Lectures 28-30

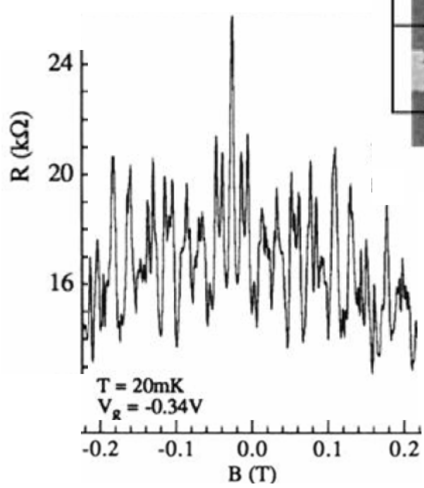
Semiconductor quantum dots

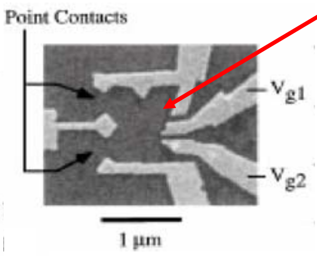
Interference effects in chaotic quantum dots

The resonance tunnelling phenomenon

The Coulomb blockade in quantum dots

Quantum transport in semiconductor dots





Group of C.Marcus (Harvard) 1992-now

Chaotic motion of electrons in dots

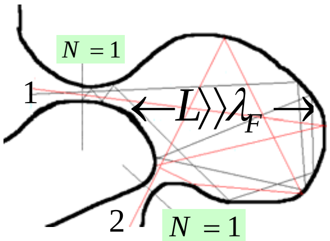
$$\vec{p}_0, \vec{x}_0$$

$$\vec{p}_0 + \delta\vec{p}$$
$$\vec{x}_0 + \delta\vec{x}$$

$$\Rightarrow$$

$$|\vec{p}_t - \vec{p}_t^{\,'}| \sim \delta p e^{t/\tau_L}$$
$$|\vec{x}_t - \vec{x}_t^{\,'}| \sim \delta x e^{t/\tau_L}$$

Lyapunov time

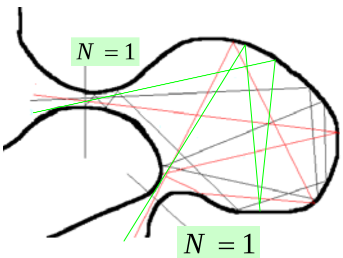


$$t > \tau_L \sim L/V_F$$
$$|\vec{p}_t - \vec{p}_t^{\,'}| \sim p_F$$
$$|\vec{x}_t - \vec{x}_t^{\,'}| \sim L$$

$$G = \frac{2e^2}{h} w_{12}(\varepsilon_F)$$

$$w_{12}^{typical} \sim \frac{1}{2}$$

Chaotic motion of electrons in dots



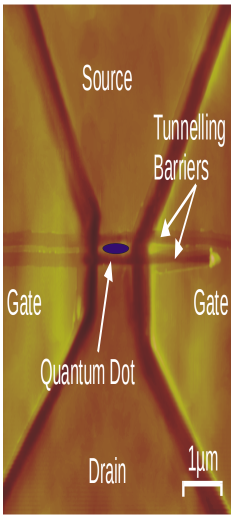
$$t_{escape} > \tau_L \sim L/V_F$$

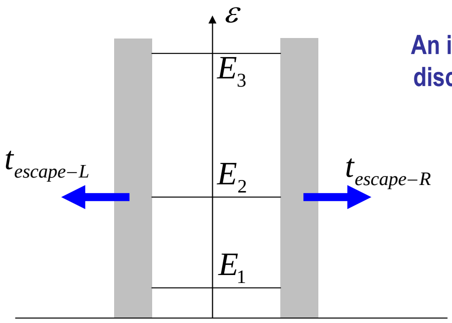
Chaotic motion of electrons in a dot resembles random motion in a disordered system, including the effect of the interference between electron waves propagating along randomly shaped trajectories.

$$G = \frac{2e^2}{h} \cdot \frac{1}{2} + \delta G(B) \qquad \langle \delta G^2 \rangle \sim \left(\frac{e^2}{h}\right)^2$$

Random magnetic field dependent part, specific to each particular shape of the dot.

Almost isolated quantum dot





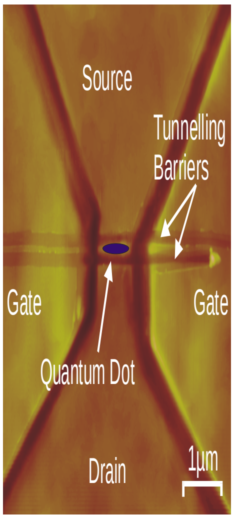
An isolated dot has a discrete spectrum of states $\mathbf{E_n}$

$$\Gamma_{L-dot} + \Gamma_{R-dot} = \Gamma$$

$$\Gamma_{L-dot} = \frac{h}{t_{escape-L}}; \quad \Gamma_{R-dot} = \frac{h}{t_{escape-R}}$$

Level broadening: due to a finite life-time of electron on the dot, its energy is not known with accuracy better than Γ

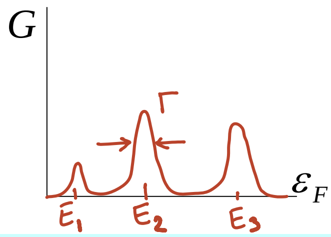
Resonance tunnelling phenomenon



$$w_{12} = \frac{\Gamma_{R-dot}\Gamma_{L-dot}}{(\varepsilon_F - E_n)^2 + \frac{1}{4}(\Gamma_{R-dot} + \Gamma_{L-dot})^2}$$

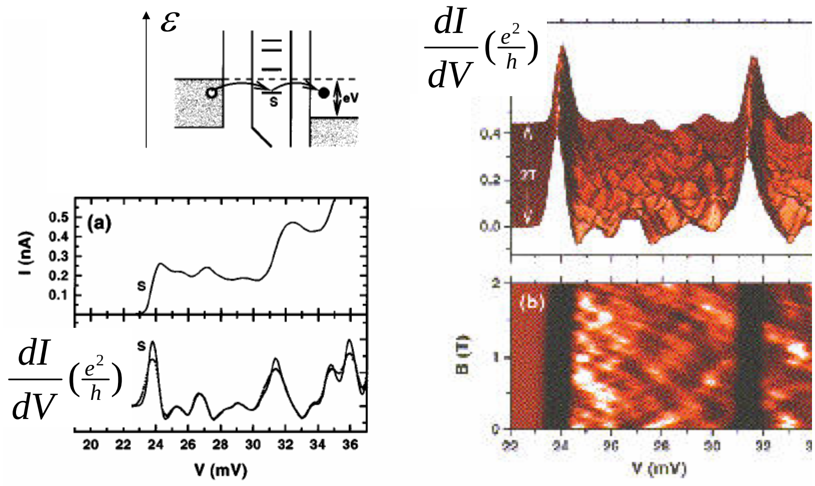
Resonance transmission (Breit-Wigner formula)

$$G = \frac{2e^2}{h} w_{12}(\varepsilon_F)$$



$$G = \begin{cases} \frac{e^2}{h}\frac{4\Gamma_{R-dot}\Gamma_{L-dot}}{(\Gamma_{R-dot} + \Gamma_{L-dot})^2} \sim \frac{e^2}{h} & if \quad \varepsilon_F = E_n \\ 0 \quad if \; |\varepsilon_F - E_n| > \Gamma \end{cases}$$

Resonant tunnelling through quantum dots



Schmidt, Haug,Falko, von Klitzing, Forster, Luth - Europhys. Lett. 36, 61 (1996)

Coulomb blockade

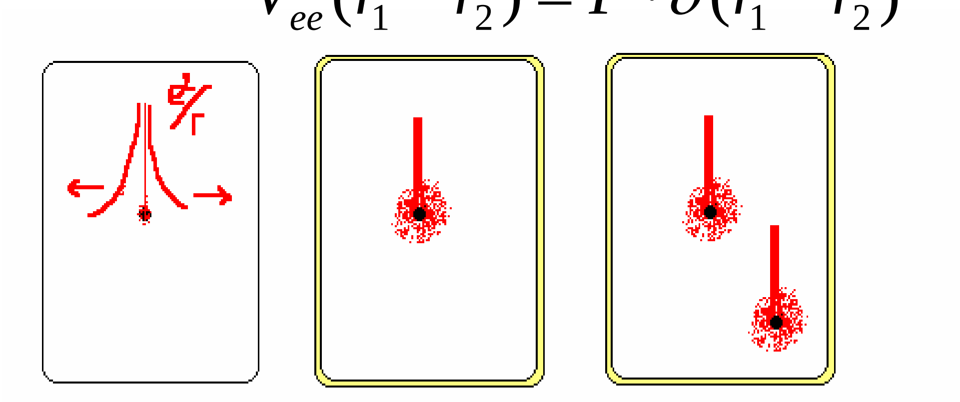
Dynamical screening and Coulomb blockade in quantum dots.

Counting electrons one by one.

Coulomb blockade in a superconducting island: 'parity effect'

Charge quantization

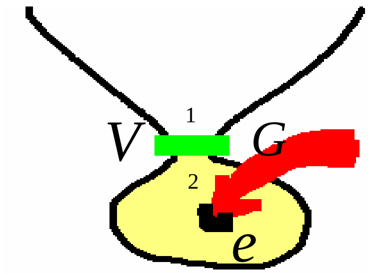
$$V_{ee}(\vec{r}_1 - \vec{r}_2) = F \cdot \delta(\vec{r}_1 - \vec{r}_2)$$



Although electron carries electric charge e, its interaction with other electrons is screened by the other electrons from the Fermi sea, so that e-e interaction is reduced.

$$\frac{dQ}{dt} = -I = -GV = -\frac{GQ}{C}$$

$$\tau_{scr}^{-1} = \frac{G}{C} = \frac{2e^2 w_{12}}{hC}$$



Decay rate of charge localised at the dot determines broadening of single-electron charged state of the dot due to dynamical screening

Charging energy

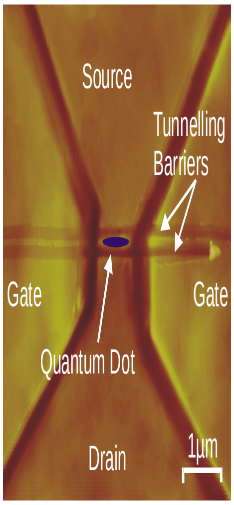
$$E_c = \frac{e^2}{2C} > h\tau_{scr}^{-1} = \frac{2e^2 w_{12}}{C}$$

if

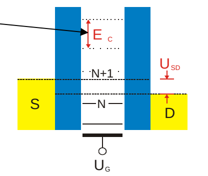
$$w_{12} < 1$$

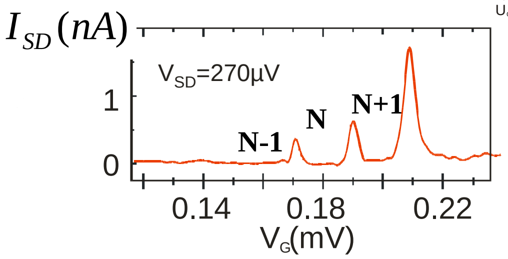
screening is blocked: Coulomb blockade

Charge quantization in isolated quantum dots



$$E_c = \frac{e^2}{2C}$$



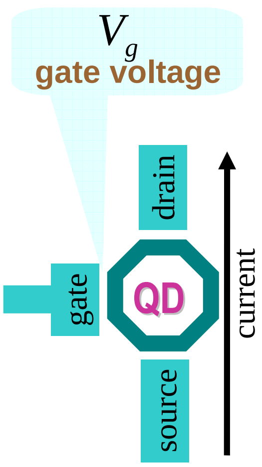


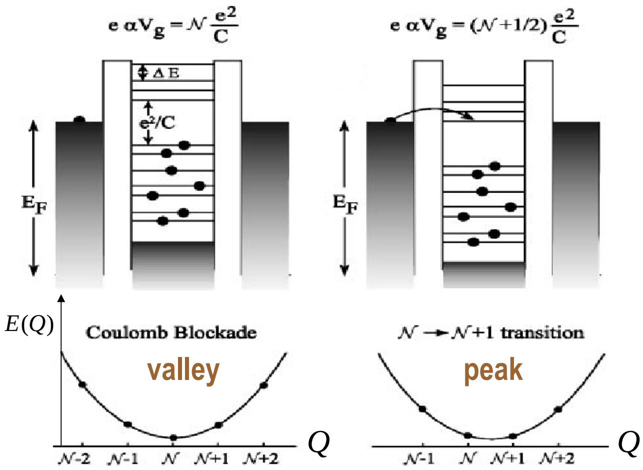
First observations: T.Fulton, G.Dolan PRL 59, 109 (1987) – Bell Labs
M.Kastner - Rev. Mod Phys. 64, 849 (1992) – MIT

Active groups:
Marcus (Harvard); Kouwenhoven TUDelft); Haug (Hannover); Enssling (ETH Zurich)

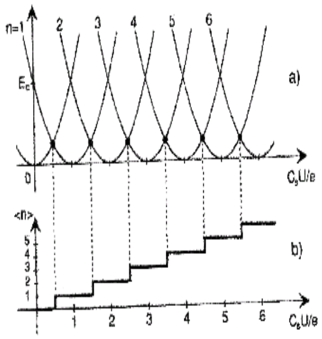
Coulomb Blockade of electron tunneling

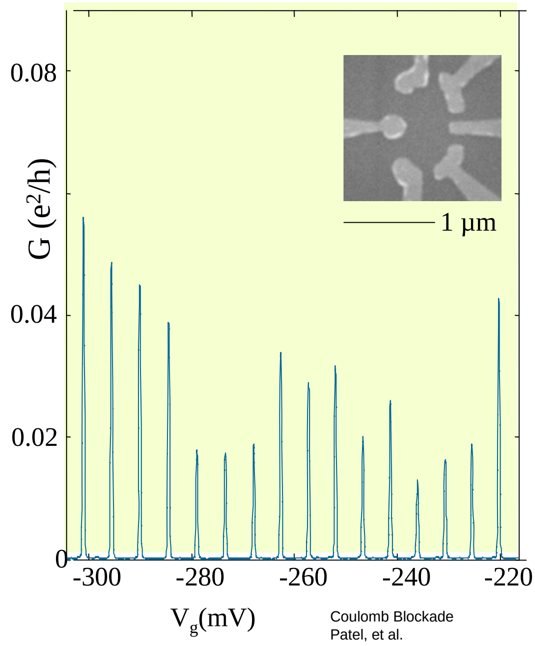
$$E = \frac{(Q - CV_g)^2}{2C}$$





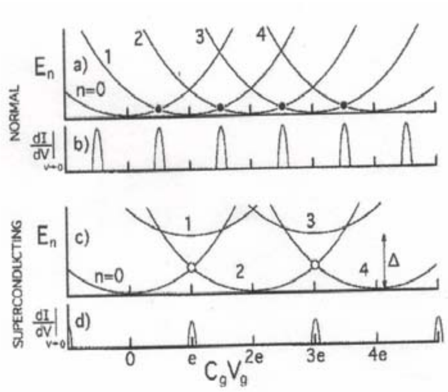
$$E_n(V_g) = \frac{(ne - CV_g)^2}{2C}$$

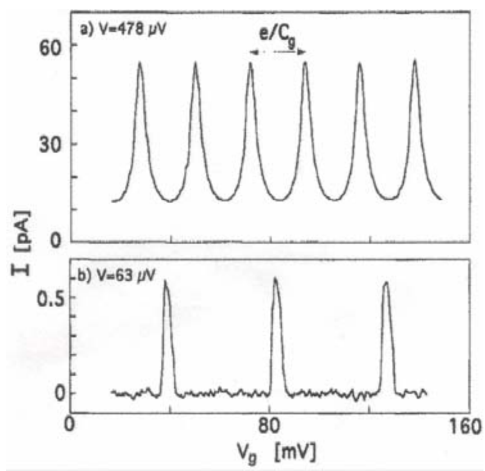


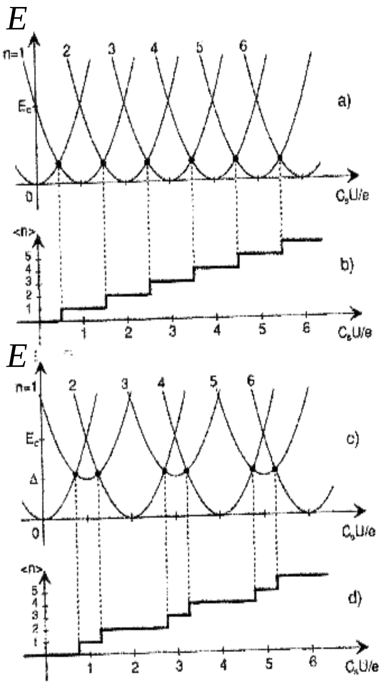


Coulomb Blockade
Patel, et al.
Phys.Rev.Lett. 80 4522 (1998)

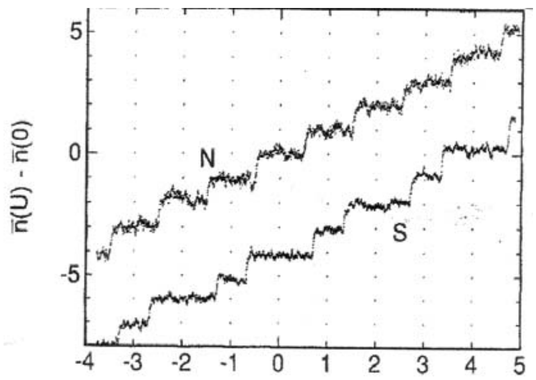
Coulomb blockade in a superconducting island: condensate of Cooper pairs creates a gap Δ in the single-particle spectrum







Parity effect
in a superconducting island



Lafarge, Joyez, Esteve, Urbina, Devoret, PRL 70, 994 (1993)