Analysis of solutions to arithmetic operations differing in the number of computational terms

Technical Report (CDRG8), Royal Holloway, University of London. 2001.

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Acknowledgement

The report is based on research funded by the Economic and Social Research Council, grant number R000222789. We are grateful to the staff and pupils at Alfred Sutton Primary School, Reading, for their co-operation.

Overview

A group of children completed arithmetic questions under speeded instructions. The stimulus set or computations were based on previous analyses (Hutton, Towse & Hitch, 1997). This report summarises the method used in the extension of previous work, describes the responses given and tabulates the results according to accuracy and also according to the time taken to respond with a solution.

Background

This study further develops a group of sums that were used as part of a working memory task by Hutton, Towse and Hitch (1997). The task itself was “operation span”, a procedure in which a series of arithmetic questions are presented for solution, and each answer forms an item used in a subsequent recall phase of the trial. In the previous study, the sums were designed so that computations involved additions and subtractions with the addends 1 and 0. The sum length, that is the number of computations required, varied from 2 to 4 terms. In the present study, these constraints were changed. (a) The digit 2 was sometimes included, both for subtraction and addition. However, it was never used as a computation more than once in a trial. (b) Sums varied in length from 2 to 5 terms. Notwithstanding these changes, the target arithmetic answer was always a single digit.

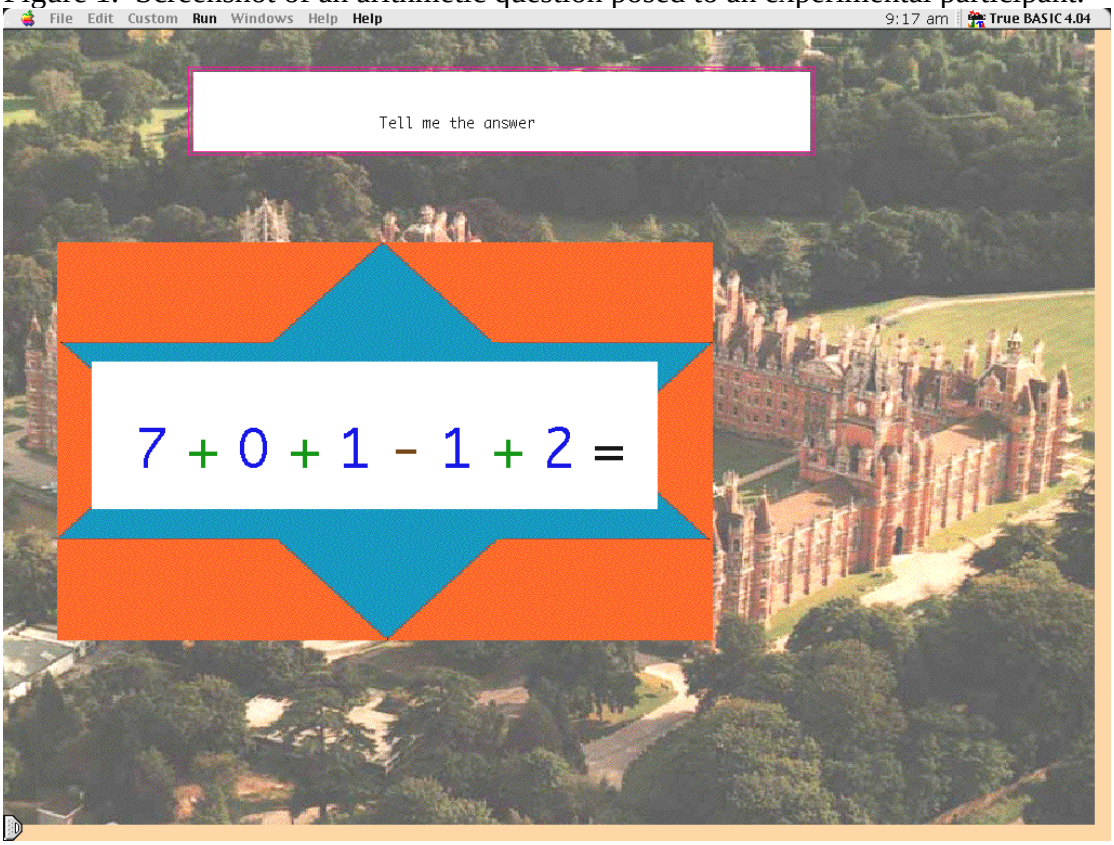
The principal motivation for incorporating longer operations was the need for a greater number of stimulus items to use within working memory tasks. By increasing the number of computations, and including the digit 2 for calculation, it might be possible to obtain a greater range of solution times. This is particularly useful for experiments that demand a variety of stimuli, which differ in completion time. Thus, the purpose of this study was to determine the performance of children in answering the sums when the task was varied by including an extra term, or the addition or subtraction of the digit 2.

Method

Participants, Design and Procedure

The subjects were selected from a primary school in Berkshire, and comprised all 36 class members from a year-4 group. Ages ranged from 8; 4, to 9; 6, with a mean age of 8; 11, and a standard deviation of 4 months. The sums were presented on the screen of an Apple Powerbook 5300c computer. They appeared within a box (see Figure 1), and participants were asked to work out the answer to the sum as quickly as possible, and then tell the experimenter the answer. Three practice sums were presented as laminated cards before the computer-based trials began. With these practice items children were reminded to pay particular attention to the addition and subtraction signs.

Figure 1. Screenshot of an arithmetic question posed to an experimental participant.



The stimulus set comprised 162 unique sums with a correct single digit answer, and 6 repeat questions that made up a complete set of 168 items. There were 42 questions at each length of 2, 3, 4 & 5 terms. With such a large collection, they were split into 2 and children were presented with one or other set, determined randomly. Each set had 84 sums and these were split into 7 blocks of 12, being shuffled into a random sequence for each participant. The computer measured the time from the presentation of the sum, to the input of the subject's answer on an external keyboard by the experimenter. After each group of 12 sums the subject was given feedback, which consisted of the mean time taken to complete the 12 sums in that block, and also the number of sums correctly answered. Following a short interval, children proceeded with the next block of sums. Administration took approximately 20 minutes.

Results

Table 1 shows, as expected, that the shortest 2-term sums were completed most quickly, and that the sums with a larger number of terms tended to take longer to solve. Results are categorised into percentile groups according to the mean time taken for each sum, only using correct responses to the problems.

Table 1: Completion times for sums, grouped by sum length.

Percentile Range (and associated mean response time band)	Term	Sum (ordered by magnitude of correct solution)
1-15	2 term	3+0; 3-0; 1+2; 2+2; 4-0; 5-1; 3+1; 4+0; 3+2; 5+0; 4+1; 5-0; 5+1; 6+0; 7-0; 7+0; 6+1; 8-0; 8+0; 8+1; 9-0; 9+0; 7+2;

(2.0-2.6 seconds)	3 term	3+0+0;
16-30 (2.7-4.0 seconds)	2 term	2+1; 4-1; 5-2; 6-2; 6-1; 6-0; 4+2; 8-2; 7-1; 5+2; 8-1; 7+1; 10-2; 9-1; 6+2; 10-1;
	3 term	1+1+1; 3+0+1; 4+1+0; 5+1+0; 5+1+1; 6+1+0; 8+0+1;
	4 term	1+1+1+1;
31-45 (4.1-5.1 seconds)	2 term	7-2; 11-2;
	3 term	3+1-1; 2+2-1; 2+1+1; 4+1-1; 3+1+1; 4+0+1; 4+1+1; 6+0+1; 7+1+0; 6+1+1; 7+1+1; 9+1-1; 6+2+1;
	4 term	1+1+1+0; 1+0+1+1; 2+1+1+0; 2+0+1+1; 3+1+1+1; 4+1+1+1; 5+1+1+1; 6+1+1+1; 8+1+0+0;
46-60 (5.1-6.1 seconds)	2 term	9-2;
	3 term	3-1+1; 4-1+1; 5+1-2; 3+2-1; 4+2-1; 5+1-1; 5+2-1; 8-1-1; 6+1-1; 7+1-1; 7+1-0; 8+1-1; 9+1-2; 7+2-1; 9-1+1
	4 term	2+1+1+1; 3+0+1+1; 5+1+1+0; 9-1+0+0; 6+1+1+0; 7+0+1+1; 7+1+1+0;
	5 term	1-1+1+1+1; 1+1+1+1-1;
61-75 (6.2-7.3 seconds)	3 term	4+1-2; 6+2-1;7+1-2; 8+1-2; 8+2-1;
	4 term	1+1+2-1; 2+1+1-1; 2-1+1+1; 2+1+2-1; 3-1+1+1; 3+1-1+2; 3-0+1+1; 4-1+1+1;5-1+1+1; 6+0+1-1; 6+1+2-1; 6+1+1-1;
	5 term	1+1+2-0-0; 2-1+1+1+1; 2+1+1+1-1; 3-1+1+1+1; 5+1+1+1-1; 6-1+1+1+1; 8+1+1+0-1;
76-90 (7.3-8.7 seconds)	3 term	6+1-2;
	4 term	1+1-1+2; 3+1+2-1; 4+1-1+2; 7+1+2-1; 5+1+2-1; 5+1-1+2; 6+1-1+2; 8-1-1+1; 7-1+1+1;
	5 term	1+1-1+2+0; 3+1+1+0-1; 3+1-1+2+0; 4+1+1+0-1; 3+1+1+1-1; 5+1+1+0-1; 4+1-1+2+0; 5-1+1+1+1; 4-1+1+1+1; 4+0+1-1+2; 4+1+1+1-1; 6+1+1+0-1; 6+1+1+1-1; 7+1+1+1-1;
91-100 (8.7-11.5 seconds)	4 term	2+1-1+2; 4+1+2-1; 7-1-1+1; 7+1-1+2;
	5 term	1+0+1-1+2; 2+1+1+0-1; 2+1-1+2+0; 2+0+1-1+2; 3+0+1-1+2; 7+1+1+0-1; 5+1-1+2+0; 5+0+1-1+2; 6+1-1+2+0; 6+0+1-1+2; 7-1+1+1+1; 7+1-1+2+0; 7+0+1-1+2;

Arithmetic operations were also sorted by accuracy of response, shown in Table 2. Response accuracy ranged between 69.2% and 100%. It should be noted each sum elicited a response from 18 children. Therefore, with only a single child responding to one question incorrectly there is an error value of 5%. Implications of this table should be drawn with this error sensitivity in mind.

In terms of 2-term sums, 52% were always answered correctly, and none of the error rates exceeded 15%. This accuracy contrasts with responses to the 5-term sums, only one of which was answered with 100% accuracy, while 58% resulted in an error rate over 15%. The 3- and 4-term sums are spread consistently across the accuracy spectrum, with errors from 0% to 20%.

Table 2: Number of sums answered incorrectly, grouped according to sum length.

Number of erroneous responses	Sum
0	2-term: 4-1; 3+1; 4+1; 7-2; 8-2; 8-1; 9-1; 9+0; 11-2; 3+0; 1+2; 2+2; 5+0; 5-0; 3+2; 6+0; 6-0; 4+2; 7-0; 8+0; 8-0; 6+2; 3-term: 3+0+0; 2+2-1; 4+1+0; 5+2-1; 6+1+0; 7+1+0; 7+2-1; 8+0+1; 1+1+1; 4+1-2; 2+1+1; 3+1+1; 4+1+1; 5+1+1; 7+1-0; 7+1+1; 4-term: 1+1+1+0; 2+0+1+1; 3+0+1+1; 3+1+1+1; 4+1+2-1; 5+1+1+1; 6+1+1+1; 1+0+1+1; 1+1+1+1; 5+1+1+0; 7+0+1+1; 5-term: 2+1-1+2+0;
1	2-term: 5-1; 2+1; 5-2; 6+1; 10-2; 6-2; 6-1; 5+1; 7-1; 8+1; 4+0; 4-0; 7+0; 10-1; 7+2; 3-term: 3+1-1; 3+0+1; 3+2-1; 4+2-1; 5+1+0; 7+1-1; 6+2-1; 6+0+1; 6+2+1; 4+0+1; 6+1+1; 8-1-1; 4-term: 2+1+2-1; 4+1+1+1; 6+1+1+0; 8+1+0+0; 6+0+1-1; 6+1-1+2; 7+1+2-1; 2+1+1-1; 7-1+1+1; 7+1-1+2; 3-0+1+1; 5-term: 1+1+2-0-0; 1+0+1-1+2; 1+1-1+2+0; 4+0+1-1+2; 5+0+1-1+2;
2	2-term: 3-0; 5+2; 9-0; 9-2; 7+1; 3-term: 8+1-2; 9+1-2; 5+1-1; 4+1-1; 8+1-1; 7+1-2; 4-term: 2+1+1+1; 3+1+2-1; 6+1+2-1; 1+1-1+2; 2+1+1+0; 2+1-1+2; 5-1+1+1; 4+1-1+2; 9-1+0+0; 7+1+1+0; 5-term: 2+0+1-1+2; 3+1-1+2+0; 5+1+1+0-1; 6+1+1+0-1; 5+1-1+2+0; 7+1+1+0-1; 8+1+1+0-1; 4-1+1+1+1; 6-1+1+1+1;3+1+1+1-1;
3	3-term: 3-1+1; 4-1+1; 6+1-2; 6+1-1; 9+1-1; 9-1+1; 8+2-1; 4-term: 4-1+1+1; 3+1-1+2; 5+1-1+2; 2-1+1+1; 5+1+2-1; 5-term: 1-1+1+1+1; 2+1+1+1-1; 4+1+1+1-1; 5+1+1+1-1; 5-1+1+1+1; 3-1+1+1+1; 4+1-1+2+0; 6+0+1-1+2; 7+0+1-1+2; 3+0+1-1+2; 7+1-1+2+0;
>3	3-term: 5+1-2; 4-term: 1+1+2-1; 3-1+1+1; 7-1-1+1; 6+1+1-1; 8-1-1+1; 5-term: 7-1+1+1+1; 2-1+1+1+1; 7+1+1+1-1; 1+1+1+1-1; 6+1+1+1-1; 2+1+1+0-1; 3+1+1+0-1; 6+1-1+2+0; 4+1+1+0-1;

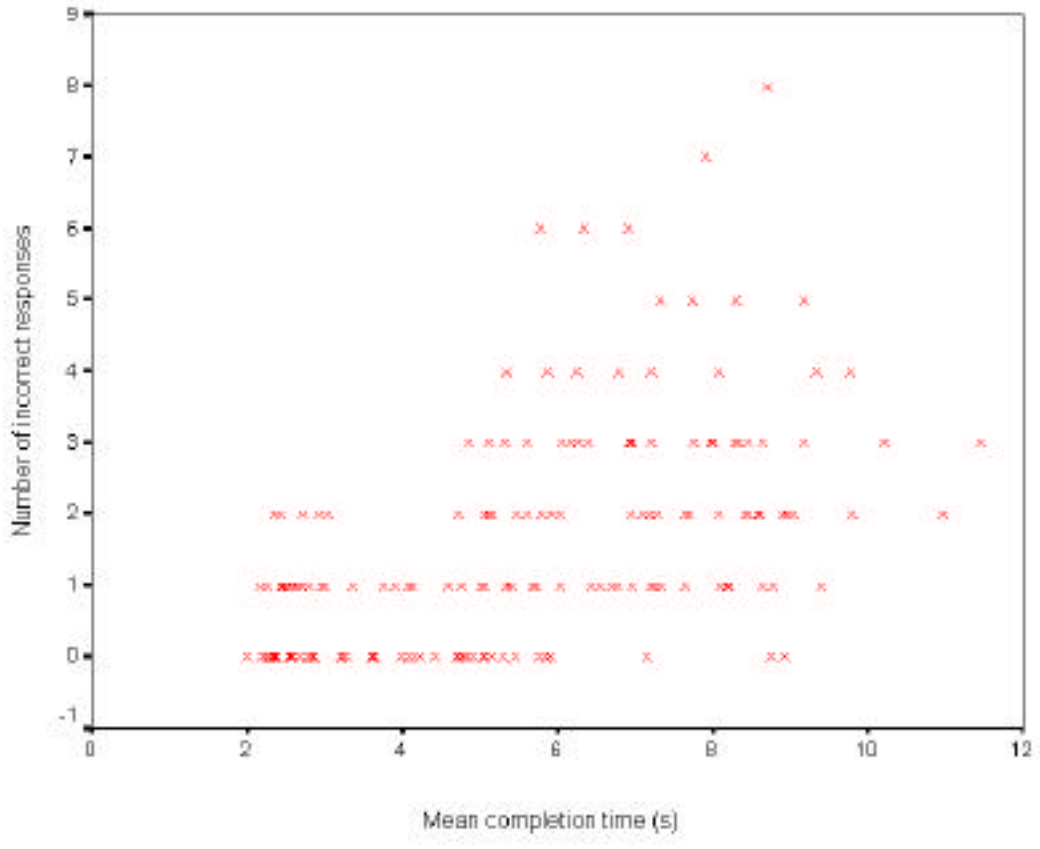
Table 3 describes the error profile in a different manner. Here, the responses have been ordered according to whether the digit 2 was included in the arithmetic operation or not. Due to the nature of the stimulus set, trials with the digit 2 were only half as frequent as trials without. For 2-term sums, the use of 2 in a computation makes little difference to the response accuracy. There is a slight increase in errors with 3-term sums using the digit 2. There is also an increase in errors on 4-term sums with the digit 2. Interestingly, among 5-term sums there are fewer errors on questions incorporating the digit 2.

Table 3: Mean percentage of sums answered incorrectly reported by digits used for addition and subtraction.

	Mean Percentage answered incorrectly	
Sum Length	Digits 1 and 0 only used for subtraction and addition.	Digit 2 included for subtraction and addition.
2-term	3.3	3.1
3-term	5.9	7.8
4-term	6.5	10.3
5-term	18.8	12.0

Figure 2 provides a scatter-plot showing the relationship between the time it took to complete the sums, and how many of them were completed incorrectly. There is a positive correlation here between mean completion time and completion accuracy, $r(34) = 0.62$, $p < .01$.

Figure 2. A visual indication of the relationship between response accuracy and response latency.



References

Hutton, U., Towse, J. N., & Hitch, G. J. (1997). *Children's competence at solving arithmetic operations in a working memory task* (Technical report CDRG2): Royal Holloway University of London.