Lecture 26-27

Transport characteristics of electrons in a metal and electron gases in 2D semiconductor structures, scattering time and momentum relaxation rate.

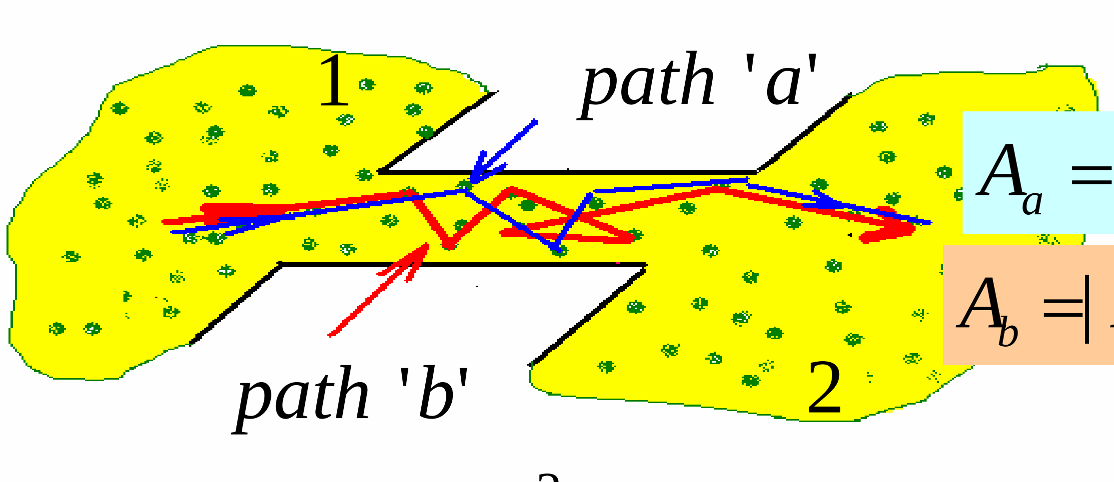
Drude formula, diffusion coefficient, Einstein relation; diffusion equation and its solution.

Interference of waves in disordered media, the phenomenon of enhanced backscattering.

Weak and strong localisation of electrons in disordered conductors.

Universal conductance fluctuations.

Quantum corrections to conductance



$$A_a = |A_a| e^{i\varphi_a}$$

$$A_b = |A_b| e^{i\varphi_b}$$

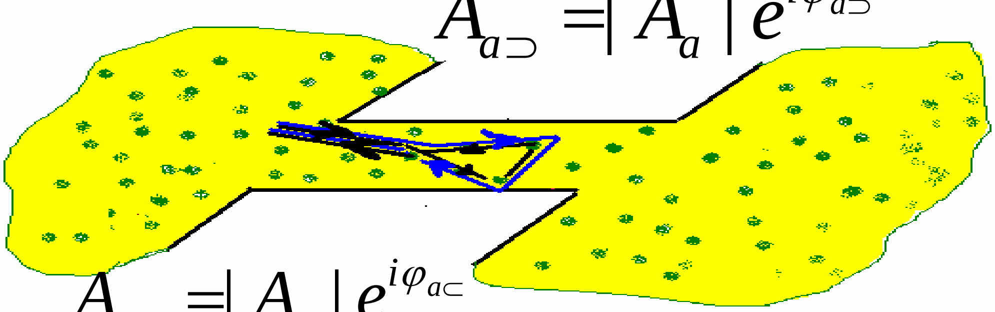
$$w_{1\to 2} = \left|\sum A_a\right|^2 = \sum |A_a|^2 + \sum A_a A_b^*$$

$$= \sum |A_a|^2$$ classical diffusion probability

$$+ \sum_{a \neq b} A_a A_b^*$$ interference pattern

$$\langle \sum_{a\neq b} A_a A_b^* \rangle = 0$$ **averaging over ensembles of impurities**

$$A_{a\supset} = |A_a| e^{i\varphi_{a\supset}}$$



$$A_{a\subset} = |A_a| e^{i\varphi_{a\subset}}$$

time-reversal symmetry,

$$\varphi_{a\subset} = \varphi_{a\supset}$$

$$\langle A_{a\subset} A_{a\supset}^* \rangle = \langle |A_a| e^{i\varphi_a} |A_a| e^{-i\varphi_a} \rangle = \langle |A_a|^2 \rangle$$

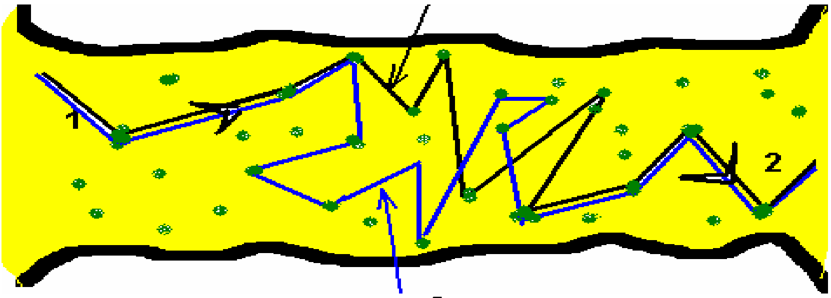
Mesoscopic conductance fluctuation

In each particular sample, there is no self-averaging.

$$\langle \sum_{a \neq b} A_a A_b^* \rangle = 0$$

$$\sum_{a \neq b} A_a A_b^* \neq 0$$

A_a



A_b

$$t_{fl} \sim \frac{L^2}{D}$$

$$\langle G^2 \rangle = \langle G \rangle^2 + \langle \delta G^2 \rangle$$

$$\langle \sum_{i \neq j} (A_a A_b^*)(A_a^* A_b) \rangle$$

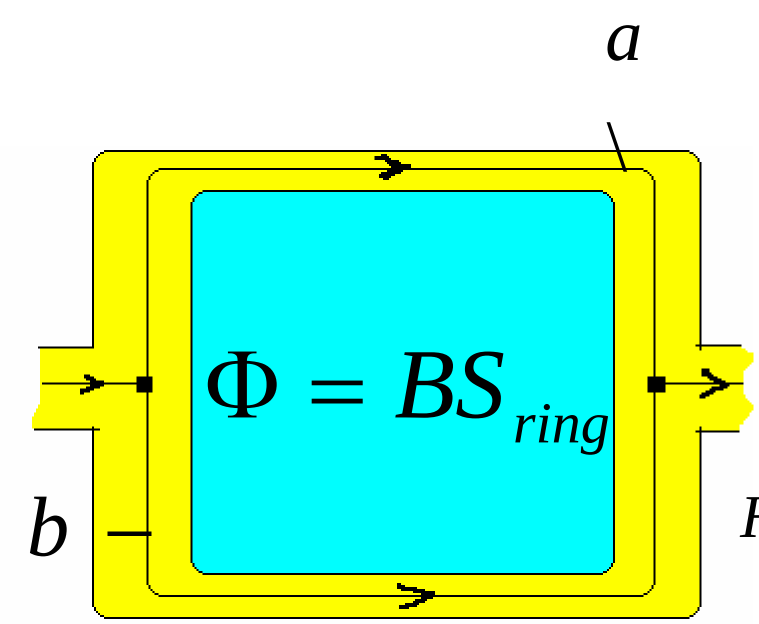
Universal conductance fluctuations (UCF's)

$$\langle \delta G^2 \rangle \sim \left(\frac{e^2}{h} \right)^2$$

Altshuler 1985
Lee & Stone 1985

Aharonov-Bohm oscillation in the ring geometry

$$w_{1\to 2} = \left| |A_a| e^{i\varphi_a} + |A_b| e^{i\varphi_b} \right|^2$$
$$= |A_a|^2 + |A_b|^2 + 2|A_a A_b| \cos(\varphi_a - \varphi_b)$$



$$\theta = \varphi_a(0) - \varphi_b(0)$$
$$A = |A| e^{i\varphi(B)}$$

$$\hat{H}_0 = \frac{1}{2m}\left[-i\hbar\vec{\nabla} - \frac{e}{c}\vec{A} \right]^2$$

$$\varphi_a - \varphi_b - \theta = \frac{e}{\hbar c}\left(\int_a - \int_b\right) d\vec{r} \cdot \vec{A} = \frac{e}{\hbar c} BS_{ring}$$

$$\delta G \sim \frac{e^2}{h} \cos\theta \times \cos\left(\frac{e}{\hbar c} BS_{ring} \right)$$

$$\phi_0 = \frac{hc}{e}$$

$$\sim \frac{e^2}{h} \cos\theta \times \cos \frac{2\pi\Phi_{ring}}{\phi_0}$$

Observation of h/e Aharonov-Bohm Oscillations in Normal-Metal Rings

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IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

$$\Delta B_{period} \frac{e}{\hbar c} S_{ring} = 2\pi$$

$$\phi_0 = \frac{hc}{e}$$

magnetic flux quantum

$$\Delta B_{period} = \frac{hc}{eS_{ring}} = \frac{\phi_0}{S_{ring}}$$

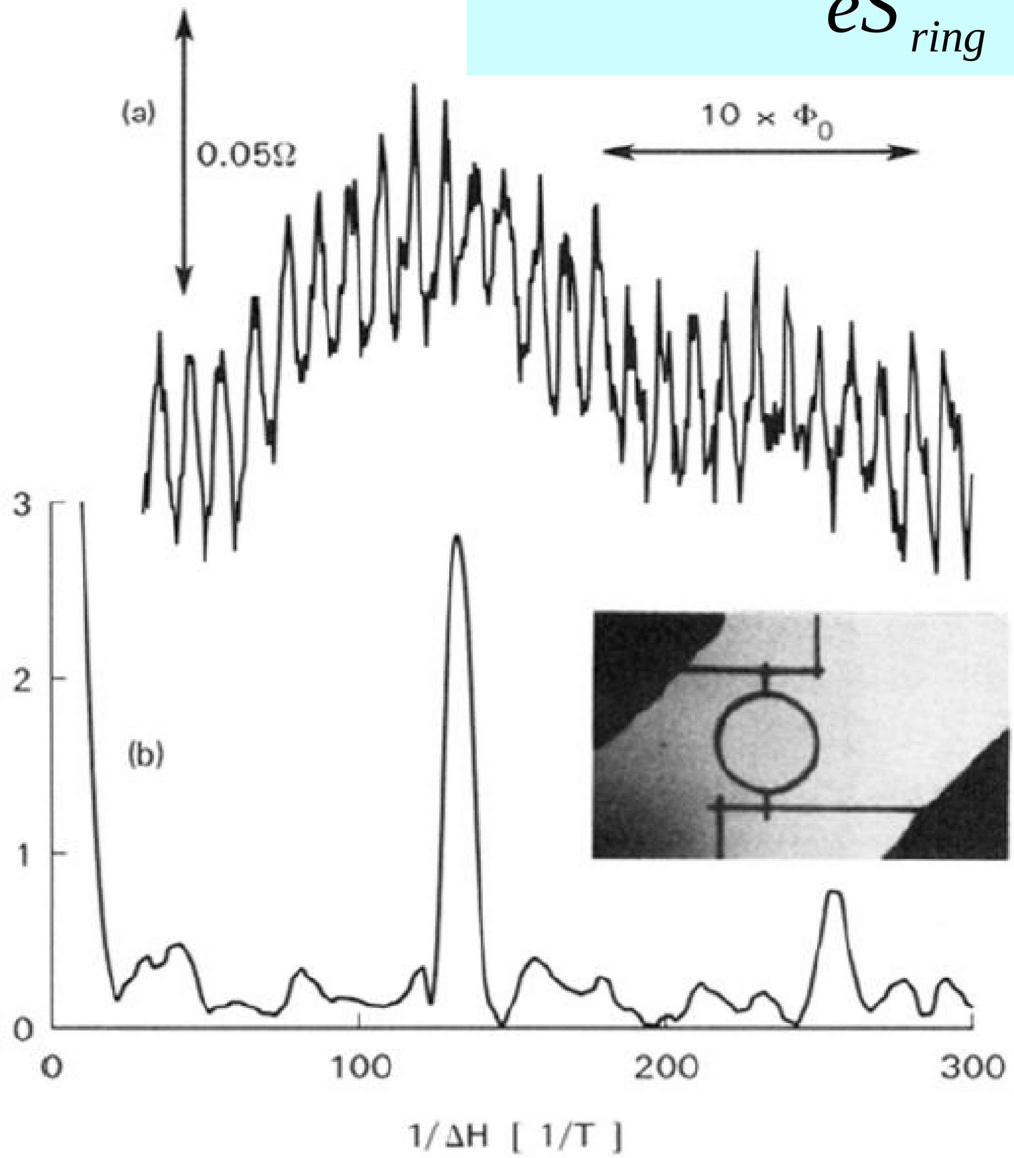


FIG. 1. (a) Magnetoresistance of the ring measured at $T = 0.01$ K. (b) Fourier power spectrum in arbitrary units containing peaks at h/e and $h/2e$. The inset is a photograph of the larger ring. The inside diameter of the loop is 784 nm, and the width of the wires is 41 nm.

Magneto-conductance 'fingerprint'

$$A_a = |A_a|\, e^{i\varphi_a(B)}$$

$$\theta_{ab} = \varphi_a(0) - \varphi_b(0)$$

$$A_b = |A_b|\, e^{i\varphi_b(B)}$$

$$\varphi_a - \varphi_b - \theta_{ab} = \frac{e}{\hbar c}\left(\int_a - \int_b\right) d\vec{r}\cdot\vec{A} = \frac{e}{\hbar c} BS_{a\bar{b}}$$

change of a magnetic field

$$\Delta B > B_c = \frac{\phi_0}{S_{wire}}$$

causes a random change in the interference pattern

$$G = \langle G \rangle + \frac{e^2}{h}\,\delta\!\left(\frac{B}{B_c}\right)$$

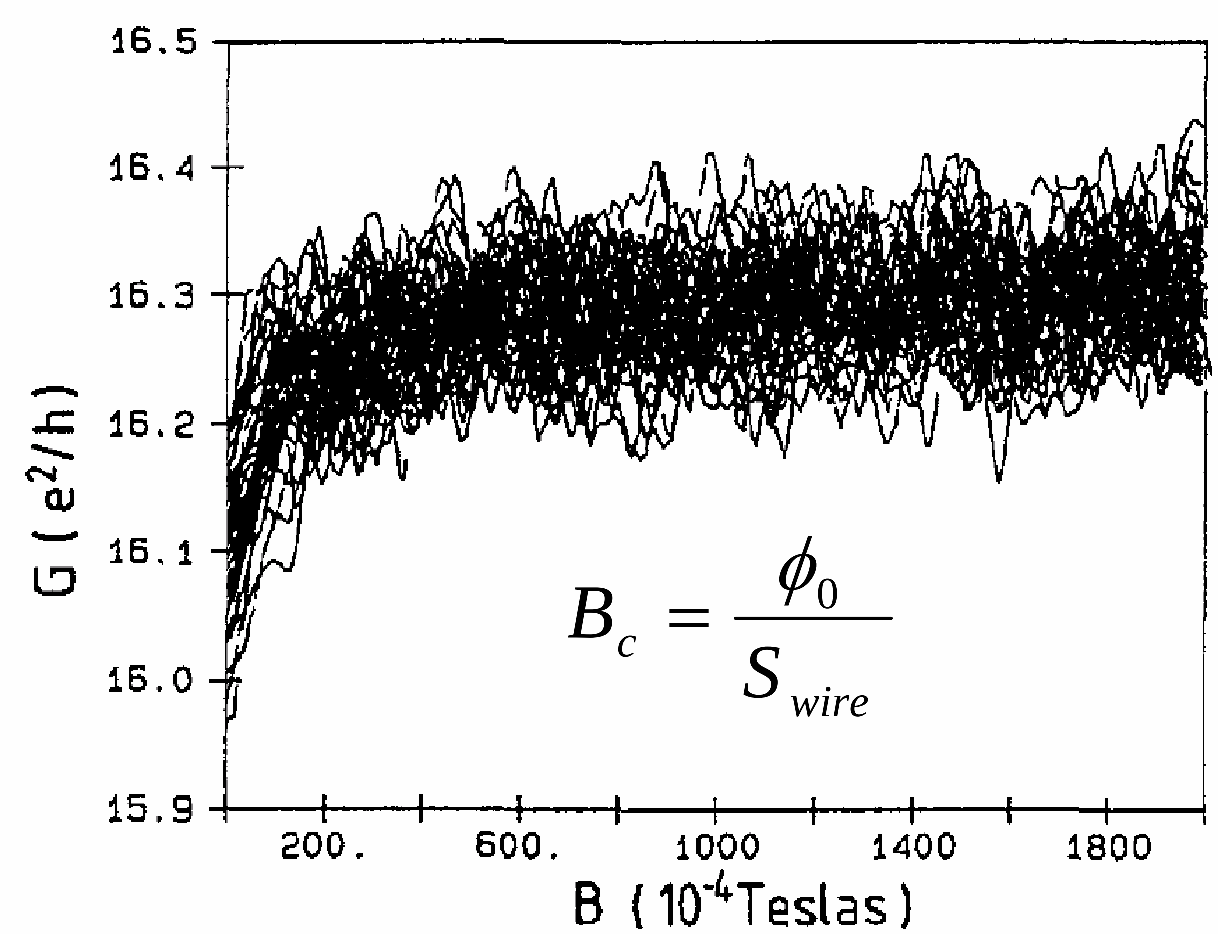
a random function of both magnetic field and electron energy

Altshuler & Khmelnitski 1985

Universal conductance fluctuations (UCFs) were discovered experimentally by S.Washburn, R.Webb (1984)

How do UCF's look like in experiments:

Micron-size wire of heavily doped n-GaAs (at low temperature).



Data show magnetoresistance of the same wire thermally cycled 100 times. Fluctuation patterns changed after each thermal cycle, due to the re-charging of deep traps which changed the microscopic realisation of disorder without changing the mean free path.

D.Mailly, M.Sanquer (1992)

Dephasing by Extremely Dilute Magnetic Impurities Revealed by Aharonov-Bohm Oscillations

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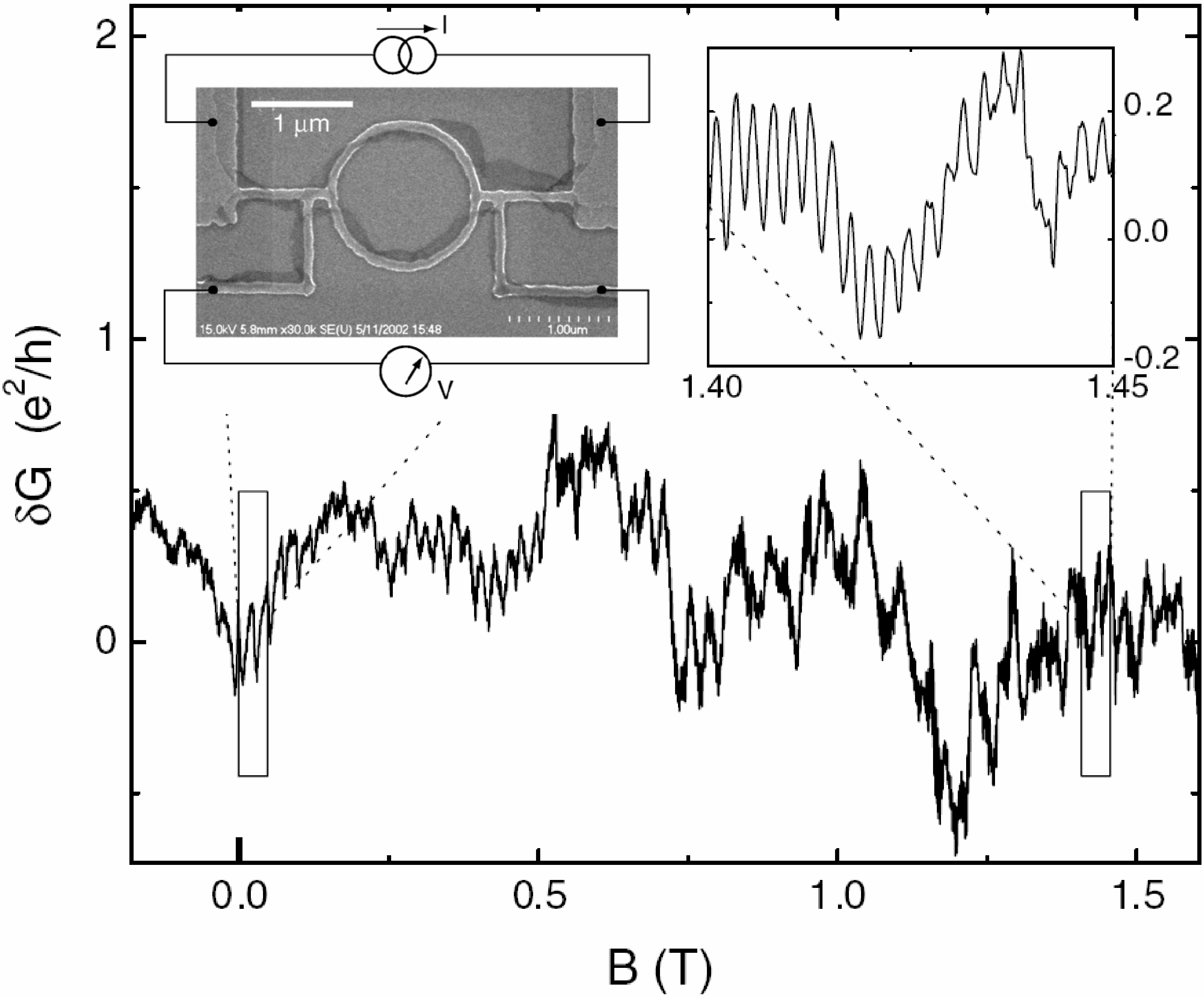
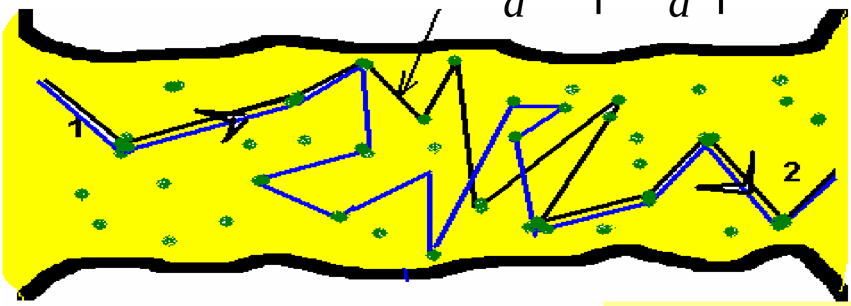


Periodic at a small scale, $$\Delta B_{period} = \frac{\phi_0}{S_{ring}}$$

Random at a larger scale, $$B_c = \frac{\phi_0}{S_{wire}}$$

Fluctuations, as a function of energy

$$A_a = |A_a| e^{i\varphi_a(\varepsilon)}$$



$$\Lambda \sim t_{fl} v_F >> \lambda_F$$

$$\varphi_a(\varepsilon) = \Lambda_a k_\varepsilon + \sum \alpha_k$$

scattering phases

semiclassical phase

$$\frac{dk}{d\varepsilon} = 1 / \hbar v_F$$

$$\varphi_a(\varepsilon') - \varphi_a(\varepsilon) = \Lambda_a [k_\varepsilon - k_{\varepsilon'}] \sim t_{fl} v_F \cdot \frac{\delta\varepsilon}{\hbar v_F}$$

$$\langle \delta G_\varepsilon \delta G_{\varepsilon'} \rangle \sim \langle \sum_{a \neq b} (A_{a\varepsilon} A^*_{b\varepsilon})(A^*_{a\varepsilon'} A_{b\varepsilon'}) \rangle$$

$$\sim \langle \sum_{a \neq b} |A_a|^2 |A_b|^2 e^{i\Lambda_{a+b}[k(\varepsilon) - k(\varepsilon')]} \rangle$$

$$\frac{\delta\varepsilon \, t_{fl}}{\hbar} \sim 2\pi \Rightarrow \delta\varepsilon \sim E_T = \frac{h}{t_{fl}} = \frac{hD}{L^2}$$

Thouless energy (D.Thouless, 1980)

δG_ε is a randomly oscillating function of electron energy ε, typically on the scale of E_T. Thus, observation of UCF's requires low temperature,

$$k_B T < E_T = \frac{hD}{L^2}$$