taxes are raised they will pay little more if poor, but significantly more if rich. This effect of a high covariance between tax and income is illustrated in figure 7.1 (for a simplified distribution with only 'poor' and 'rich' states of equal probability).
We are interested in the change in the 'certainty equivalent' of each man's future net income after tax, by this we mean the sure income that would yield him the same utility as the income possibilities he actually faces. Before the bond transfer and consequent future tax liability, let him have an expected net income of Ey, the average of his 'poor' and 'rich' states; the expected utility of this is EU_0 and the certainty equivalent is y_0. After the bond transfer, everyone's expected net income remains the same because the tax payments averaged across the two states must equal the bond interest receipts for the government's budget to balance in the future. But now each person will be better off than before when poor (he received the bond interest but his extra tax burden is less than this), and worse off than before when rich (his extra tax burden exceeds the bond interest); the arrowed line on figure 7.1 joins these two states, and, because of the insurance he receives in effect, his expected utility rises to EU_1 and his certainty equivalent income to y_1 so that the bond transfer

increases private wealth. Barro (1974) has further pointed out that, if the tax rate is raised when taxes go up to pay for the bond interest, then this increases the progressiveness of the tax system and this insurance effect is enhanced, as illustrated by the dashed line with higher expected utility EU_2 and certainty equivalent income y_2. Clearly this insurance effect of bond issue would be eliminated if the private insurance market already provided full insurance. However, this is unlikely because of incentive incompatibility (see e.g. Hart, 1983); full insurance gives the insured person an incentive to lie about his poverty or fraudulently to avoid trying to be rich.

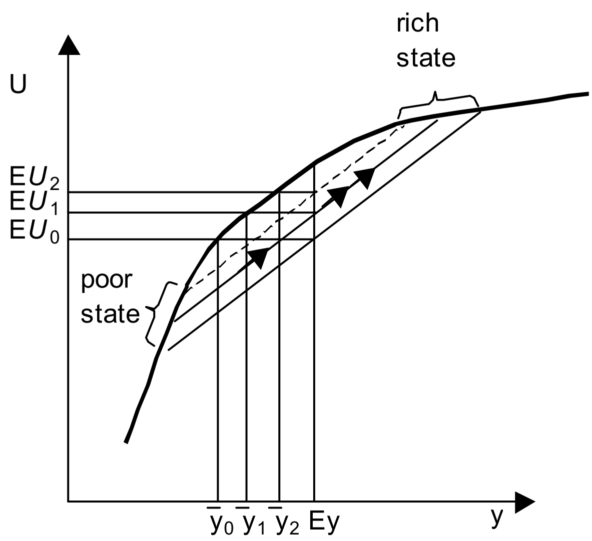

Figure 7.1: Certainty equivalent and expected net income after tax

Thirdly, we may drop the assumption of infinite life; this in practice is the most important relaxation. Plainly current households' members do not live for ever. But Barro (1974) has shown in a model of overlapping generations that, if each person leaves a bequest, then everyone acts as if he is infinitely lived. The reason is that the transfer he receives diminishes the utility of his heirs while raising his current resources. Yet he has already decided, having left them a bequest, that his optimal course is to give them their previous utility at the expense of his current resources; therefore he will raise his bequest in order to offset the government's action, in effect saving the whole of the transfer for his heirs. The transfer, by not changing his opportunity set, leaves his net wealth and behaviour unchanged.

If he has not planned a bequest, then the level of utility he desires for his heirs cannot be established; the transfer may or may not push their utility below his desired level. Hence we cannot establish how much, if at all, he will offset the transfer by saving.

It is of interest that the lack of bequests might not matter, because a 'bequest' could be negative (parents could receive transfers from their children) as well as positive, a continuous variable; hence the government transfer would always be offset. What matters is that one generation cares about the other, so that the government's redistribution from one to the other is offset (accidental bequests, e.g. because death comes unexpectedly early, are beside the point).

Barro's argument can be shown formally by simply setting out each generation's lifetime budget constraint. Suppose first of all that there is no government borrowing or taxation; then generation 1 has a life-time budget constraint viewed at $t+1$ (when this generation is old) of

$$y^1_{t+1} + y^1_t(1+r) = c^1_{t+1} + c^1_t(1+r) + q^1_{t+1} \tag{9}$$

where q^1_{t+1} is its bequest to generation 2, made at the end of $t+1$ period; $y =$ income, $c =$consumption, both in real terms, and r is the real rate of interest (assumed constant over time in this base-case situation). Superscripts denote the generation. Generation 2's lifetime budget constraint viewed from period $t+1$ is:

$$\frac{y^2_{t+2}}{1+r} + y^2_{t+1} + q^1_{t+1} = \frac{c^2_{t+2}}{1+r} + c^2_{t+1} + \frac{q^2_{t+2}}{1+r} \tag{10}$$

Now let the government transfer the amount b_t in t to generation 1, then young, and announce the plan to tax generation 2 by $T^2_{t+1} = b_t(1+r)$ when it is young, in $t+1$, meanwhile offering b_t bonds at t to the loan market. Then the two generations' constraints become:
(gen. 1)

$$y^1_{t+1} + y^1_t(1+r) + b_t(1+r) = c^1_{t+1} + c^1_t(1+r) + q^1_{t+1} \tag{11}$$

(gen. 2)

$$\frac{y^2_{t+2}}{1+r} + y^2_{t+1} + q^1_{t+1} - b_t(1+r) = \frac{c^2_{t+2}}{1+r} + c^2_{t+1} + \frac{q^2_{t+2}}{1+r} \tag{12}$$

If generation 1 had already optimally planned a bequest designed to give generation 2 a certain lifetime consumption possibility while leaving itself with a certain other one, then the government's action, if not offset somehow, will raise its own consumption while reducing that of generation 2, which is by implication suboptimal. However, if generation 1 raises its bequest exactly by the amount of the transfer grossed up by interest received from putting it into savings, then both generations will be exactly as well off as it originally planned; hence the transfer is saved by generation 1, which thereby buys the bonds the government offers.

The Model of Selfish Overlapping generations: Unpleasant Monetarist Arithmetic

Whether generations are linked with each other in Barro's sense is an empirical matter. It may vary across and within societies. It seems reasonable to explore a model which is at the other end of the spectrum from the Ricardian and assumes each generation cares only about itself. Such a model is for example used by Sargent and Wallace (1981) in a paper designed to investigate the upper limit of deficits when money supply growth is being held down by 'monetarist' policies.

In their overlapping-generations (OLG) model, debt is held by the young, to be redeemed from them when old (the old will only wish to consume, in the absence of heirs). In such a model the government clearly cannot borrow more from the young than their current income. We will consider below (Chapter 12) the detailed workings of such a model. But plainly the pattern of taxation can have large effects in it; for example a tax cut financed by borrowing throughout the lives of the current generation, so that taxes are raised on the next generation, or perhaps indefinitely deferred, must raise the consumption of the current generation as it will raise its permanent income.

Let us turn to the connection of fiscal with monetary policy. As we will see in Chapter 12, borrowing in an OLG model is limited by the capacity of the younger generation to lend; the old will not lend since they wish to consume all their wealth before they die. The young will lend as much as they can be induced to through higher interest rates; let us assume that the government is deterred from raising interest rates above some normal level, r, by the higher debt servicing costs this will imply. Then at this interest rate there will be some maximum amount the young are willing to lend: call this the government's debt ceiling.

Now consider how the ceiling on debt would affect a simple macro model. Suppose that GDP, N (also standing for the population by an appropriate choice of indices), grows at the rate n per year and that the real rate of interest, r, is constant and greater than n. Let $H^* =$ rate of growth of money, $' =$ per capita value, $\pi =$ rate of inflation, $P =$ price level, b_t = the value of one-year indexed bonds. Write the demand for high-powered money, H, in the quantity theory manner as:

$$H_t = hP_tN_t \tag{13}$$

so that $H_t^* = \pi_t + n$. Suppose now that at time $t = 1$ new intentions are announced and carried out, for future fiscal and monetary policy; the announcement (fully believed) changes the present real value of bonds to b_1 as prices and interest rates react. There are then two phases of policy: 'transition' from $t = 2,\ ...,\ T-1$, and 'terminal' from $t = T,\ ...,$. During the transition phase policies may be different from their terminal phase when they must be in steady state. The government budget constraint is from $t = 2$ onwards:

$$b_t - b_{t-1} = d_t + rb_{t-1} - \frac{H_t - H_{t-1}}{P_t} \tag{14}$$

Expressing (14) in per capita terms, dividing all though by N_t, gives:

$$\frac{b_t}{N_t} = \frac{d_t}{N_t} - \frac{H_{t-1}}{P_tN_t}\frac{H_t - H_{t-1}}{H_{t-1}} + \frac{(1+r)b_{t-1}}{(1+n)N_{t-1}} \tag{15}$$

Using $P_tN_t = (1 + H_t^*)P_{t-1}N_{t-1}$, this becomes

$$b_t' = d_t' - \frac{hH_t^*}{1 + H_t^*} + (1 + r - n)b_{t-1}' \tag{16}$$

Since $r > n$ this is an explosive difference equation if interest-exclusive deficits and monetary targets are pursued independently. Suppose that during transition a constant H^* and d are chosen: these policies are monetarist so that H^* is 'low' but fiscal policy is 'expansionary', so that $d' > \frac{hH^*}{1+H^*}$. The set limit on this per capita stock of debt b', discussed above, is now bound to be reached at some point. At the date when this occurs, $b_t' = b'$, then policies have to change so as to ensure:

$$0 = b_{t+1}' - b_t' = d' - \frac{hH^*}{1 + H^*} + (r - n)b_t' \tag{17}$$

that is, that real bonds do not change any more. However, the policies could have been chosen so as to change before this, so that $b_t' < b'$ at this point. In general, the terminal date T, for the switch to a sustainable policy with unchanging per capita debt can be chosen freely from $t = 2$ onwards, so that H_T and d_T, the terminal or steady state policies are governed by:

$$0 = b_T' - b_{T-1}' = d_T' - \frac{hH_T^*}{1 + H_T^*} + (r - n)b_{T-1}'(b_T' \leq b') \tag{18}$$

The point is that there is a trade-off between the transitional policies (including the length of time they are pursued) and the terminal policies because the transitional policies affect the terminal stock of debt:

$$b'_{T-1} = (1+r-n)^{T-2}b'_1 + \sum_{i=2}^{T-1}(1+r-n)^{i-2}\left(d' - \frac{hH^*}{1+H^*}\right) \quad (19)$$

The trade-off implies each of the following:

1. If the government wishes to maintain a constant interest-exclusive deficit $d' = d'_T$, then the smaller is current (transitional) money supply growth, the larger will future money supply growth H^*_T have to be. Given fiscal 'profligacy', there is therefore a trade-off between current and future inflation.

2. If the government wishes to maintain constant money supply growth H^*, then the higher are the transitional deficits the larger are the future surpluses that will be required. Given monetary discipline, there is therefore a trade-off between current and future fiscal discipline.

The message is, in short, that tough monetary policies require tough fiscal policies (called by Sargent and Wallace 'unpleasant monetarist arithmetic').

The role of long-dated nominal bonds

We have so far neglected the term in b'_1, the value of real bonds after the policies are announced; implicitly we have suggested that b'_1 was quite small. This then allows scope for policy makers to choose between trade-offs 1 and 2 above. If, however, b'_1 is large, and close to b', then there is little scope for choice; the policy makers are forced to go rapidly to steady state fiscal-monetary policies, hard as these must be.

If the government bonds are nominal and short-dated, then the revaluation due to policies of lower inflation will be small unless the change is drastic; hence for example on a one-year nominal bond b_0, the change in value will be $-b_0(H^* - H_0^{*e})$ where H_0^{*e} is the money supply growth expected before the policy change. If the bonds are indexed as we assumed above, then the revaluation will be nil (given the fixed real interest rate assumption).

If the bonds are long-dated, the revaluation effect can be very large. For example, take a bond paying a fixed money amount, M_K, on maturity at $t = K$. The present value of this at $t = 1$ expected at $t = 0$ was $b_0 = \frac{M^K}{(1+R_0)^{K-1}}$ where $R_0 = H_0^{*e} + r - n$ was the nominal interest rate at $t=0$; the actual present value at $t=1$ is then $\frac{M^K}{(1+R_1)^{K-1}}$ where $R_1 = H_K^* + r - n$ and $H_K^* = \frac{T-1}{K-1}H^* + \frac{K-T}{K-1}H_T^*$ (that is, a weighted average of the transitional and the terminal H^*). Hence the unanticipated capital revaluation on such a bond is $-b_0(K-1)(H_K^* - H_0^{*e})$. For a credible anti-inflation policy where $H_K^* = H_T^* = n$, this revaluation will be a $K-1$ multiple of the terminal fall in inflation, and create the necessity for harsh fiscal discipline with much greater rapidity.

Definitions of the 'deficit'

Several definitions of the government 'deficit' are in use: inclusive or exclusive of debt interest and, if inclusive, inclusive of either nominal or real debt interest. We can re-express the restrictions placed on fiscal policy in terms of these different definitions.

Consider a version of the government budget constraint expressed in nominal terms in the steady state:

$$\Delta H_t + \Delta B_t = (g-\tau)P_t N_t + R_t B_t \tag{20}$$

B is the nominal market value of bonds (in steady state interest rates will not be changing), R is the market nominal interest rate, τ the tax rate and g the share of government spending in GNP (both assumed constant); H, P, π and N are respectively money supply, prices, inflation and GNP as before.

From our previous analysis, we constrain the ratio of debt to GDP to a constant in steady state so that:

$$\frac{\Delta B_t}{B_t} = \pi + n (\text{the growth of nominal GNP}) \tag{21}$$

Equation (20) can be written as:

$$\frac{\Delta H_t}{H_t}\frac{H_t}{P_t N_t} + \frac{\Delta B_t}{B_t}\frac{B_t}{P_t N_t} = g - t + \frac{B_t}{P_t N_t}R_t \tag{22}$$

So that using (21):

$$\frac{\Delta H_t}{H_t} = \overline{v}[g - \tau + \overline{b}(r-n)] \tag{23}$$

where $\overline{v}$ is the equilibrium velocity of money and $\overline{b}$ the equilibrium ratio of debt to GNP.

Equation (23) says that the steady state growth rate of money depends upon the ratio to GNP of the steady state deficit inclusive of real debt interest (sometimes called the 'inflation-adjusted real deficit'), minus an allowance for growth, $\bar{b}n$. We can also note that since in equilibrium $\frac{\Delta H_t}{H_t} = \pi + n$,

$$(\pi + n)(H_t + B_t) = R_t B_t + (g - \tau) P_t N_t \tag{24}$$

or

$$\frac{\Delta H_t}{H_t} = \pi + n = \frac{R_t B_t + (g - \tau) P_t N_t}{H_t + B_t} = \frac{PSBR_t}{P_t N_t} \frac{P_t N_t}{H_t + B_t} \tag{25}$$

which says that money growth equals the public sector borrowing requirement (PSBR) to GNP ratio times the 'velocity' of government net financial liabilities ('outside money', discussed in the next section).

Equations (23) and (25) are of course exactly equivalent, although one uses the inflation-adjusted deficit while the other uses the unadjusted deficit. However, when (23) is used to assess what fiscal policy must be used to validate a certain counter-inflationary monetary policy (e.g one to reduce $\Delta H_t/H_t$ to n from some high level), great care must be taken to include in $\bar{b}$ the effects of falling inflation and interest rates on the value of outstanding bonds; this adjustment can be very large as we saw above when a large proportion of these bonds are non-indexed and of long maturity, so that large cuts in the government deficit excluding interest may be necessary. When (25) is used, the implications are more transparent since nominal debt interest will not change except for short maturity stocks, which are rolled over before inflation comes down. These remarks are relevant to the debate on the Thatcher government's Medium Term Financial Strategy, which did not always carefully observe this point (for example, Buiter and Miller (1981) incorrectly argued that the fiscal policies were 'unnecessarily' restrictive using a crude adjustment for current inflation on debt at current market value).

Given the overall policy requirement of fiscal-monetary consistency, the application in any situation will be largely a question of what is politically feasible. This is particularly true of the short-run time path immediately on announcement. It may well be wise, for example, to cut public spending rapidly as money supply growth is cut, even though this implies a 'real' budget surplus (that is, inclusive of real debt interest) in the first few years because during this period debt interest on long-dated stock is still offset by high inflation. This real surplus will disappear as soon as inflation comes down, because the debt interest on long-dated stock will fall away only very slowly. Then, with the spending cuts done, the budget deficit will be at steady state levels and some of the debt revaluation (on the long-dated stock) will have been worked off by the previous real surpluses; this is an illustration of the previous section's discussion.

A Neo-Keynesian model with wealth effects

We can embed the logic of the OLG model above with its wealth effects of government bonds within a conventional IS-LM model of the type we used in earlier chapters; the advantage of doing so is that while the conclusions of the OLG analysis follow also in this conventional model, it can additionally be used for analysis of short-term stabilization policy. To illustrate these points we represent the government budget constraint in a different but convenient way. Private sector net financial wealth consists, in a closed economy (which we continue to assume), of government bonds and high powered money (the 'monetary base', consisting of the notes and coins issue plus commercial banks' deposits with the central Bank), that is, government net financial liabilities. These are known as 'outside money' (following an earlier literature — e.g. Patinkin, 1965; Metzler, 1951; Gurley and Shaw, 1960). Bank deposits are 'inside money' in that banks are a private sector institution; their deposits are therefore both assets and liabilities of the private sector, cancelling out in net terms.

Let f be the stock of government net financial liabilities (hence government debt) in real terms (that is, deflated by the consumer price index). f will rise for two reasons: first, a government deficit will create new liabilities, and second, the existing stock of liabilities will be subject to capital gains, as the price of bonds rises or the consumer price index falls. We write:

$$\Delta\theta_t = \frac{\Delta f_t}{f_{t-1}} = \frac{d_t}{\overline{b}} - (q\Delta R_t + \Delta p_t) \qquad (26)$$

where d is the (total, inclusive of debt interest) government deficit as a fraction of GDP, $\overline{b}$ is the ratio of government debt to GDP, $q =$ is the proportionate response of long-term bond prices to the long-term interest rate (R); and $\theta = log f$. The unit coefficient on p_t (the log of the price level) reflects our assumption that all government liabilities are denominated in money terms. We will view (26) with $\overline{b}$, q held constant at some average value as an appropriate approximation.

To (26) we add a relationship determining the supply of high-powered money (in logs), which we shall write as m (which we can also treat as total money, reflecting the convenient and conventional assumption that there is a fixed 'money multiplier' between the two). We write:

$$\Delta m_t = \frac{\Phi}{\overline{b}}(d_t - \overline{d}) + \Delta\overline{m} + \epsilon_t \qquad (27)$$

where $\overline{d}$ is the equilibrium (steady state) government deficit as a fraction of GDP, ϵ is an error term, and $\Delta\overline{m}$ is the equilibrium rate of growth of money. Equation (27) states that out of equilibrium money supply will have an independent random component (to which we could add other independent temporary determinants of money if we wished) as well as a component responding to the temporary component of the deficit. Given (26), (27) implicitly also determines the supply of nominal bonds as the difference between nominal financial assets and the monetary base.

Equation (27) focuses on two aspects of monetary policy with which we shall proceed to deal. First, how far does the equilibrium growth rate of money reflect the government deficit? Secondly how far should the money supply growth rate be varied (over the 'cycle') as budgetary financing needs change? In other words, what are the links between fiscal and monetary policy, first in, and second out, of steady state? This distinction is an important one in rational expectations models, as we shall see.

We begin with behaviour out of steady state.

Stability and Bond-Financed Deficits Out of Steady State

One issue that was given great prominence from the early 1970s until the general acceptance of rational expectations was the possibility of instability in models with wealth effects, if budget deficits are bond-financed. This can be illustrated in a simple fixed-price *IS-LM* model without rational expectations (a log-linear adaptation of Blinder and Solow's 1973 model). We use non-stochastic continuous time and abstract from the steady-state relationship between money and deficits by setting $\overline{d} = \Delta\overline{m} = 0$:

$$y_t = k\theta_t - \alpha r_t + \phi d_t \tag{28}$$

$$m_t = \overline{p} + \delta y_t - \beta r_t + \mu\theta_t \tag{29}$$

$$\dot{\theta}_t = \frac{d_t}{\overline{b}} - q\,\dot{r}_t \tag{30}$$

$$d_t = \overline{g} - \tau y_t \tag{31}$$

$$\dot{m}_t = \frac{\Phi}{\overline{b}} d_t \tag{32}$$

$\overline{g}$, government expenditure as a fraction of GDP, includes debt interest: this formulation assumes that other expenditure is reduced as debt interest rises. If it were not, the instability under bond finance discussed below would be severely worsened. τy_t measures marginal tax receipts as a fraction of GDP and hence τ is the income elasticity of taxation minus one: initial average tax receipts are netted out of $\overline{g}$. $\dot{}$ denotes the time derivatives. Because prices are fixed at $\overline{p}$, r_t is both the nominal and real interest rate. Equations (28) and (29) are the *IS* and *LM* curves with wealth effects; (30)–(32) are the budget constraint and money supply relationship for this model.

The model solves for y_t and r_t given θ_t. Using (32) and (30) gives $\dot{m}_t = \Phi\ (\dot{\theta}_t + q\ \dot{r}_t)$ so that $m_t = \Phi(\theta_t + qr_t) + K_m$, where K_m is an arbitrary constant. Substituting for m_t from this into (29) and for d_t from (31) into (28) yields the equations for y_t and r_t. Substituting the solution of them for y_t and r_t into (30) yields the equation of motion for θ_t as:

$$\dot{\theta}_t = \frac{-\frac{\tau}{b}(k\beta - \alpha\mu + \Phi kq + \Phi\alpha)}{(1+\phi\tau)(\beta + q\mu) + \delta(\alpha + qk)}\theta_t \tag{33}$$

For stability we require $(k\beta - \alpha\mu + \Phi kq + \Phi\alpha) > 0$. Clearly if money supply is held constant regardless of the deficit, i.e $\Phi = 0$ (bond financed deficits), then we must have $k\beta > \alpha\mu$, which raises the possibility of instability if there are relatively strong wealth effects in the LM curve. $\Phi > 0$ reduces the possibility; Blinder and Solow and others have accordingly advocated money-financed deficits as a means of avoiding possible instability.

This instability is illustrated in Figure 7.2. In addition to the IS and LM curves we have drawn in a $'WW'$ curve, which is the equation of the budget constraint, (30), showing the level of output where $\dot{\theta}_t = 0$, that is. there are no changes in wealth. For $\dot{\theta}_t = 0$, we must have both $d_t = 0$ and $\dot{r}_t = 0$; $\dot{r}_t = 0$ automatically when $\dot{\theta}_t = 0$ because as we have seen, r_t and y_t (the intersection of the IS and LM curves) depend on θ_t and cease to move when θ_t stops moving. This level is $\overline{y} = \frac{\overline{g}}{\tau}$since from (31) $d_t = 0$ at this point. To the right of the WW curve, $\dot{\theta}_t$ is falling (and rising to the left). The IS curve shifts leftward as θ_t falls but the LM curve shifts rightwards. Instability under bond-financing ($\Phi = 0$) occurs when the intersection moves rightwards (and down) that is, $k\beta < \alpha\mu$, as the effect of θ_t on y_t (from equations (28) and (29)) is

$$\frac{k\beta - \alpha\mu}{\beta + \alpha\delta}$$

This can be seen diagramatically by noting that the rightward shift of the LM is $\mu\theta$, the leftward shift of the IS is $k\theta$; but the flatter the IS curve (the higher α) the more the LM curve shift dominates the output movement, and vice versa the flatter the LM curve (the higher β).

We pointed out that there is additional instability if one assumes government expenditure is fixed and does not fall to offset debt interest. The interested reader can work this case out, using in place of (31):

$$d_t = \overline{g} - \tau y_t + \overline{r}\overline{b}\theta_t + \overline{b}r_t \tag{30'}$$

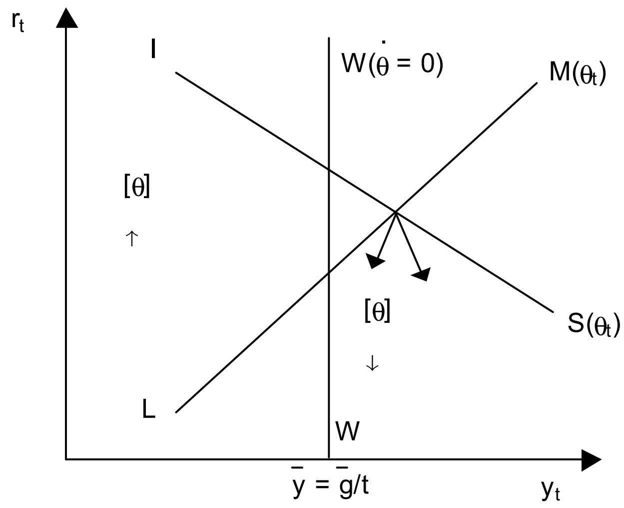

Figure 7.2: Instability with bond-financed deficits

where the last two terms approximate debt interest around some average interest rate, $\overline{r}$, and an average financial asset to GDP ratio, $\overline{b}$. The reader will find that the equivalent of (33) contains a number of extra positive terms in the numerator, increasing the chances of instability.

Under rational expectations, however, this Blinder-Solow argument carries less force. To convert their model into a rational expectations form, it is sufficient to recognize that the valuation of financial assets is forward-looking, that is, it depends on expectations of future interest rates; for convenience, now use discrete time. Returning, for example to the model given by (28) to (32), replace (30) and (32) by:

$$\triangle\theta_t = \frac{1}{\overline{b}}d_t - q(E_t r_{t+1} - E_{t-1}r_t) \tag{29'}$$

$$\triangle m_t = \frac{\Phi}{\overline{b}}d_t \tag{31'}$$

If the model is now solved by the methods of chapter 2, we obtain (if we drive the forward root backwards) a second-order characteristic equation in which one of the roots (the inverted forward root) should be absolutely greater than unity for a unique stable (saddlepath) solution (i.e. having both forward and backward roots stable). The roots involve all the coefficients and there is no general condition to ensure the saddlepath property.

If $\Phi = 0$, the characteristic equation $x_t + ax_{t-1} + bx_{t-2} = 0$ with roots σ_1, σ_2 has:

$$\begin{aligned} a &= -(\sigma_1 + \sigma_2) = -1 + \frac{D + \frac{\tau}{b}(k\beta - \alpha\mu)}{q[\delta k + (1 + \phi\tau)\mu]} \\ b &= \sigma_1\sigma_2 = -\frac{D}{q[\delta k + (1 + \phi\tau)\mu]} \end{aligned} \tag{34}$$

where $D = (1 + \phi\tau)\beta + \delta\alpha$.

Since $b < 0$, the roots cannot be complex, and at least one of the roots must be negative (alternating motion) and the other positive (monotonic motion) which is consistent with a saddlepath. For example, take the following parameter values, which approximate those of the Liverpool model of the UK:

$\beta = 2,\ \phi = k = \delta = 1,\ \tau = 0.3,\ \alpha = 0.5,\ q = 3,\ \mu = 0,\ \frac{1}{b} = 2$

These give $b = -1.03$, $a = 0.43$. Hence the roots are σ_1, $\sigma_2 =$ 0.82, −1.25: the monotonic saddlepath (figure 7.3a). Had we found by contrast $\mu = 5$ so that $k\beta < \alpha\mu$, we would have had σ_1, $\sigma_2 =$ 1.015, −0.135; again a saddlepath, but this time with alternating motion (figure 7.3b).

Interestingly, in the particular example here, the $k\alpha < \alpha\mu(\mu = 5)$ is actually more stable in that, although it is alternating, the absolute value of the stable root is much lower than in the case of $\mu = 0$. Computer examination of a wide range of values for the parameters suggests that problems with saddlepath stability arise whether $k\beta >$ or $< \alpha\mu$.

If $k\beta > \alpha\mu$, there is saddlepath stability when q or μ are low (otherwise the roots have a tendency to be both less than unity in absolute value: a 'non-uniqueness' problem). If $k\beta < \alpha\mu$, there is saddlepath stability when q is high (or in some cases when μ is high, even though q is low); otherwise the roots tend to be both greater than unity. The two typical cases of saddlepath stability are shown in figure 7.3

We conclude this section negatively: there is no compelling reason within rational expectations models to believe that the cyclical or short-run component of monetary policy ought to be influenced by fiscal policy. The decision, for example, whether to pursue a constant money supply growth rate through the cycle even though the budget deficit will be moving cyclically, can be taken on other grounds, notably those appropriate to stabilization policy. We now turn to the steady state component of the money supply rule, where the situation is quite different.

Long-term monetary targets

We now take the previous model, but abstract from short-run behaviour while reinstating the possibility of steady state inflation. We have in

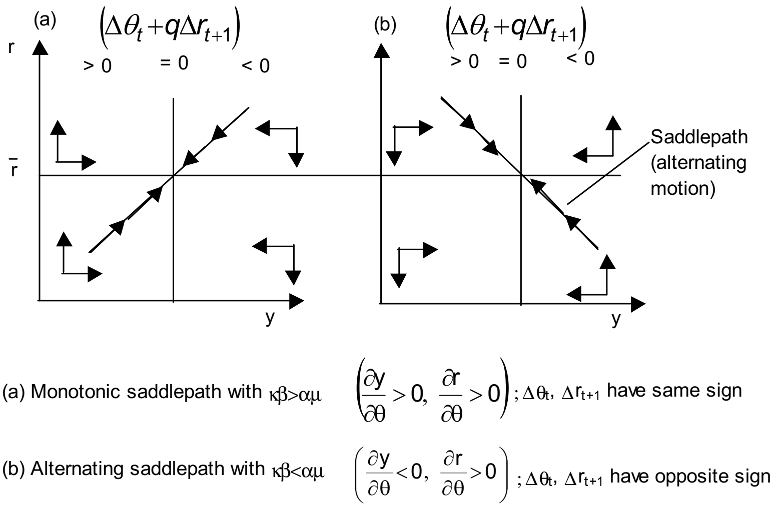

Figure 7.3: Two possible types of motion under rational expectations

equilibrium (* values are equilibrium ones):

$$y^* = k\theta^* - \alpha r^* + \phi\overline{d} \tag{35}$$
$$m^* = p^* + \delta y^* - \beta R^* + \mu\theta^* \tag{36}$$
$$\triangle\theta^* = \frac{1}{b}\overline{d} - q\Delta R^* - \triangle p^* \tag{37}$$
$$\overline{d} = \overline{g} - \tau y^* \tag{38}$$
$$R^* = r^* + \Delta p^* \tag{39}$$
$$\triangle m^* = \triangle\overline{m} \tag{40}$$

We assume y^* is exogenously given as the natural rate in an (omitted) Phillips Curve.

The question we wish to ask is: can $\triangle\,\overline{m}$ be chosen independently of $\overline{d}$? For simplicity assume a steady state growth rate of zero ($\triangle y^* = 0$), although this does not affect the argument. Assume also that $\overline{d}$ and $\Delta\overline{m}$ are chosen to be constants.

The first thing to notice is that, if in steady state both real interest rates and inflation are constant, then at once we have $\triangle R^* = 0$ from (39), $\triangle\theta^* = \frac{\alpha}{k}\,\triangle\,r^* = 0$ from (35); then from (36) and (40) we have $\Delta\overline{m} = \Delta m^* = \Delta p^*$, while from (37) we have $\frac{1}{b}\overline{d} = \triangle p^*$. It then follows that $\frac{1}{b}\overline{d} = \Delta\overline{m}$; monetary and fiscal policy have to be 'consistent', that is the rate of money supply growth has to be equal to the deficit as a fraction of financial assets, $\frac{1}{b}\overline{d}$. This in turn is equal to the rate at which nominal financial assets are growing ($\triangle\theta^* + \triangle p^*$); hence if money is growing at this rate, so also are bonds. Consistent monetary and fiscal policy hence implies that money and nominal bonds must be growing at the same rate.

What happens if they do not? Suppose for example we place a terminal condition on inflation reflecting a government inflation target achieved via money supply growth. So inflation will be constant in steady state. We may then derive by a similar procedure to the one already used:

$$\triangle r^* = \frac{k}{k(\beta + q) + \alpha(1 - \mu)}(\frac{1}{b}\overline{d} - \triangle\overline{m}) \tag{41}$$

In other words, if the government (given its terminal condition on inflation) fails to pursue consistent fiscal and monetary policy, real interest rates will take the strain and eventually either (rising r^*) government expenditure would have to contract to zero (taxes rise to absorb the whole of GNP) or (falling r^*) real interest rates would become negative — neither of which is possible. Of course a sensible government will wish to stop any such tendency well before any such stage is threatened.

This argument has been conducted on the assumption that $\overline{d}$ is set, with changing interest payments on debt being offset by changes in government spending or taxes; if one assumes instead that government spending and tax rates are unaltered, then the steady state inflation rate under consistent policy, $\frac{1}{b}\overline{d} = \triangle\overline{m}$, depends on the level of government spending and taxes chosen; the algebra of this is more complex (see Minford et al., 1980), and the inflation resulting from any initial rise in the deficit is much greater than in the analysis above, because the eventual deficit is compounded by the rise in interest payments, but the essential message remains that there must be consistency between fiscal and monetary policy in steady state.

To conclude, wealth effects imply a constraint across the steady state components of fiscal and monetary policy, though not the short-run components. Hence in (27) $\triangle\ \overline{m} = \frac{1}{b}\overline{d}$, whereas we are free to write any short-run response function or error process besides. What we have done in this section is to show within a conventional *IS-LM* model with wealth effects essentially the same result as derived in the overlapping generations framework of Sargent and Wallace — that there are close connections between fiscal and monetary policy choices in the long run, but that in the short run there is considerable flexibility in their relationship.

A model with distortionary taxation: the optimal pattern of borrowing

We have seen that if a government wishes to achieve a certain money supply growth rate (to reach a desired inflation rate), then it is limited in the steady state (or 'average') deficit it can pursue at the same time. Nevertheless, provided it is willing to make up temporary deficits in excess of this with future deficits that fall short of it (even running into surpluses), then as we have seen it can still achieve its monetary objectives. The question we now ask is: what is a desirable pattern of such temporary deficits and surpluses?

In chapter 4 we looked at this issue from the point of view of stabilization policy ('demand management'). We concluded first, that tax rates acted as an effective automatic stabilizer and that this was desirable, if the tax rates were an unavoidable microeconomic distortion and if unemployment was distorted upwards in cyclical troughs by the benefit system; secondly that activist, 'feedback', fiscal policy was also effective and could also be beneficial on similar grounds. But we now consider the issue abstracting from the business cycle and stabilization policy. We use the criterion of optimal public finance alone: that is, we consider the distortionary costs of taxation. In order to do this, we shall drop the assumption that taxes are lump sum, and assume instead that each period there is an average (= marginal) tax rate, T_t. Such distortionary taxation implies that even in an economy with infinitely-lived households (or where each generation cares about the others) the pattern of deficits and borrowing affects consumer surplus and so private wealth, by creating variation in the extent of the tax distortions over time and hence in their present discounted value. It turns out that there is an optimum pattern which minimizes this present discounted loss of surplus.

Suppose that output, N, is (apart from temporary effects of tax rate variation) continuously at $\overline{N}$, so that stabilization is not of interest, and for simplicity suppose further that $\overline{N}$ is constant, real government spending is constant at $\overline{G}$, and money supply growth is constant at μ. As in the Sargent-Wallace example, let the demand for money be given by the quantity theory as:

$$H_t = hP_t\overline{N} \tag{42}$$

and the budget constraint be given by:

$$b_t - b_{t-1} = G - T_t(1-\epsilon)\overline{N} + rb_{t-1} - \frac{H_t - H_{t-1}}{P_t} \tag{43}$$

where b_t are indexed one-period bonds with a constant real interest rate, r, as at the start of this chapter and note that $T_t(1-\epsilon)\overline{N}$ deducts tax revenue lost as output responds to the tax rate with an elasticity ϵ (assumed constant). We can rewrite the budget constraint, following (4) as:

$$b_{t-1} = \frac{1}{1+r}\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^{i}\left(T_{t+i}(1-\epsilon)\overline{N} - \overline{G} + \frac{h\overline{N}\mu}{1+\mu}\right)\right]$$
$$= \frac{\frac{h\overline{N}\mu}{1+\mu} - \overline{G}}{r} + \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^{i+1} T_{t+i}(1-\epsilon)\overline{N} \quad (44)$$

This says that the outstanding value of bonds must be equal to the present value of taxation less the present value of government spending (net of inflation tax revenue). This is the constraint on the pattern and total of taxation — due to the solvency constraint.

Now examine the distortion costs of taxation. By the usual consumer surplus triangle analysis, we can write the present value of these costs at the end of $t-1$, C_{t-1}, as:

$$C_{t-1} = \sum_{i=0}^{\infty} \frac{1}{2} T_{t+i}^{2} \varepsilon \overline{N} \left(\frac{1}{1+r}\right)^{i+1} \quad (45)$$

The optimal tax rates are discovered by minimizing C_{t-1} subject to the tax constraint. Form the Lagrangean:

$$L = C_{t-1} + m[K - \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^{i+1} T_{t+i}(1-\epsilon)\overline{N}] \quad (46)$$

where $K = b_{t-1} + \frac{\overline{G} - \frac{h\overline{N}\mu}{1+\mu}}{r}$. The first-order condition is:

$$0 = \frac{\partial L}{\partial T_{t+i}} = T_{t+i}\epsilon - m(1-\epsilon) \quad (47)$$

Since ε and m are fixed constants, this yields

$$T_{t+i} = T = \frac{m(1-\epsilon)}{\varepsilon} \quad (48)$$

This result, due to Lucas and Stokey (1983), extends Ramsey's (1927) rule that commodity tax rates should be inversely proportional to their demand elasticities. In other words, the optimal tax rate, T, is a constant if the output elasticity is constant over time. We can work out what it must be by solving for $T_{t+i} = T$ in the taxation constraint as:

$$T(1-\epsilon) = \frac{rb_{t-1}}{\overline{N}} + \frac{\overline{G}}{\overline{N}} - \frac{h\mu}{1+\mu} \quad (49)$$

The constant tax yield is that which will pay the interest on the outstanding stock of debt plus the government spending bill net of the inflation tax. Hence the stock of debt is held constant by optimal taxation. (If GDP was growing and government spending constant as a fraction of it, then the constant tax rate formula would imply that debt would also be constant as a fraction of GDP. The reader may like to rework the optimal tax rate problem with GDP growing at the rate n, using the budget constraint formulation of equation (16). He should find that the steps are identical, except that $r - n$ is substituted for r and bonds and government spending are expressed as fractions of GDP; the optimal tax rate remains constant.)

To put optimal tax rates another way, remember that optimality requires the intertemporal rate of transformation between revenues to equal that between welfare costs; that is,

$$\left(\frac{\delta C_{t-1}}{\delta T_{t+1}}\right) / \left(\frac{\delta R_{t-1}}{\delta T_{i+1}}\right) = \left(\frac{\delta C_{t-1}}{dT_{t+i+1}}\right) / \left(\frac{\delta R_{t-1}}{\delta T_{t+i+1}}\right) \tag{50}$$

where $R_{t-1} = \sum_{t=0}^{\infty} (\frac{1}{1+r})^{i+1} T_{t+i}(1-\epsilon)\overline{N}$ is the present value of revenues. In words this states that the (discounted) marginal cost per unit of (discounted) revenue gained from raising the tax rate in period $t+i$ must equal that from raising the tax rate in period $t+i+1$. The former (LHS) is $\frac{T_{t+i}\varepsilon}{m(1-\epsilon)}$, the latter (RHS) is $\frac{T_{t+i+1}\varepsilon}{m(1-\epsilon)}$ — in each the numerator is the contemporaneous welfare cost, the denominator the contemporaneous tax yield, of a penny rise in the tax rate. Notice that since both the welfare cost and the tax yield are discounted, the discount factor drops out.

Figure 7.4 illustrates this. It is assumed that workers have a constant marginal product MPL. The supply curve, SS, shows the output they produce as their marginal real wage, w, rises, increasing their hours input into the overall production function. The tax rate, T, depresses their marginal take home pay by Tw. The triangles of consumer surplus are minimized in total area when they are equal: as one increases from this point, the other decreases by less because their area depends on the square of the tax rate. The height of the triangle is measured by T_{t+i}, which is also a measure of the loss per unit of extra revenue raised.

CONCLUSIONS

We have considered the role of fiscal policy in three basic models: with Ricardian equivalence (where there are complete markets, infinitely-lived

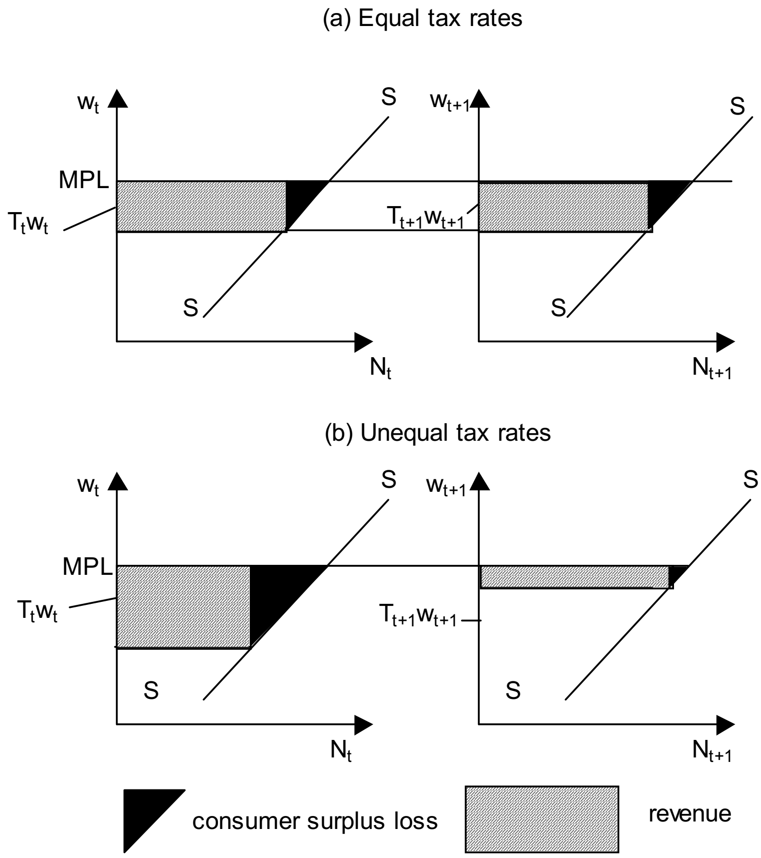

Figure 7.4: The optimality of a constant tax rate

households and non-distorting, i.e. lump-sum, taxes), and then in two main cases without it, the first with overlapping generations with finite life that do not care about each other, and the second where they do (and so behave like infinitely-lived households) but there are distorting taxes.

In the first Ricardian equivalence model the pattern of taxes and borrowing does not matter. The only constraint on the government is its solvency; if it violates this condition with its tax, spending and monetary policies then the markets will mark its debt down in value until the condition is met (in other words the debt value factors in expected default). Monetary policy sets the rate of inflation and should if properly conducted also fix the price level. Fiscal policy then must set taxes and spending so that solvency is ensured.

In the second model with unconnected overlapping generations we saw that there are important links between fiscal and monetary policy due to the fact that patterns of public borrowing affect the current generation's wealth. These links do not involve short-run behaviour, that is, over the economic cycle; independent fiscal and monetary responses to the cycle do not in general cause instability (or non-uniqueness) under rational expectations. By contrast, with adaptive expectations and wealth effects, coordination may be necessary even in the short run to avoid instability. It is in steady state that fiscal and monetary policy must be 'consistent', that is, the growth rate of real government bonds must be reduced to the rate of growth of GDP (assuming this is the rate at which the demand for bonds will grow in steady state at constant real interest rates).

In the final model, with distortionary taxation and infinitely-lived agents, the pattern of borrowing and taxation affects consumer surplus (and so private wealth) and we find that the optimum tax rate that minimises this loss of surplus is planned to be constant over time from the present state, given past and current shocks ('tax-smoothing'). As new shocks arrive, the tax rate adjusts to maintain the same future constancy, with borrowing taking up the resulting difference between taxes and spending.

So, summarising, we can say that given the choice of monetary targets to set prices fiscal policy is significantly constrained, to achieve solvency, to ensure long-run consistency between debt and money holdings, and to achieve optimality in tax patterns. We have moved a long way from the models of the 1970s where fiscal policy could be freely set, the only result being 'crowding out' via interest rate movements.