Word Embeddings for Language Modelling

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24th February 2021

Background - Language Models

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A key issue in language modelling is the large **vocabulary size** - we must model the meanings and interactions of millions of unique words. It would be useful to exploit similarities between words.

Background - Word Embeddings

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We will focus on **skip-gram with negative sampling**, which obtains embeddings by assuming that words with similar meanings appear in similar contexts.

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- We randomly generate **negative samples** from the rest of the vocabulary. For each positive sample (w, w^+), we generate k negative samples (w, w_j^-).

Example sentence: “*Sing, and the hills will answer.*”

If we set $l = 2$ the observed context words for “hills” are “the” and “will”. We make the assumption that the order of the context words is not important, representing these target-context pairs as the **skip-grams** (*hills*,*the*) and (*hills*,*will*).

In our example, (*hills*,*sing*) and (*hills*,*answer*) are possible negative samples since “sing” and “answer” were not observed within our context window for “hills”.

We define two sets of embeddings, the **target embeddings** $\boldsymbol{v}_{w_i}$ and the **context embeddings** $\boldsymbol{c}_{w_j}$. The dot product $\boldsymbol{v}_{w_i} \cdot \boldsymbol{c}_{w_j}$ will be used to determine similarity.

The probability that w_j is a true context word for w_i is modelled by passing this into the **logistic** function, σ:

$$\mathbb{P}(w_j | w_i) = \sigma(\boldsymbol{v}_{w_i} \cdot \boldsymbol{c}_{w_j}) = \frac{1}{1 + e^{-\boldsymbol{v}_{w_i} \cdot \boldsymbol{c}_{w_j}}}$$

σ takes inputs in $(-\infty, \infty)$ and outputs values in $(0, 1)$, so this is a valid probability. The probability of w_j not being a context word for w_i is simply:

$$1 - \sigma(\boldsymbol{v}_{w_i} \cdot \boldsymbol{c}_{w_j}) = \sigma(-\boldsymbol{v}_{w_i} \cdot \boldsymbol{c}_{w_j})$$

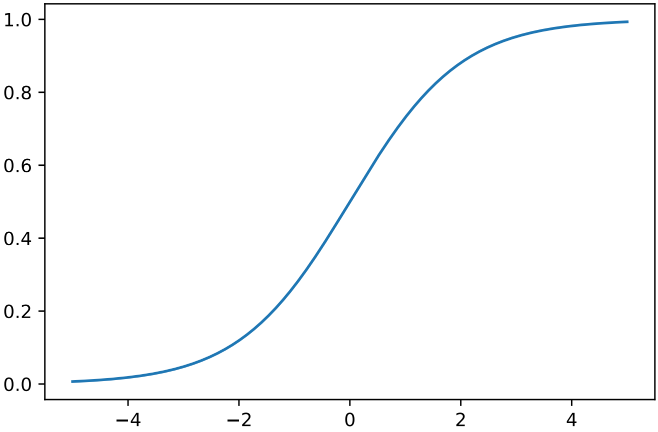


Figure: The logistic function, $y = \frac{1}{1+e^{-x}}$.

Making the assumption that context words occur independently of each other, the likelihood for one positive observation (w, w^+) and the corresponding k negative (w, w_j^-) is:

$$\mathbb{P}(w^+|w)\prod_{j=1}^{k}(1-\mathbb{P}(w_j^-|w)) \;=\; \sigma(\boldsymbol{c}_{w^+}\cdot\boldsymbol{v}_w)\prod_{j=1}^{k}\sigma(-\boldsymbol{c}_{w_j^-}\cdot\boldsymbol{v}_w)$$

The total **log likelihood** for a dataset is obtained by summing the log of this expression over each positive sample (w, w^+) in the set of observed context words C_w , for each word in the vocabulary V:

$$\sum_{w\in V}\;\sum_{w^+\in C_w}\left(\log\sigma(\boldsymbol{c}_{w^+}\cdot\boldsymbol{v}_w)+\sum_{j=1}^{k}\log\sigma(-\boldsymbol{c}_{w_j^-}\cdot\boldsymbol{v}_w)\right)$$

Note that this has **no unique maximum** (rotating the vectors preserves the log likelihood), but can still be optimised numerically.

Gradient Descent

Gradient descent aims to find the parameters $\boldsymbol{\theta} \in \mathbb{R}^d$ that minimise a **loss function** $L(\boldsymbol{\theta})$ by updating θ in the opposite direction to the gradient ∇L.

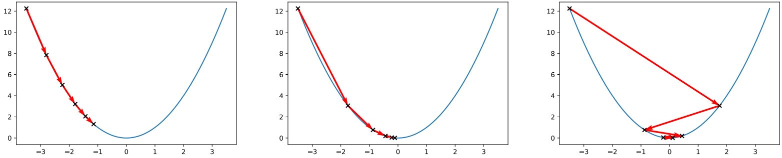


Figure: Using gradient descent to find the minimum of $y = x^2$.

For **likelihood maximisation**, we set the loss function to be $-\frac{1}{N}$ times the log likelihood:

$$L(\boldsymbol{\theta}) = -\frac{1}{N}\ell(\boldsymbol{\theta}) = -\frac{1}{N}\sum_{i=1}^{N}\ell(\boldsymbol{\theta}; x^{(i)}, y^{(i)})$$

Then the gradient is:

$$\nabla L(\boldsymbol{\theta}) = -\frac{1}{N}\sum_{i=1}^{N}\nabla_{\theta}\,\ell(\boldsymbol{\theta}; x^{(i)}, y^{(i)})$$

Computing ∇L is very computationally expensive for large datasets.

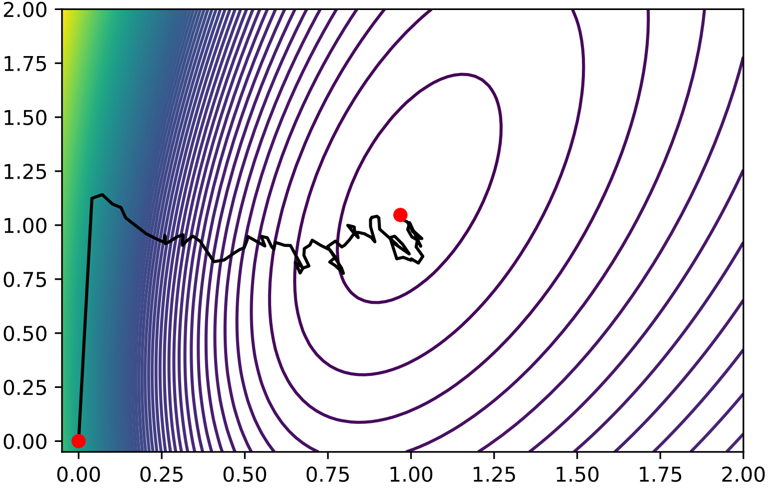
Stochastic Gradient Descent

An efficient method is **stochastic gradient descent** (SGD), which uses a random sample at each iteration to estimate the gradient:

Algorithm: SGD

Set starting value $\boldsymbol{\theta}_0$ and step size $\eta > 0$. Iterate the following for $t \geq 1$:

1. Take random sample b of size m from the dataset
2. Estimate the gradient at $\boldsymbol{\theta}_t$ by :
$$\boldsymbol{g}_t \leftarrow \frac{1}{m} \sum_{x,y \in b} \nabla_\theta L(\boldsymbol{\theta}_{t-1}; x, y)$$
3. Update parameters: $\boldsymbol{\theta}_t \leftarrow \boldsymbol{\theta}_{t-1} - \eta \, \boldsymbol{g_t}$


2.00
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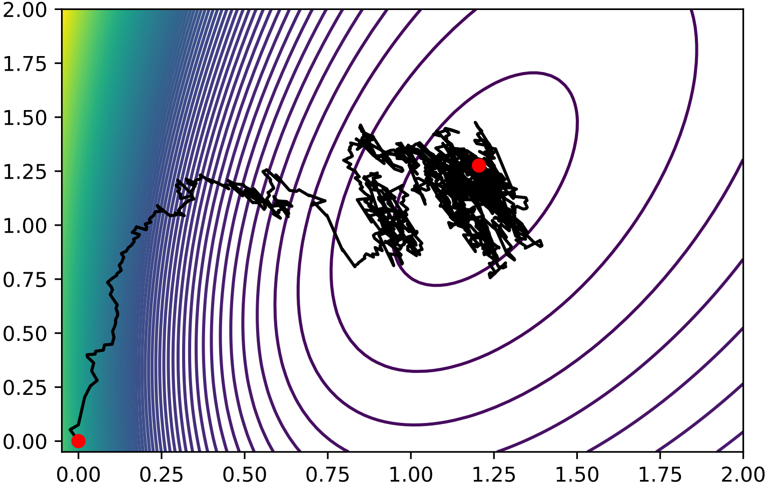
1.75
2.00

(a) $m = 50$


2.00
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(b) $m = 1$

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- A small embedding dimension $d = 100$ was chosen, with $k = 5$ and $l = 10$
- Fitted model several times with different random seeds

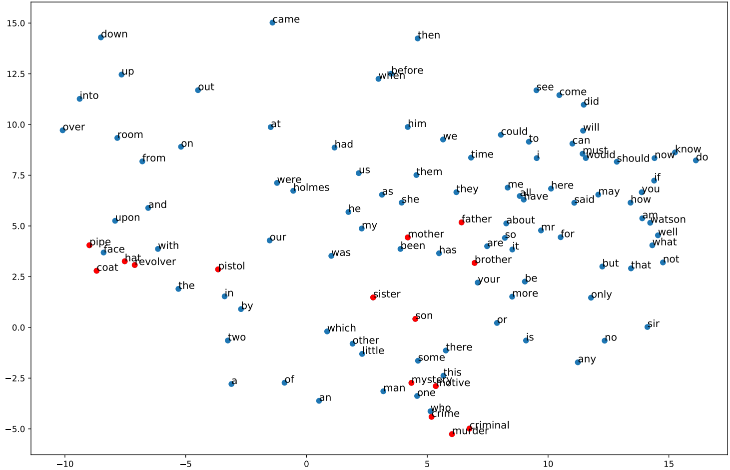

Figure: The embeddings for 110 common words from Sherlock Holmes, normalised and projected into 2D with principal component analysis

Conclusions

- The model did well at grouping words with similar meanings together
- e.g. **you** and **yourself**, **brother** and **son**, and **crime** and **murder**
- This worked best for more common words - embeddings for rarer words were still quite random
- More training data would be required to unlock the full potential of word embeddings - state-of-the-art word vectors are trained on datasets with over a billion words

Future Work

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Future Work

- An embedding layer is typically included as the first layer in **neural network** models for language, which is what this project will focus on next
- Embeddings can be trained as part of the network or trained separately to cut down on training time - skip-gram with negative sampling is an efficient way of doing this
- It is also common to use a set of **pre-trained** word embeddings out of the box if the dataset is small or computational power is limited

References

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Any Questions?