Evaluating Methods for Making Decisions Under Uncertainty

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Motivation

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- However these decisions are often made with uncertainty.
- Because of this uncertainty we cannot always find the best solution but are instead looking for ones that perform well under these uncertainties.
- Stochastic programming looks at incorporating the uncertainty into making decisions but these methods can be computationally expensive.

Newsvendor Problem

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- The newsvendor owns a stall at which they sell two different newspapers, paper A and paper B.
- Before each day, they need to order the stock that they want to sell.
- Their aim is to maximise the profits they make.

Parameter Values

- The buying and selling prices for each item are known before making the decision.
- In the deterministic case, the demand is also known.

	Newspaper A	Newspaper B
Buying Price	£1.50	£1.20
Selling Price	£3.00	£4.20
Known Demand	80	70

Table: Buying and selling prices per newspaper, and the known demands for each product.

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As there is a £1.50 profit on newspaper A and a £3 profit on newspaper B, the maximum profit that the person can make is:

$$\pounds 1.50 \times 80 + \pounds 3.00 \times 70 = \pounds 330.$$

Adding Uncertainty

This is not a realistic scenario as in reality the number of customers is not known before.

- To model this we can add introduce two more scenarios:
  - Low popularity with a demand of 40 for paper A and 35 for paper B.
  - High popularity with a demand of 120 for paper A and 105 for paper B.
- Each of these scenarios along with the demand from the deterministic example will occur with a probability of 1/3.

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In real life there would often be more than 3 scenarios for number of customers, and demand for each item does not have to be dependent, but this will make the problem easier to solve.

Expected Value of Perfect Information

- One way to find the best solution is to maximise the expected profit using the assumption that the probability of each of the scenarios is a third.
- This gives an optimal strategy of ordering 80 of paper A and 105 of paper B for an expected profit of £248.

Expected Value of Perfect Information

- One way to find the best solution is to maximise the expected profit using the assumption that the probability of each of the scenarios is a third.
- This gives an optimal strategy of ordering 80 of paper A and 105 of paper B for an expected profit of £248.
- This is smaller than the expected profit for if the true demand was known before the decision was made:
  - $\frac{1}{3} \times £495 + \frac{1}{3} \times £330 + \frac{1}{3} \times £165 = £330.$

We can then calculate the $EVPI$ which for a maximisation problem is calculated as:

- $EVPI = WS - RP$.
  - WS represents the expected profit of the optimal solution, where we know the scenario we are going to encounter.
  - RP represents the solution achieved corresponding to the recourse problem. This is where we take into account the probabilities of each of the different scenarios.

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In the newsvendor problem, the $EVPI$ is:

$$£330 - £248 = £82.$$

This represents the maximum amount a decision maker would be happy to pay to get accurate and complete information about the uncertainty.

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- One solution is to instead solve the easier problem where each value with uncertainty is replaced with their mean value.
- The Expected Result of Using an Expected Value Solution (EEV) can be found by using this solution over all the possible scenarios.
- For the newsvendor problem, this corresponds to ordering 80 of paper A and 70 of paper B for a profit of £241.

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This can be thought of as the cost of ignoring the uncertainty when making a decision.

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- If VSS is low then this means not much profit is lost when solving the deterministic problem compared to the stochastic problem with the uncertainty.
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If both are small, then using the Expected Value solution provides a good solution to the problem, with not much extra profit available by using the more computationally expensive methods.

Predicting $EVPI$ and VSS

Being able to estimate the values of $EVPI$ and VSS for a stochastic problem, without having to solve the full problem would greatly simplify the process of stochastic programming.

- If we were able to know if these values for a problem were going to be high or low, we would only need to find the solution to the stochastic problem if one of these were large.

- In practice this is difficult to do and there is no general rule for when low or high values occur.
- It has been thought using stochastic programming is more valuable when there is high amounts of randomness leading to higher values for the metrics however this is not always the case.

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Summary

- We have seen that being able to predict the relative sizes of the values of $EVPI$ and VSS is useful when finding good decisions when making decisions under uncertainty.
- This would allow computational time to be saved when the benefit gained from the new information is small and complexity of the model is large.
- In practice, these values are not possible to estimate, and it can be difficult to predict patterns in the values, even if they have been calculated for similar problems.
- However one solution to this can be to look at bounds for these two values

Thank you for listening!
Any questions?

References

Birge, J. (1982).The Value of the Stochastic Solution in Stochastic Linear Programs with Fixed Recourse. *Mathematical Programming*, *24*. https://doi.org/10.1007/BF01585113

Birge, J. R., & Louveaux, F. (2011). *Introduction to Stochastic Programming*. Springer Science & Business Media.

Chao, H.-P. (1981).Exhaustible Resource Models: The Value of Information. *Operations Research*, *29*(5), 903–923.

Higle, J. L. (2005). Stochastic Programming: Optimization When Uncertainty Matters. In *Emerging theory, methods, and applications* (pp. 30–53). Informs.

Manne, A. S. (1974).Waiting For the Breeder. *The Review of Economic Studies*, *41*(5), 47–65.