Dynamic localization in quantum dots: analytical theory

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What everybody knows…

- $\hat{H} = \hat{H}_0 + \hat{V} \cos \omega t$
- (Quasi)continuous spectrum
- Absorption and emission of quanta $\hbar^*$
  random walk up and down
- Diffusive evolution of the electron distribution function
What some people know…

Kicked rotor:

\[ \hat{H}(t) = -\frac{\partial^2}{\partial \theta^2} + V(\theta) \sum_{n=-\infty}^{\infty} \delta(t - nT) \]

\[ \psi_m^{(0)}(\theta) = e^{im\theta}, \quad E_m^{(0)} = m^2 \]

Dynamic localization in the energy space:
after some time the rotor stops absorbing

(G. Casati, B. V. Chirikov, J. Ford, and F. M. Izrailev, 1979)
Historical developments

1. Quantum interference – analogous to the Anderson localization (Fishman, Grempel, and Prange, 1982)

2. Incommensurate periods $T_1$, $T_2$, $T_3$ – 3D localization (Casati, Guarneri, Shepelyansky, 1989)

3. Particle in a box: just $\psi(0) = \psi(2\pi) = 0$ instead of the periodic $\psi(0) = \psi(2\pi)$ – no localization (Hu, Li, Liu, Gu, 1999)

4. Mapping to a quasi-$1d$ $\sigma$-model (Altland, Zirnbauer, 1996)

What do these observations mean and how general are they?
Spatial localization

Quantum correction to the diffusion coefficient of electrons in disorder:

\[ D - D_0 \sim - \frac{D_0}{\nu} \int_0^{1/l} \frac{d^d \vec{k}}{D_0 k^2 + 1/t_\phi} \]

Density of states

Mean free path

Dephasing time

Change variables \( D_0 k^2 = 1/t \):

\[ D - D_0 \sim -\frac{1}{\nu} \int_\tau^{t_\phi} \frac{D_0 dt}{(D_0 t)^{d/2}} \]

Localization:

\[ d = 1 : \quad L_{loc} \sim \nu D_0 \sim l \]

\[ d = 2 : \quad L_{loc} \sim l \exp(\nu D_0) \quad (?) \]

\[ d \geq 3 : \quad \text{no localization in weak disorder} \]
Random matrix theory

\[ \hat{H}(t) = \hat{H}_0 + \hat{V}\phi(t) \]

real symmetric \( N \times N \) Gaussian random matrices with statistically independent elements

Mean level spacing at the center

Density of states of \( \hat{H}_0 \)

In the end let \( N \to \infty \)
Chaotic systems

Ballistic systems:

\[ \tau_{\text{erg}} = \frac{L}{v_F} \quad \text{ergodic time} \]

RMT is valid at low energies:

\[ E \ll E_{\text{Th}} = \frac{\hbar}{\tau_{\text{erg}}} \quad \text{(Thouless energy)} \]

Diffusive systems:

\[ \tau_{\text{erg}} = \frac{L^2}{D} \]
Technicalities

- Time-dependent RMT
- Keldysh non-equilibrium formalism
- Diagrammatic technique
- Nonlinear $\sigma$-model
- Perturbative (loop) expansion
Zero order (diffusion)

\[ \Gamma \equiv \left\langle V_{\text{ll}}^2 \right\rangle / \delta \]  – one photon absorption rate
(measure of perturbation strength)

Long-time, period-averaged dynamics:

\[
\left[ \frac{\partial}{\partial t} - D \frac{\partial^2}{\partial E^2} \right] f(E, t) = 0
\]

\[ D = \Gamma \left( \frac{d\phi}{dt} \right)^2 \]  – energy diffusion coefficient

\[ W_0 \equiv \frac{\partial}{\partial t} \int E f(E, t) \, dE = \frac{D}{\delta} \]  – energy absorption rate
One-loop correction

\[ W(t) = \frac{D}{\delta} + \frac{\Gamma}{\pi} \int_0^t \phi(t) \phi(t - \tau) C_{t-\tau/2}(\tau,-\tau) \, d\tau \]

Cooperon keeps track of the quantum interference:

\[ C_t(\tau_1, \tau_2) \equiv \theta(\tau_1 - \tau_2) \exp \left[ - \int_{\tau_2}^{\tau_1} \frac{\Gamma}{2} \left[ \phi(t + \tau/2) - \phi(t - \tau/2) \right]^2 \, d\tau \right] \]

deephasing rate
Periodic perturbation

\[ \phi(t) = \sum_{n=1}^{\infty} A_n \cos(n \omega t - \varphi_n) \]

\[ W_0 = \frac{\Gamma \omega^2}{2 \delta} \sum_n n^2 A_n^2 \]

\[ C_t(\tau_1, \tau_2) \approx \exp \left[ -\Gamma (\tau_1 - \tau_2) \sum_{n=1}^{\infty} A_n^2 \sin^2(n \omega t - \varphi_n) \right] \]

If \( \varphi_n = n \varphi \) the exponent can vanish at \( t_k = \frac{\varphi + k \pi}{\omega} \)

No-dephasing points give a large negative contribution to the integral:

\[ W(t) - W_0 \sim -\omega^2 \sqrt{\Gamma t} \]
Time-reversal symmetry

\[ \varphi_n = n \varphi \iff \phi(t - t_0) = \phi(-t - t_0) \]

Average dephasing rate versus time:

- **T-symmetry: yes**
- **T-symmetry: no**

Monochromatic perturbation: **T-symmetry always** – a very special case
Two loops

There is a contribution from diffusons:

\[ D_\tau(t_1, t_2) \equiv \theta(t_1 - t_2) \exp \left[ -\int_{t_2}^{t_1} \Gamma [\phi(t + \tau / 2) - \phi(t - \tau / 2)]^2 \, dt \right] \]

For a periodic perturbation:

\[ D_\tau(t_1, t_2) \approx \exp \left[ -2\Gamma (t_1 - t_2) \sum_{n=1}^{\infty} A_n^2 \sin^2 n\omega\tau \right] \]

No-dephasing points are always present, regardless of the time-reversal symmetry…
Incommensurate periods

\[ \phi(t) = \sum_{n=1}^{d} A_n \cos(\omega_n t - \varphi_n) \]

\[ W_0 = \frac{\Gamma}{2\delta} \sum_n \omega_n^2 A_n^2 \]

dephasing rate:

Phase relationships do not matter that much

Almost-no-dephasing points contribute:

\[ W(t) - W_0 \sim -\omega^2 \int_{1/\Gamma}^{t} \frac{\Gamma dt_1}{\sqrt{(\Gamma t_1)^d}} \]

– large for \( d<3 \)
A glance at the reality

GaAs dot:
- size $L \sim 1 \mu m$
- mean level spacing $\delta \sim 1 \mu eV$
- Thouless energy $E_{Th} \sim 100 - 1000 \mu eV$
- dephasing time $t_{\phi} \sim 1 \text{ ns}$

Microwave field:
- $V \sim$ several $\mu eV$ (field $\sim$ several $V/m$)
- $\hbar \omega \sim 10 - 100 \mu eV$ ($\sim 10^{10}$ Hz)

Dynamic localization:
- $t_{loc} \sim 10 \text{ ns}$, $E_{loc} \sim \sqrt{Dt_{loc}} \sim 100 - 1000 \mu eV \sim 1 - 10 \text{ K}$
Conclusions…

1. A quantum-mechanical system under a time-dependent perturbation may be subject to *dynamic localization* in energy space.

2. It *depends* both on the model for the unperturbed system and the perturbation.

3. We have studied *one-loop correction* to the usual Fermi-Golden-Rule dissipation rate for a chaotic system described by RMT.
...conclusions

4. For a perturbation with $d$ incommensurate frequencies the correction can grow arbitrarily with time if $d=1,2$ (analogously to spatial localization in $d$-dimensional disorder)

5. For commensurate frequencies phase relationships matter:

6. Time-reversal symmetry: the “dimensionality” is effectively lowered

7. No time-reversal: the correction is small