Cooper Pair Tunneling and Coulomb Blockade

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Outline

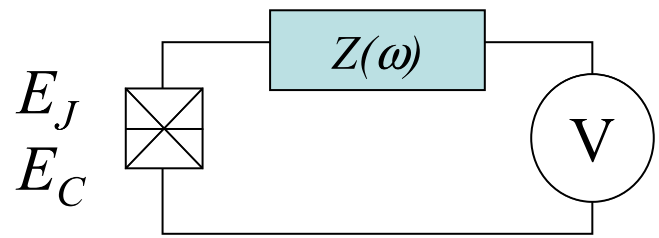
1. Cooper Pair Tunneling, Impedance of Environment

2. Josephson Junction Transmission Line (SQUID array)

3. Experiment 1: Single JJ biased by JJ Array – High Impedance Environment

4. Experiment 2: JJ Array in Low Impedance Environment.

Quantum Fluctuations



"P(E) Theory" (Devoret et al. 1990, Girvin et al. 1990)

$Z << R_Q = h/4e^2 = 6.45\ \mathrm{k}\Omega$

Josephson effect + quantum fluctuations of the phase

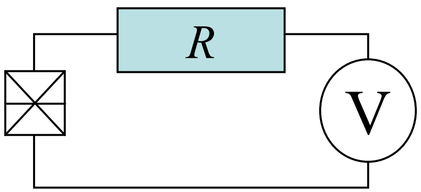
Perturbation theory, $E_J/E_C << 1$

$Z >> R_Q = h/4e^2 = 6.45\ \mathrm{k}\Omega$

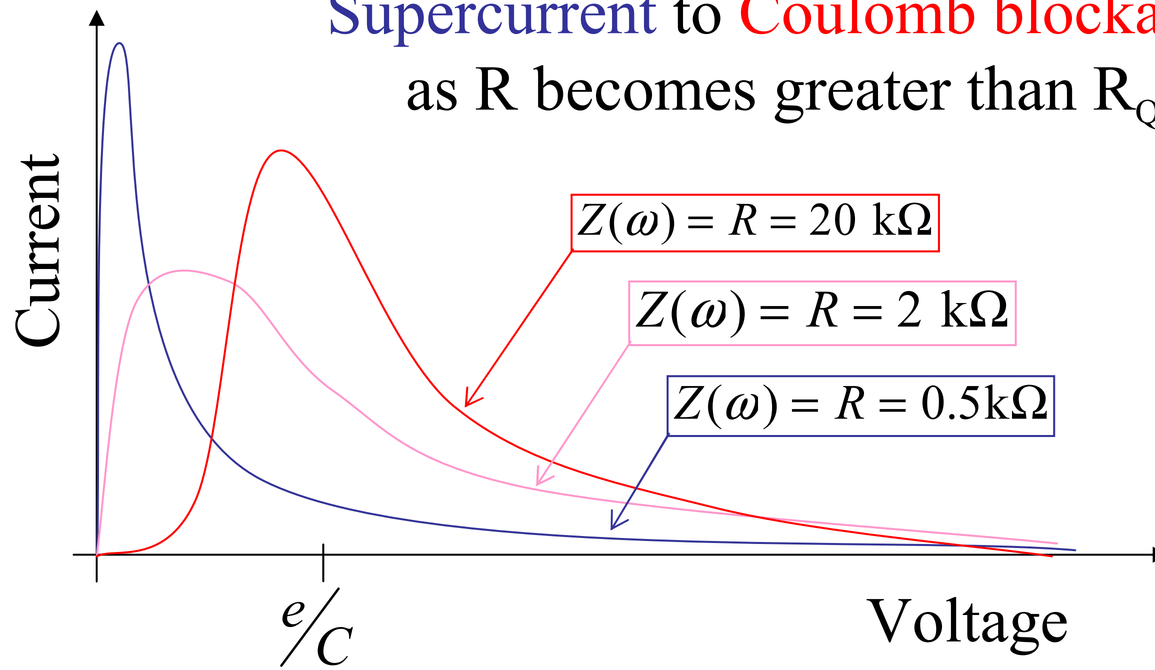
Coulomb blockade + charge fluctuations (uncorrelated single C.P.tunneling events)

Perturbation theory $E_J/E_C << (R_Q/Z)^{1/2}$

Results of Perturbation Theory



Supercurrent to Coulomb blockade as R becomes greater than R_Q



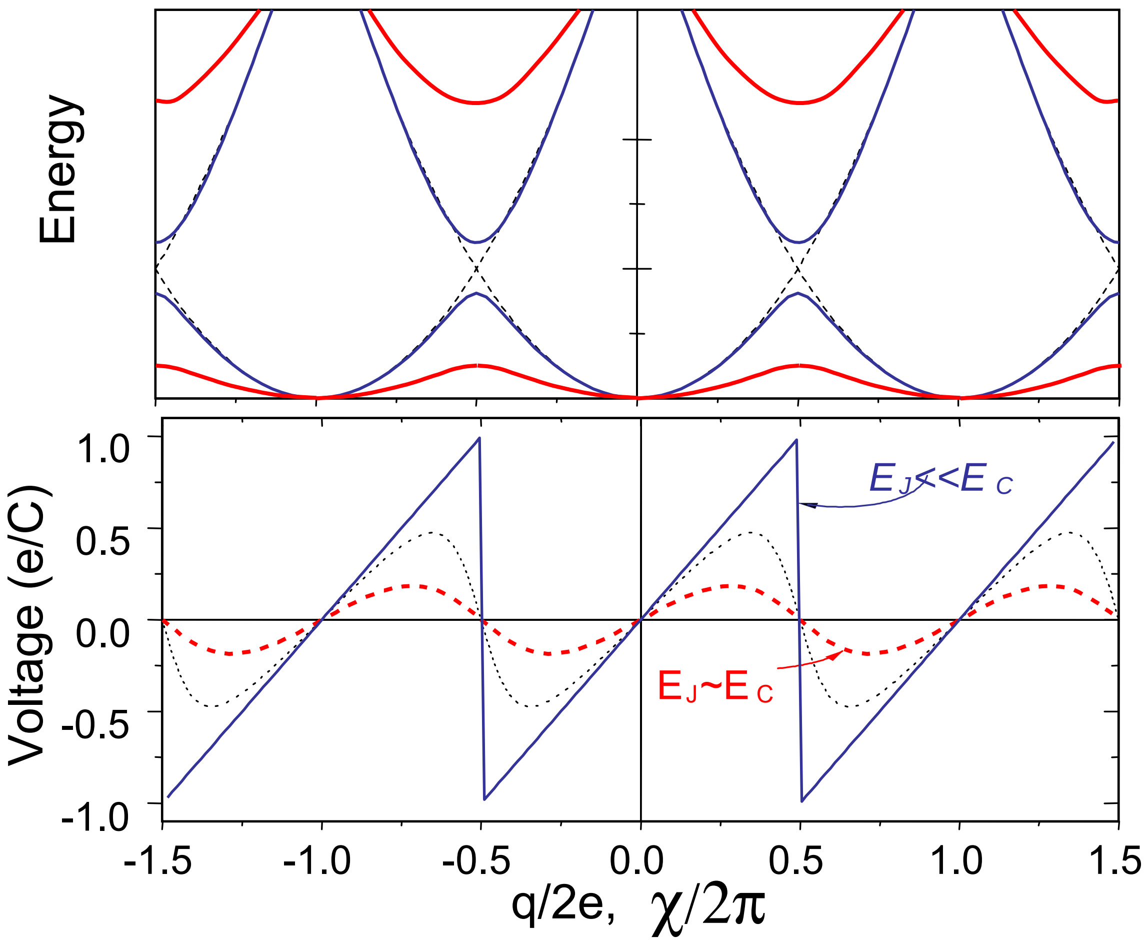
Interesting Case outside P(E) Theory:

- $E_J \sim E_C$ and $Z >> R_Q$
- Coulomb Blockade + Current Bias

→ **Correlated Cooper Pair Tunneling**

Quasi Charge description of Josephson Junction

Averin, Likharev and Zorin 1985



$$H = \frac{Q^2}{2C} - E_J \cos\left(2\pi\frac{\Phi}{\Phi_0}\right)$$

$$V = \frac{dE_0}{dq} = V_C \operatorname{saw}\chi$$

Critical Voltage: $V_C(E_J / E_C)$

dimensionless quasi-charge

$$\chi = \frac{2e}{2\pi} q$$

$$I_{\text{ext}} = \frac{2e}{2\pi}\dot{\chi}$$

External source provides current bias

How to make $Z \gg R_Q$

- Large Resistors (physically small): heating and noise.
- Small Capacitor in series with single junction → Cooper pair box.

✓Large Inductance in series:

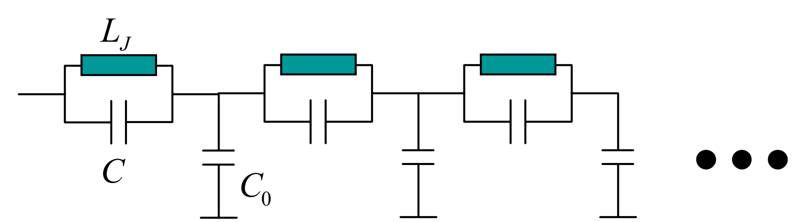
- Advantage: non disipative
- Electromagnetic inductance too small ($Z_0/R_Q=8\alpha<<1$)

Use Josephson Inductance: $$L_J = \frac{\Phi_0}{2\pi I_C}$$

$$\frac{\Phi_0}{2\pi} = \frac{\hbar}{2e} = 0.33 \ \mu\text{H - nA}$$

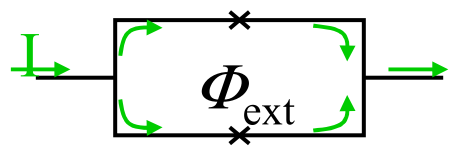
Infinite 1D, JJ array

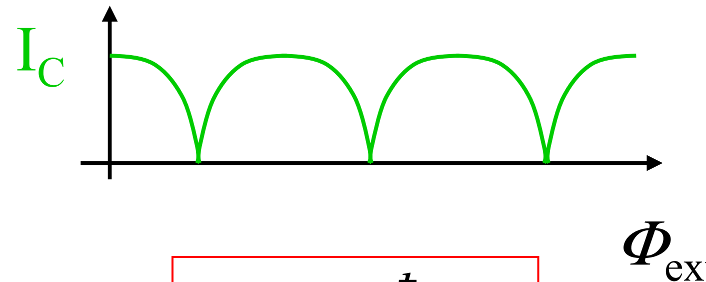
Linear model (small phase, $I<<I_C$)



$$\text{for} \quad \omega < \omega_p = \frac{\sqrt{4E_C E_J}}{\hbar}, \qquad Z_A = \sqrt{\frac{L_J}{C_0}} = R_Q\sqrt{\frac{4E_C}{E_J}}\sqrt{\frac{C}{C_0}}$$

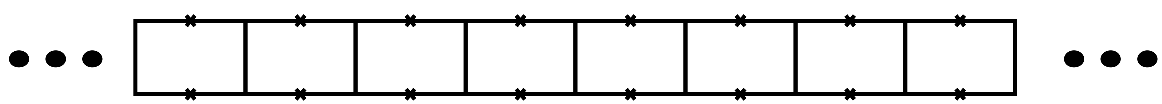
1D SQUID array as tunable Josephson transmission line

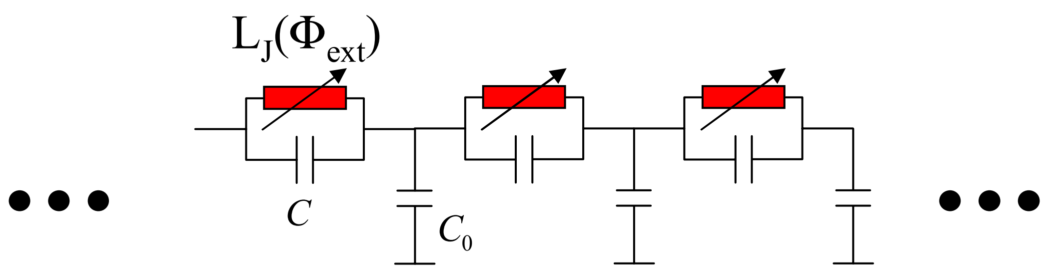




Tunable Josephson Inductance

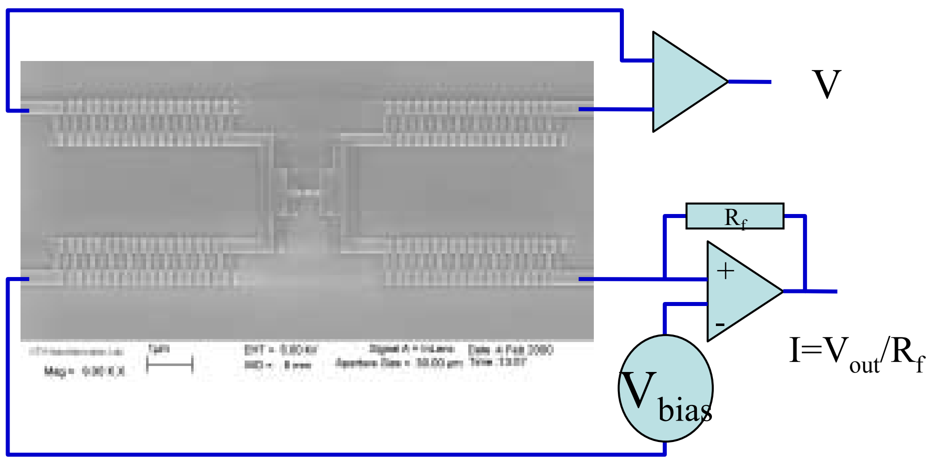
$$L_J = \frac{\hbar}{2eI_C(\Phi_{ext})}$$

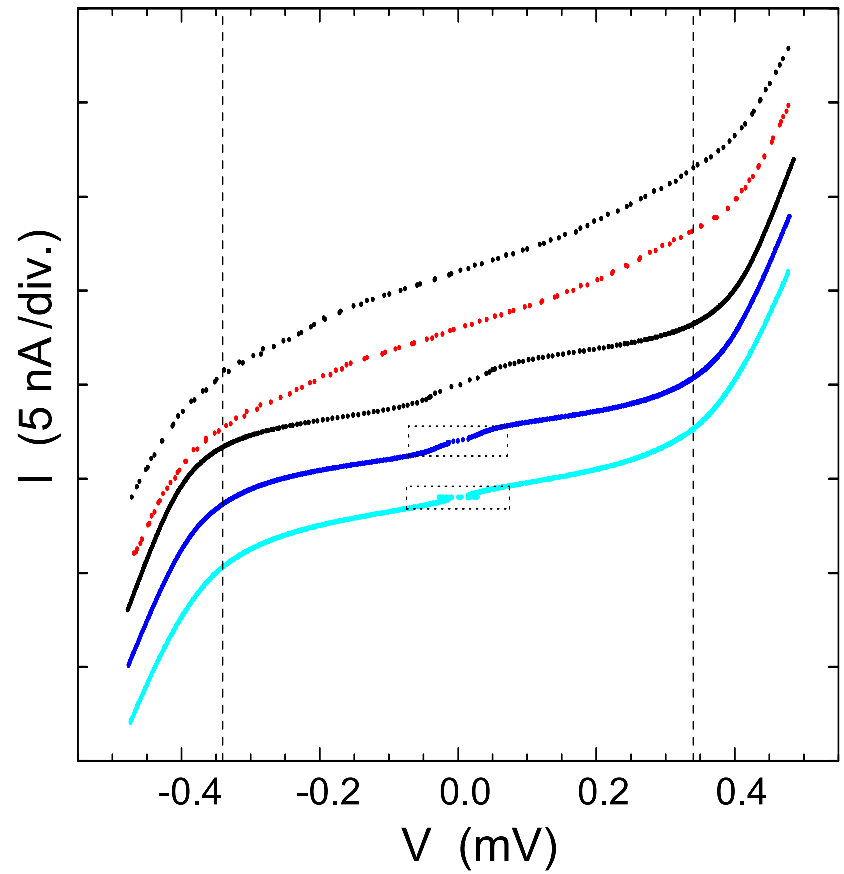




Experiment: Array Bias of Single Junction

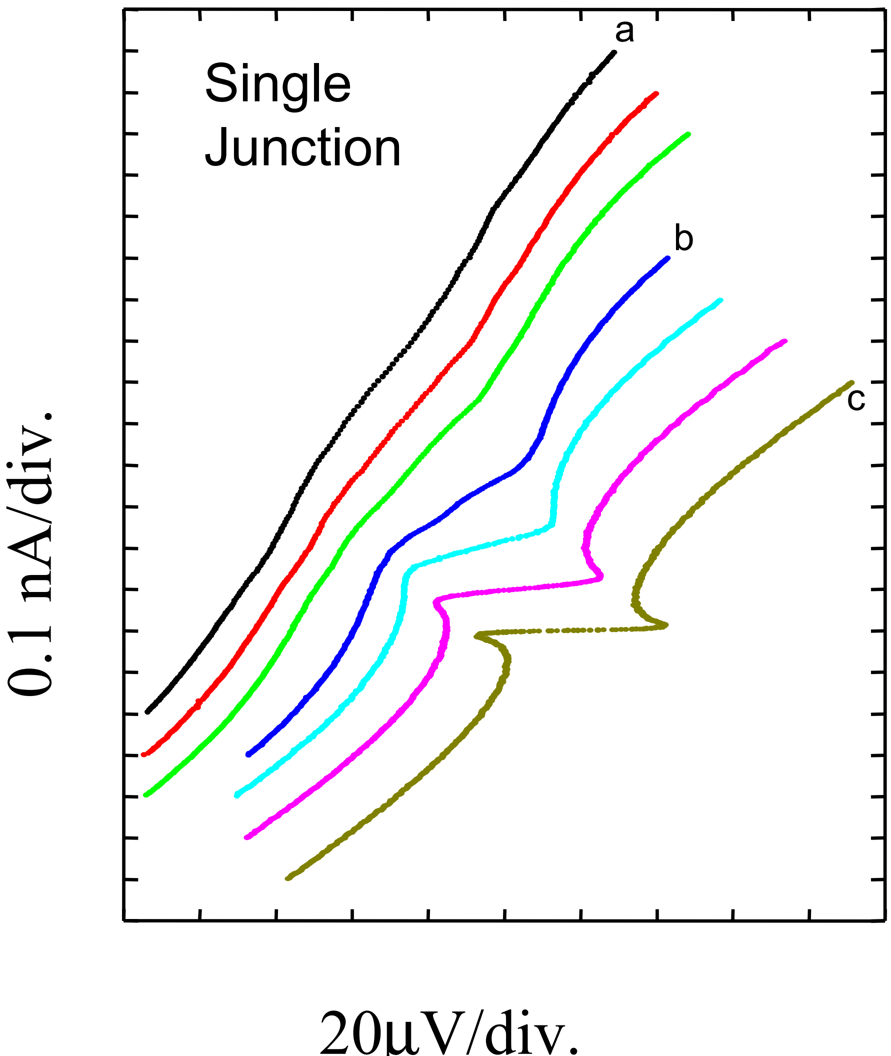
Measure IVC of single junction
While tuning impedence of
SQUID array leads

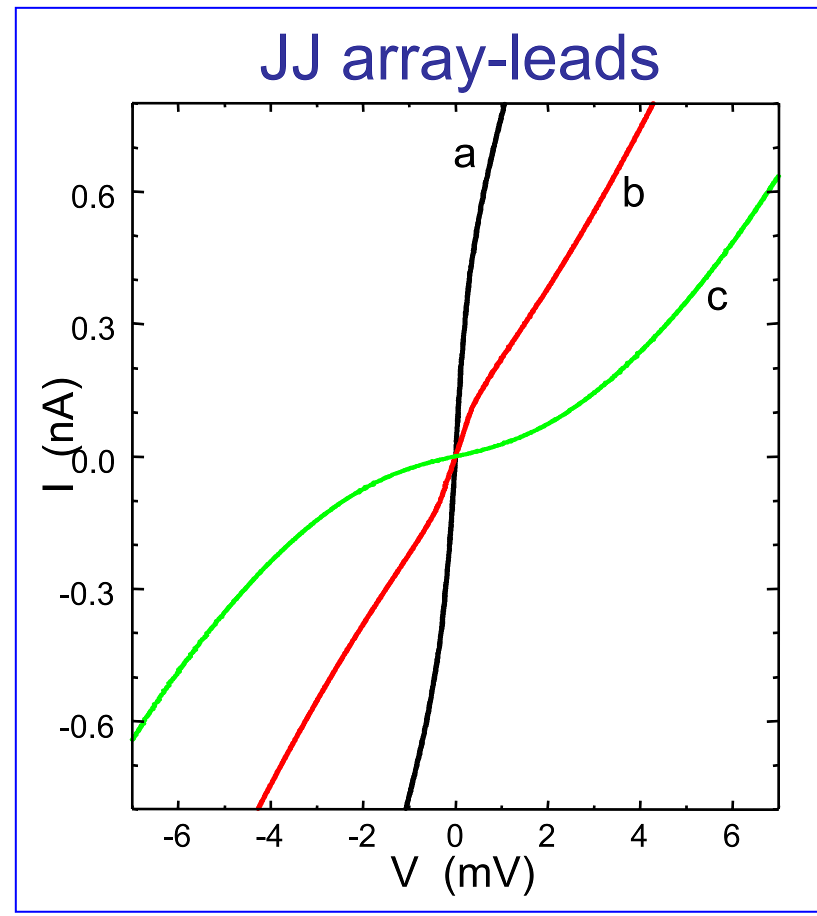




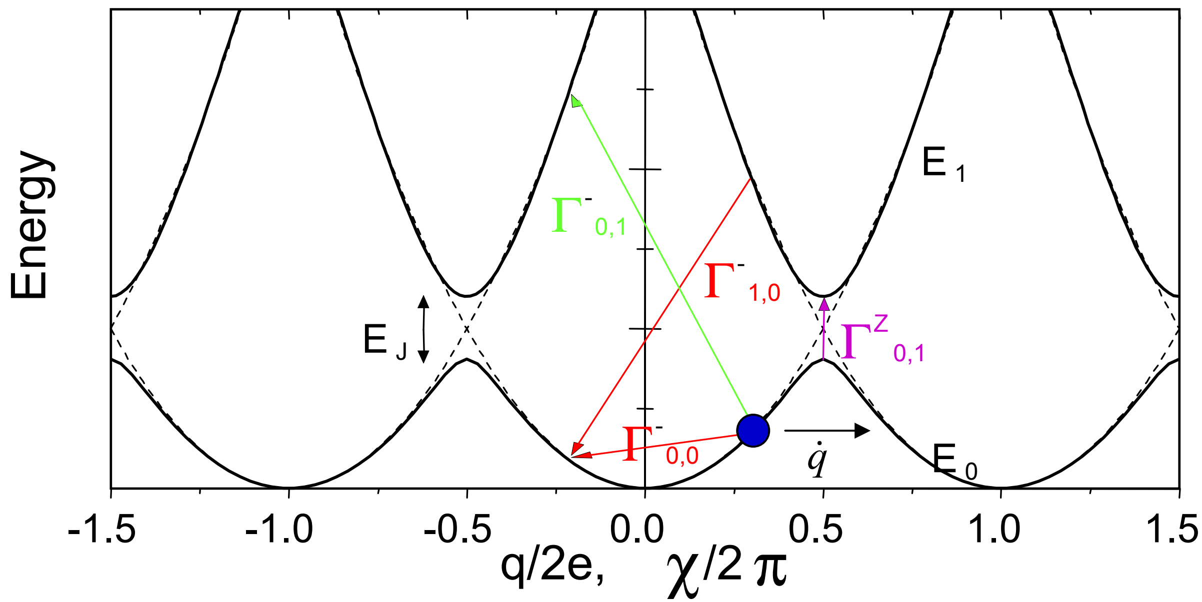
Coulomb blockade appears in single junction as *environment* is tuned

Watanabe and Haviland, PRL 2000





Theory: Quasi-particle (single electron) relaxation



$$I_{\text{ext}} = \frac{dq}{dt} = 2e\frac{d\chi}{dt}$$

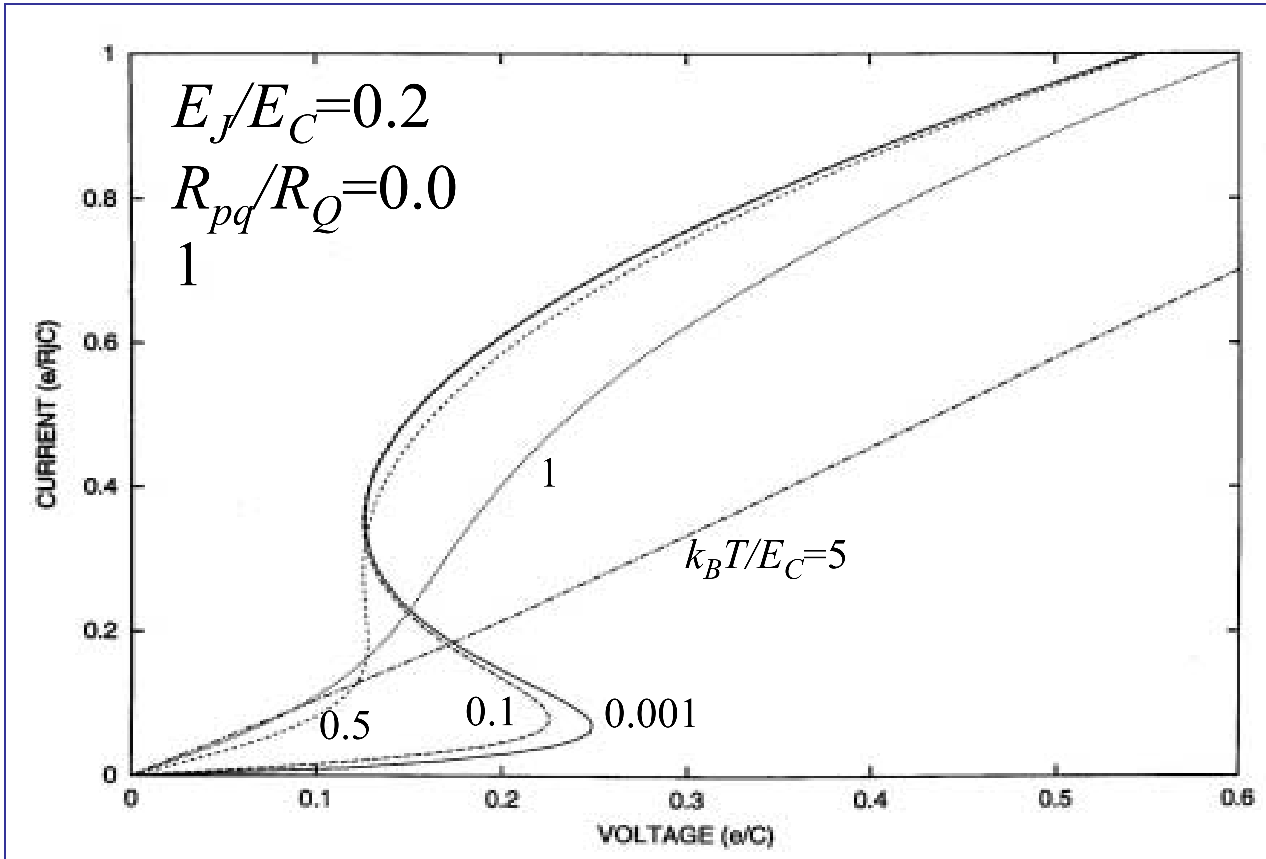
$$V = \frac{dE_o}{dq} = V_C \text{saw}\chi$$

- Single electron tunneling: rate depends on energy change, dissipative process
- Zener tunneling between bands: dissipative when relaxing to lower band
- Theoretical calculation of IVC: Master Equation for probability distribution in χ

Theoretical Calculation of IV

Averin and Likharev (1986),Geigenmüler and Schön (1988)

I-V simulator for arbitrary E_J, E_C, T and R_{qp} , Dick Kautz (NIST Boulder)



Comparison with Theory

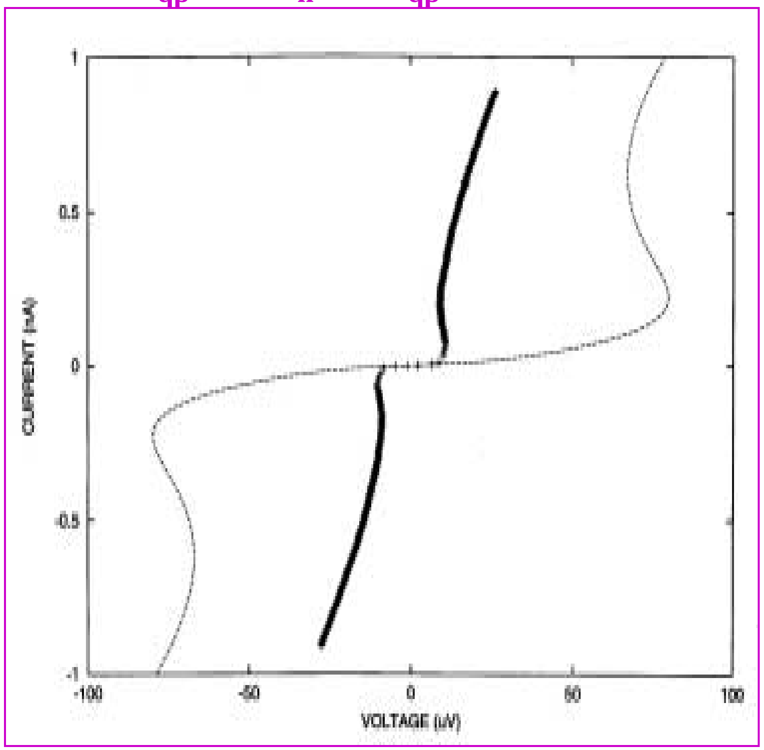
Dick Kautz (NIST, Boulder)

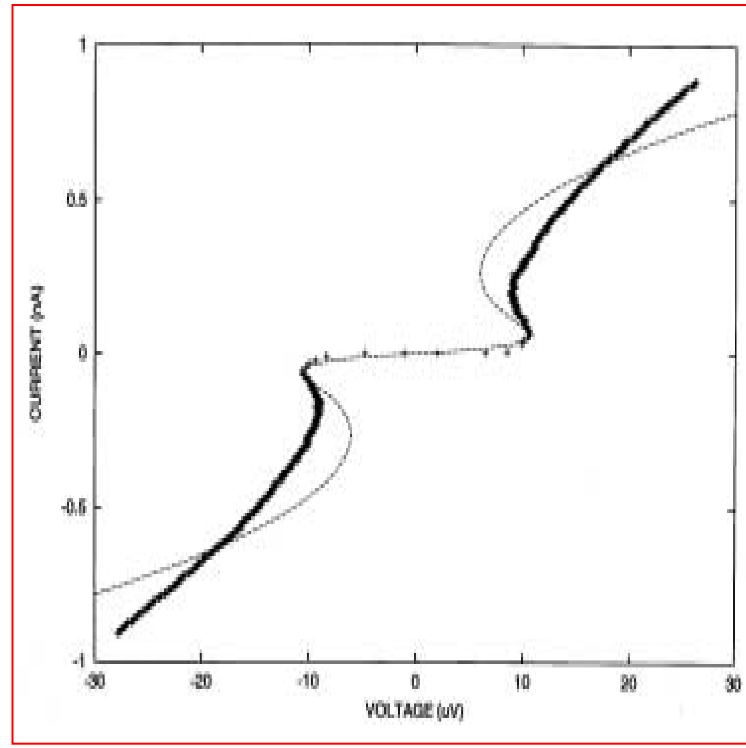
Measured : R_n=9kΩ $\rightarrow$ E_J=72μeV , T=40mK

Estimated: A=0.01 μm^2 $\rightarrow$ C~0.5fF

R_{qp} ~10R_n $\rightarrow$ R_{qp}=100k Ω

Adjust Parameters:
C=3.0fF, E_J=24μeV, R_{qp}=70k Ω



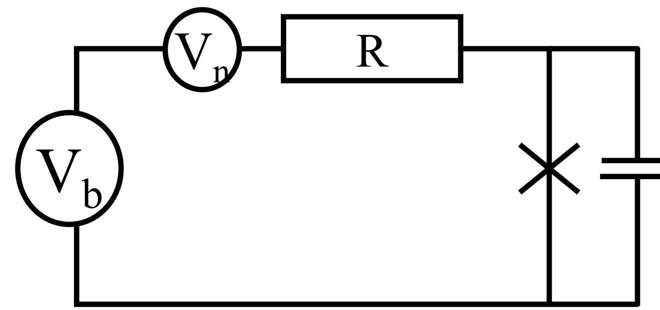


Theory is missing fluctuation effects due to environment

Classical Fluctuations - Phase

noise term (→ V_n)

$$V_b + V_n = \frac{\hbar}{2e}\dot{\phi} + RI_0 \sin\phi \; + \cancel{RC\frac{\hbar}{2e}\ddot{\phi}}$$



Overdamped limit $\beta = \dfrac{R^2 C}{\hbar / 2eI_0} << 1$

Analytical Solution exists for delta correlated noise term

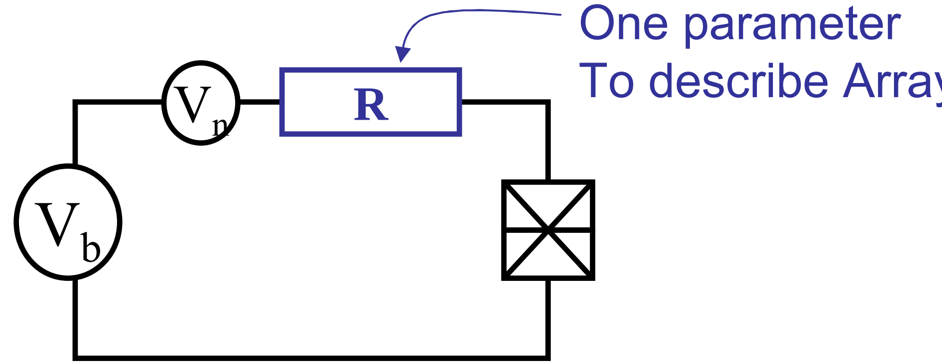
Ivanchenko and Zilberman, 1969

$$\langle V_n(0) V_n(t) \rangle = 2k_B TR\delta(t)$$

$$\frac{\langle I_J \rangle}{I_0} = \langle \sin\phi \rangle = \mathrm{Im}\left[\frac{I_{1-i\alpha\upsilon}(\alpha)}{I_{i\alpha\upsilon}(\alpha)}\right] \quad \text{where } \alpha = \frac{V_b}{RI_0} \;,\; \upsilon = \frac{E_J}{k_B T}$$

Classical Fluctuations - Quasicharge

$$V_b + V_n = R\frac{2e}{2\pi}\dot{\chi} + V_C \mathrm{saw}\chi$$



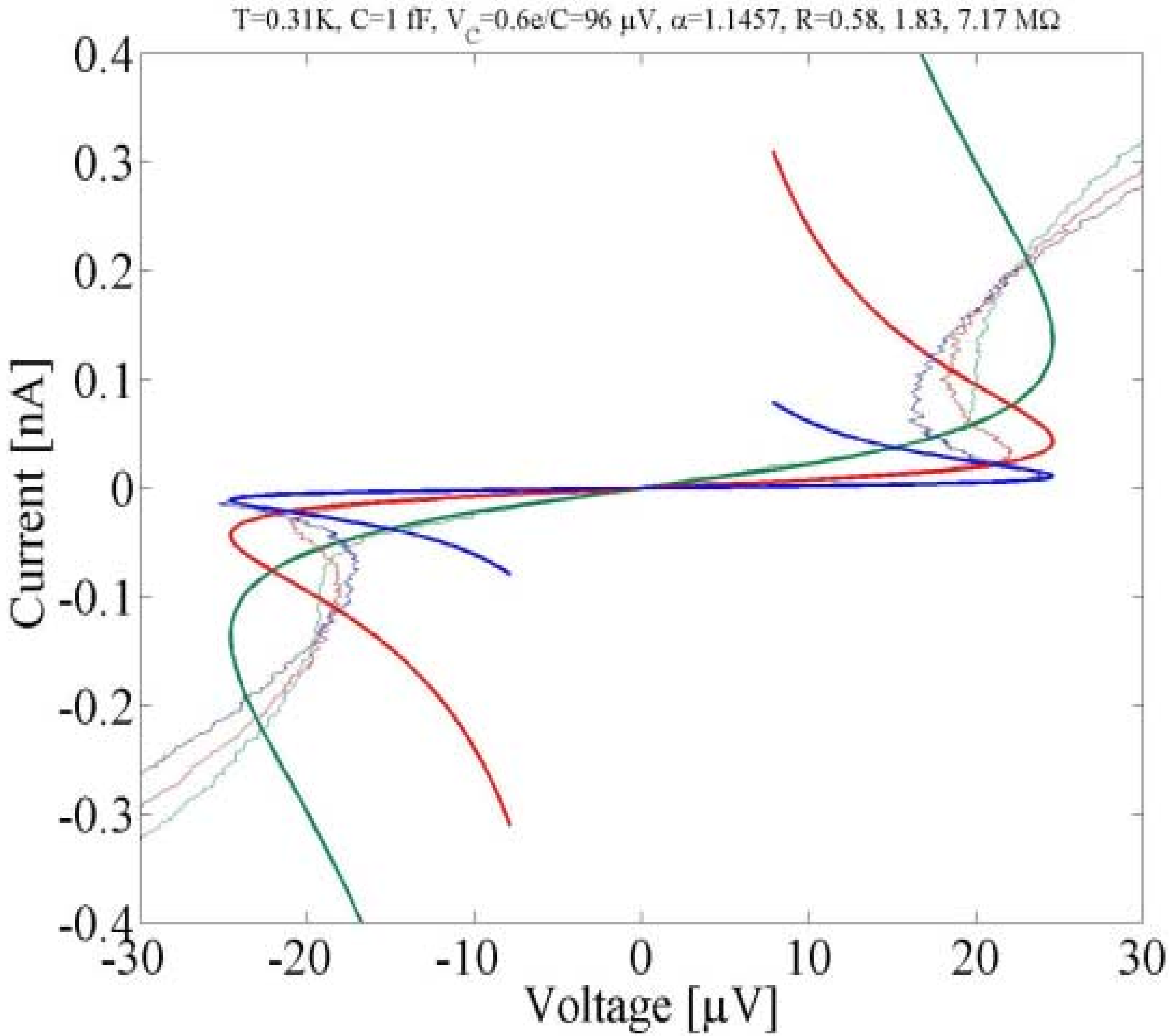
approximate $\mathrm{saw}\chi \cong \sin\chi$

Apply Ivanchenko and Zilberman solution for delta correlated noise term

$$\langle V_n(0)V_n(t)\rangle = 2k_B TR\delta(t)$$

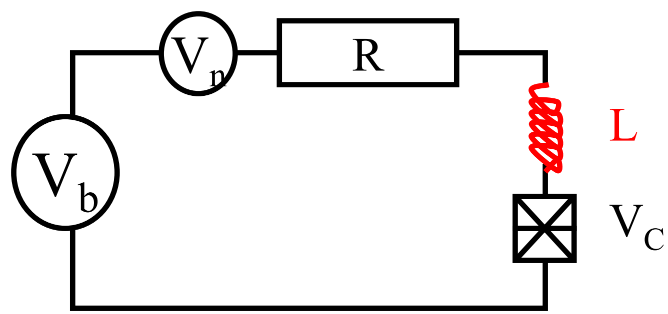
$$\frac{\langle V_J\rangle}{V_C} = \langle \sin\chi\rangle = \mathrm{Im}\left[\frac{I_{1-i\alpha\upsilon}(\alpha)}{I_{i\alpha\upsilon}(\alpha)}\right] \quad \text{where } \alpha = \frac{eV_C(E_J/E_C)}{\pi k_B T},\ \upsilon = \frac{V_b}{V_C}$$

Quick Fit of Ivanchenko and Zilberman



Duality: Phase - Quasicharge

$$V_b + V_n = R\frac{2e}{2\pi}\dot{\chi} + V_C \mathrm{saw}\chi + L\frac{2e}{2\pi}\ddot{\chi}$$



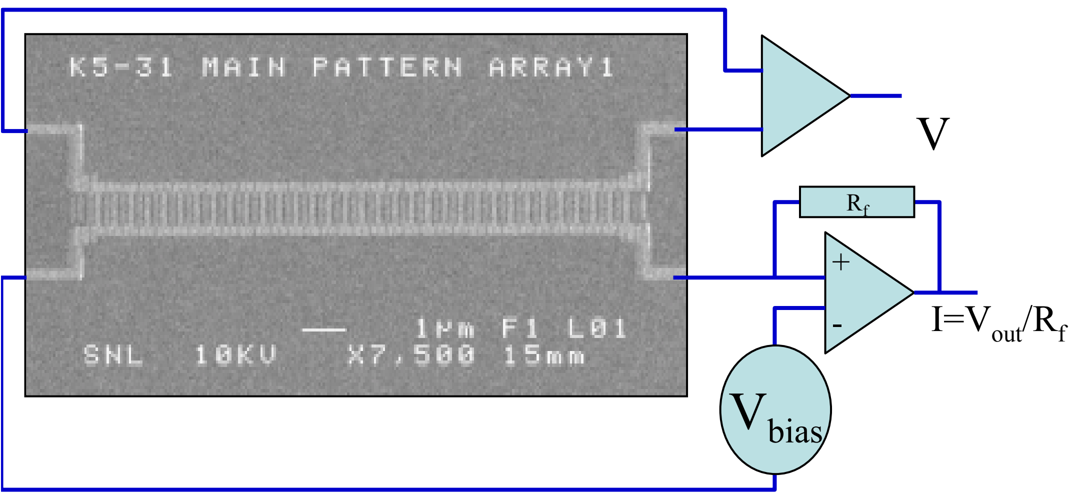
•Series inductance gives “mass term” in dynamical model

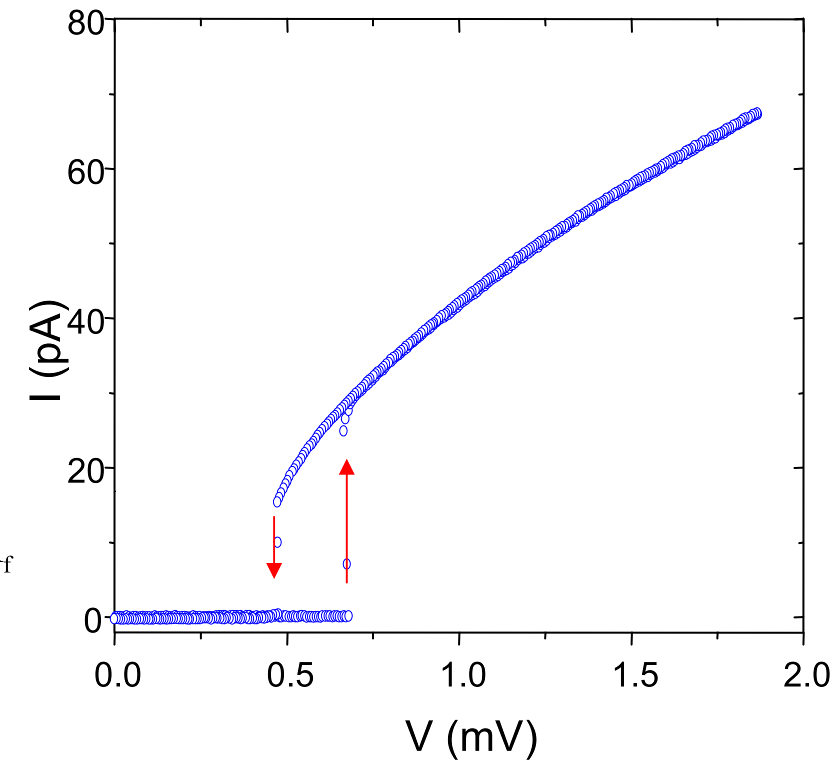
•Electromagnetic inductance completely negligible (thickness barrier 20 Å)

➡ Quasicharge dynamics always over damped for single junction

Experiment: Voltage biased array

Hysteresis (Back-bending) IV curve
for low impedance environment!



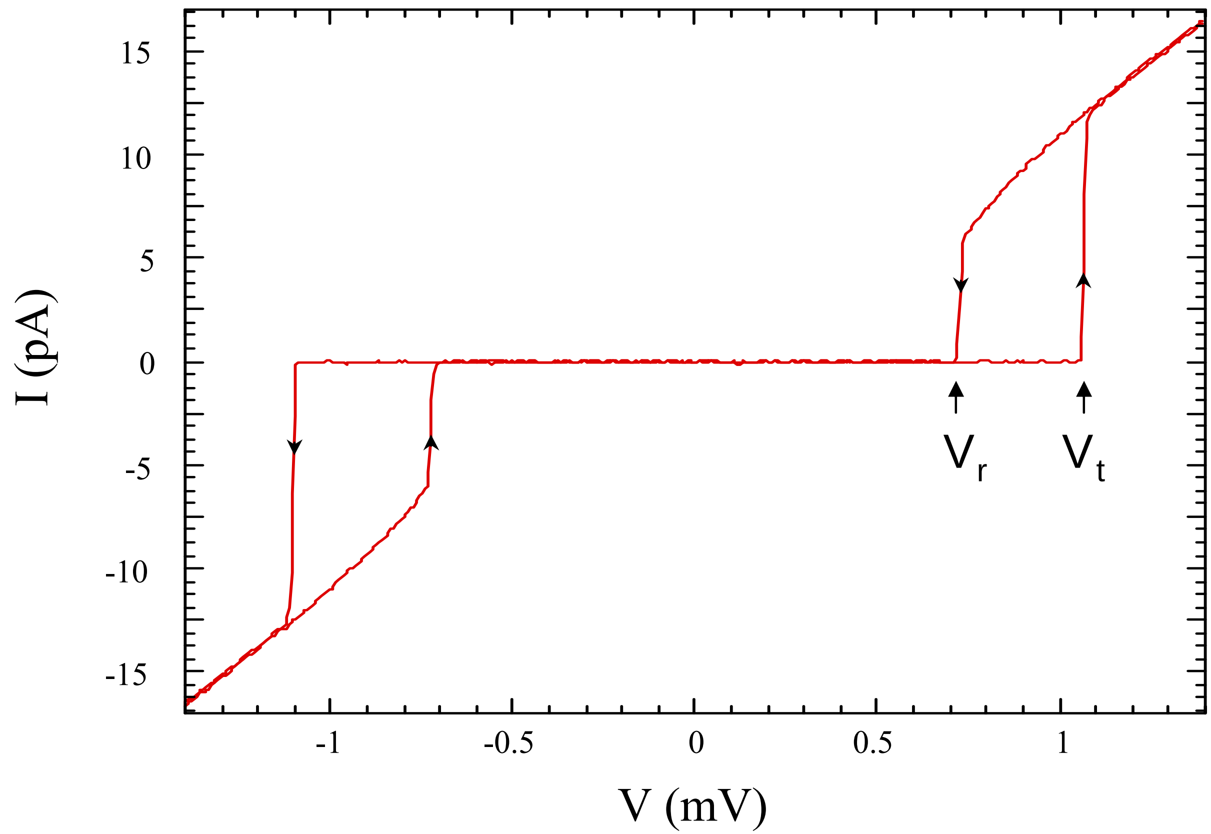


•Can not explain hysteresis (backbending) with overdamped dynamics!

-Kinetic inductance of one Cooper Pair soliton? (*P. Ågren et. al, JLTP, 2001*)

-Heating in finite current state?

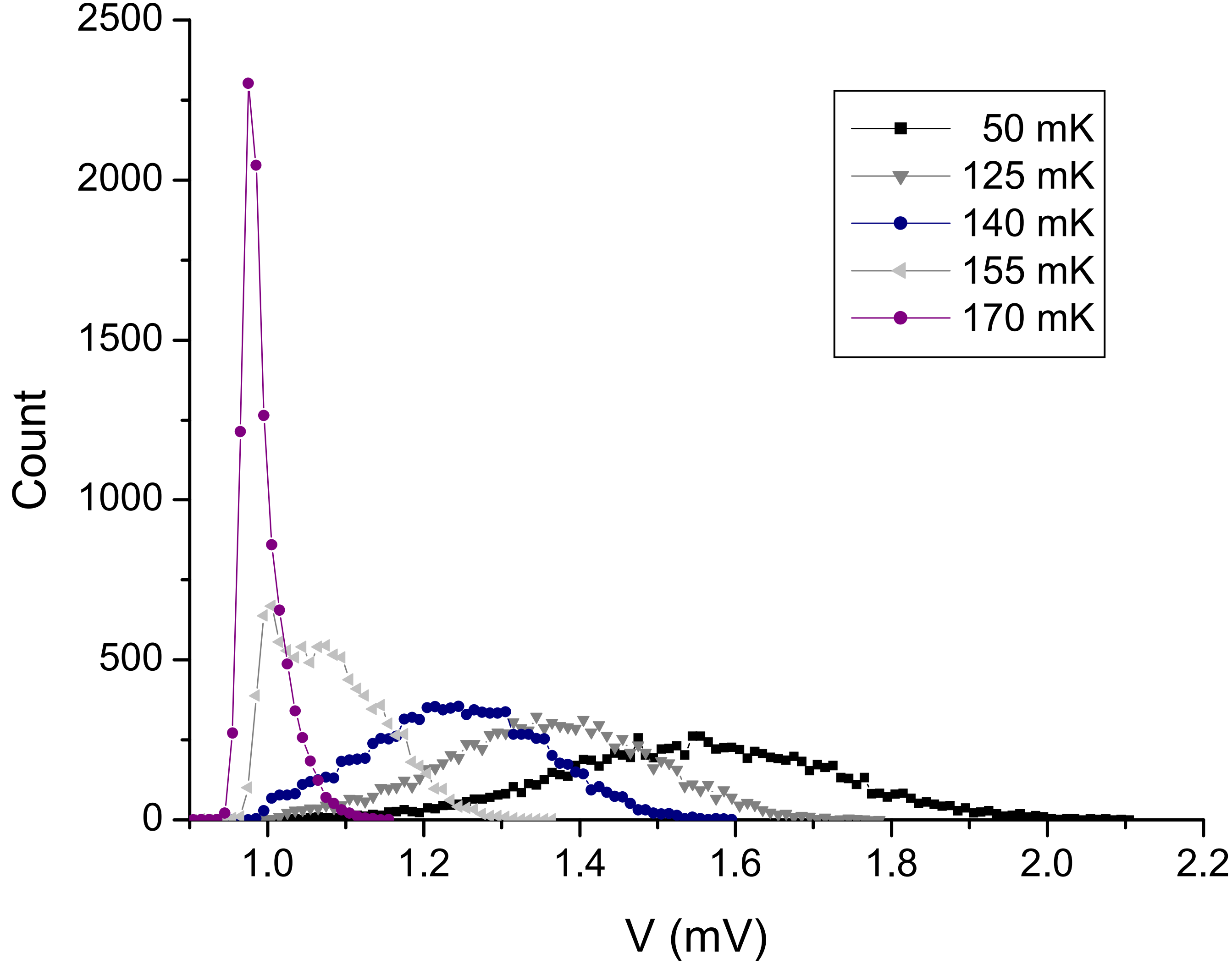
Fluctuations of Threshold Voltage



Switching from Zero-current State → No heating effects

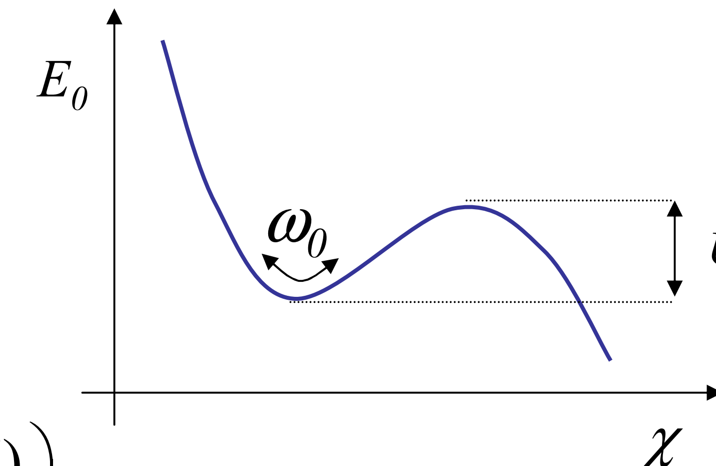
K. Andersson and D. B. Haviland, submitted to PRB

Switching Histograms vs. temperature



Kramers Model: Thermal escape from a bias dependant local minima

$$\frac{V_b}{V_C} - \mathrm{saw}\,\chi = \frac{1}{\gamma}\dot{\chi} + \frac{1}{\omega_0{}^2}\ddot{\chi}$$



Escape rate $\Gamma(V) = \frac{\omega_0}{2\pi}\exp\left(\frac{U(V)}{\mathrm{k_B}T}\right)$

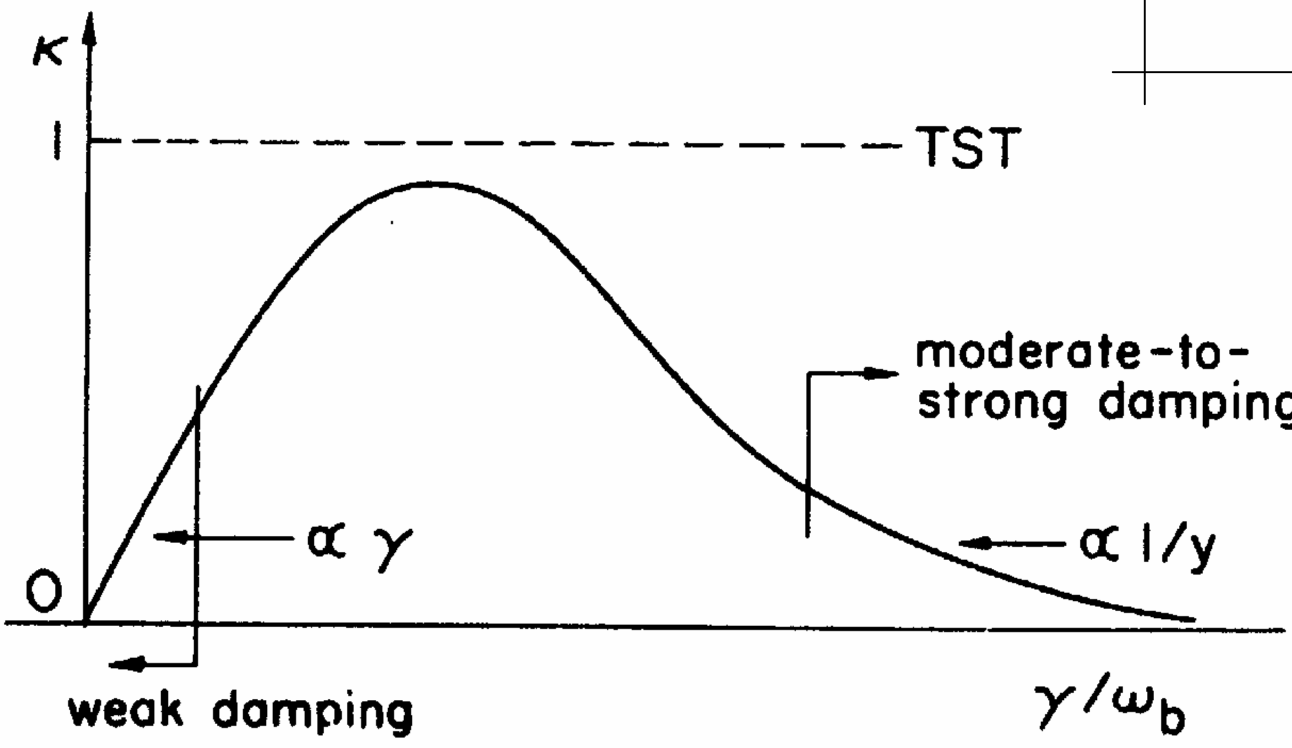
Escape probability density *P(V)* measured in experiment

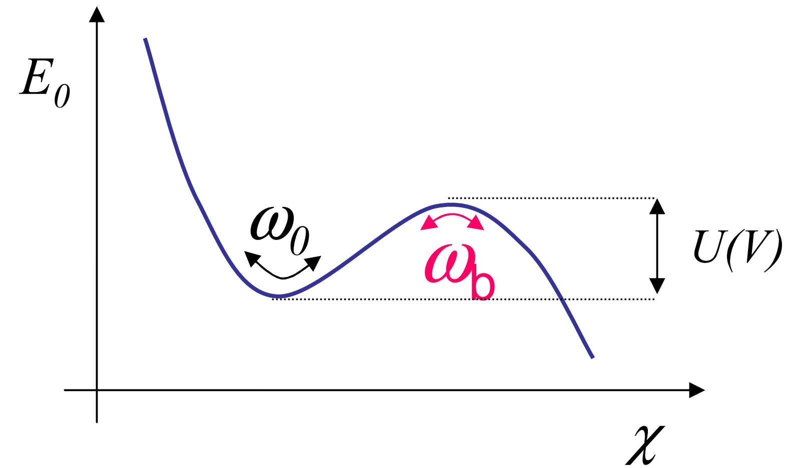
$$\Gamma(V) = \frac{\dot{V}}{\Delta V}\ln\left[\frac{\sum_{V'\geq V} P(V')}{\sum_{V'\geq V+\Delta V} P(V')}\right]$$

Effect of Damping on Escape

$$\Gamma = \kappa \frac{\omega_0}{2\pi} \exp\left(- \frac{U}{k_B T}\right)$$

ω_A

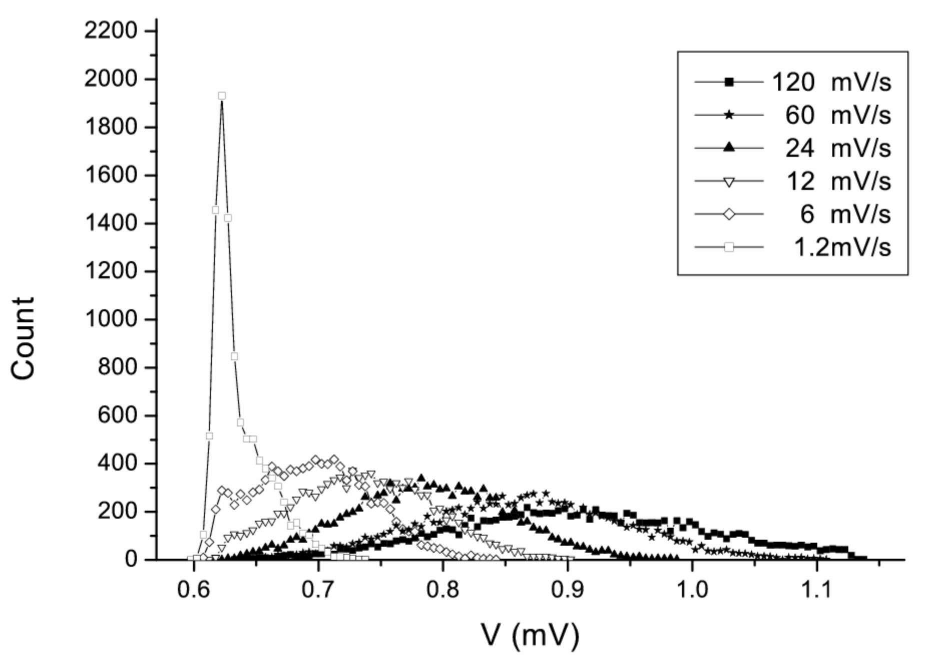


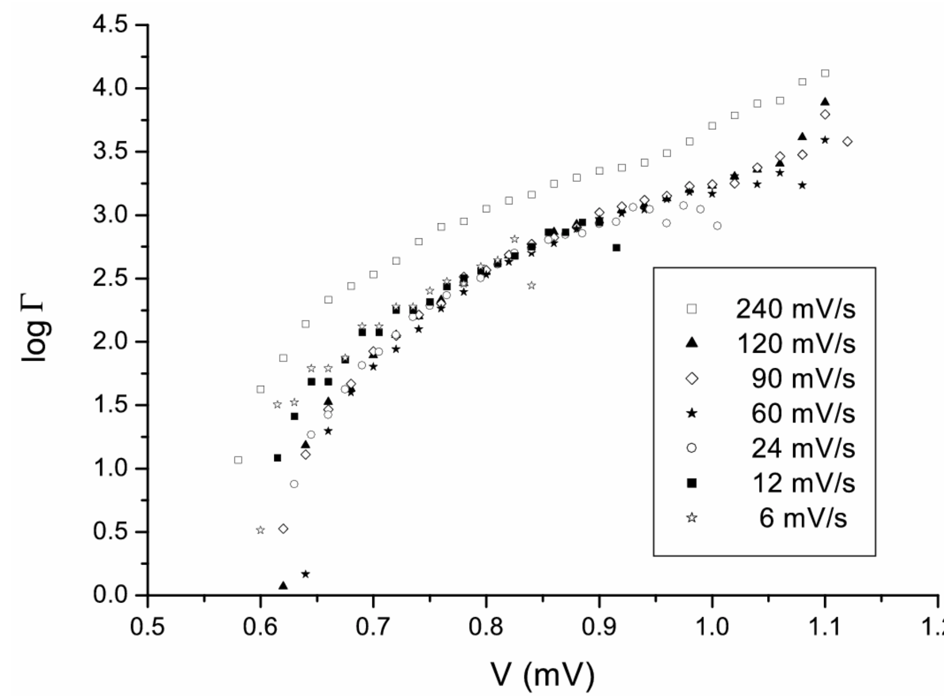


Review Article:
Hänggi, Talkner and Borkovec
Rev. Mod. Phys, 1990

Sweep speed dependence

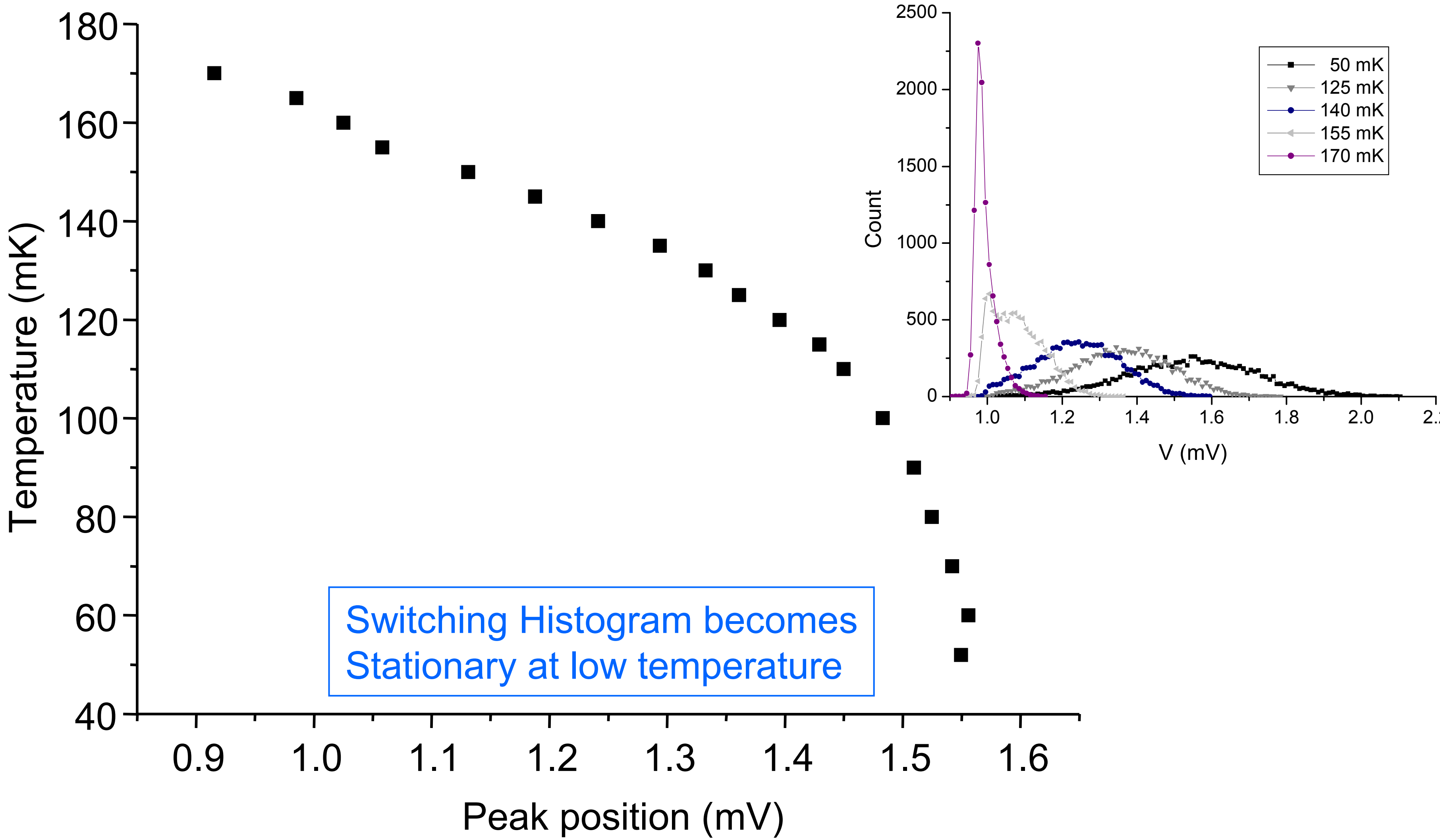
Switching Histogram → Escape rate vs. bias Voltage



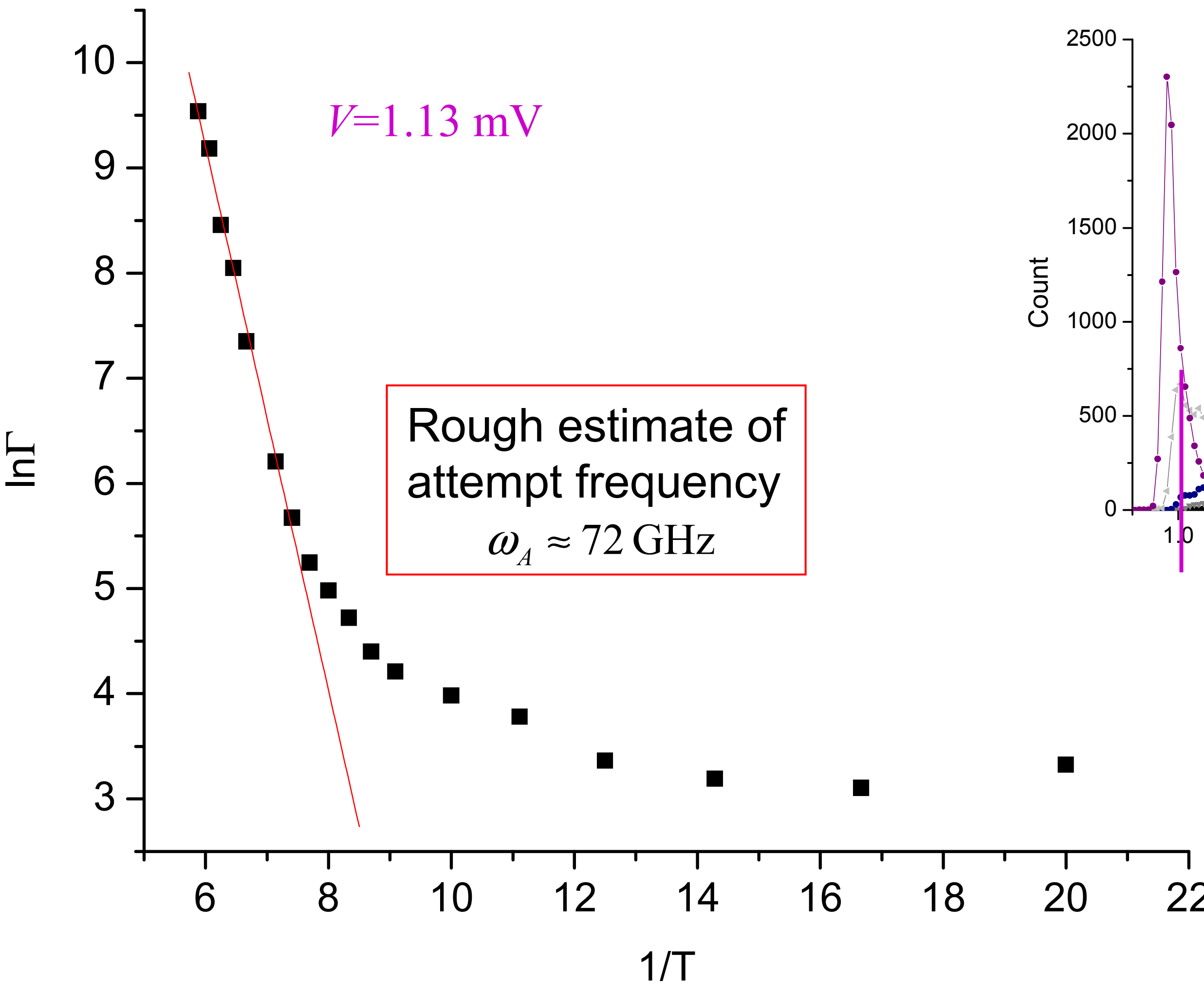


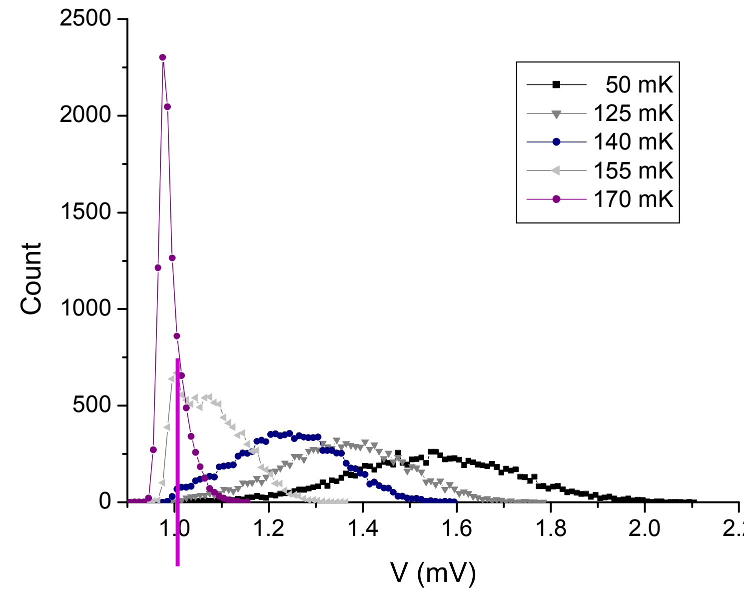
Sweep speeds less than 120 mV/s give consistent escape rate vs. V_b

Temperature dependence

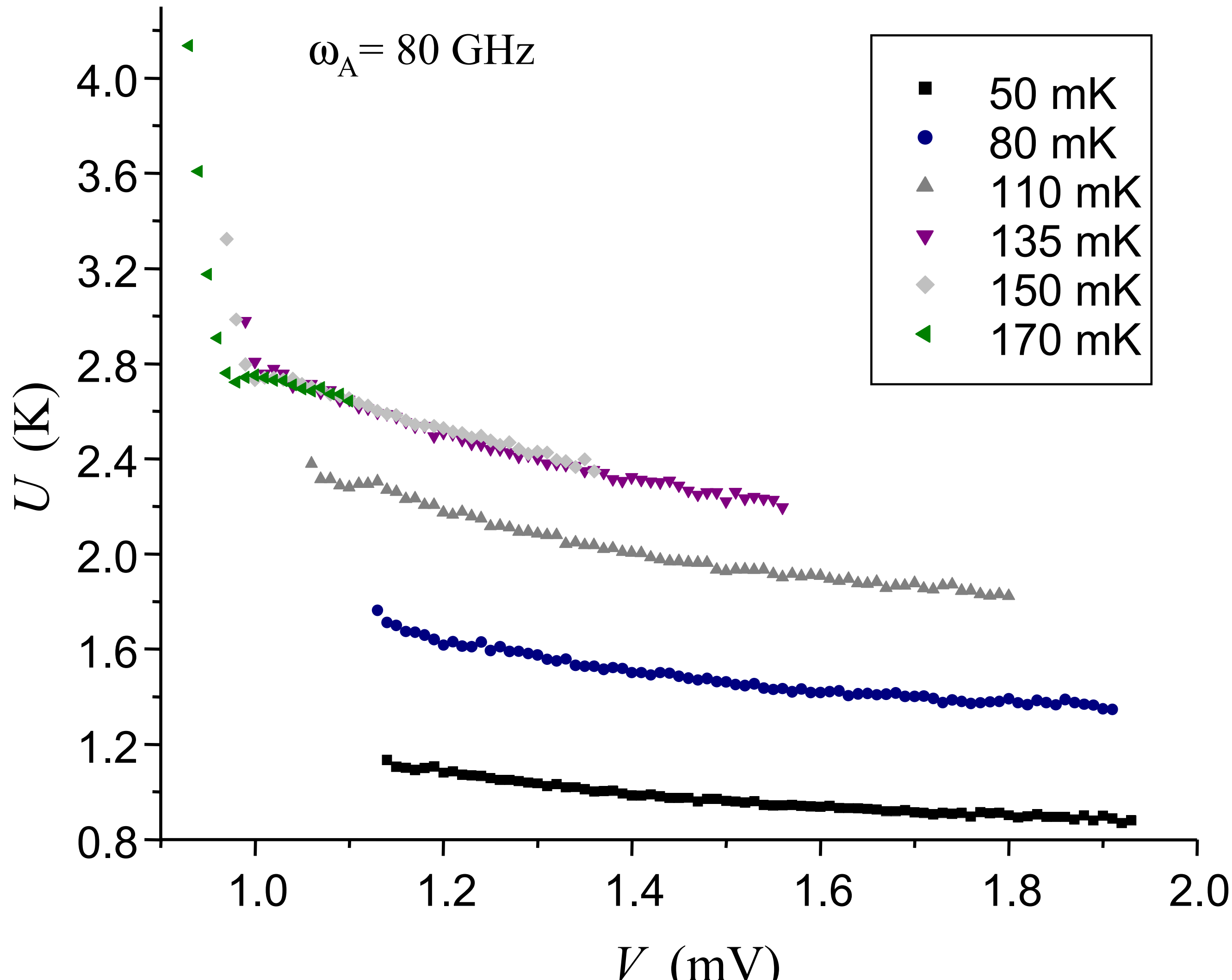


Kramers model fits at high temperature

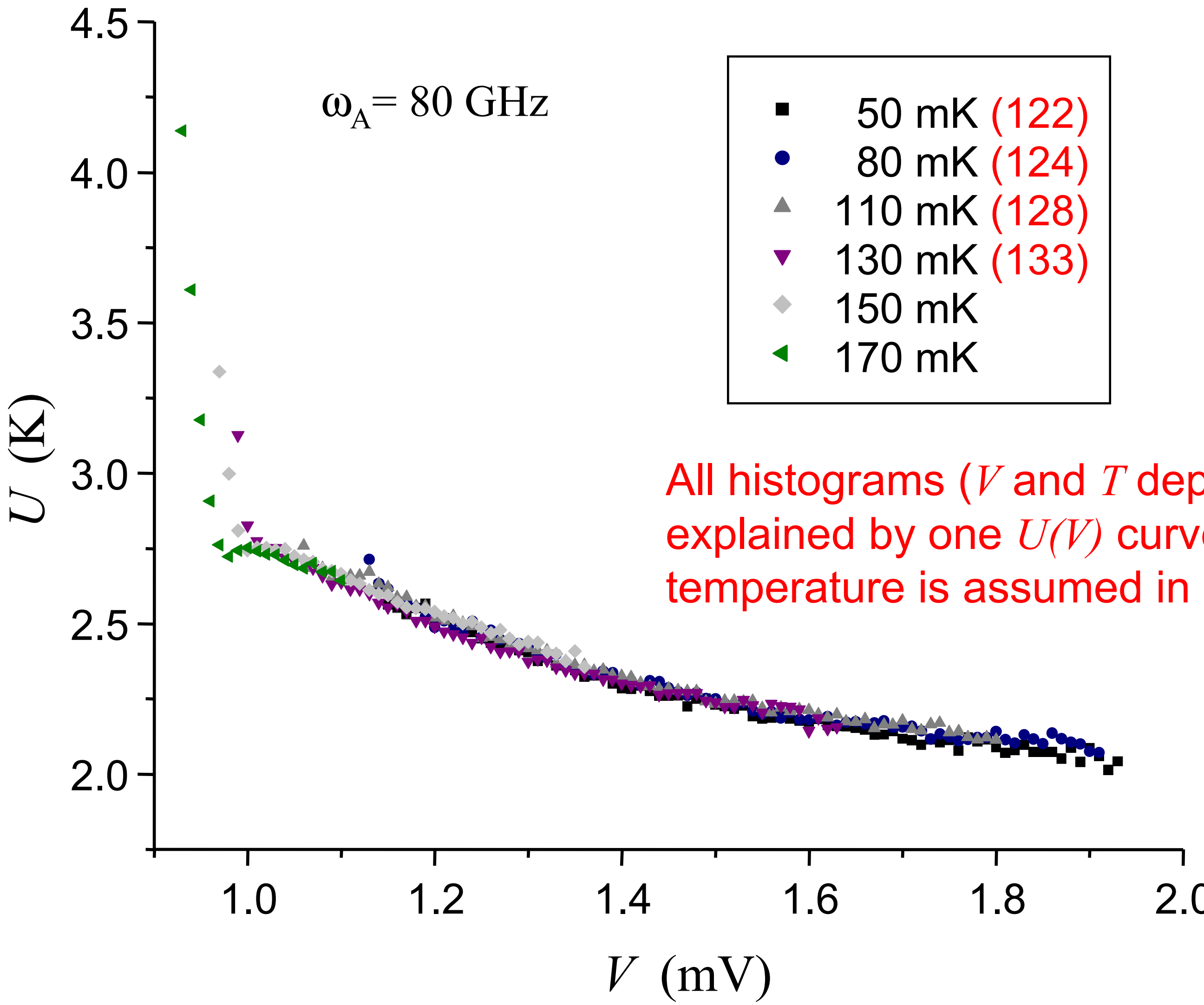




Energy Barrier vs. Bias voltage

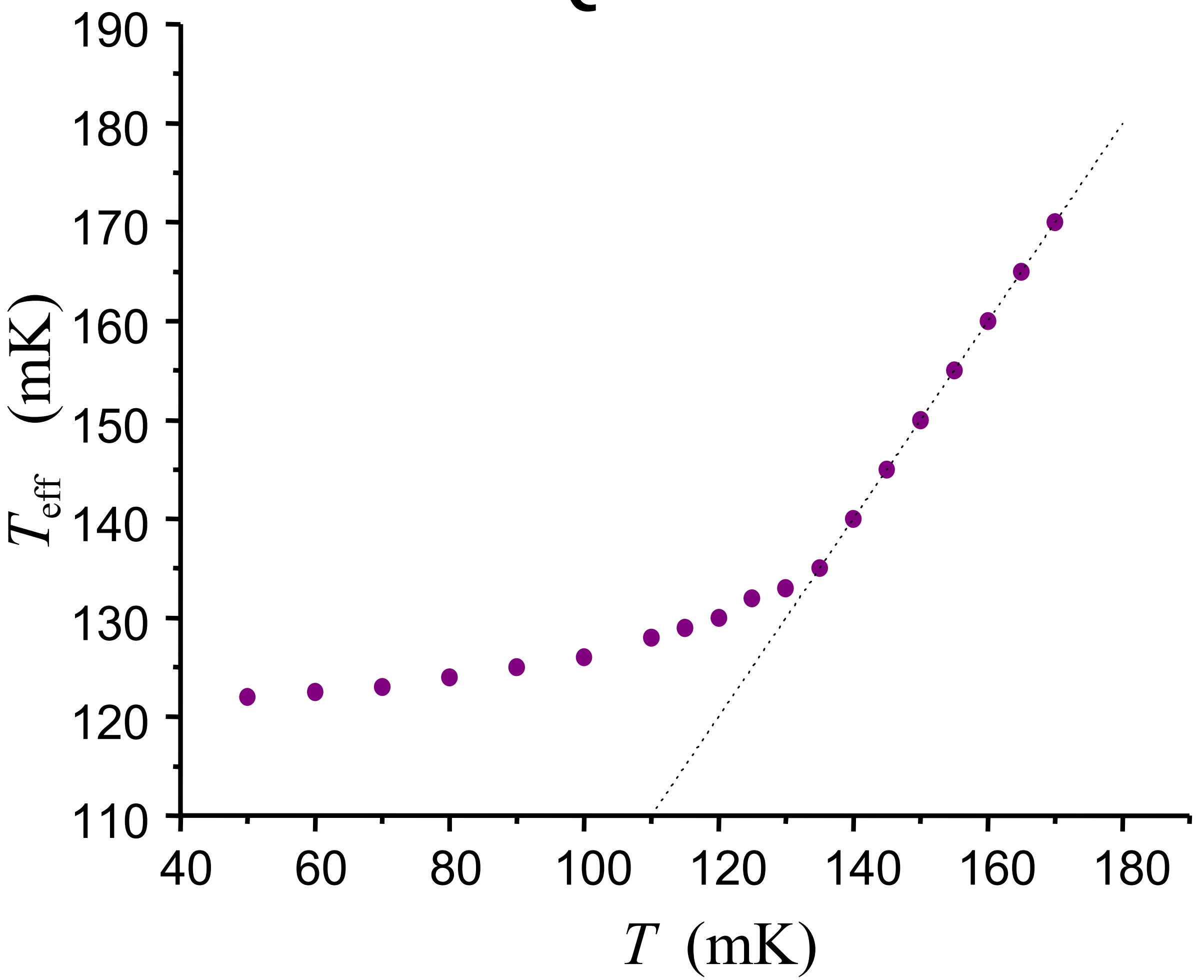


Effective Temperature for Low T region.



All histograms (*V* and *T* dependence) explained by one *U(V)* curve if effective temperature is assumed in low *T* region.

MQT or External Noise?



MQT Theory:
Escape Temperature

$$T_{\mathrm{esc}} = \frac{\hbar\omega_0 / k_B T}{7.2(1+0.87/Q)}$$

Assume: Q<1, κ~1

$T_{\mathrm{esc}} \cong 85$ mK

MQT not unreasonable, but hard to rule out external noise source!

Summary

- Small Capacitance SQUID arrays provide a tunable electromagnetic environment with $Z_e >> R_Q$
- Single JJ biased with Array:
  - Fluctuations due to non-infinite impedance of environment appear to explain “Bloch Nose”.
  - Overdamped dynamics, or negligible mass (inductance).
- Voltage Biased Array:
  - Hysteretic IV
    - if dynamic effect → non-negligible inductance
    - Heating effect due to finite current
  - Fluctuations of Switching voltage
    - Kramers model explains data, $U(V)$
    - Effective temperature: MQT? External Noise?