Lecture 24-25

Transport characteristics of electrons in a metal and electron gases in 2D semiconductor structures, scattering time and momentum relaxation rate.

Drude formula, diffusion coefficient, Einstein relation; diffusion equation and its solution.

Interference of waves in disordered media, the phenomenon of enhanced backscattering.

Weak and strong localisation of electrons in disordered conductors.

Macroscopic classical systems

$$L >> l >> \lambda_F \, (and \;\; \lambda_F \to 0)$$

electrons propagation is described using classical laws.

$$l \sim L_\varphi$$

Mesoscopic quantum systems

$$L \sim L_\varphi > l >> \lambda_F$$

Regime of multiply scattered phase-coherent waves which interference affects transport characteristics.

$$\tau_\varphi >> \tau$$

phase-coherence time and length

$$L_\varphi = \sqrt{\tau_\varphi D} >> l$$

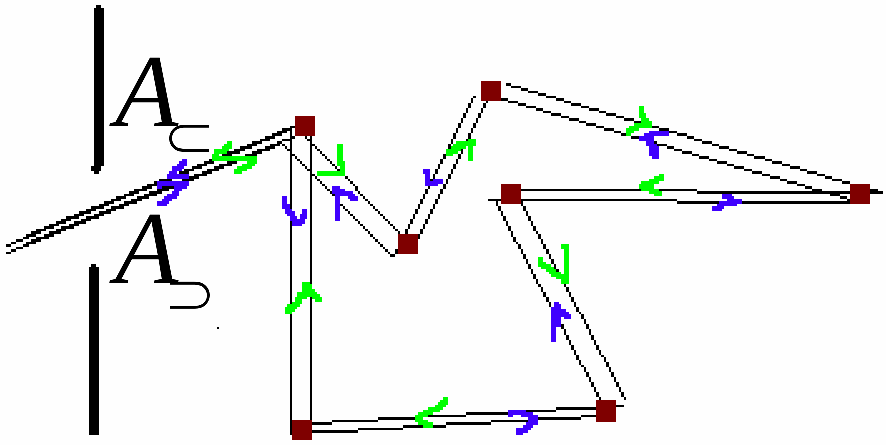
Ballistic – 'nanoscopic' systems

(discussed in the previous lectures)

$$L \sim \lambda_F << l$$

Enhanced back-scattering from a disordered medium

$$\theta = \pi$$



classical contribution $P_{classical}$

$$w \sim |A_{\circlearrowleft} + A_{\circlearrowright}|^2 = |A_{\circlearrowleft}|^2 + |A_{\circlearrowright}|^2$$
$$+ [A_{\circlearrowleft}^* A_{\circlearrowright} + A_{\circlearrowleft} A_{\circlearrowright}^*]$$

quantum (interference)

time-reversal symmetry
(no magnetic field)

$$A_{\circlearrowright} = A_{\circlearrowleft}$$

$$\langle A_{\circlearrowleft}^* A_{\circlearrowright} + A_{\circlearrowleft} A_{\circlearrowright}^* \rangle = \langle |A_{\circlearrowleft}|^2 + |A_{\circlearrowright}|^2 \rangle > 0$$

$$w \sim \langle |A_{\circlearrowleft}|^2 + |A_{\circlearrowright}|^2 + [|A_{\circlearrowleft}|^2 + |A_{\circlearrowright}|^2] \rangle$$
$$\sim w_{classical} + w_{classical} \sim 2 w_{classical}$$

Back-scattering is enhanced

Enhanced backscattering of light
from plastic beads suspended in water

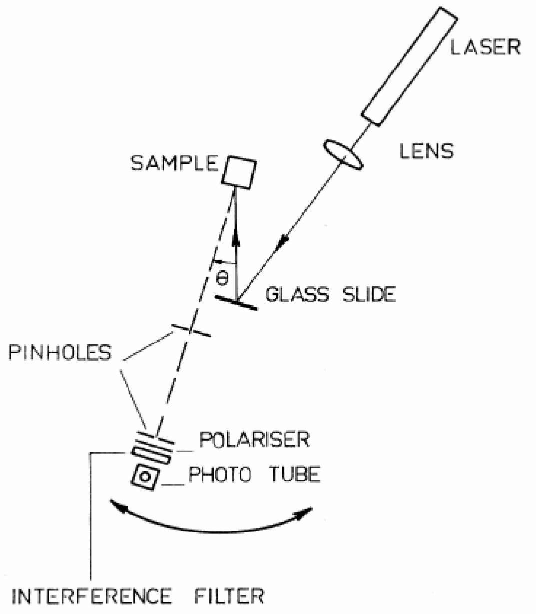


FIG. 1. Experimental setup for the study of backscatter-

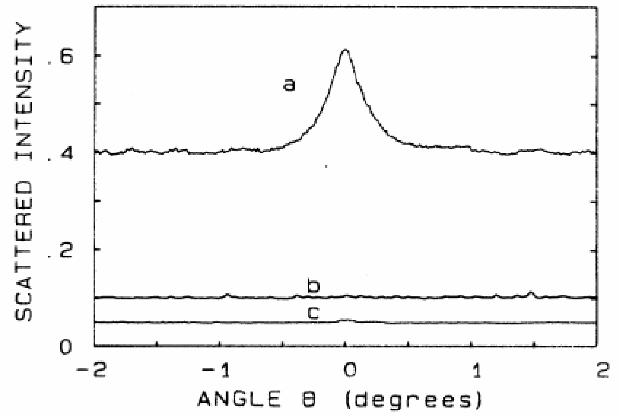
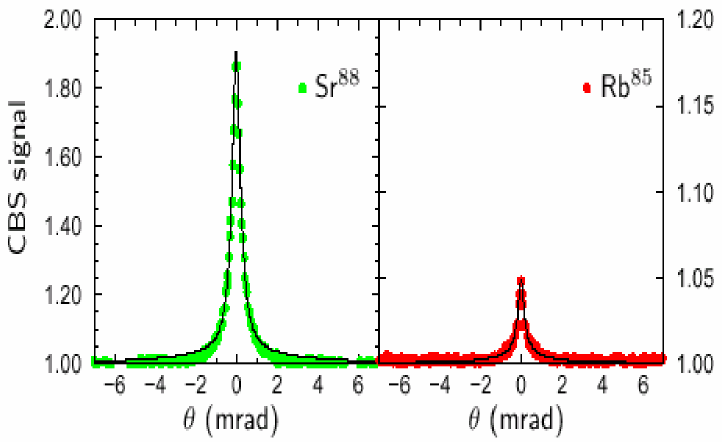


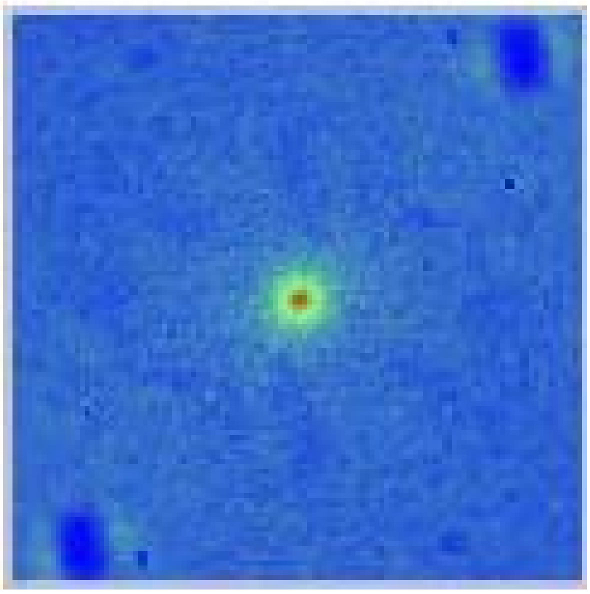
FIG. 2. Angular dependence of the scattered light intensity (curve *a*) by an aqueous suspension of 0.46-μm-diam polystyrene beads (solid fraction 10%), (curve *b*) by the same cell filled with water, and (curve *c*) in the absence of any cell. For these curves, no analyzer was used; scales are identical, but curves *b* and *c* are shifted by 0.1 and 0.05 vertical units, respectively.

P.Wolf, G.Maret, 1985

or from a cloud of cold atoms
(scattering cross-section from a single atom is large due to the resonance value of the light frequency).

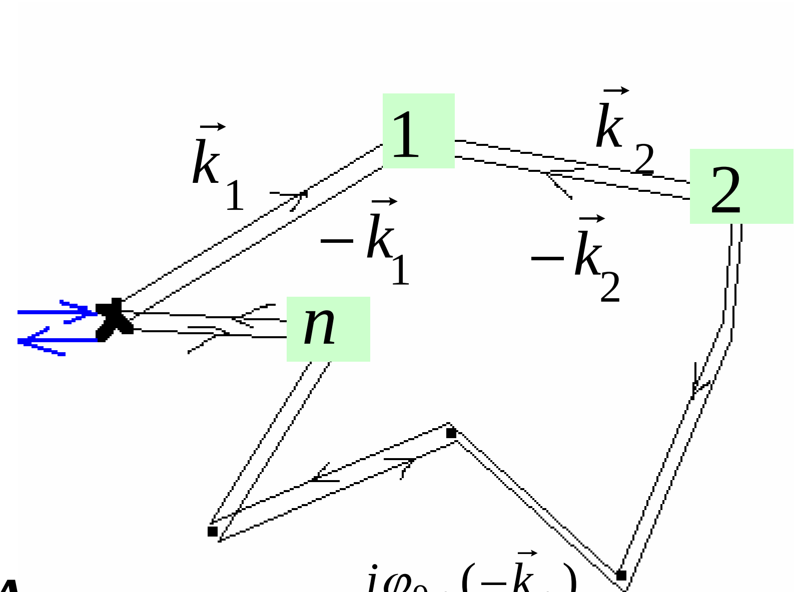


[Bidel et al., PRL **88**, 203902 (2002)] [Labeyrie et al., EPL **61**, 327 (2003)]



clockwise path

$$A_{\supset} \sim a_{\vec{k},\vec{k}_1} e^{i\varphi_{12}(\vec{k}_1)} a_{\vec{k}_1,\vec{k}_2} \cdots e^{i\varphi_{n0}(\vec{k}_n)} a_{\vec{k}_n,-\vec{k}}$$



$$\varphi_{i,i+1} = l_{i,i+1} k_F$$

$$A_{\subset} \sim a_{\vec{k},-\vec{k}_n} e^{i\varphi_{0n}(-\vec{k}_n)} \cdots a_{-\vec{k}_1,-\vec{k}_2} e^{i\varphi_{21}(-\vec{k}_1)} a_{-\vec{k}_1,-\vec{k}}$$

anti-clockwise path

Enhanced back-scattering is limited by the de-coherence time,

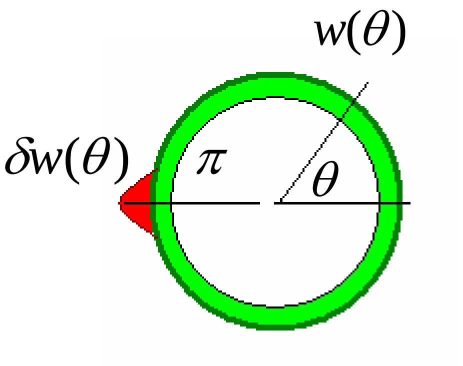
$$\tau_{\varphi}(T)$$

scattering involving energy transfer:

e-phonon scattering, inelastic e-e scattering

Momentum relaxation rate

The momentum relaxation rate is sensitive to the efficiency of back-scattering (scattering to the angle of π)

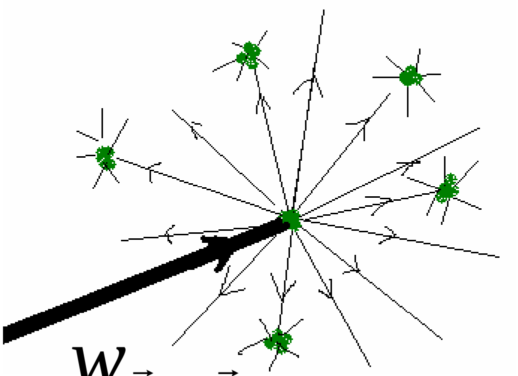


$$w(\theta) = n_{imp} \frac{\pi \gamma_F^{-1}}{\hbar} + \delta w(\theta)$$

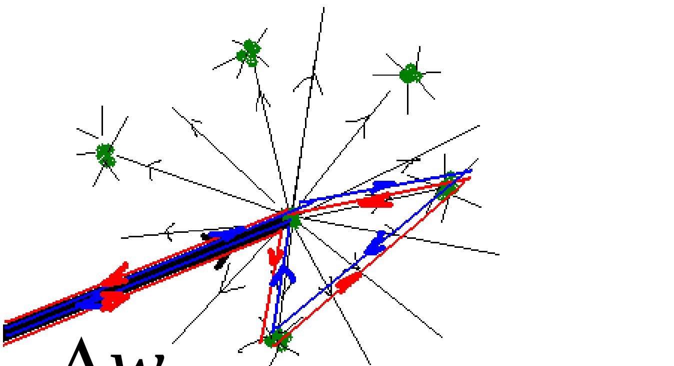
$$\tau^{-1} = \int d\theta \, [1 - \cos\theta] w(\theta) \approx \frac{1}{\tau} + \int d\theta \, \delta w$$

$$\delta\tau^{-1} = \int d\theta \delta w(\theta) \times [1 - \cos\theta] \approx 2\int d\theta \delta w(\theta)$$

Scattering cross-section of a single-impurity is enhanced by the presence of surrounding scatterers.



$w_{\vec{k} \to \vec{k}'}$



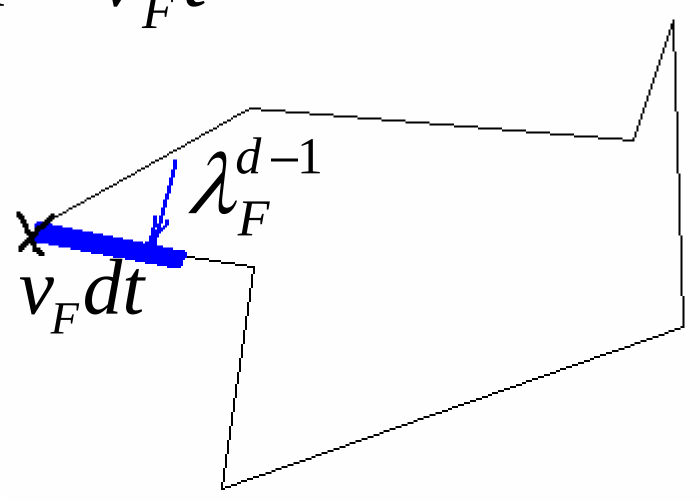
$\Delta w_{\vec{k} \to -\vec{k}}$

$$\frac{\Delta\rho_{WL}}{\rho_0} \sim \frac{\Delta\tau^{-1}}{\tau^{-1}} \sim \frac{\Delta w_{\vec{k} \to -\vec{k}}}{w^1_{\vec{k} \to \vec{k}'}}$$

Estimations

Closed loop with the length $\Lambda = v_F t$

$$\frac{\Delta w_{\vec{k}\to -\vec{k}}}{w_{\vec{k}\to\vec{k}'}} \sim \frac{N_{loops}}{N_{all}}$$



$$P(t;\vec{r}_0,\vec{r}_0) = \frac{1}{(Dt)^{d/2}}$$

Describes the proportion of loops among all random walk paths with the length $\Lambda = v_F t$

$$\frac{N_{loops}}{N_{all}} \sim \int_{\tau}^{\tau_\varphi} \lambda_F^{d-1} v_F \, dt \times P(t;\vec{r}_0,\vec{r}_0)$$

$$\sim \int_{\tau}^{\tau_\varphi} \frac{\lambda_F^{d-1} v_F \, dt}{(Dt)^{d/2}} \sim \begin{cases} \dfrac{\lambda_F v_F}{D} \ln \dfrac{\tau_\varphi}{\tau}, & d = 2 \\ v_F \sqrt{\tau_\varphi / D}, & d = 1 \end{cases}$$

$$\frac{\Delta\sigma_{WL}}{\sigma_0} \sim -\frac{\Delta\rho_{WL}}{\rho_0} \sim -\frac{\Delta w_{\vec{k}\to -\vec{k}}}{w_{\vec{k}\to\vec{k}'}} \sim -\frac{N_{loops}}{N_{all}}$$

$$\sigma_0 = e^2\nu_F D = \frac{e^2 n_e \tau}{m}$$

$$\gamma_F \sim \frac{\lambda_F^{-d}}{\varepsilon_F}$$

Einstein's relation between conductivity

Two-dimensional electron gas in Si-MOSFET's or GaAs/AlGaAs heterostructures (*d*=2)

$$\Delta\sigma_{WL} \sim -e^2\gamma_F D \times \frac{\lambda_F v_F}{D}\ln\frac{\tau_\varphi}{\tau}$$

$$\varepsilon_F \sim \frac{v_F h}{\lambda_F}$$

$$\sim -e^2\frac{\lambda_F^{-2}}{\varepsilon_F}\lambda_F v_F \ln\frac{\tau_\varphi}{\tau} \sim -\frac{e^2}{h}\ln\frac{\tau_\varphi}{\tau}$$

$$\sigma_{2D} = \sigma_0 - \frac{e^2}{h}\ln\frac{\tau_\varphi}{\tau}$$

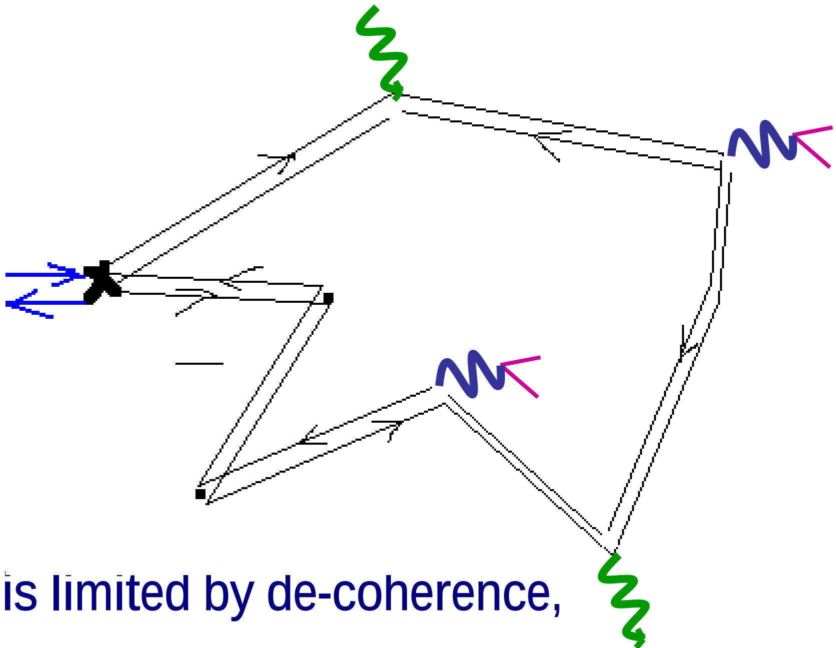
quantum conductance unit

B.Altshuler, D.Khmelnitski, A.Larkin, P.Lee (1980-1982)

$$\frac{\Delta\rho_{WL}}{\rho_0} = -\frac{\Delta\sigma_{WL}}{\sigma_0}$$

de-coherence time characterising the loss of phase-memory, 'phase breaking'

$$\Delta\rho_{WL} = -\frac{\Delta\sigma_{WL}}{\sigma_0^2} = \frac{e^2/h}{\sigma_0^2}\ln\frac{\tau_\varphi}{\tau}$$



Enhanced back-scattering is limited by de-coherence, scattering involving energy transfer: e-phonon scattering, inelastic e-e collisions

$$\varphi_{i,i+1}(t_i) = l_{i,i+1}k_F$$

$$\varphi_{i+1,i}(t - t_i) = l_{i,i+1}k(E_F + \delta\varepsilon) = l_{i,i+1}k + l_{i,i+1}\delta k = \varphi_{i,i+1} + \delta\varphi$$

Results in the temperature-dependent de-coherence time, $\tau_\varphi(T)$.

One-dimensional electrons in a disordered quantum wire (*d=1*)

$$\varepsilon_F \sim \frac{v_F h}{\lambda_F}$$

$$\Delta\sigma_{WL} \sim -e^2 \gamma_F D \times v_F \sqrt{\tau_\varphi / D}$$

$$\sim -e^2 \frac{\lambda_F^{-1}}{\varepsilon_F} v_F \sqrt{\tau_\varphi D} \sim -\frac{e^2}{h} \sqrt{\tau_\varphi D}$$

$$\sigma_{1D} = \sigma_0 - \frac{e^2}{h} \sqrt{\tau_\varphi D} = \sigma_0 - \frac{e^2}{h} L_\varphi$$

From weak to strong localisation in *d=1*

$$\sigma_0 = \frac{e^2 n_e \tau}{m} = \frac{e^2 \lambda_F^{-1} \tau}{m h^2} \sim \frac{e^2}{h} l$$

$$n_e^{1D} = \lambda_F^{-1}$$

$$l = v_F \tau = \frac{p_F}{m} \tau$$

$$\sigma_{1D} = \frac{e^2}{h} l - \frac{e^2}{h} L_\varphi \xrightarrow[L_\varphi \sim l]{} 0$$

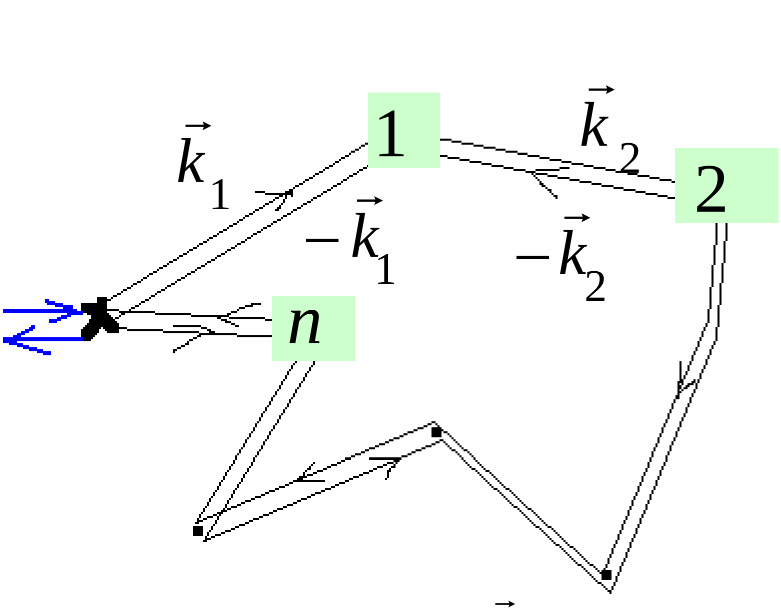
In a phase-coherent disordered 1D wire all states are localised at the length scale $L_{loc} \sim l$ (localisation length)

$$\sigma(L_\varphi >> l) \to 0 \Rightarrow \rho \to \infty$$

time reversibility !!!!!!

clockwise path

$$A_{\supset} \sim a_{\vec{k},\vec{k}_1} e^{i\varphi_{12}(\vec{k}_1)} a_{\vec{k}_1,\vec{k}_2} \cdots e^{i\varphi_{n0}(\vec{k}_n)} a_{\vec{k}_n,-\vec{k}}$$



$$\varphi_{i,i+1} = l_{i,i+1} k_F = \frac{l_{i,i+1} p_F}{\hbar}$$

$$A_{\subset} \sim a_{\vec{k},-\vec{k}_n} e^{i\varphi_{0n}(-\vec{k}_n)} \cdots a_{-\vec{k}_1,-\vec{k}_2} e^{i\varphi_{21}(-\vec{k}_1)} a_{-\vec{k}_1,-\vec{k}}$$

anti-clockwise path

Enhanced back-scattering is limited by the de-coherence time, $\tau_{\varphi}(T)$ due to scattering involving energy transfer: e-phonon scattering, inelastic e-e scattering

$$\varphi_{i,i+1}(t_i) = l_{i,i+1} k_F$$

$$\varphi_{i+1,i}(t - t_i) = l_{i,i+1} k(\varepsilon_F + \delta\varepsilon) = \varphi_{i,i+1} + \delta\varphi$$

or by $t \to -t$ symmetry breaking due to a magnetic field,

$$\varphi_{i,i+1}(\vec{k}_i) \neq \varphi_{i+1,i}(-\vec{k}_i)$$

Quantum magneto-resistance and the Aharonov - Bohm effect

$$rot\vec{A} = \vec{B} = B\vec{l}_z$$

$$\hat{H} = \frac{1}{2m}\left[-i\hbar\vec{\nabla} - \frac{e}{c}\vec{A}\right]^2 + u(\vec{r})$$

impurities

$$\oint d\vec{r}\cdot\vec{A} = BS = \Phi$$

circulation theorem

magnetic field flux

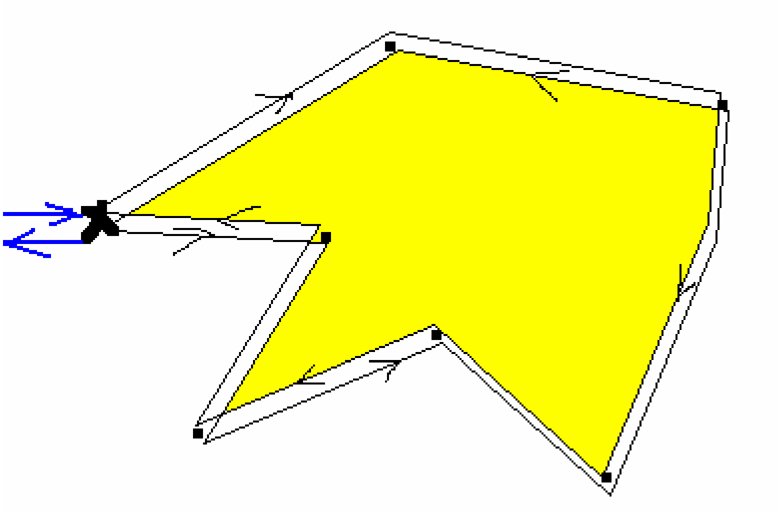
$$\varphi_{i,i+1} = l_{i,i+1}k_F + \frac{e}{\hbar c}\int_i^{i+1} d\vec{r}\cdot\vec{A}$$

$$\varphi_{i+1,i} = l_{i,i+1}k_F - \frac{e}{\hbar c}\int_i^{i+1} d\vec{r}\cdot\vec{A}$$

$$\varphi_{\supset} - \varphi_{\subset} = \frac{2e}{\hbar c}\oint d\vec{r}\cdot\vec{A} = \frac{2e}{\hbar c}BS$$

Weak localisation magneto-resistance

$$\varphi_{\supset} - \varphi_{\subset} = \frac{2e}{\hbar c} BS = 2\pi \frac{2BS}{\Phi_0}$$



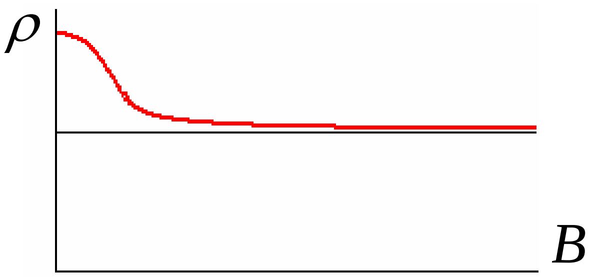
$$\Lambda = v_F t$$

$$L \sim \sqrt{Dt}$$

$$S \sim L^2 \sim Dt$$

random phase difference $$|\varphi_{\supset} - \varphi_{\subset}| \geq 2\pi$$

$$\langle A^*_{\subset} A_{\supset} + A_{\subset} A^*_{\supset} \rangle \to 0$$



$$t < \tau_B \sim \frac{\Phi_0}{BD}$$

$$\frac{BDt}{\Phi_0} \leq 2\pi$$

Magnetic field suppresses weak localisation

$$\rho_{2D} - \rho_0 = \frac{e^2/h}{\sigma_0^2} \ln \frac{\tau_B}{\tau} = -\frac{e^2/h}{\sigma_0^2} \ln \frac{Bl^2}{\Phi_0}$$

B.Altshuler, D.Khmelnitski, A.Larkin, P.Lee (1982)

Weak localisation magneto-resistance

$$\rho_{2D} - \rho_0 = \frac{e^2/h}{\sigma_0^2} \ln \frac{\tau_B}{\tau} = -\frac{e^2/h}{\sigma_0^2} \ln \frac{Bl^2}{\Phi_0}$$

was observed in all disordered conductors at low temperatures: Electrons in SiMOSFET's, heterostructures, heavily doped semiconductors, thin metallic films and wires.

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Magnetoresistance in Si metal-oxide-semiconductor field-effect transistors: Evidence of weak localization and correlation

D. J. Bishop, R. C. Dynes, and D. C. Tsui
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 20 January 1982)

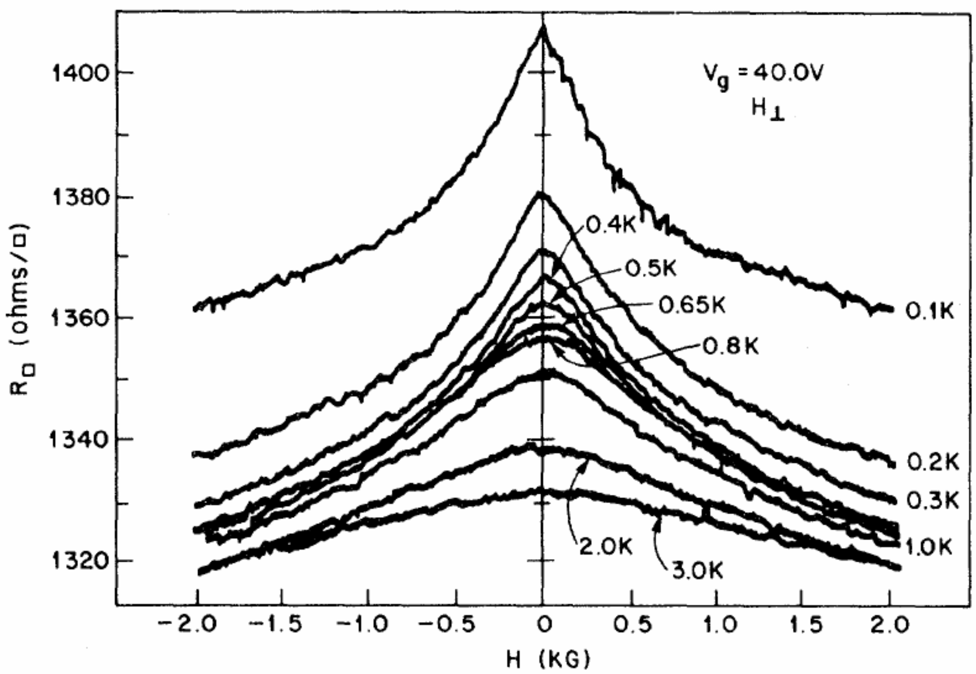

FIG. 2. Low-field magnetoresistance of a Si(111) MOSFET in a perpendicular field for various tempera-