

# *Cooper pair transport in an array of Josephson nanojunctions with dice lattice*

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## Outline

### ✓ Introduction

- localisation effect in the dice lattice

### ✓ classical superconducting arrays :

- $T_c$ ,  $I_c$  suppression, glassy vortex state

### ✓ quantum arrays

- S-I transition, metallic phase

### ✓ the dice family of JJ arrays

- exotic superconducting phase

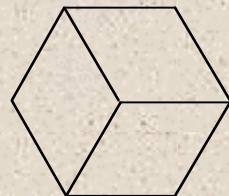
### • Collaborators

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- P.Bataud, J. Vidal
- O. Buisson, K. Hasselbach
- Th. Fournier

### • Acknowledgements

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- " CEA-LETI-PLATO"

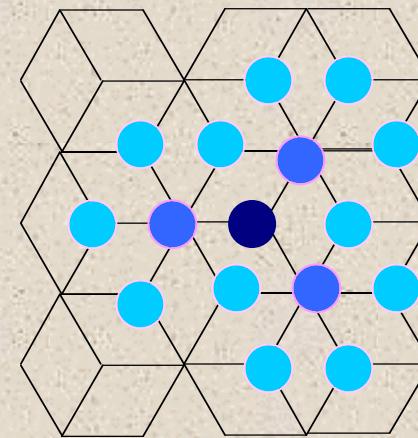
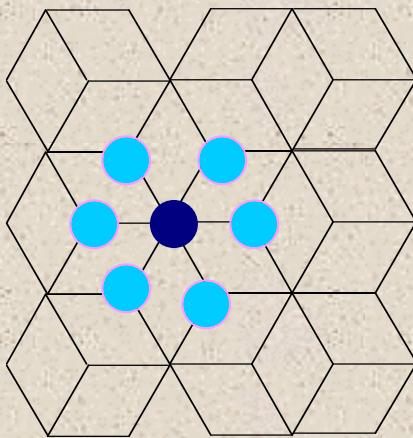
# *localisation effect in the « dice » lattice*



Bravais cell

$$f = \phi / \phi_0$$

$\phi_0$  : flux quantum



- non-interacting tight binding electrons (*Vidal, PRL81, 1998, p5888*)

$f=1/2$                        $\Rightarrow$                       localisation due to quantum interferences (AB cages)  
cage effect suppressed by : disorder, edge states , interaction.

- GaAs quantum wires (electrons : fermions e) *C. Naud et al. PRL86, 5104 (2001)*

•  $h/e, h/2e$  magnetoresistance oscillations           Aharonov Bohm cages

- Superconducting arrays (Cooper pairs : bosons 2e)

• wire networks : Schrödinger equation (1 particle)  $\equiv$  linearized GL equations  
for the macroscopic superconducting state (fluctuations neglected)

# Josephson Junction arrays

- classical dice JJ array

highly frustrated state with thermal fluctuations :  $\cos(\phi_i - \phi_j - A_{ij}) \Rightarrow J(f) S_i S_j$   
 vortices on the Kagomé (dual) lattice

- quantum JJ array

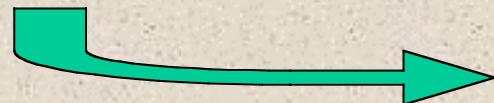
Josephson coupling + Coulomb blockade



$$\frac{1}{2} \sum n_i U_{ij} n_j$$

Hubbard

$$H = -E_J \sum \cos(\phi_i - \phi_j - A_{ij}) + \frac{(2e)^2}{2} \sum n_i C_{ij}^{-1} n_j$$

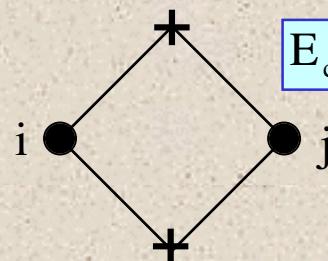


$$-\frac{t}{2} \sum (e^{-iA_{ij}} b_i^+ b_j + h.c.)$$

hopping of Cooper pairs

- role of elementary rhombuses ("dimers")

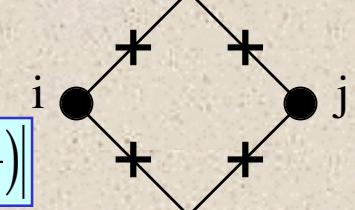
SQUID :  $E_J$  suppressed at  $f=1/2$



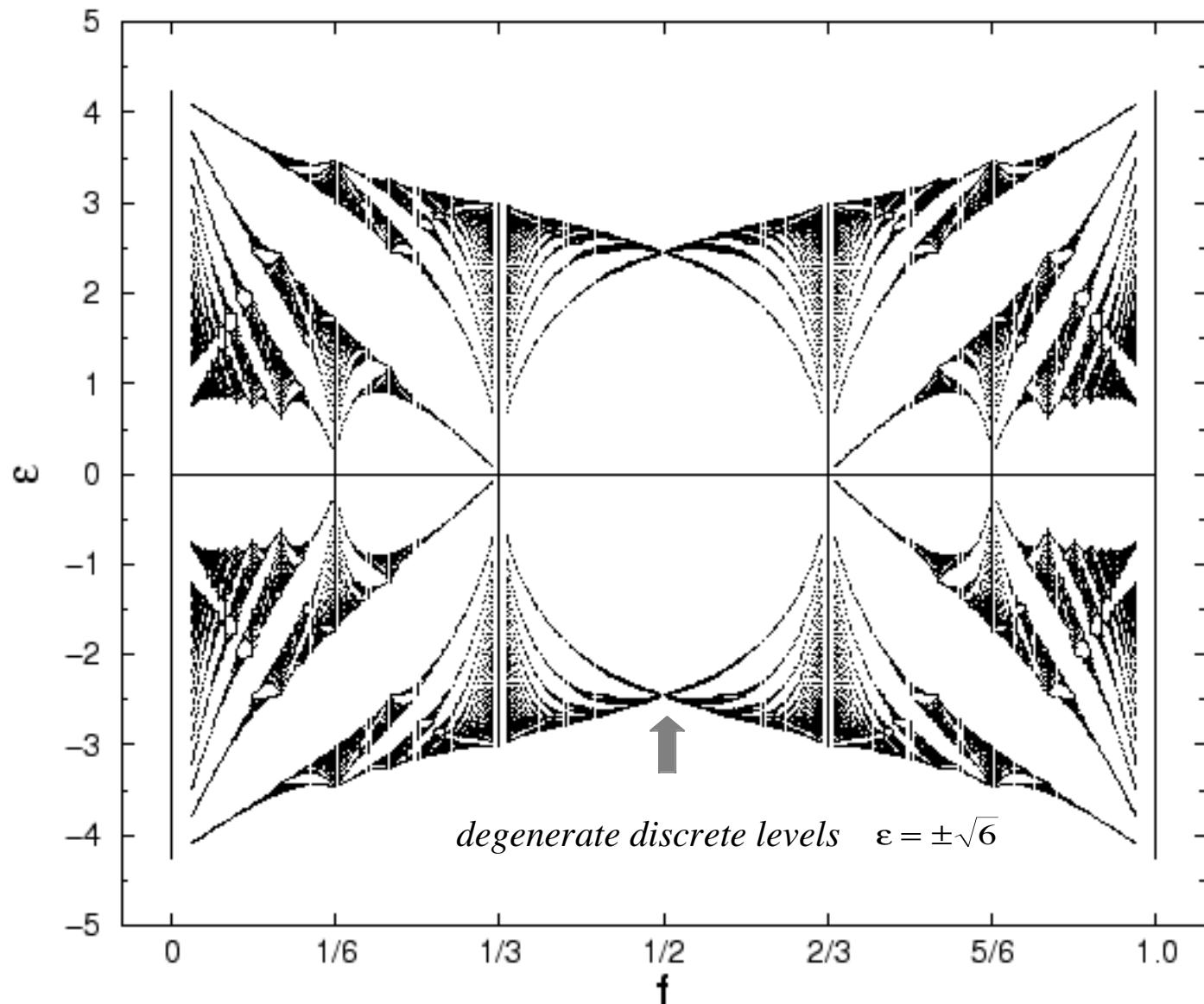
$$E_{\text{class}} = -2 \cos \theta_{ij} |\cos \pi f|$$

Rhombus : additionnal degree of freedom : phase of intermediate island

$$E_{\text{class}} = -2 \left| \cos \left( \frac{\theta}{2} - \frac{\pi f}{2} \right) \right| - 2 \left| \cos \left( \frac{\theta}{2} + \frac{\pi f}{2} \right) \right|$$

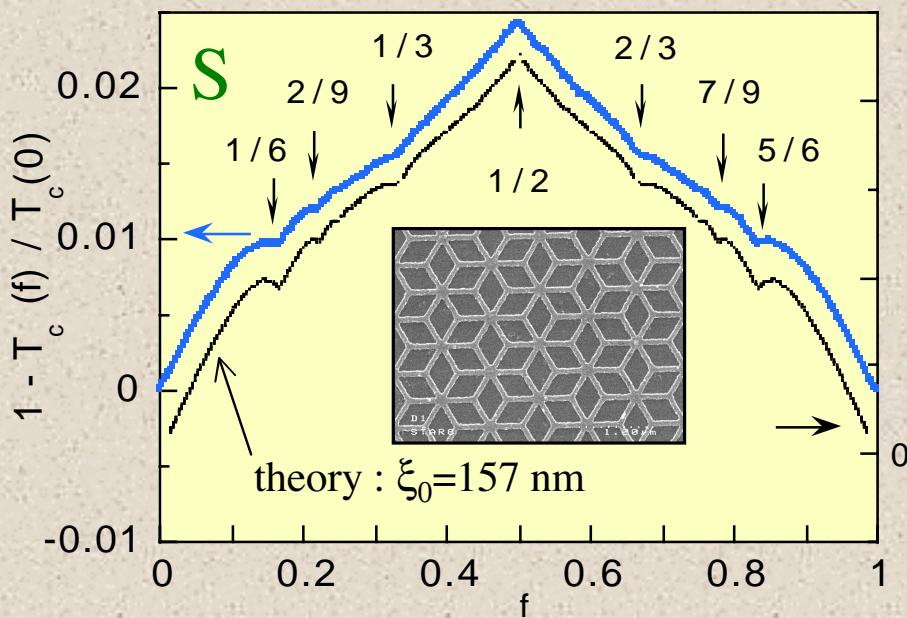


# *Landau levels of the ‘dice’ array*



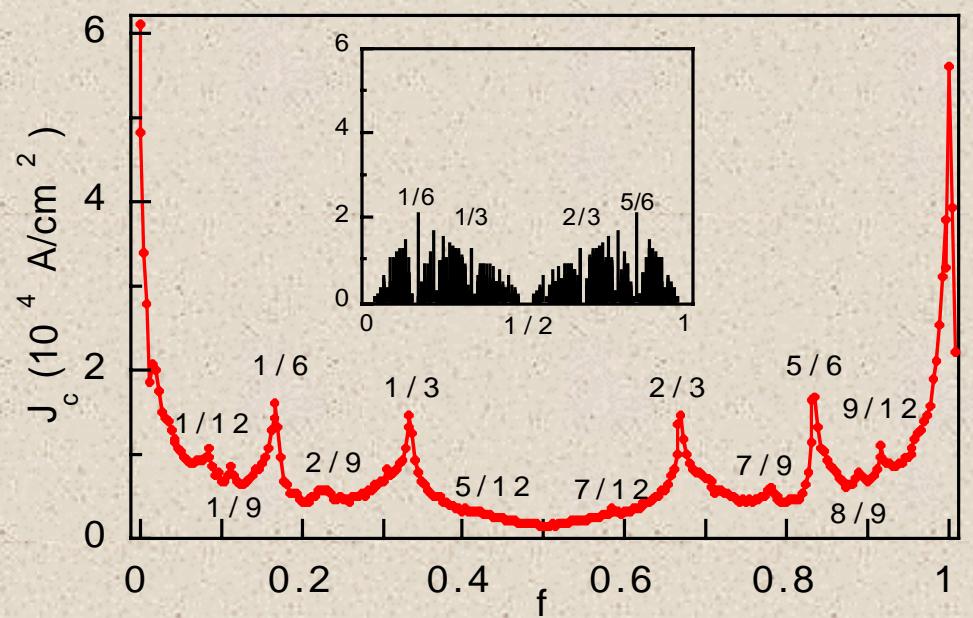
# Superconducting Al wire network

- Critical temperature



$$T_c(0) = 1.234 \text{ K}$$

- Critical Current

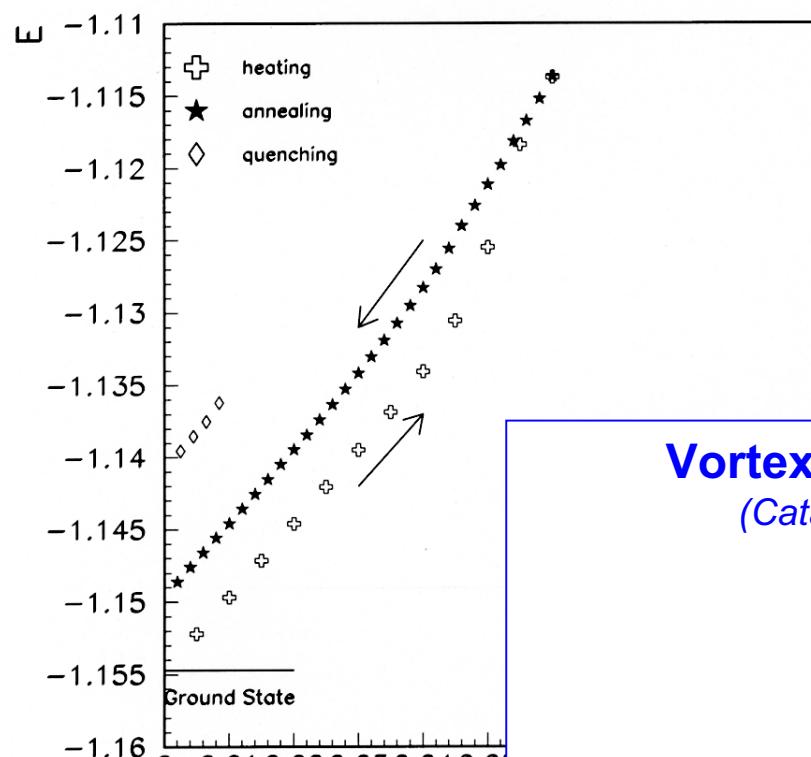


C. Abilio et al. PRL83, (1999) 5102

→ at  $f=1/2 \Rightarrow \text{« suppression » of superconducting order}$

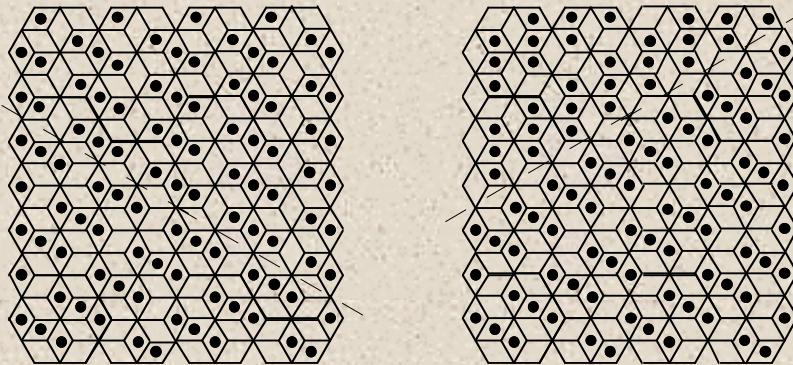
# The superconducting ground state at $f=1/2$

- Classical spins on Kagomé lattice  
disordered à  $T=0$   
(Huse PRB45, 1992 p7536)
- Josephson « dice » array :  
highly degenerate metastable states



## Theoretical Prediction:

S.Korshunov PRB, 63, 134503 (2001)



ground state  $\Rightarrow$  vortex triads

with zero energy domain walls

$\hookrightarrow S=(N+M)\ln 2$  : non-extensive entropy

## Vortex glass phase at $T < T_{KTB}$

(Cataudello and R. Fazio, 2002)

$T_{KTB} = 0.03 E_J$   
thermal hysteresis  
slow dynamics

## Magnetic imaging:

## Observed Configurations at $f=1/2$

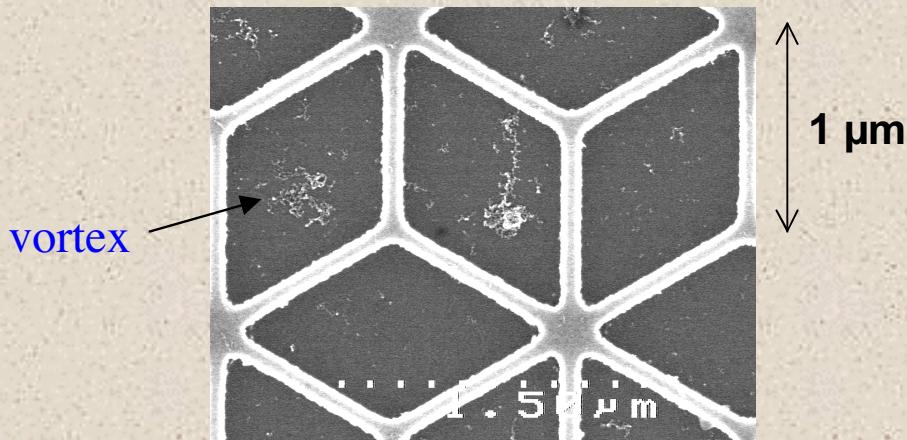
### Magnetic decoration of vortices

*Europhys. Lett.* 59, 225 (2002)

field cooled epitaxial Nb wire array

$T_c=9\text{K}$

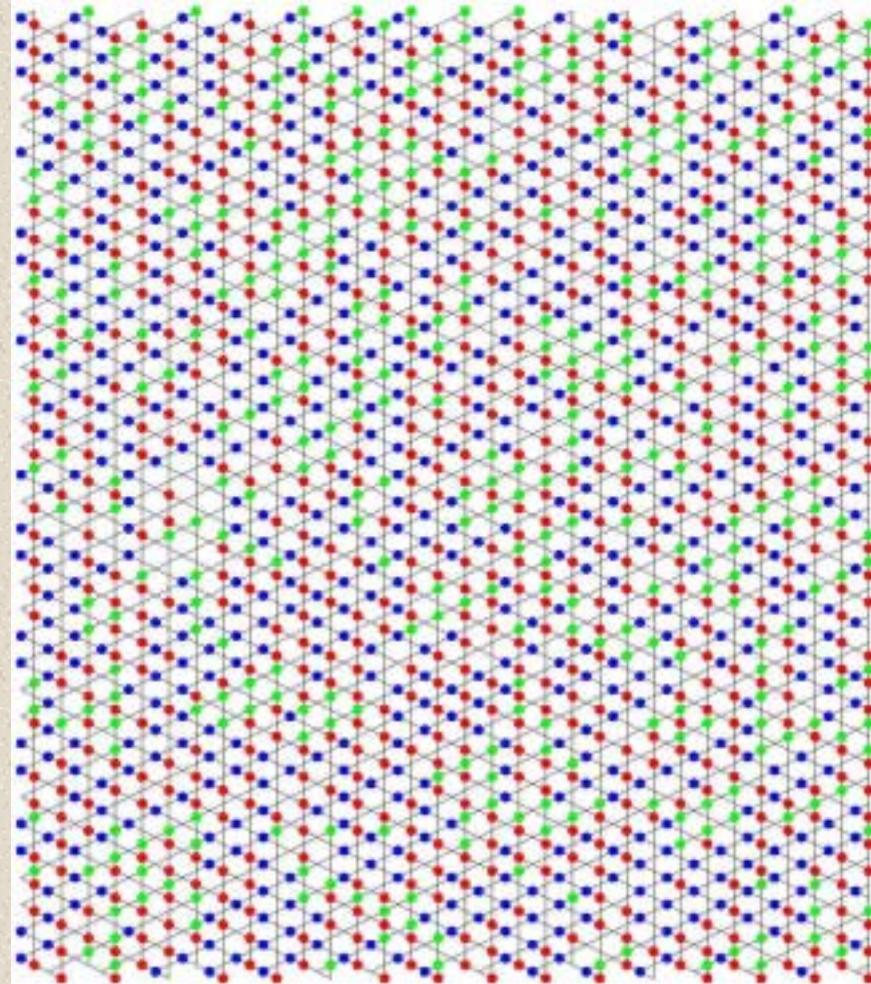
$B = 11.93 \text{ Gauss for } f=1/2$



♦ No commensurate phase  
⇒ disordered configuration?

### Observed configuration

(reconstructed on the Kagomé lattice)



3 456 sites containing 1 725 vortices

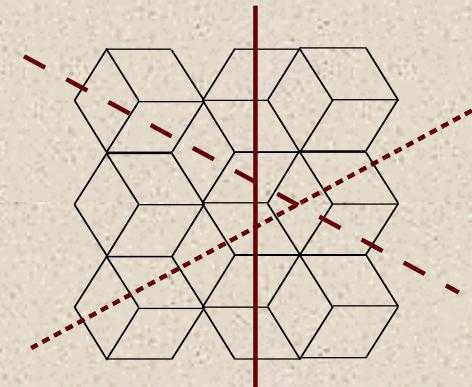
→  $f=1/2-0.001$

## Magnetic imaging:

## Correlation function calculation

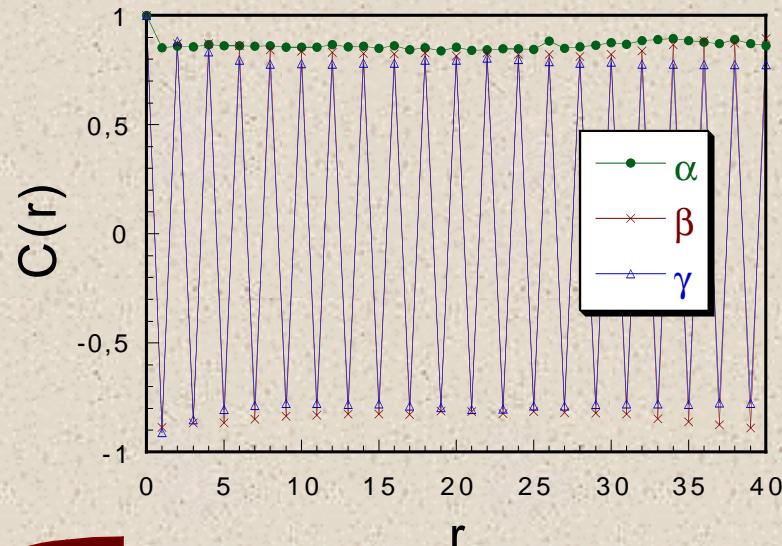
$$C_{\alpha, \beta, \gamma}(r) = \langle V_i \cdot V_{i+r} \rangle$$

$V_i$ : « vortex » variable  
 = 1 if a vortex is in the  $i$  cell  
 = -1 if not

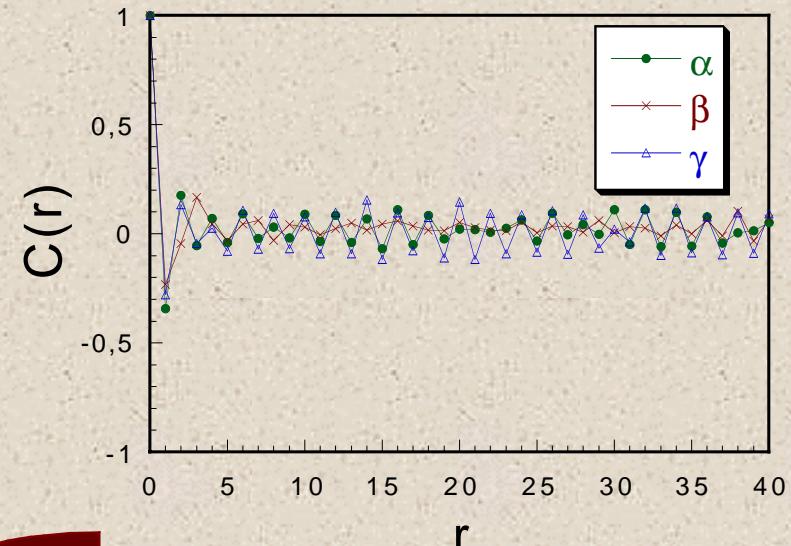


Collaboration. P. Butaud

♦  $f=1/3$



♦  $f=1/2$



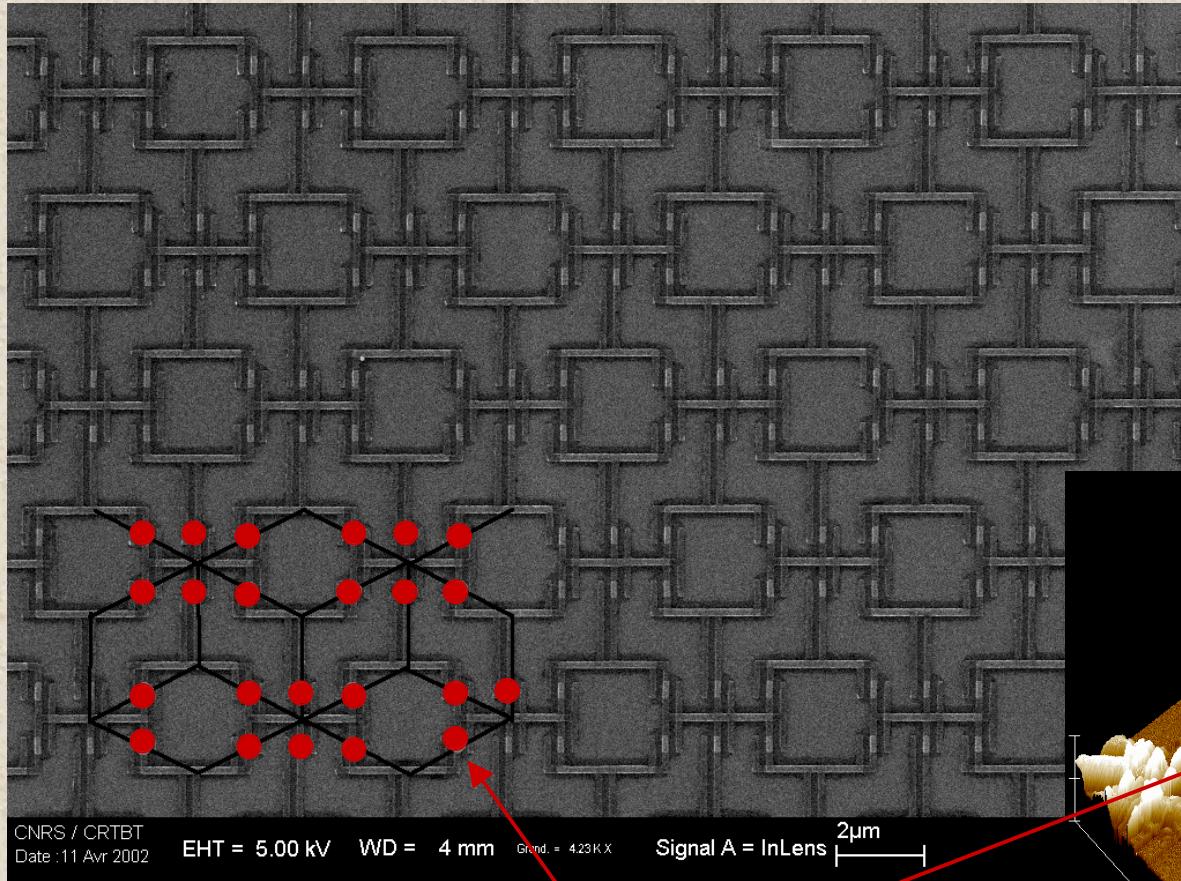
Long range order:  
 $C(r) > 0,8$  until  $r = 40$

No long range order:  
 $\xi \approx 1,5$

# Nanofabrication:

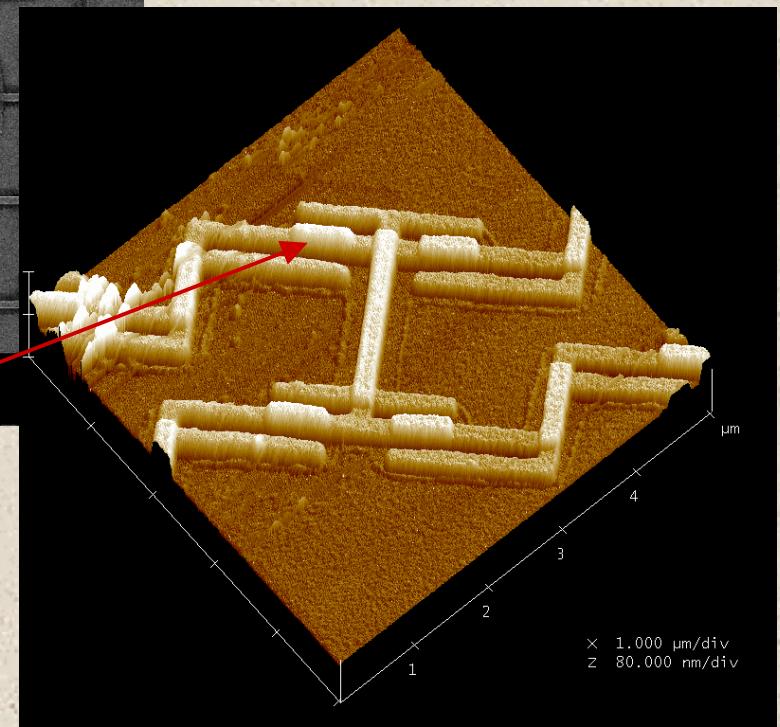
# Samples

(Array containing more than 127 000 junctions)



Josephson junctions

Chain of « cages »



## Transport:

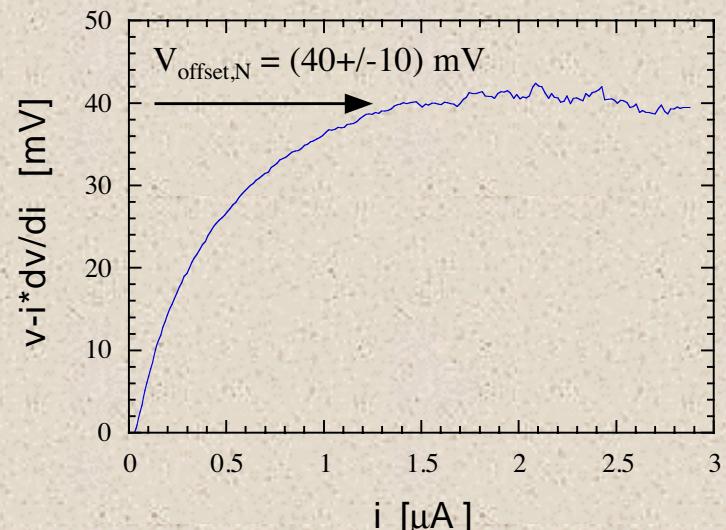
→ Estimation of  $E_J$  with  $R_n$  measured at 4K

→ Estimation of  $E_c$  with the offset voltage:

$$V_{offset} = N \frac{e}{2C}$$

i.e. about  $50\text{fF}/\mu\text{m}^2$

## Samples overview



Sample	$S$ [ $\mu\text{m}^2$ ]	$R_n$ [ $\text{k}\Omega$ ]	$E_J$ [ $\mu\text{eV}$ ]	$C$ [ $\text{fF}$ ]	$E_c$ [ $\mu\text{eV}$ ]	$E_J/E_c$	Regime
A	0,06	4,93	130	3	27	4,9	Classical
B	0,023	20,4	32	1,3	61	0,5	Quantum
C	0,015	53,3	12	0,5	160	0,05	Charge

Cell area =  $5,57 \mu\text{m}^2 \Rightarrow f=1 \equiv B=0,3716 \text{ mT}$

## Transport:

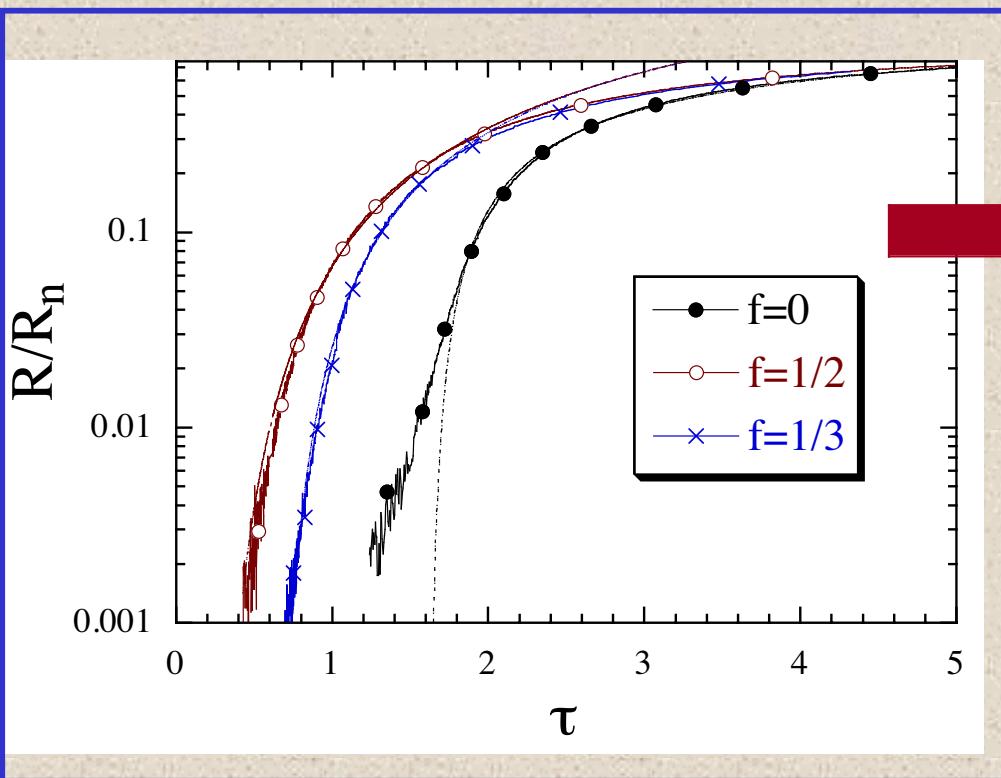
Classical array with  $E_J/E_C=4,9$

### Study of the KTB transition:

if  $T > T_{KTB}$ ,

$$\frac{R(T)}{R_n} = b_1 \exp \left[ \frac{-b_2}{\sqrt{\tau - \tau_{KTB}}} \right]$$

with  $\tau = k_B T / E_J(T)$ , and  $b_1, b_2 \approx 1$



	$\tau_{KTB}$ theoretical	$\tau_{KTB}$ measured
$f=0$	1,8	1,68
$f=1/3$	0,36	0,57
$f=1/2$	0,108	0,155

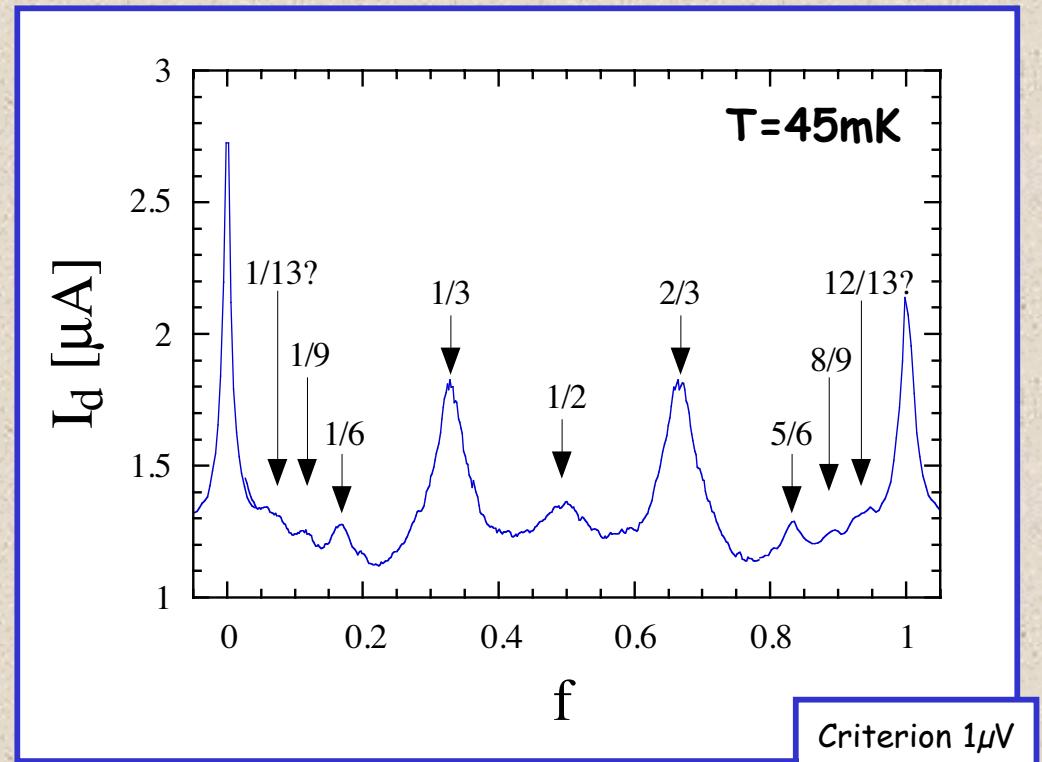
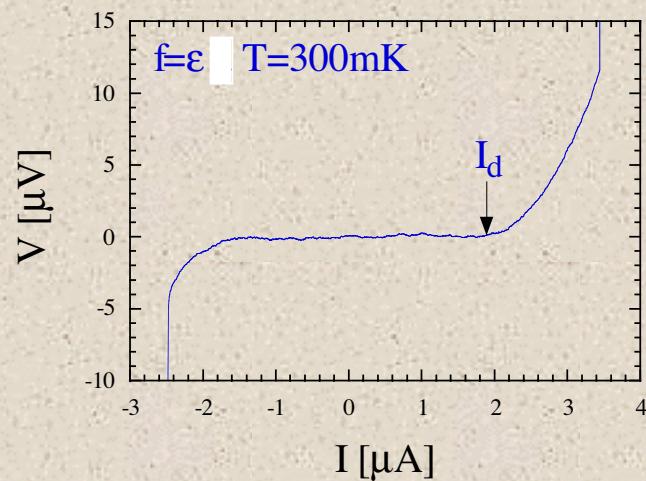
- ◆ At  $f=1/2$ , the array undergoes a KTB transition
- ◆ Relatively good agreement with theory (uncertainty on  $\Delta(T)$ )

Is the  $f=1/2$  state a vortex glass?

## Transport:

Classical array with  $E_J/E_C=4,9$

Study of the superconducting phase at  $f=1/2$   
⇒ vortex configuration pinning force measurement



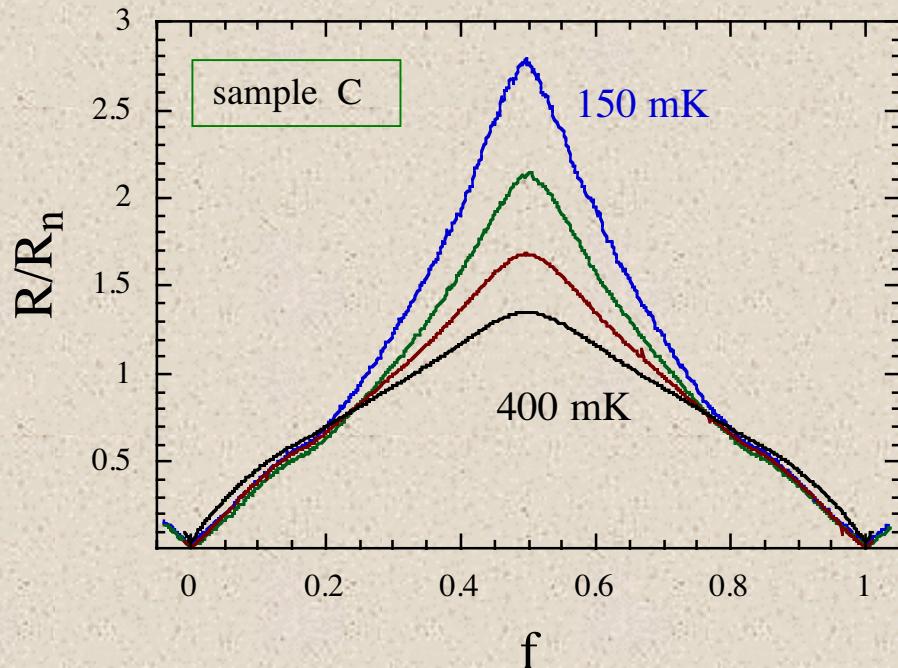
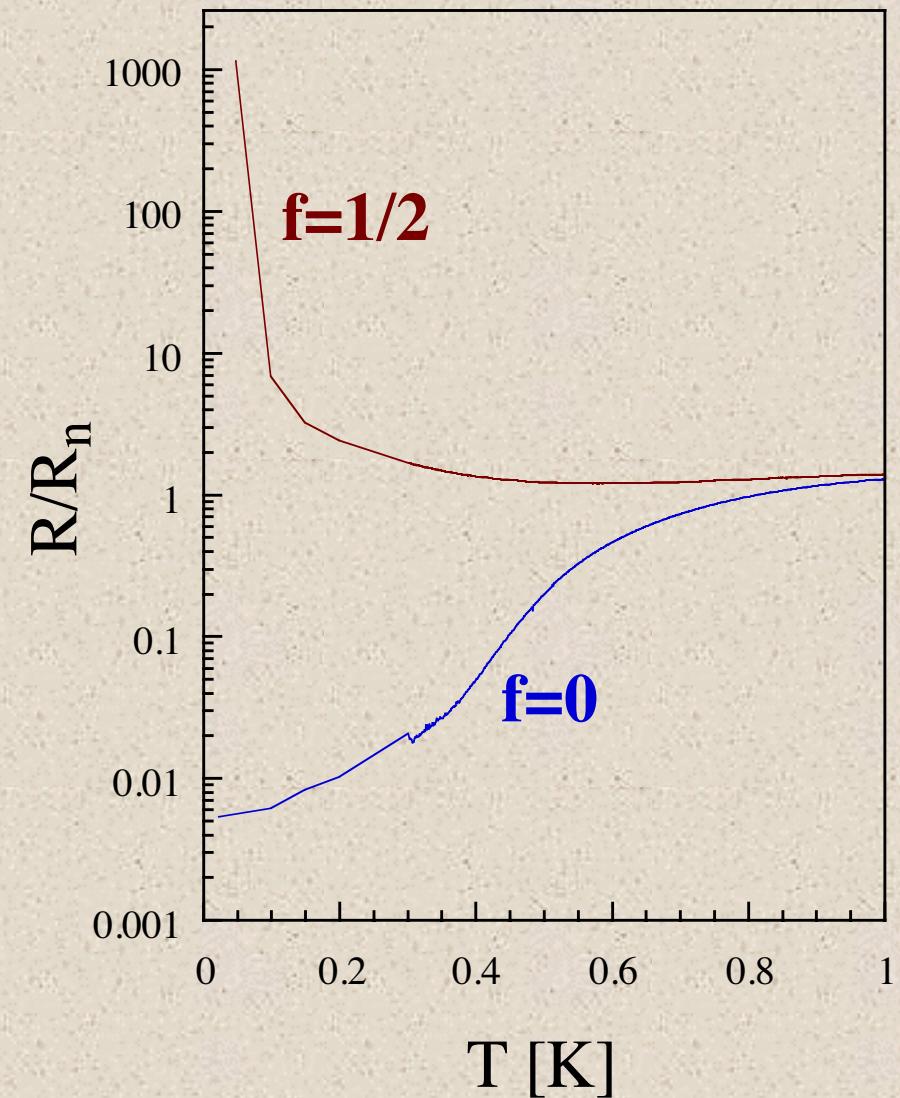
1. Max of  $I_d$  at  $f = 0, 1/13, 1/9, 1/6, 1/3, 2/3, 5/6, \dots$
2. Max of  $I_d$  at  $f=1/2 \neq$  wire arrays



Commensurate state at  $f=1/2$

## Transport:

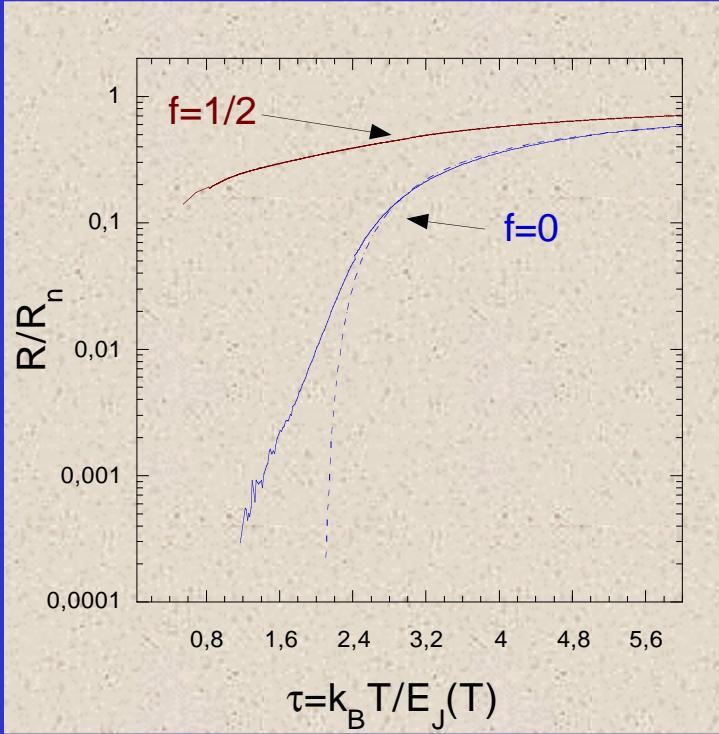
Charge array with  $E_J/E_C=0,05$



Metal-Insulator transition induced by  $B$   
at  $f_{c,1} = 0,23$  and  $f_{c,2} = 0,76$

At  $f=1/2$ : insulator ( $R > 50M\Omega$ )  
At  $f=0$ : low resistance state

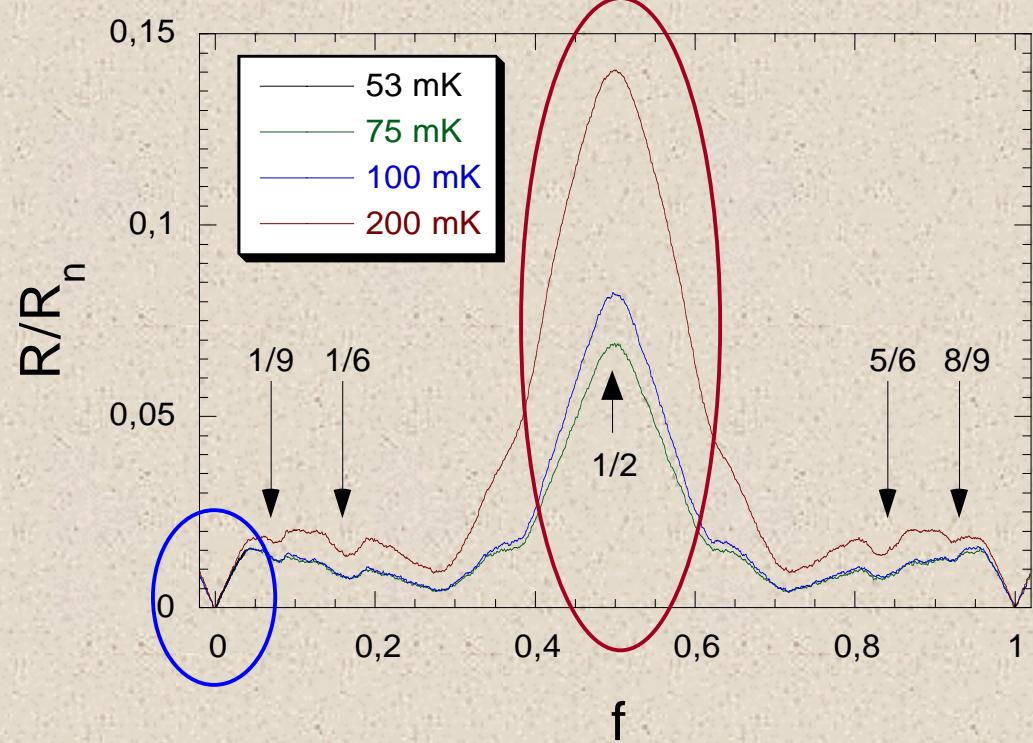
## Transport:



- ◆ both near  $f=0$  and  $1/2$  :
- differential Resistance is :
  - T- independent
  - proportionnal to  $f$

## Quantum array with $E_J/E_C=0,5$

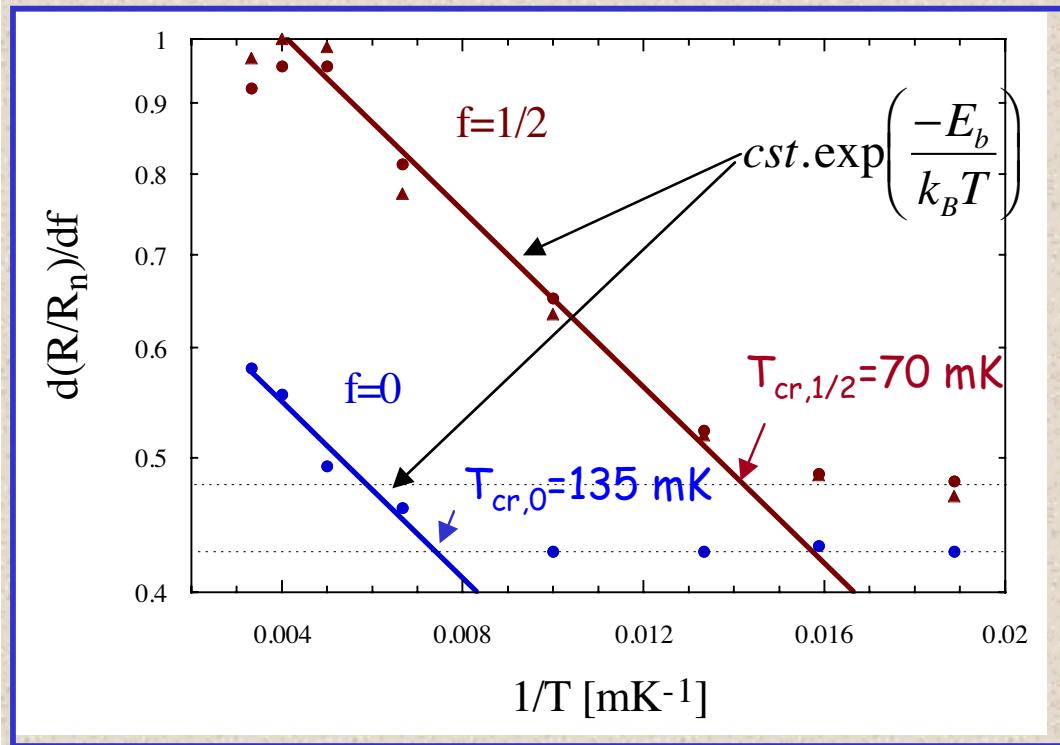
- ◆ At  $f=0$ : KTB transition  
fit  $R_0(T)$  =>  $\tau_{\text{KTB,mes}}=1,58$   
theory with quantum fluctuations =>  $\tau_{\text{KTB,th}}=1,47$
- ◆ At  $f=1/2$  : saturation of  $R_0(T)$



## Transport:

## Quantum array with $E_J/E_C=0,5$

Study of the resistive phase at  $f=1/2$   
 ⇒ Behavior comparison between  $f=0$  and  $f=1/2$



♦ If  $T > T_{cr}$ , thermal activation behavior:

Same energy barrier at  $f=0$  and  $1/2$ :

$$E_b = 73 \text{ mK} = 0,2 E_J \\ (\text{theory : } 0.19 E_J)$$

♦ Theoretical prediction for  $T_{cr}$ :

$$T_{cr,th} \approx \sqrt{E_b E_c} = 230 \text{ mK}$$

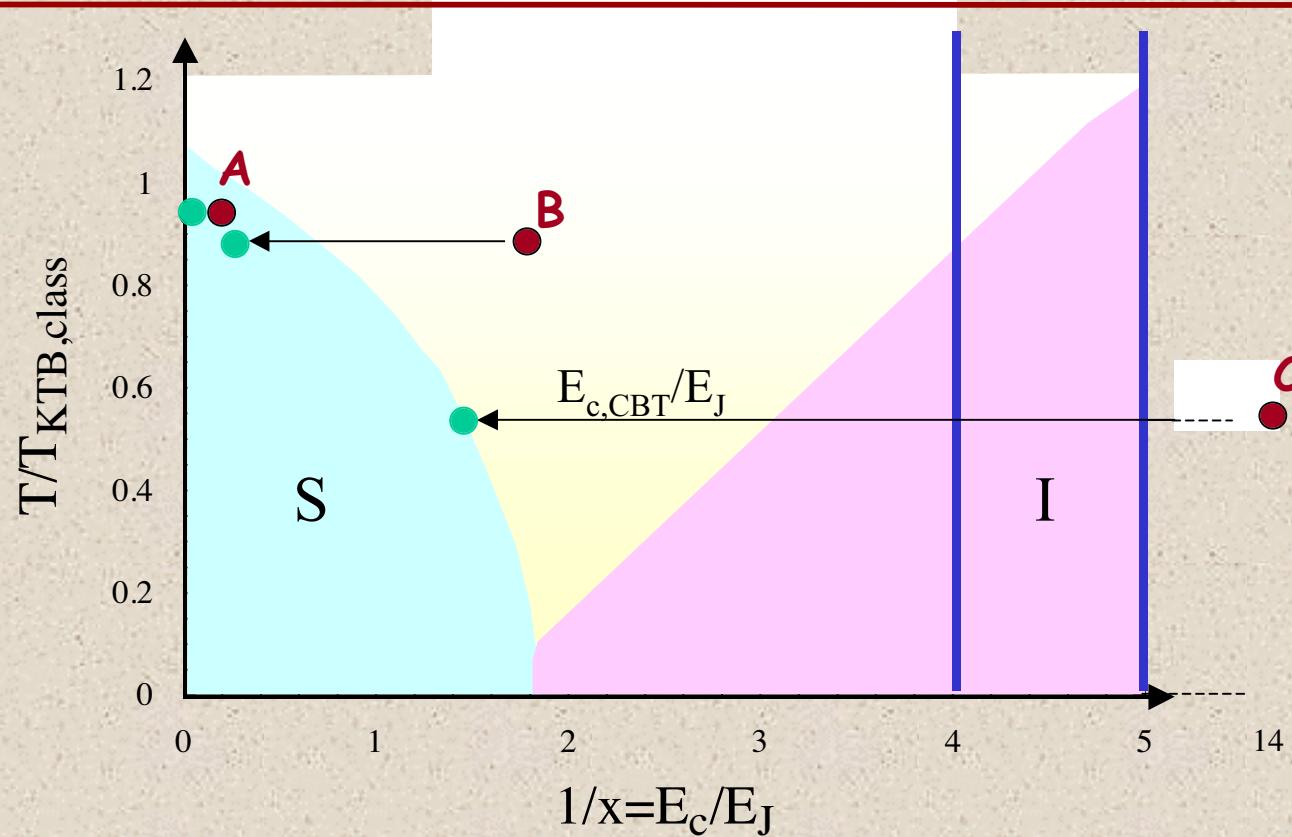
Observed  $T_{cr}$  is smaller  
 (dissipation effect)

At  $f=1/2$ , resistive phase at  $T \rightarrow 0$  :

evidence of a vortex liquid induced by the quantum fluctuations

## Phase diagram:

At  $f=0$



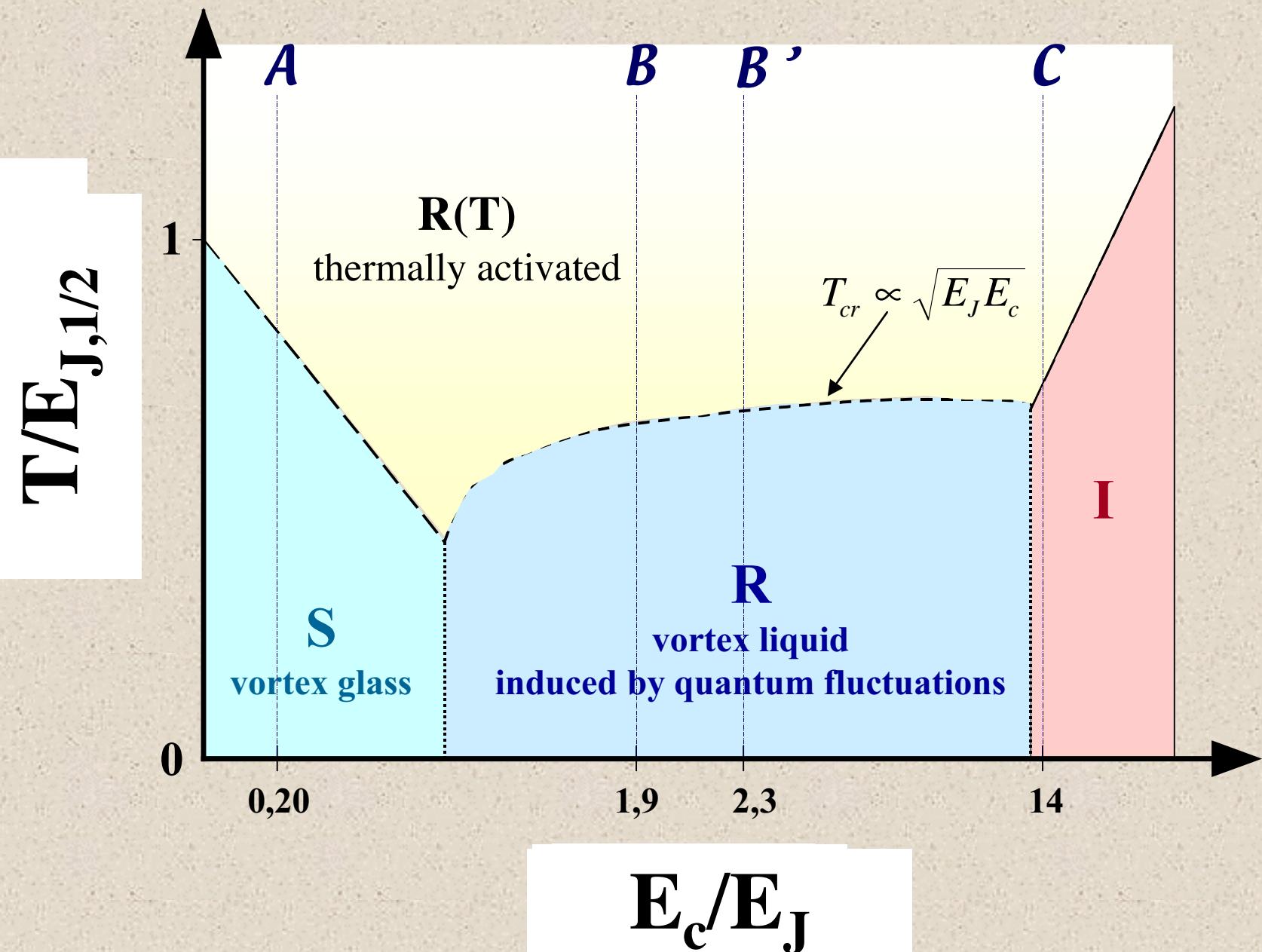
disagreement between dice and square phase diagrams

- ◆  $E_{c,\text{eff}}$  measure with CBT for sample C  $\Rightarrow E_{c,CBT} = E_c / 10$  ➡ what is  $E_{c,\text{eff}}$  ?
- ◆ fabrication and measurements of other samples:  $E_{c,CBT} \approx E_c$   
and for  $1/x=4$  et  $5 \Rightarrow$  superconducting at  $f=0$

➡ Suppression of the quantum fluctuations in the dice array at  $f=0$  !

## Phase diagram:

At  $f=1/2$



# Conclusions

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**Imaging:** at  $f=1/3$  observation of a commensurable state  
at  $f=1/2$  very short range order (triades)

**Transport at  $f=0$ :** suppression of the quantum fluctuations in the dice array  
compared to the square array

**Transport at  $f=1/2$ :**

- charge array:
  - observation of an insulating phase
- classical array:
  - evidence of a commensurate phase at  $f=1/2$
  - no thermal hysteresis
  - no ordering induced by electrical excitation ( $\neq f=1$ )
- quantum array:
  - evidence of a vortex liquid induced by quantum fluctuations