International trade and the division of labour

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Abstract

This paper develops a model of international trade based on the division of labour under perfect competition. International trade, by expanding the size of the market, eliminates duplication of coordination costs and leads to a greater variety of intermediate goods, each produced at a larger scale than in autarky. The model predicts an *upper bound* on the volume of trade; however, the gains from trade are independent of the trade volume.

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1 Introduction

"I always say there are two and a half theories of trade".

Paul Krugman to Peter Neary, as quoted in Neary (2010).

There are three main approaches to theoretical modelling of international trade: the approach based on comparative advantage and perfect competition from Ricardo to Heckscher and Ohlin, the approach based on monopolistic competition as in Krugman (1979, 1980), and the approach based on oligopoly as initially developed by Brander (1981)[1]. This paper develops a new model of international trade which takes a different approach to the preceding literature, by focussing on the division of labour as the reason for international trade. The role of the division of labour in raising per capita incomes has been recognised since at least Adam Smith (1776). The model we develop is based on trade in intermediate inputs, which constitutes over half of total goods trade, as documented by Miroudot et al (2009) and Sturgeon and Memedovic (2010), and shares features of both the comparative advantage and monopolistic competition approaches. From the comparative advantage literature, it uses a perfectly competitive market structure; from the monopolistic competition literature, countries are ex ante identical to each other, so there is no comparative advantage reason for international trade.

In our model, the division of labour is limited by both the extent of the market, and by coordination costs. International trade eliminates the duplication of coordination costs across countries, which encourages greater division of labour, and hence higher levels of output and welfare. Thus, similarly to models of trade based on monopolistic competition, we endogenise the number of varieties of intermediate goods produced; however, this is done under perfectly competitive markets. Because countries are assumed to have identical technologies in producing intermediate goods, both the volume and direction of trade are indeterminate beyond the existence of an upper bound on the volume of trade; however, the gains from trade are independent of the volume of trade.

The role of the division of labour in international trade has been developed especially by Ethier (1979, 1982a). In the earlier paper, the distinction is not made between external and

[1] The oligopolistic approach is what Neary (2010) refers to as the half theory of trade, since it is not as widely used as the other two approaches, despite the efforts of Neary (2009).

internal scale economies, while the later paper is explicit in its use of both internal and external scale economies. Francois (1990a, 1990b) makes use of the production function developed by Edwards and Starr (1987) to develop a model of international trade in which scale economies arise from producer services in a monopolistic competition model. More recently, Chaney and Ossa (2013) open up the black box of the production function in the Krugman (1979) model of monopolistic competition, modelling the production process as a series of stages produced by teams. Becker and Murphy (1992) develop a closed economy model in which the extent of the division of labour is limited by the cost of coordinating inputs. This is similar to that used by Francois (1990a, 1990b), and is the approach adopted in the present paper.

Because the present paper makes use of a perfectly competitive framework, it is different from the literature above (apart from Becker and Murphy (1992), who do not consider international trade). Swanson (1999) develops a different model of the division of labour under perfect competition, in which a larger market enables greater specialisation and hence higher skill levels and output per worker via the endogenous development of comparative advantage. More closely related is Soo (2013), who develops a model of international trade based on the division of labour and comparative advantage in a perfectly competitive framework. Unlike Soo (2013), who makes use of comparative advantage to pin down the structure of production, in the present paper we focus on the cost of coordinating inputs that limits the extent of the division of labour.

In order to close the model, we assume that the production of intermediate goods takes place under what Ethier (1979) refers to as international scale economies which are external to the firm. As discussed below, the fact that the external scale economies are international in nature is what generates the indeterminacy in trade patterns and volumes, and is similar to the result obtained in Ethier (1979). Most related work in this area assumes scale economies which are national in nature, for instance Markusen and Melvin (1981) and Panagariya (1981). Helpman (1984) provides an insightful survey of this literature, while Grossman and Rossi-Hansberg (2010) offer a recent treatment. Nevertheless, whilst assuming international scale economies avoids the necessity of making additional assumptions in order to close the model, we show that it is not crucial to the main results of the paper.

The next section outlines the main building blocks of the model. Section 3 discusses the autarkic equilibrium, while Section 4 discusses the implications of international trade, and Section 5 discusses the patterns of trade. Section 6 discusses the implications of assuming international scale economies, while Section 7 provides some concluding comments.

2 The model

The model is set up with two countries, $i \in H, F$ for Home and Foreign, although the solution method allows for easy extension to many countries. There is a single final good which is used in consumption. Let the representative consumer's utility function be:

$$U_i = C_i^{\theta}, \qquad 0 < \theta < 1 \tag{1}$$

All markets are perfectly competitive. There are many possible intermediate goods, $j = 1, \dots, N$. The final good is assembled from intermediate goods using the following production function:

$$Q_i = \left(\sum_i n_i\right)^{\beta} \min\{x_{ij}\}, \qquad 1 < \beta < 2 \tag{2}$$

Where n_i is the number of intermediate goods actually produced, and x_{ij} is the quantity of each intermediate good j used in country i. The production function is such that the intermediate inputs are perfect complements, so that in equilibrium x_i is always the same across intermediate inputs. This is quite a strong assumption, but simplifies the analysis and ensures that the equilibrium is both stable and unique (see Appendix A). That $\beta > 1$ indicates the gains from the division of labour; the more the production process is divided into different stages, the larger the output of the final good[2]. Thus, firms will, in the absence of coordination costs, want to divide the production process into as many steps as possible; it is the coordination cost that constrains the division of labour.

Intermediate goods are produced using labour as the only factor of production using a production function that exhibits external scale economies which are international in nature (Ethier, 1979). That is, output of an intermediate good depends on the total employment in that intermediate input in the world:

$$q_j = \left(\alpha l_j\right)^{\gamma}, \qquad \alpha < 1, \quad 1 < \gamma < \beta \tag{3}$$

[2] The model is isomorphic to one in which consumers consume the intermediate goods directly. However, while it may be reasonable to assume division of labour in the production process, it is more difficult to justify on the consumption side.

Where $\alpha < 1$ is labour productivity, and $\gamma > 1$ indicates external scale economies. The reason for assuming external scale economies instead of constant returns to scale, as will be shown below, is to pin down the number of intermediate goods actually produced. Our main results below do not change if constant returns to scale are assumed in the production of intermediate goods; this is further discussed in Section 6 below. Under perfect competition, normalising the wage rate to unity, and assuming average cost pricing (see Ethier, 1979), the zero profit condition implies that the price of each intermediate good is given by:

$$p_j = \left(\alpha^\gamma l_j^{\gamma-1}\right)^{-1} \tag{4}$$

Since $\gamma > 1$, the larger the employment in sector j, the lower the price. Also, the higher is labour productivity α, the lower the price. Appendix A shows how equations (3) and (4) can be obtained from the production function for each perfectly competitive firm and the firm's profit-maximising condition, respectively.

Each country has an endowment of L_i units of labour. In assembling the final good from the intermediate goods, there is a coordination cost that depends on the number of intermediate goods used in the production process:

$$c_i = \psi n_i^\rho, \qquad \rho > \beta > 1 \tag{5}$$

The assembly process uses real resources in the sense that final output is reduced by the assembly cost (analogously to the "iceberg" trade costs in other papers). This cost is assumed to be shared by all firms producing the final good, and may be thought of as the cost of maintaining a production network; the more intermediate inputs there are, the more difficult and expensive it becomes to coordinate all the inputs. As we will see below, the restriction that $\rho > \beta$ ensures that a larger country not only has a larger number of intermediate goods, but also that each intermediate good is produced at a larger scale.

3 Autarkic equilibrium

In autarky, all domestically produced intermediate goods are used in producing the domestic final good, and since all intermediate goods are produced in equal quantities, we have:

$$x_{ij} = q_{ij} = \left(\alpha l_{ij}\right)^\gamma = \left(\frac{\alpha L_i}{n_i}\right)^\gamma. \tag{6}$$

Substituting this into the production function (2) and subtracting the assembly cost (5) gives the production function for final goods net of assembly cost:

$$Q_i = n_i^{\beta-\gamma}(\alpha L_i)^{\gamma} - \psi n_i^{\rho} \tag{7}$$

Each firm in the final good sector chooses the number of intermediate inputs to maximise profits. All firms are identical to each other, so total industry profits are:

$$\pi_i = P_i Q_i - p_i n_i x_i \tag{8}$$

Where P_i is the price of the final good, and is taken as given by the perfectly competitive firms. Substituting from equations (4), (6) and (7), we can rewrite the profit function (8) as:

$$\pi_i = P_i\left[n_i^{\beta-\gamma}(\alpha L_i)^{\gamma} - \psi n_i^{\rho}\right] - L_i \tag{9}$$

Differentiating equation (7) with respect to n_i allows us to solve for the number of intermediate goods produced in each economy (ignoring integer constraints) [3]:

$$n_i = \left[\frac{(\alpha L_i)^{\gamma}(\beta-\gamma)}{\psi\rho}\right]^{\frac{1}{\rho+\gamma-\beta}} \tag{10}$$

Equation (10) shows that the assumption made above that $\gamma < \beta$ is required to generate positive values of n_i. In principle, each final good producing firm could demand different intermediate inputs. However, because production of intermediate inputs occurs under external scale economies, the total number of intermediate goods produced will be the minimum number that will satisfy equation (10). That is, all final good producers will use the same intermediate goods. This is the role played by the assumption of external scale economies.

Since from equation (6) $q_i = (\alpha L_i/n_i)^{\gamma}$, we also have:

$$q_i = (\alpha L_i)^{\frac{\gamma(\rho-\beta)}{\rho+\gamma-\beta}}\left(\frac{\psi\rho}{\beta-\gamma}\right)^{\frac{\gamma}{\rho+\gamma-\beta}} \tag{11}$$

Equation (11) shows that the assumption that $\rho > \beta$ implies that $dq_i/dL_i > 0$. Similarly, from equation (10), as long as $\rho + \gamma > \beta$ (which always holds since we assume that $\rho > \beta$), we have $dn_i/dL_i > 0$. That is, a larger country produces a larger number of distinct intermediate goods, and produces each of these intermediate goods at a larger scale. This is in accord with the empirical evidence presented in Hummels and Klenow (2005) and Hanson (2012). Following the terminology of the literature, a larger country expands both in terms of the intensive margin (more output of each intermediate is produced) and in terms of the extensive margin (more types of intermediates are produced). This gives similar results to Krugman (1979), and contrasts with the monopolistic competition literature based on the CES utility function (e.g. Krugman 1980), in which a larger country has a larger variety of

[3] It can be verified that $d^2\pi_i/dn_i^2 < 0$, so that equation (10) is indeed the profit-maximising expression for n_i.

goods, but not larger sectors. The extent of the division of labour depends on the size of the market as in Smith (1776), but also on the coordination cost as in Becker and Murphy (1992).

We can also obtain the price of the final good. Setting the profit function (9) equal to zero and solving gives:

$$P_i = \frac{L_i}{n_i^{\beta-\gamma}(\alpha L_i)^{\gamma} - \psi n_i^{\rho}} \tag{12}$$

From equation (10) above, a country with a larger labour force will produce a larger number of distinct intermediate goods. This reduces the cost of production of the final good because of the division of labour, and hence reduces the price of the final good relative to intermediate goods in equilibrium.

Substituting from the number of intermediate goods (10) into the production function for final goods (7) and then into the consumer's utility function (1), making use of $C_i = Q_i/L_i$ gives autarkic utility as a function of the model's parameters:

$$U_i^A = \left\{\frac{1}{L_i}\left[n_i^{\beta-\gamma}(\alpha L_i)^{\gamma} - \psi n_i^{\rho}\right]\right\}^{\theta} \tag{13}$$

Larger countries have a higher level of utility under autarky, since a larger economy enables greater division of labour; $dU_i^A/dL_i > 0$.

Note that the market equilibrium as described above is efficient, since it yields the same outcome as would be obtained by a benevolent central planner, whose objective is to maximise the country's utility by choosing the optimal number of intermediate inputs to maximise net output. The reason for this is that the assumptions we have made above mean that firms internalise the effects of increasing numbers of intermediate inputs on their profits, as shown in equation (9). More intermediate inputs imply greater division of labour, but also higher coordination costs, and final goods firms take both effects into account when choosing the number of intermediate inputs.

4 International trade

In this section we allow for international trade in both intermediate and final goods between the two countries. We start by considering free international trade between the two countries, and then extend the model to include trade frictions.

4.1 Free trade

Similarly to Krugman (1979, 1980), international trade is equivalent to an increase in the size of the economy, since countries have identical technologies and there is only one factor of production. The crucial assumption here is that when international trade is allowed, the coordination cost is shared between the two countries, since the two countries effectively become one market. Following the same steps as for the autarkic equilibrium, the number of intermediate goods that is consistent with profit maximisation by all final goods firms is:

$$n^T = \left\{\frac{[\alpha(L_H+L_F)]^{\gamma}(\beta-\gamma)}{\psi\rho}\right\}^{\frac{1}{\rho+\gamma-\beta}} \tag{14}$$

And the output of each intermediate good is:

$$q^T = [\alpha(L_H + L_F)]^{\frac{\gamma(\rho-\beta)}{\rho+\gamma-\beta}}\left(\frac{\psi\rho}{\beta-\gamma}\right)^{\frac{\gamma}{\rho+\gamma-\beta}} \tag{15}$$

These expressions also indicate how the model can be extended to allow for many countries, and the implications of such an extension. We can establish that:

$$n_H^A, n_F^A < n^T < n_H^A + n_F^A \tag{16}$$

And

$$q_H^A, q_F^A < q^T < q_H^A + q_F^A \tag{17}$$

That is, the number of intermediate goods and the output of each intermediate good both increase compared to the autarkic number and output of each intermediate good. However, the increase is less than proportional to the expansion in market size resulting from trade liberalisation. International trade leads to an expansion on both the intensive and extensive margins.

The representative consumer's utility with free international trade is given by:

$$U_i^T = \left\{\left(\frac{1}{L_H+L_F}\right)\left[(n^T)^{\beta-\gamma}\big(\alpha(L_H + L_F)\big)^{\gamma} - (\psi(n^T)^{\rho})\right]\right\}^{\theta} \tag{18}$$

It can be shown that $U_i^T > U_i^A$; that is, there are always gains from free international trade. These gains arise from the saving in the coordination cost; whereas in autarky the coordination cost is shared only by domestic firms, in international trade it is shared by both domestic and foreign firms. This cost saving enables firms to increase the division of labour, thus yielding a productivity gain in the output of the final consumption good. Note also that since the free trade welfare is the same for all consumers in both countries whereas autarkic utility is higher in the larger country, we get the usual result that a smaller country gains more from trade than does a larger country.

4.2 Trade frictions

Now suppose that there are trade frictions that increase the cost of coordination in the presence of international trade, so that the coordination cost becomes:

$$c^{TF} = \tau\psi(n^{TF})^{\rho} \tag{19}$$

Where the superscript TF denotes the outcome with trade frictions, and $\tau \geq 1$ is the additional coordination cost due to the frictions that arise from international trade (for instance, different languages or legal systems). Unlike trade costs, which affect only imported goods but not domestically produced goods, we assume that the trade friction affects both imported and domestically-produced intermediates, so has no impact on relative prices or demands. The additional trade friction incurred because of international trade must be less than the gain from spreading the coordination cost across countries. Following the same steps as above, the equilibrium number of intermediate goods is:

$$n^{TF} = \left\{\frac{[\alpha(L_H+L_F)]^{\gamma}(\beta-\gamma)}{\tau\psi\rho}\right\}^{\frac{1}{\rho+\gamma-\beta}} \tag{20}$$

And the output of each intermediate good is:

$$q^{TF} = [\alpha(L_H + L_F)]^{\frac{\gamma(\rho-\beta)}{\rho+\gamma-\beta}}\left(\frac{\tau\psi\rho}{\beta-\gamma}\right)^{\frac{\gamma}{\rho+\gamma-\beta}} \tag{21}$$

The representative consumer's utility with trade frictions is given by:

$$U_i^{TF} = \left\{\left(\frac{1}{L_H+L_F}\right)\left[(n^{TF})^{\beta-\gamma}\big(\alpha(L_H + L_F)\big)^{\gamma} - (\tau\psi(n^{TF})^{\rho})\right]\right\}^{\theta} \tag{22}$$

Comparing the trade-friction outcome with the free trade outcome, the free trade outcome has a larger number of intermediate goods, and each intermediate good is produced on a smaller scale. Intuitively, the trade friction increases the cost of coordinating inputs, so reduces the incentive for firms to divide the production process into more intermediate components. As a result, utility in the presence of trade frictions is lower than utility in free trade.

Comparing the trade-friction outcome with the autarkic outcome, the number of intermediate goods is larger with trade frictions than in autarky if:

$$\tau < \left(\frac{L_H+L_F}{L_i}\right)^{\gamma} \tag{23}$$

The output of each intermediate good is greater with trade frictions than in autarky if:

$$\tau > \left(\frac{L_i}{L_H+L_F}\right)^{\rho-\beta} \tag{24}$$

And utility is greater than in autarky (there are gains from trade) if:

$$\tau < \left(\frac{L_H+L_F}{L_i}\right)^{\frac{\rho+\gamma-\beta-\rho\gamma}{\gamma-\beta}} \tag{25}$$

Condition (24) is always satisfied provided $\tau \geq 1$ as we have assumed, while conditions (23) and (25) are satisfied provided τ is not too large. If conditions (23) and (24) hold, then we can also establish that, similarly to the case for free trade, that the number of intermediate goods and the output of each intermediate good both increase less than proportionally to the expansion in market size resulting from international trade.

Note also the role of the trade friction term τ in the analysis above. It can be shown that $dn^{TF}/d\tau < 0$, $dq^{TF}/d\tau > 0$, and $dU^{TF}/d\tau < 0$. That is, the higher the trade friction, the smaller the number of intermediate goods, the larger the output of each intermediate good, and the lower the utility from international trade. Equivalently, trade liberalisation which reduces τ would increase the number of intermediate goods, reduce the output of each intermediate good, and increase consumer welfare. The increase in welfare may be attributed to greater division of labour resulting from the increased number of intermediate goods; Broda and Weinstein (2006) show empirically that increased import variety has contributed substantially to US national welfare.

5 Trade patterns

The pattern of trade may be described as follows. There is no trade in the final good, since each country can assemble the final good using the same technology. There may or may not be trade in the intermediate goods. Since production of each intermediate good exhibits international scale economies and technologies are identical across countries, each intermediate good can be produced in either country with equal efficiency. However, it is also possible that the two countries specialise in different intermediate goods. In this model, unlike models of international trade with monopolistic competition, there is no presumption that each variety of intermediate good is unique to a firm or country, so simultaneous production of the same intermediate good in both countries is a possible outcome. Because of the external scale economies which are international in nature, total world output of each intermediate good is independent of the location of production.

The two possible extreme outcomes are the follows. First, it is possible that both countries produce all the varieties of intermediate goods n^{TF}. In this case, no trade occurs. The other extreme outcome is that both countries may end up specialising in different intermediate inputs, and exchange them. In this case, the volume of trade is maximised. Here, since the output of each intermediate good is the same and technologies are identical across countries, the number of intermediate goods produced by each country is proportional to its share of world labour supply: $L_i/(L_H + L_F)$. And since prices are the same across countries and preferences are homothetic, each country's demand for each intermediate good is proportional to its national income. Hence the maximum volume of trade is equal to (since we normalise $w = 1$):

$$\max VT = \frac{L_H L_F}{L_H + L_F} \tag{25}$$

Therefore, we have established that when international trade is allowed, the trade volume may take on any value between 0 and $L_H L_F/(L_H + L_F)$. The upper bound of this range is identical to the expression for the volume of trade in Krugman (1979, 1980), and for the same reason: there are gains from increased variety (intermediates in the present paper, final goods in Krugman 1979, 1980). To further draw the parallel with the literature on monopolistic competition, whereas in models of monopolistic competition the volume of trade is determinate but the direction of trade is indeterminate, in this model, neither the volume of trade nor the direction of trade is determinate. The best that we can do is to obtain an upper bound to the volume of trade, which from equation (25) is maximised for given total country sizes when the two countries are identical in size.

Note that we have also shown that the gains from trade do not depend on the volume of trade. In this model, the gains from trade arise from the fact that international trade eliminates the duplication of the coordination network of intermediate inputs, which lowers the coordination cost, results in more intermediate inputs being produced in equilibrium, and hence higher levels of utility. This gain would exist even in situations where the two countries do not trade with each other, because of the savings made to the coordination cost.

6 Alternative assumptions for the production of intermediate goods

In developing the model, we have made use of the assumption that production of intermediate goods takes place under conditions of international scale economies which are external to the firm. Because of the external scale economies, the fewest possible varieties of intermediate goods are produced which is consistent with the equilibrium. However, because the scale economies are international in nature, it does not matter in which country production takes place. As a result, as argued in Section 5 above, the best that we can do in terms of the trade pattern is to obtain an upper bound on the volume of trade; both the trade pattern and the trade volume are indeterminate. In this section we discuss the implications of making alternative assumptions for the production of intermediate goods.

Perhaps the most natural alternative assumption to make on the production of intermediate goods is to assume constant returns to scale. That is, let

$$q_j = \alpha l_j \tag{26}$$

This of course is equivalent to setting $\gamma = 1$ in equation (4). Careful examination of the results in the previous sections will show that, apart from simplifying the expressions somewhat, all the main results remain valid. In addition, we would obtain exactly the same result for the pattern of trade: with identical technologies across countries and constant returns to scale, the location of production of each intermediate good does not matter. Hence the pattern of trade remains indeterminate, and only an upper bound on the volume of trade identical to that in Section 5 can be obtained. What then is the value in assuming external scale economies?

The answer is the following. If constant returns to scale is assumed, then *the number of intermediate goods produced is indeterminate*. Take for example the case of autarky. Each final-good-producing firm uses a number of intermediate goods as defined in equation (10). However, nothing constrains all firms to use the same intermediate goods. With constant returns to scale in intermediate goods production, it is entirely possible that each final-good-producing firm uses different intermediate goods. What would be required is an additional assumption, that all final goods firms use the same intermediate goods in production. Therefore, by assuming external scale economies, we do not have to make this additional assumption; the presence of external scale economies automatically limits the economy to the minimum possible number of intermediate goods consistent with equation (10).

The other possible assumption to make about intermediate goods production is that it takes place under *national* as opposed to *international* scale economies. Once again this does not change any of the expressions in the previous sections. However, there are two important implications. First, if scale economies are national in nature, then Ethier (1982b) among others shows that there may exist multiple, inefficient, and possibly unstable equilibria. For instance, under national scale economies the efficient equilibria occur when each country is specialised in different intermediate goods. However, it is also possible that there exists equilibria in which both countries are producing some of the intermediate goods. To make progress, it would be necessary to either consider all these possible equilibria, or to assume that only efficient equilibria exist and to focus only on these equilibria.

Even if we focus only on the efficient equilibria, there is still one important difference between the model with international scale economies as compared with national scale economies. With national scale economies, as noted above, each country is specialised in different intermediate goods. As a result, the volume of trade is determinate, and is equal to the maximum volume of trade predicted by the model with international scale economies, equation (25) above.

7 Conclusions

This paper develops a simple model of international trade based only on the division of labour; there is no comparative advantage or imperfect competition. Firms assemble final goods from intermediate inputs, and there are gains to having a larger variety of intermediate inputs. The extent of the division of labour is limited by the cost of coordinating intermediate inputs and the size of the market. International trade eliminates the duplication of coordination costs, resulting in an increased variety of intermediate inputs, greater division of labour, and hence to gains from trade.

A novel feature of the model is that, rather than predicting the volume of trade given country characteristics, the model predicts the presence of an *upper bound* to the volume of trade between countries. This is interesting since much has been made of “missing trade” (Trefler 1995); the present model would interpret such missing trade as falling within the bounds of the predicted trade volume. Hence an avenue for future research would be to more carefully characterise the properties of this upper bound and take it to the data, analogously to what Sutton (1991, 1998) has done in the industrial organisation literature.

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Appendix A: Further details of the production function for intermediate goods

First we establish that the production function for an intermediate good given in equation (3) can be derived from the production function of each firm producing that intermediate good (see Panagariya, 1981). The production function for a firm k producing intermediate good j depends on the total output of that intermediate good:

$$q_{jk} = \alpha q_j^{\delta} l_{jk} \qquad \text{(A1)}$$

Total output of intermediate good j is:

$$q_j = \sum_k q_{jk} = \alpha q_j^{\delta} l_j = \left(\alpha l_j\right)^{\frac{1}{1-\delta}} = \left(\alpha l_j\right)^{\gamma} \qquad \text{(A2)}$$

Where $\gamma = 1/(1-\delta)$.

Next, we solve for the prices of the intermediate goods. Under perfect competition, each firm employs labour so that the value marginal product of labour is equal to the wage rate. Differentiating equation (A1) with respect to l_{jk} gives:

$$MPL_{jk} = \frac{dq_{jk}}{dl_{jk}} = \alpha q_j^{\delta} \qquad \text{(A3)}$$

Hence, we have:

$$w = p_j \alpha q_j^{\delta} \qquad \text{(A4)}$$

Setting the wage rate equal to unity, we can solve for the price of each intermediate good as:

$$p_j = \left(\alpha q_j^{\delta}\right)^{-1} = \left[\alpha^{\gamma} l_j^{\gamma-1}\right]^{-1} \qquad \text{(A5)}$$

Which is equation (4) in the text. These results hold in both autarky and international trade, with the only difference being that the labour used in each intermediate good, l_j, differs between autarky and international trade. Note that equation (A5) also shows that, although each firm practices marginal cost pricing, at the industry level, average cost pricing is being practiced; average cost for the industry is (substituting from equation (A2)) $wl_j/q_j = \left[\alpha^{\gamma} l_j^{\gamma-1}\right]^{-1}$.

To show that the equilibrium is unique and stable, we follow the approach used in Ethier (1979, 1982b); see also Helpman (1984). In particular, Helpman (1984) shows that when the elasticity of substitution between varieties is sufficiently low (such as in the case of Cobb-Douglas utility), the industry demand curve is always steeper than the industry supply curve, and the two curves intersect each other only once. In our model, from equation (A5), industry supply is:

$$p_j^S = \left(\alpha x_j^{\delta}\right)^{-1} = \alpha^{-1} x_j^{\frac{1-\gamma}{\gamma}} \qquad \text{(A6)}$$

Differentiating with respect to x_j gives:

$$\frac{dp_j^S}{dx_j} = \alpha^{-1}\left(\frac{1-\gamma}{\gamma}\right) x_j^{\frac{1-2\gamma}{\gamma}} < 0 \qquad \text{(A7)}$$

$$\frac{d^2 p_j^S}{dx_j^2} = \alpha^{-1}\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-2\gamma}{\gamma}\right) x_j^{\frac{1-3\gamma}{\gamma}} > 0 \qquad \text{(A8)}$$

That is, the industry supply curve for an intermediate good j is downward sloping, and its slope decreases in x_j. On the other hand, industry demand for each intermediate good is, from the fact that intermediate goods are perfect complements in the production of the final good:

$$x_j = \frac{TC}{\Sigma p_j} \qquad \text{or} \qquad p_j^D = \frac{TC}{x_j} - \sum\nolimits_{-j} p_j \qquad \text{(A9)}$$

Where TC is the total cost of a final-good-producing firm, and $\sum_{-j} p_j$ is the sum of the prices of all intermediate goods apart from j. Differentiating with respect to x_j gives:

$$\frac{dp_j^D}{dx_j} = -\frac{TC}{x_j^2} - \sum\nolimits_{-j} p_j < 0 \qquad \text{(A10)}$$

$$\frac{d^2 p_j^D}{dx_j^2} = \frac{2TC}{x_j^3} - \sum\nolimits_{-j} p_j > 0 \qquad \text{(A11)}$$

Hence, similarly to industry supply, industry demand for each intermediate good j is also downward sloping, and its slope also decreases in x_j.

Note that the industry supply curve never intersects the horizontal axis; if $p_j^S = 0, x_j = \infty$. However, industry demand intersects the horizontal axis at $x_j = TC/\sum_{-j} p_j$. Hence, for a sufficiently large value of x_j, the demand curve will be below the supply curve. On the other hand, for very small values of x_j, $p_j^S < p_j^D$; therefore, the demand and supply curves must intersect. But do they intersect only once; is the equilibrium unique? Comparing the slopes of the demand and supply curves in (A7) and (A10) is difficult. Instead, we perform the following exercise. First, consider a given quantity of the intermediate good x_j. Then, increase the quantity of x_j by some factor $\lambda > 1$. Solve for the relative supply and demand prices for the two quantities. If the relative supply price exceeds the relative demand price for the two quantities for all values of x_j, then we can conclude that the supply curve is always flatter than the demand curve.

That is, let the initial price be:

$$p_j^{S1} = \alpha^{-1} x_j^{\frac{1-\gamma}{\gamma}} \qquad \text{and} \qquad p_j^{D1} = \frac{TC}{x_j} - \sum_{-j} p_j \tag{A12}$$

And let the new price be:

$$p_j^{S2} = \alpha^{-1} (\lambda x_j)^{\frac{1-\gamma}{\gamma}} \qquad \text{and} \qquad p_j^{D2} = \frac{TC}{\lambda x_j} - \sum_{-j} p_j \tag{A13}$$

Dividing the new price by the initial price gives:

$$\frac{p_j^{S2}}{p_j^{S1}} = \left(\frac{1}{\lambda}\right) \lambda^{\frac{1}{\gamma}} \qquad \text{and} \qquad \frac{p_j^{D2}}{p_j^{D1}} = \left(\frac{1}{\lambda}\right) \frac{TC - \lambda x_j \sum_{-j} p_j}{TC - x_j \sum_{-j} p_j} \tag{A14}$$

Comparing p_j^{S2}/p_j^{S1} with p_j^{D2}/p_j^{D1}, we can establish that, for $\lambda > 1$:

$$\frac{p_j^{S2}}{p_j^{S1}} > \frac{p_j^{D2}}{p_j^{D1}} \qquad \forall\, x_j \tag{A15}$$

That is, we have showed that the supply curve of intermediate goods is always flatter than the demand curve for intermediate goods, hence the two curves intersect only once and the equilibrium is unique. Finally, assuming Marshallian adjustment (if demand price exceeds supply price, then quantity increases; see Ethier 1979, 1982b), the equilibrium defined by the intersection is stable; see Figure A1.

Figure A1: Market equilibrium of an intermediate good.