Influence of a measurement on coherent charge transfer in an adiabatic Cooper pair pump

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Outline

– Introduction
  - Three junction pump
  - Single Cooper pair pump

– Cooper pair pump
  - Principle
  - Contributions to pumped charge

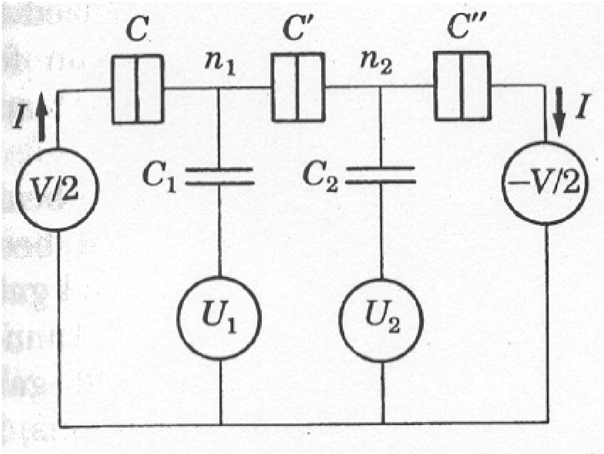
– Influence of a measuring environment

– Examples
  - Resistive environment
  - SQUID
  - Escape measurement

– Conclusions

Normal three-junction pump (Pothier et al. '92)

Set-up:

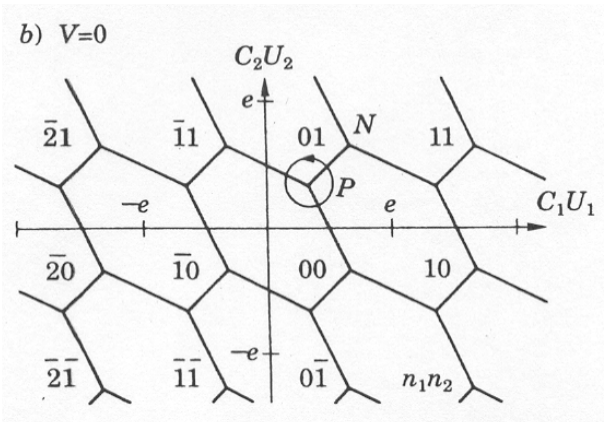


- tunnel junctions: tunnel resistance R_T
- islands: charging energy E_C
- gates: time-dependent voltages U_1, U_2

Coulomb blockade: excess island charges n_1, n_2 well defined

$$k_BT, eV \ll E_C \; , \; R_T \gg R_K$$

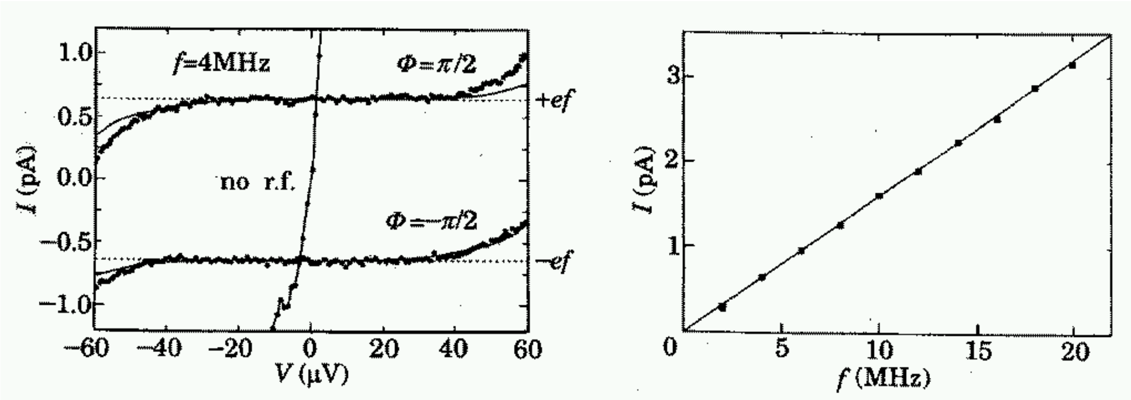
Ground state configurations:



Central idea: charge transfer upon gate modulation

$$I = ef$$

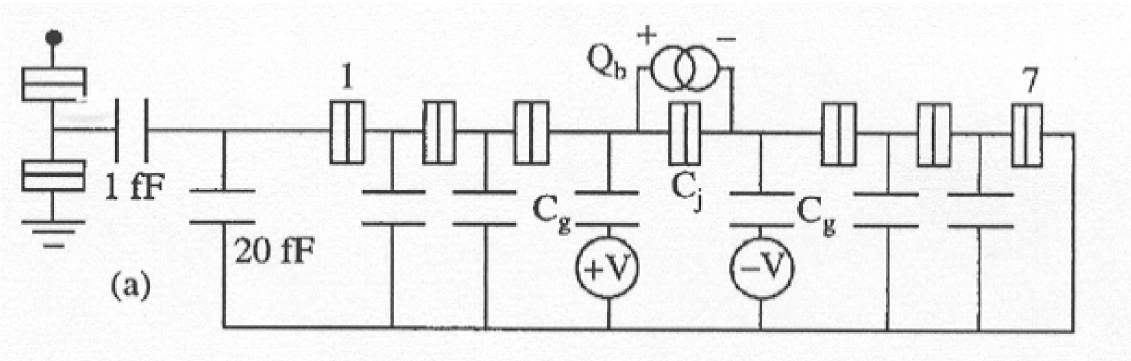
Experiment:



- accurate transfer of charge: $\Delta I \sim 0.05$ pA
- deviations due to cotunneling

Metrological applications: long arrays (Keller, Martinis et al., NIST)

$\rightarrow$ error 10^{-8} per electron



Pumps with other systems:

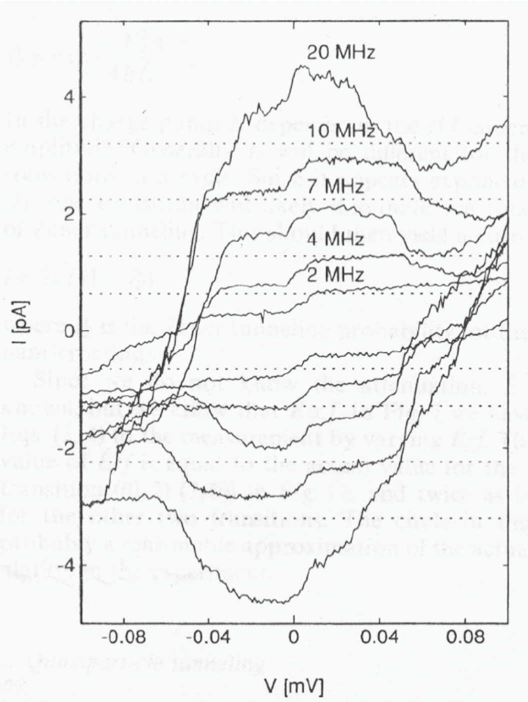
– semiconductor quantum dots (Kouwenhoven, Marcus)

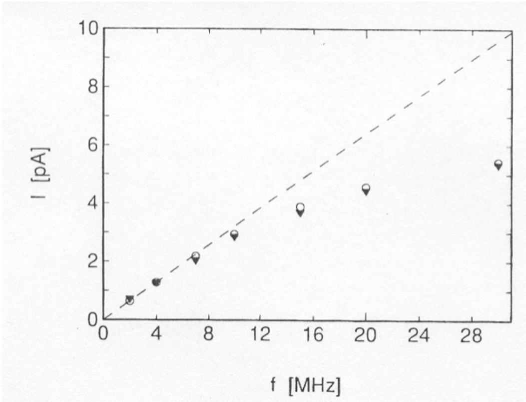
– ballistic channels (Pepper)

– ...

Single Cooper pair pump (Geerligs et al. 1991)

Central idea: replace normal metals by superconductors
→ single charges e → Cooper pairs $2e$
→ pumped current $I = ef$ → $I = 2ef$





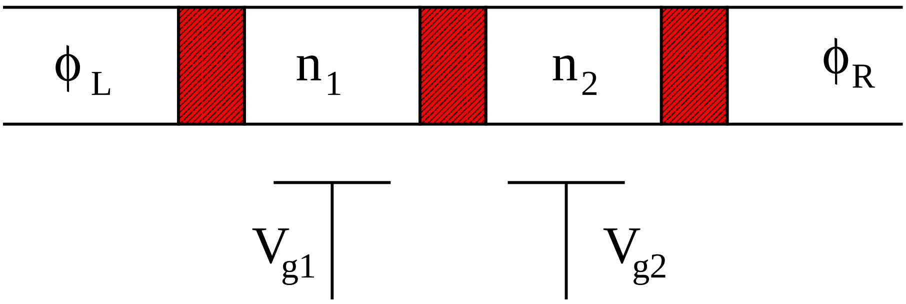
Problems: single electron tunneling → Cooper pair tunneling (Josephson effect, coherence!)

- cotunneling of Cooper pairs
- Zener tunneling
- quasiparticle tunneling

Recent work

- Pekola et al. (1999): theory of coherent charge transfer
- Zorin et al. (2000): experiment on a resistive pump
- Bibow et al. (2002): experiment on resonant tunneling

Superconducting three-junction pump



Set-up :

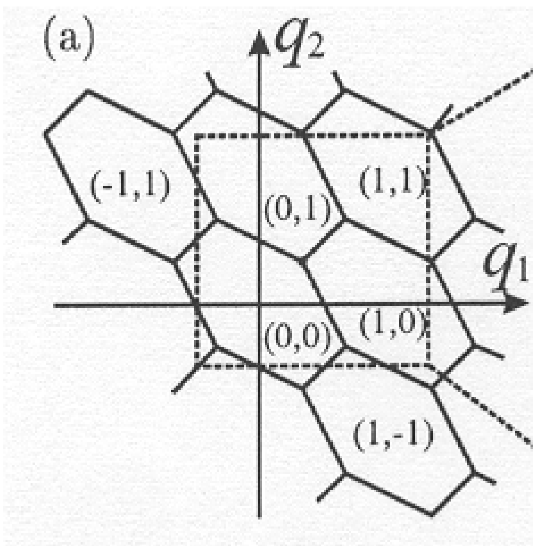
- superconducting reservoirs: phase difference $\phi_0 = \phi_L - \phi_R$
- Josephson junctions: Josephson energy E_J
- islands: charging energy E_C
- gates: time-dependent voltages V_{g1}, V_{g2}

Hamiltonian $\hat{H} = \hat{H}_C + \hat{H}_J$

Important energies

$$k_B T \ll E_J \ll E_C \ll \Delta$$

Ground state configurations (charging part)

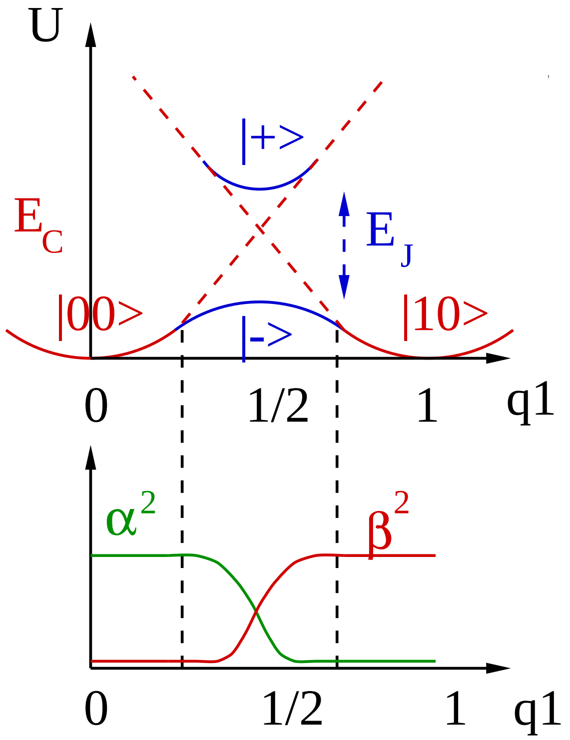


Effect of Josephson coupling: coherent mixing of charge states

Josephson coupling

$$\hat{H}_J = -E_J\left[\cos(\hat{\phi}_1+\phi_0/3) + \cos(\hat{\phi}_2-\hat{\phi}_1+\phi_0/3)+\cos(\phi_0/3-\hat{\phi}_2)\right]$$

Near degeneracy $|00\rangle \leftrightarrow |10\rangle$



$$|+\rangle = \beta|00\rangle - e^{i\phi_0/3}\alpha|10\rangle$$

$$|-\rangle = \alpha|00\rangle + e^{i\phi_0/3}\beta|10\rangle$$

$$\alpha^2 = 1 - \beta^2$$

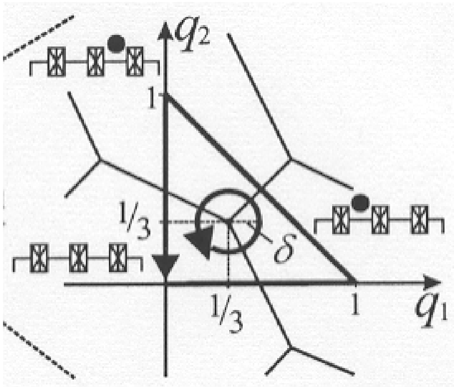
Interesting features:

- Formation of "band structure"
- Eigenstates are coherent superpositions of charge states
- Dependence on phase difference ϕ_0

Consequences:

- Possibility for Zener tunneling if V_g swept rapidly; condition for adiabaticity $\omega_0 \ll E_J^2/E_C$
- Possible dependence of pumped charge on phase ϕ_0

Pumped charge (Pekola et al. '99)



Adiabatically pumped charge

$$Q_P = 2\hbar \Im\mathrm{m} \left[\sum_{n\neq 0} \oint \frac{(\hat{I}_l)_{0n}}{E_0 - E_n} \langle n | \partial_{\vec{q}} 0 \rangle \cdot d\vec{q} \right]$$

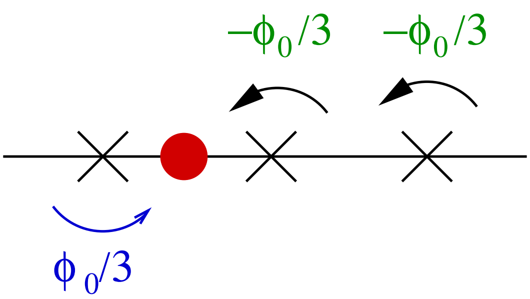
$$\hat{I}_l = I_J \sin(\hat{\phi}_1 + \phi_0/3)$$

Incoherent part: first order in Josephson coupling
→ charge pumped per cycle $Q_p = 2e$ (1 pair)
→ result independent of phase difference ϕ_0!

Coherent correction: second order in Josephson coupling

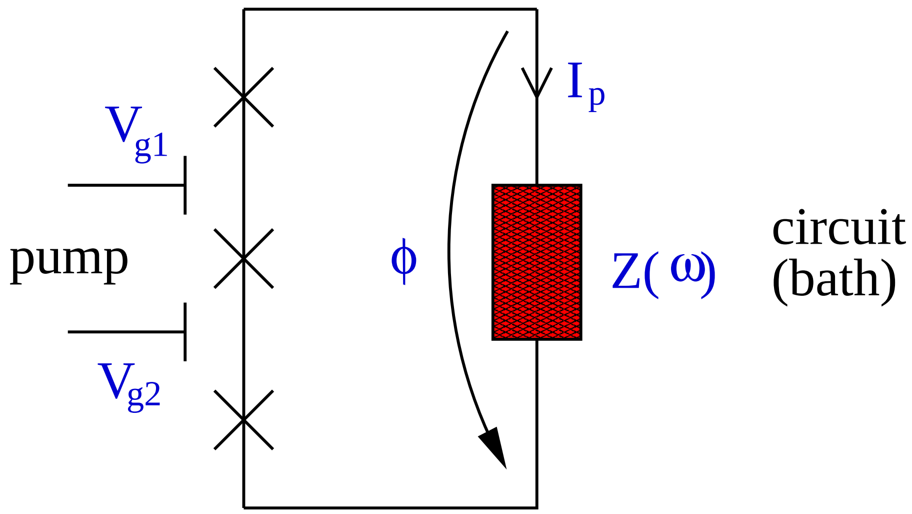
$$\frac{\delta Q_p}{2e} = -9 \frac{E_J}{E_C} \cos \phi_0$$

Interference due to coherent higher order process:



Measurement circuit: the model

Generic set-up:



Circuit: Total phase difference across pump fluctuates

$$\phi(t) = \phi_0 + \delta\phi(t)$$

Action: $S_{\rm total} = S_{\rm pump} + S_{\rm bath}$, where $S_{\rm pump} = S_{\rm charge} + S_J$ with

$$S_J = E_J \sum_{a=1}^{3} \int_0^\beta d\tau \cos[\varphi_a(\tau) + \phi(\tau)/3]$$

$$\varphi_1 = \phi_1; \varphi_2 = \phi_2 - \phi_1; \varphi_3 = -\phi_2$$

and

$$S_{\rm bath} = \frac{T}{2}\frac{\hbar}{4e^2}\sum_n \frac{|\omega_n|}{Z(i|\omega_n|)}|\phi(\omega_n)|^2$$

Partition function:

$$Z_{\rm total} = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \mathcal{D}\phi \exp\{-S_{\rm total}\}$$

Effective theory

Integrate out the circuit degrees of freedom:

$$\begin{aligned} Z_{\text{total}} &\simeq \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \mathcal{D}\phi e^{-S_{\text{charge}}-S_{\text{bath}}}(1 - S_J + S_J^2/2) \\ &= \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 e^{-S_{\text{charge}}}(1 - \langle S_J \rangle_{\text{bath}} + \langle S_J^2 \rangle_{\text{bath}}/2) \end{aligned}$$

First order: renormalization of E_J

$$\langle S_J \rangle_{\text{bath}} = E_{J,\text{eff}} \sum_{a=1}^{3} \int_0^\beta d\tau \cos[\varphi_a(\tau) + \phi_0/3]$$

where

$$E_{J,\text{eff}} = E_J e^{-G_\phi(\tau=0)/18} < E_J$$

with

$$G_\phi(\tau) = T \left(\frac{2e}{\hbar}\right)^2 \sum_n \frac{Z(i|\omega_n|)}{|\omega_n|} e^{-i\omega_n \tau}$$

→ suppression of coherent part of pumped charge δQ_p

Second order: "interaction term"

$$\langle S_J^2 \rangle_{\text{bath}} = E_{J,\text{eff}}^2 \int_0^\beta d\tau_1 d\tau_2 \sum_{a,b=1}^{3} \times$$
$$\sin[\phi_a(\tau_1) + \phi_0/3] G_\phi(\tau_1 - \tau_2) \sin[\phi_b(\tau_2) + \phi_0/3]$$

→ higher order tunneling modifies coherent part of pumped charge δQ_p

Resistive environment at low T

The case $Z(i|\omega_n|/|\omega_n|) \to R/\omega$, up to a frequency ω_c

$\to$ logarithmic divergence!

Weakly resistive case $R/R_Q \ll 1$: $E_{J,\mathrm{eff}} \lesssim E_J$

Strongly resistive case $R/R_Q \gg 1$:

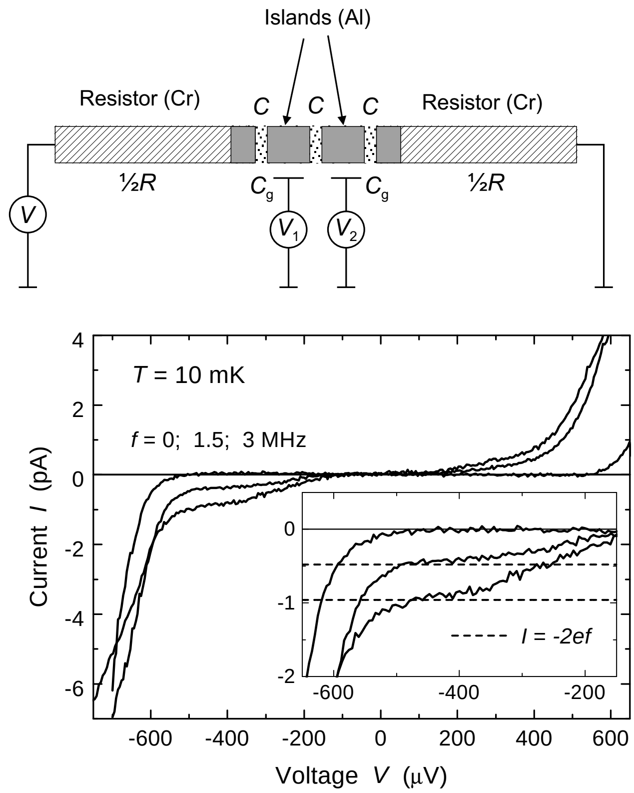
$$E_{J,\mathrm{eff}} = E_J \left(\frac{\omega_o}{\omega_c}\right)^{\alpha}$$

where

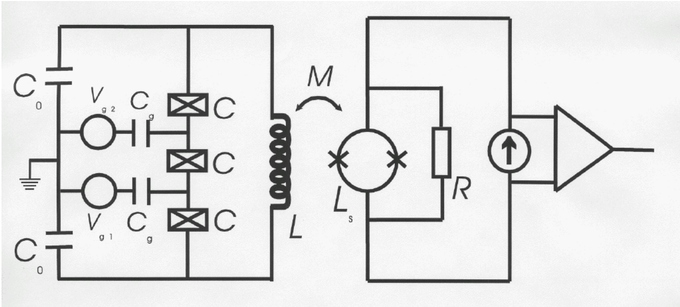
- $\alpha = 2R/9R_Q$
- ω_o frequency at which the pump is operated

$\to$ suppression of δQ_p depends on ω_o

Experiment (Zorin et al., 2000)



Measurement with a SQUID loop



Frequency-dependent impedance:

$$Z(i|\omega|)/|\omega| = L\frac{1+\tau(1-\gamma)|\omega|}{1+\tau|\omega|+LC_p|\omega|^2+\tau LC_p(1-\gamma)|\omega|^3}$$

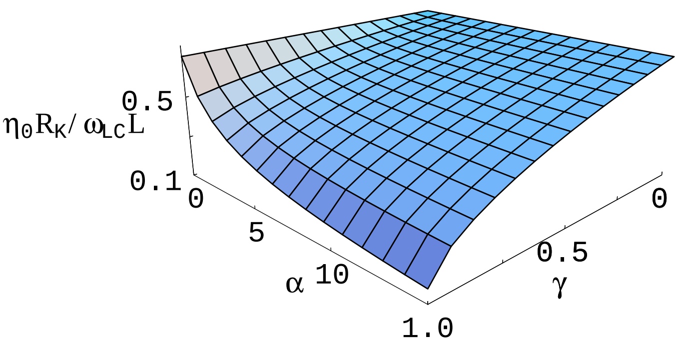
with $\tau = L_s/R$ and $\gamma = M^2/(L_sL)$

→ no low-frequency divergence
→ suppression of δQ_p independent of operating frequency!

Effective Josephson energy $E_{J,\mathrm{eff}} = E_J e^{-\eta}$, where

$$\eta \simeq \begin{cases} \frac{1}{R_Q}\sqrt{\frac{L}{C_p}} \text{ if } T \ll \omega_{LC} \\ \frac{LT}{R_Q} \text{ if } T \gg \omega_{LC} \end{cases}$$

Low temperatures $T \ll \omega_{LC}$



Here, $\alpha = L_s\omega_{LC}/R$

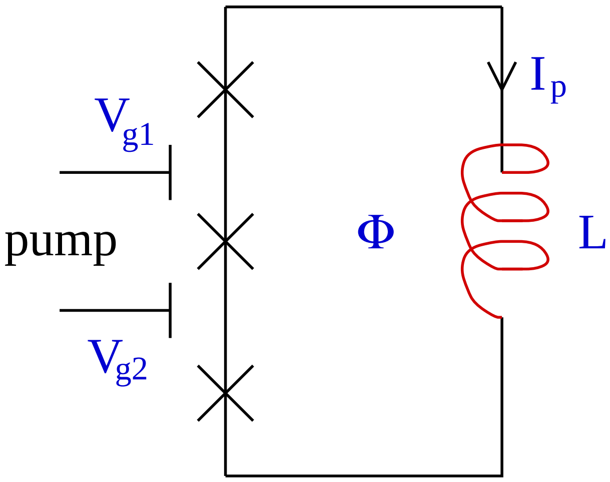
Interaction term: current-current correlations

Effective theory:

- renormalized $E_J \to E_{J,\text{eff}}$
- interaction term

$$\langle S_J^2 \rangle_{\text{bath}} = E_{J,\text{eff}}^2 \int_0^\beta d\tau_1 d\tau_2 \sum_{a,b=1}^{3} \times$$
$$\sin[\phi_a(\tau_1) + \phi_0/3] G_\phi(\tau_1 - \tau_2) \sin[\phi_b(\tau_2) + \phi_0/3]$$

Example: pump coupled to a SQUID with $\omega_{LC} \gg E_J$



Instantaneous current-current correlation:

$G_\phi(\tau) \to G_\phi(\omega_n = 0)\delta(\tau)$ leads to interaction term

$$\hat{H}_{\text{int}} = L(\pi E_{J,\text{eff}}/\Phi_0)^2 \sum_{a,b=1}^{3} \sin[\phi_a + \phi_0/3] \sin[\phi_b + \phi_0/3]$$

Inductance correlates tunneling at different junctions!

$$\frac{Q_p}{2e} = 1 - 9\frac{E_{J,\text{eff}}}{E_C} \cos\phi_0 + \frac{2\pi^2}{3} \frac{L E_{J,\text{eff}}}{\Phi_0^2} \cos\phi_0$$

Conclusions

– Cooper pair pump

  - Principle
  - Two contributions to pumped charge: coherent and incoherent

– Influence of a measuring environment

  - renormalizes Josephson energy
  - induces higher order tunneling

– Examples

  - Resistive environment: effectively suppresses coherent part at low operating frequencies
  - Inductive environment (SQUID and escape junction): suppression independent of operating frequence