Multi-Echelon Inventory Optimisation

Niharika Peddinenikalva [1] Anna-Lena Sachs [1] Robert Shone [1] Gueorgui Mihaylov [2]

[1]Lancaster University [2]Haleon UK plc

1. Motivation

Controlling the flow of products through a supply chain is crucial to ensure that products reach customers **on time** and **efficiently**.

In inventory optimisation, we aim to make **order decisions** so that **customer demand can be met efficiently**.

Some challenges which make this problem non-trivial are

- Uncertainty in demand and lead times,
- Complex supply chain structures which serve markets globally,
- Industry-specific requirements such as batching, minimum order quantities, adherence to regulations, etc.

2. Modelling a Multi-Echelon Problem

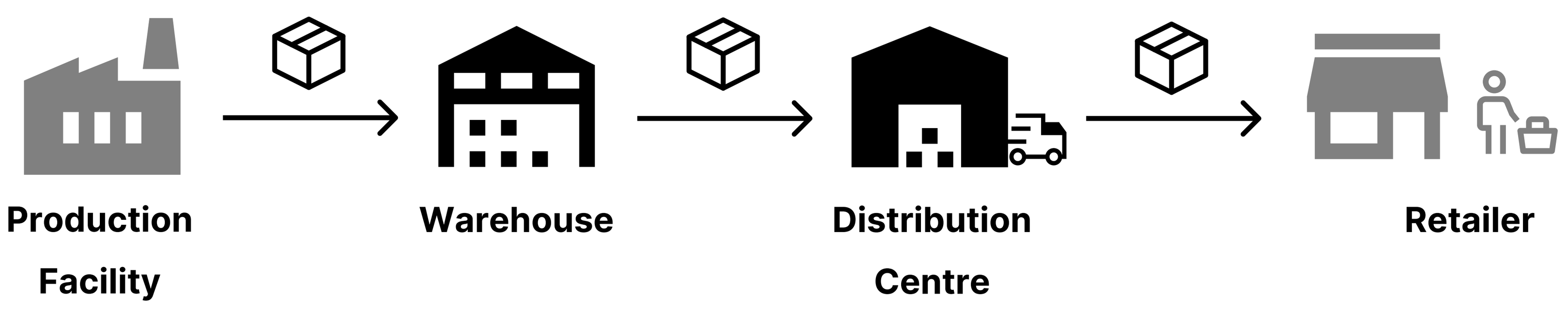


Figure 1. Multi-echelon supply chain

Given the supply chain in Figure 1, we want to find an order policy (when and how much to order) for the warehouse (W) and distribution centre (DC) so that the retailer's demand can be met at minimal costs. We assume

- The production facility has infinite supply,
- Retailer demand is stochastic,
- Unmet demand is backlogged,
- The warehouse and DC make ordering decisions to replenish stock,
- With lead time L, an order placed now arrives L time periods later.

Inventory Dynamics

In a time period (e.g., a week), consider the sequence of events at each site as illustrated in Figure 2.

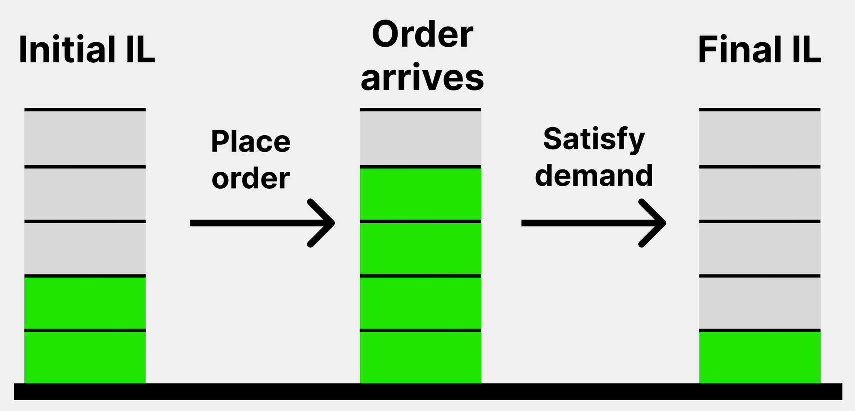


Figure 2. Inventory dynamics at a site in the supply chain

We model this problem as a Markov Decision Process over an infinite time horizon. We have

$$\text{Inventory level (IL)} = \text{on-hand stock} - \text{backlogs},$$

$$\text{Inventory position (IP)} = \text{IL} + \text{outstanding orders},$$

$$\text{Echelon DC IP} = \text{DC IP}, \quad \text{Echelon W IP} = \text{DC IP} + \text{W IP}.$$

Objective Function - Minimising Total Expected Discounted Costs

- At time t, taking action $\mathbf{q}_t$ given initial IL $\mathbf{x}_t$ yields an immediate cost $C(\mathbf{x}_t, \mathbf{q}_t)$.
- $C(\mathbf{x}_t, \mathbf{q}_t)$ is a sum of holding costs (per unit of on-hand stock) and penalty costs (per unit of backlogged demand), as per the final IL.
- Let π and γ be an ordering policy and the discount factor for future costs, respectively. Then, the objective function is given by

$$\min_{\pi} \sum_{t=0}^{\infty} \gamma^t \mathbb{E}^{\pi}[C(\mathbf{x}_t, q_t)].$$

3. Results from an Example

Using dynamic programming approaches, we solve two simple problems: (1) no lead times, and (2) lead time of one time period at the DC only.

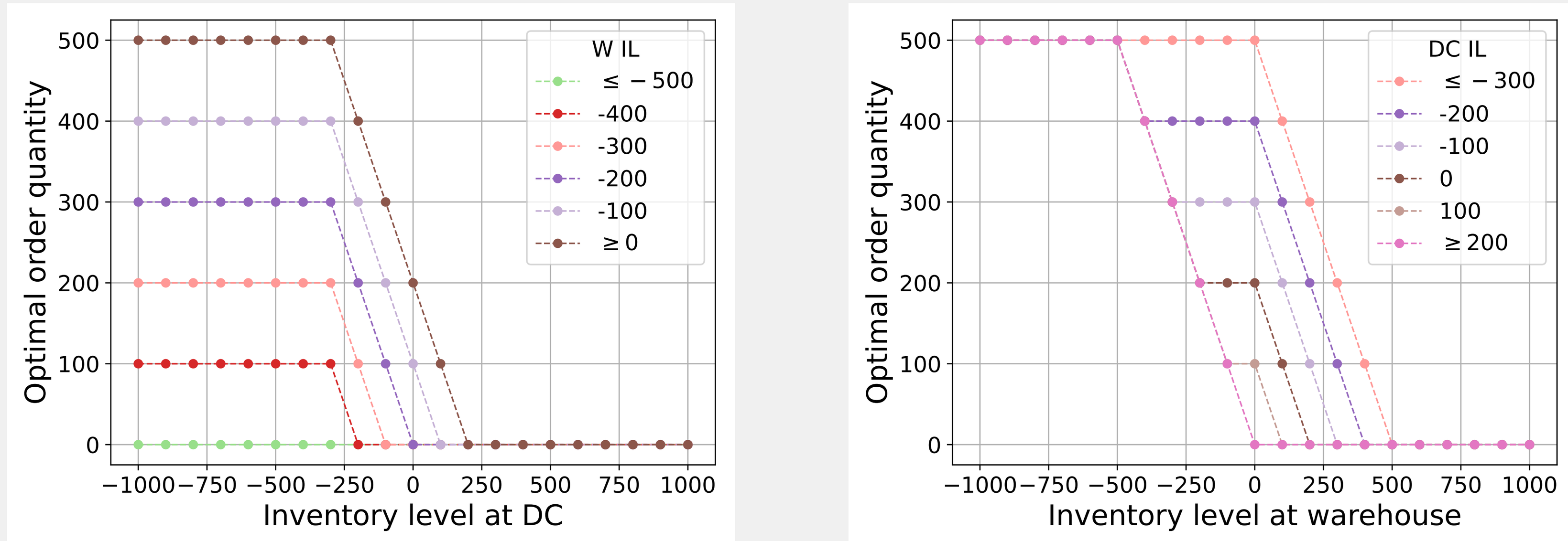
In both problems, the retailer has a discretised normal demand.

The optimal policy in both cases is

- An **echelon base-stock (or order-up-to-S) policy** (if echelon IP $< S$, place sufficiently large order so that echelon IP $= S$) [2].
- Aiming to achieve a **service level** $\geq$ 98%.

Optimal Policy without Lead Times

Figure 3 illustrates that the DC and W place orders so that the DC has an IL (=IP) ≥ 200 units before retailer demand is known.

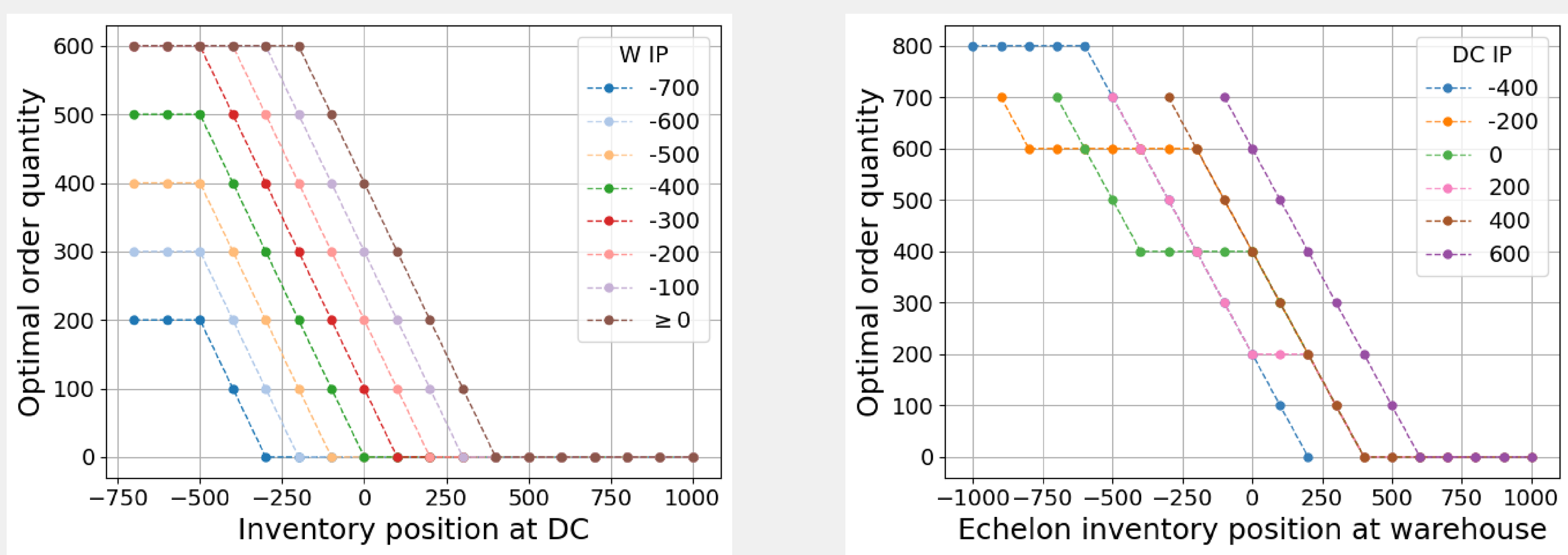


(a) DC policy (b) Warehouse policy

Figure 3. Optimal Policy at the DC and warehouse without lead times

Optimal Policy with Lead Times at DC

- When $L = 1$ at the DC, the DC's and W's echelon base-stock level is 400 units, respectively (see Figure 4).
- The DC and W place orders so that the W echelon IP is 400 units with 400 units at the DC before retailer demand is known.
- This makes it easier for the DC to cover maximum demand (200 units) in the current and following time period.



(a) DC policy (b) Warehouse policy

Figure 4. Optimal Policy at the DC and warehouse when DC experiences lead times

4. Further Work

- Model and solve problems with stochastic lead times
- Consider more complex supply chain structures
- Explore the inclusion of industry-specific requirements: minimum order quantities, incremental order quantities, order expediting, etc.
- Solve large-scale problems using other methods such as deep reinforcement learning [1], approximate dynamic programming, etc.

References

[1] R. N. Boute, J. Gijsbrechts, W. van Jaarsveld, and N. Vanvuchelen. Deep reinforcement learning for inventory control: A roadmap. *European Journal of Operational Research*, 298(2):401–412, 2022.

[2] A. J. Clark and H. Scarf. Optimal policies for a multi-echelon inventory problem. *Management Science*, 6(4):475–490, 1960.

https://www.lancaster.ac.uk/stor-i-student-sites/niharika-peddinenikalva/ n.peddinenikalva@lancaster.ac.uk