ENTANGLED ELECTRON CURRENT THROUGH NS INTERFACES

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OUTLINE

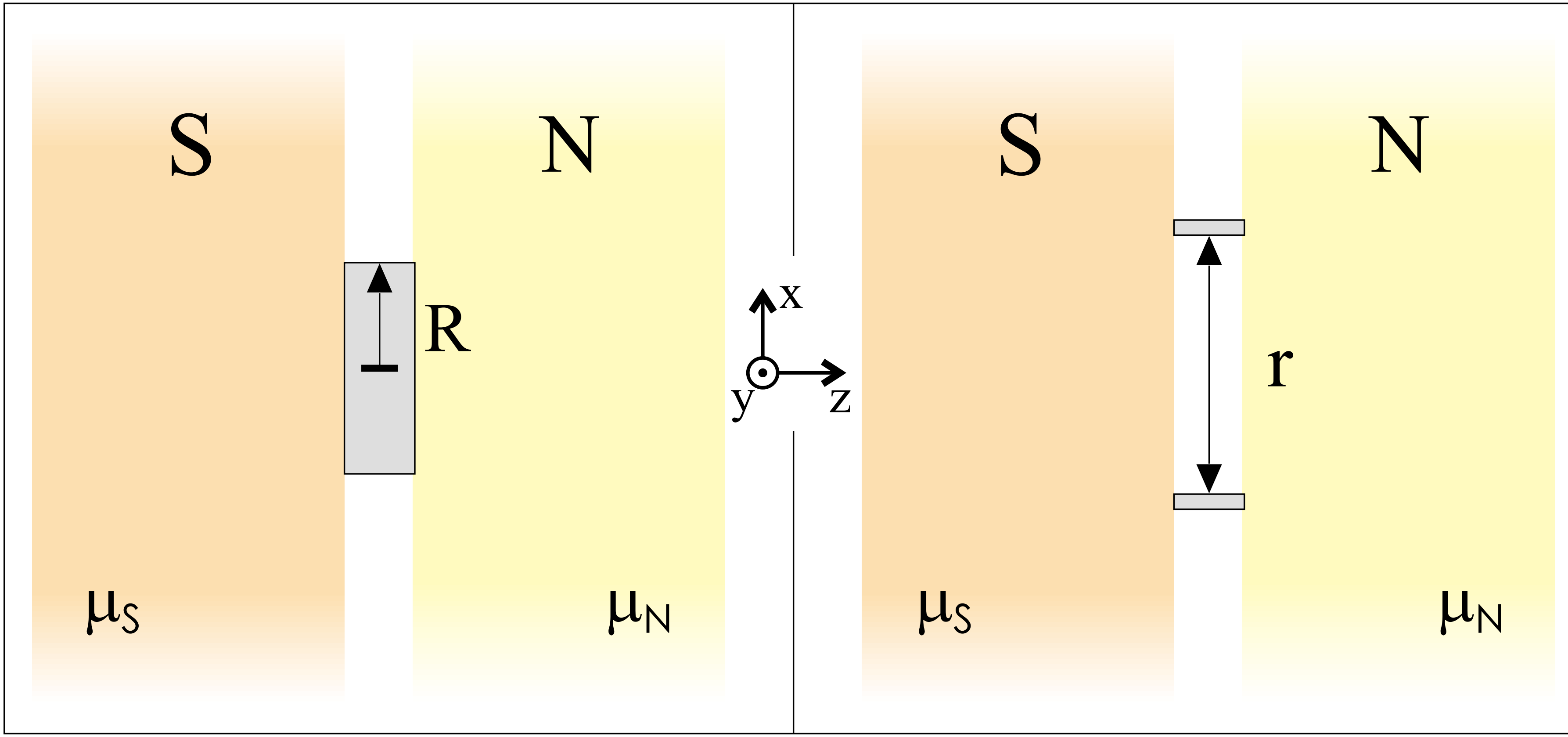
- Motivation
- Andreev reflection vs. two-electron emission
- Real space tunneling 3D Hamiltonian
- Angular distribution of current
- Circular interface of arbitrary radius
- Two-point interface
- Failure of energy-independent hopping
- Conclusions

Motivation

- Interest in superconductors as source of entangled electrons for use in quantum communication
- Understand detailed structure of transport through NS interface
- Is two-electron emission equivalent to hole Andreev reflection?
- How is angular distribution of current?
- How does transport depend on interface size? How do we recover the thermodynamic limit?
- How is entangled current through two distant point-contacts?

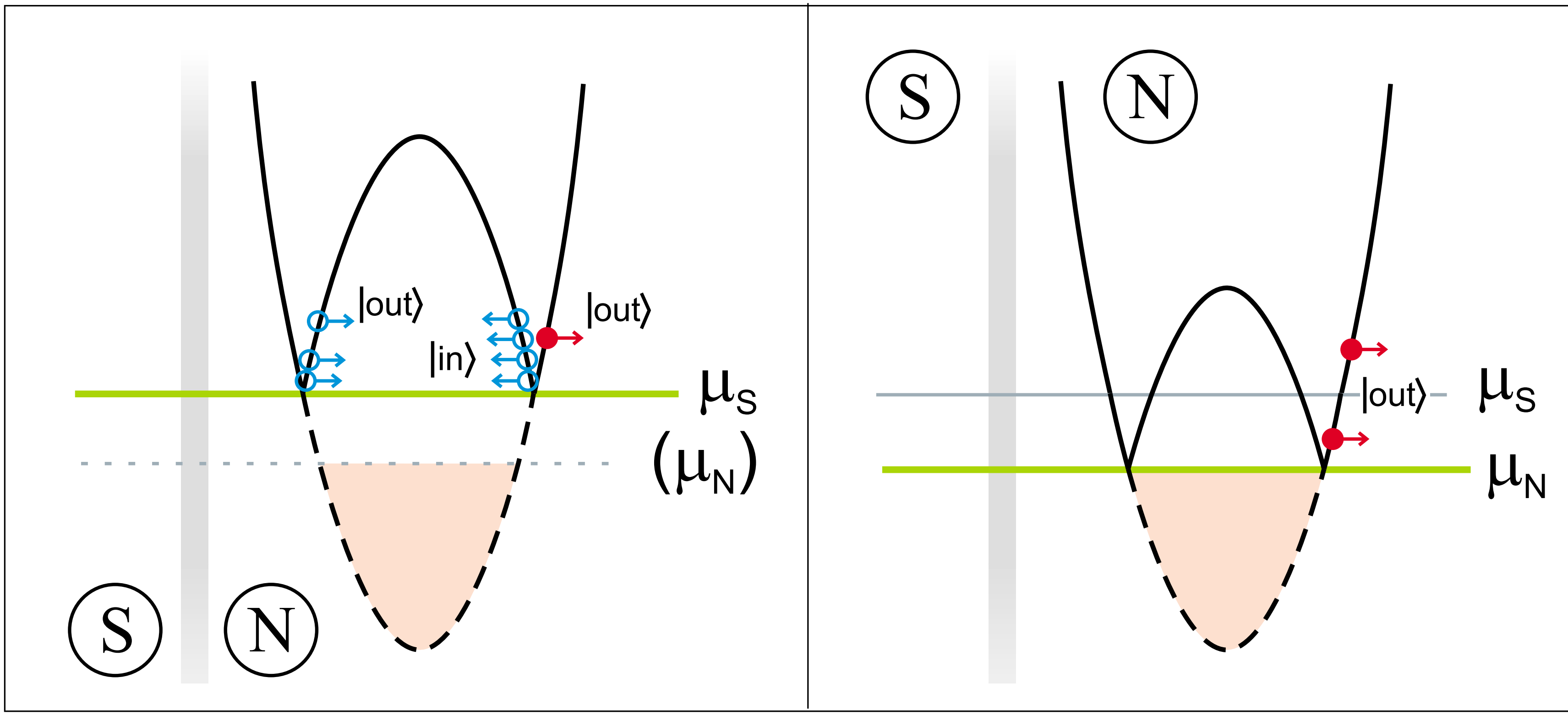
Related work:

- P. Recher, E. V. Sukhorukhov, D. Loss, Phys. Rev. B 63, 165314 (2001)
- G. B. Lesovik, T. Martin, G. Blatter, Eur. Phys. J. B 24, 287 (2001)
- N. M. Chtchelkatchev, G. Blatter, G. B. Lesovik, T. Martin,
  Phys. Rev. B 66, 161320 (2002)



circular interface

two point-like holes

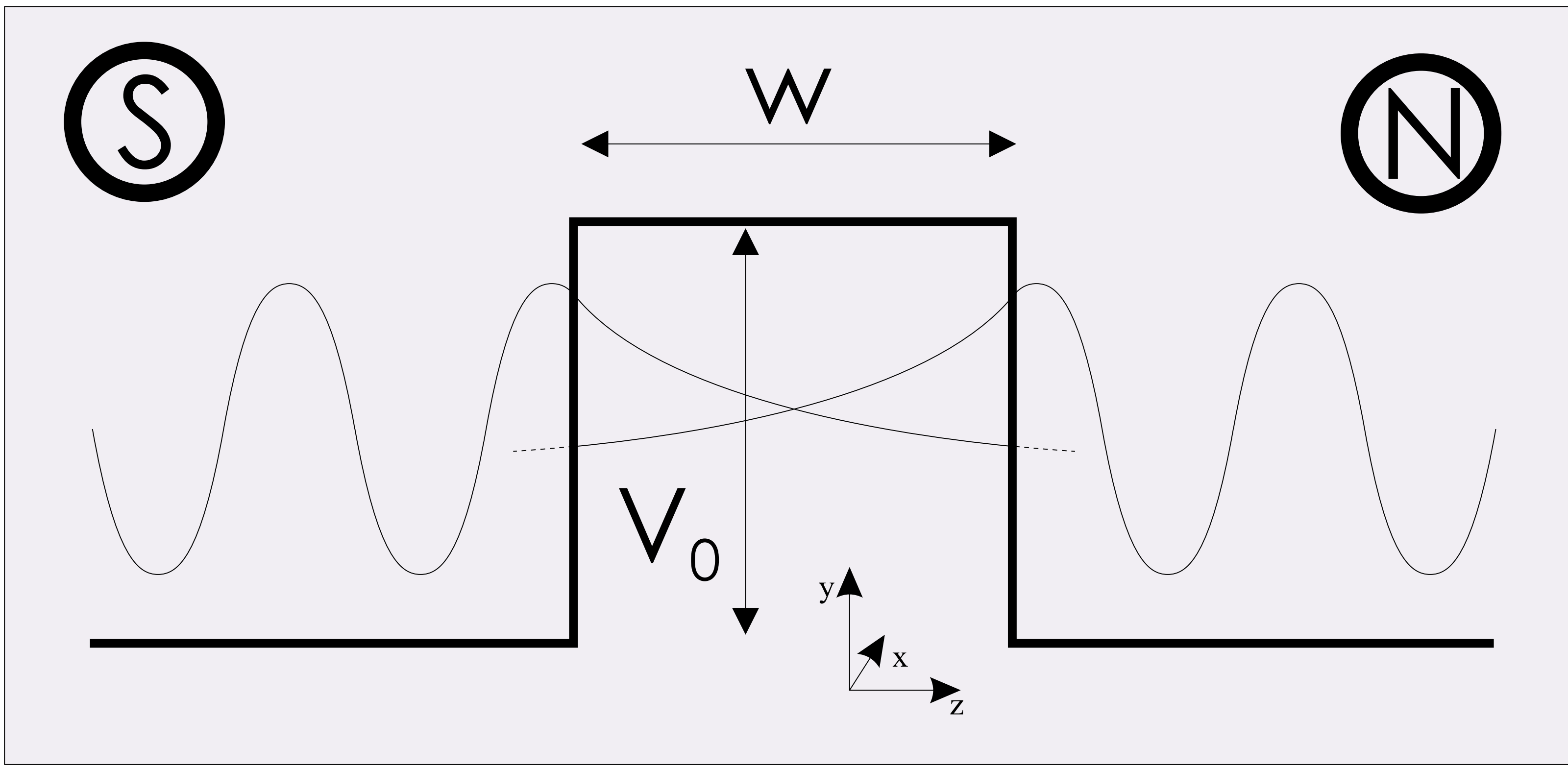


Andreev reflection of incident hole

Two-electron emission

Different choice of reference chemical potential for N

Tunneling structure



Tunneling Hamiltonian

$$V = \sum_{\mathbf{kq}} T_{\mathbf{kq}}\, c^{+}_{\mathbf{k}} c_{\mathbf{q}} + \text{H.c.}$$

$$T_{\mathbf{kq}} = \frac{\tau}{\Omega N(0)} \delta^{2}(\mathbf{k}_{||} - \mathbf{q}_{||})\; F(k_z, q_z)$$

Transparency $\tau \equiv$ transmission probability at (longitudinal) Fermi Energy for large interface

Delta barrier: $F(k_z, q_z) \sim k_z\, q_z$

Local 3D tunneling Hamiltonian

(delta barrier)

$$V = \text{cnst} \times \frac{\tau}{N(0)} \int_A d^2 r \left(\frac{\partial \psi_L^+(\mathbf{r}, z)}{\partial z} \right)_{z=0^-} \left(\frac{\partial \psi_R(\mathbf{r}, z')}{\partial z'} \right)_{z'=0^+}$$

Section *A* may have arbitrary shape and size

Perturbative approach

$$T = V + VG_0T = V + VG_0V + \dots$$

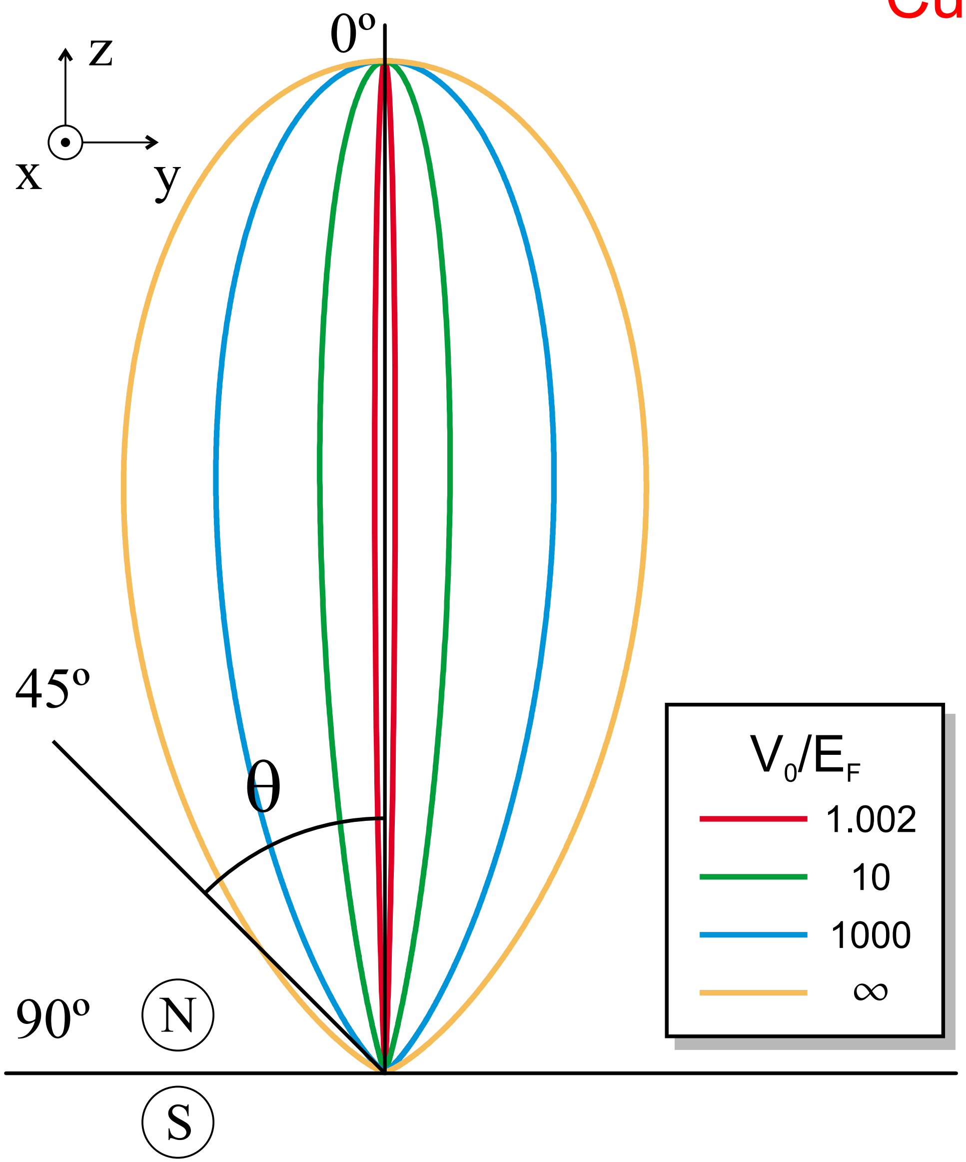
Low voltage, low temperature: $T \approx VG_0V$

$$\left| i \right\rangle = \left| \mathrm{BCS} \right\rangle \otimes \left| \mathrm{F} \right\rangle$$

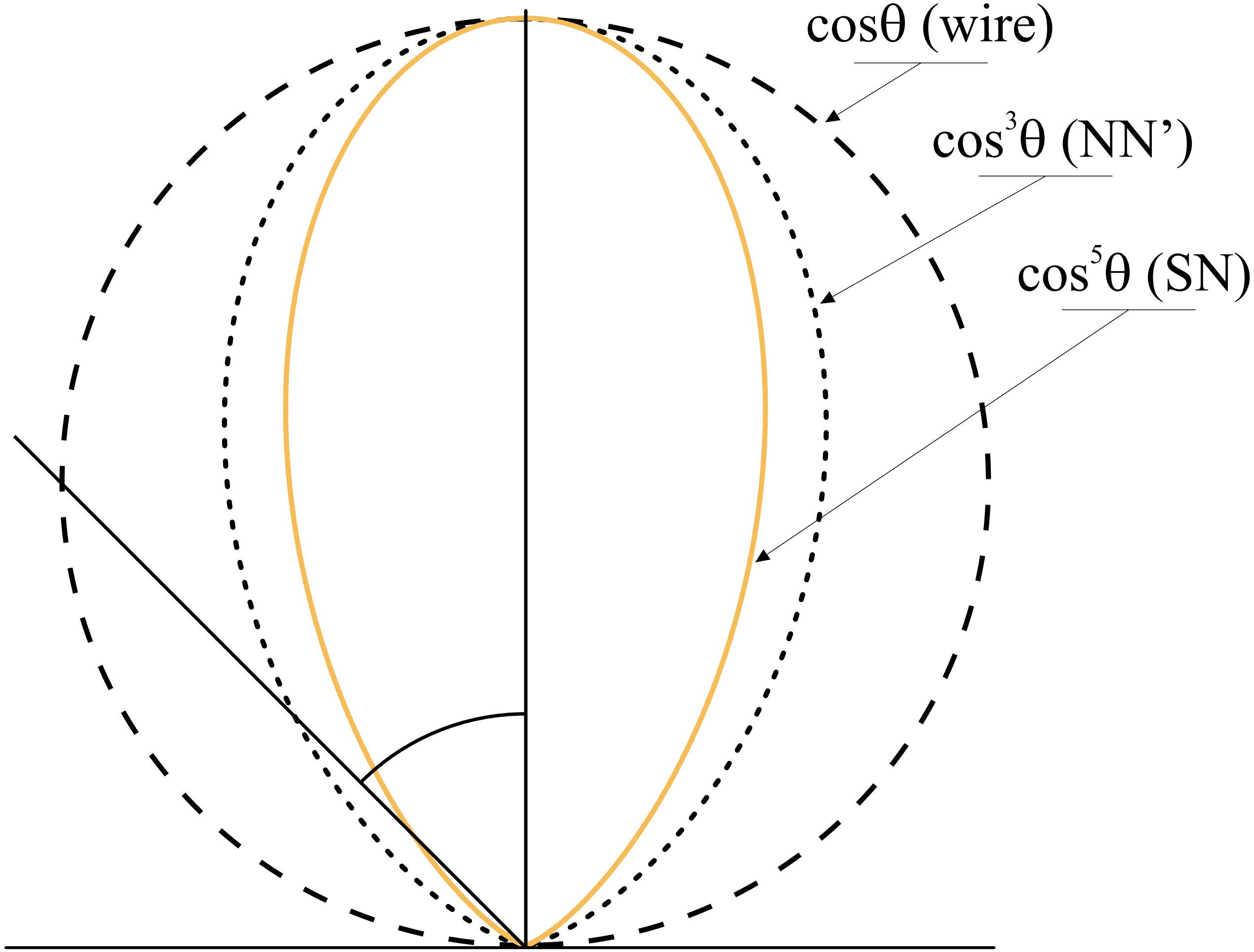
$$\left| f \right\rangle = \left| \mathrm{BCS} \right\rangle \otimes \left(c^+_{k_1\uparrow} c^+_{k_2\downarrow} - c^+_{k_1\downarrow} c^+_{k_2\uparrow} \right) \left| \mathrm{F} \right\rangle$$

apply Fermi golden rule

Current angular distribution



Focussing greater for lower barrier



Focussing for NS greater than for NN'

$A \sim R^2 \to \infty$ Delta barrier

$$V(z) = Z\hbar v_F\, \delta(z)$$ BTK ‘82 $\tau = 1/Z$

$$I_{SN} = \frac{1}{2} I_0 \tau^4 \int_0^{\pi/2} d\theta\ \sin\theta \cos^5\theta$$

$$= \frac{1}{12} I_0 \tau^4$$

$$I_0 \equiv e^2 V N(0) v_F A$$

Agrees with Kupka ‘9:

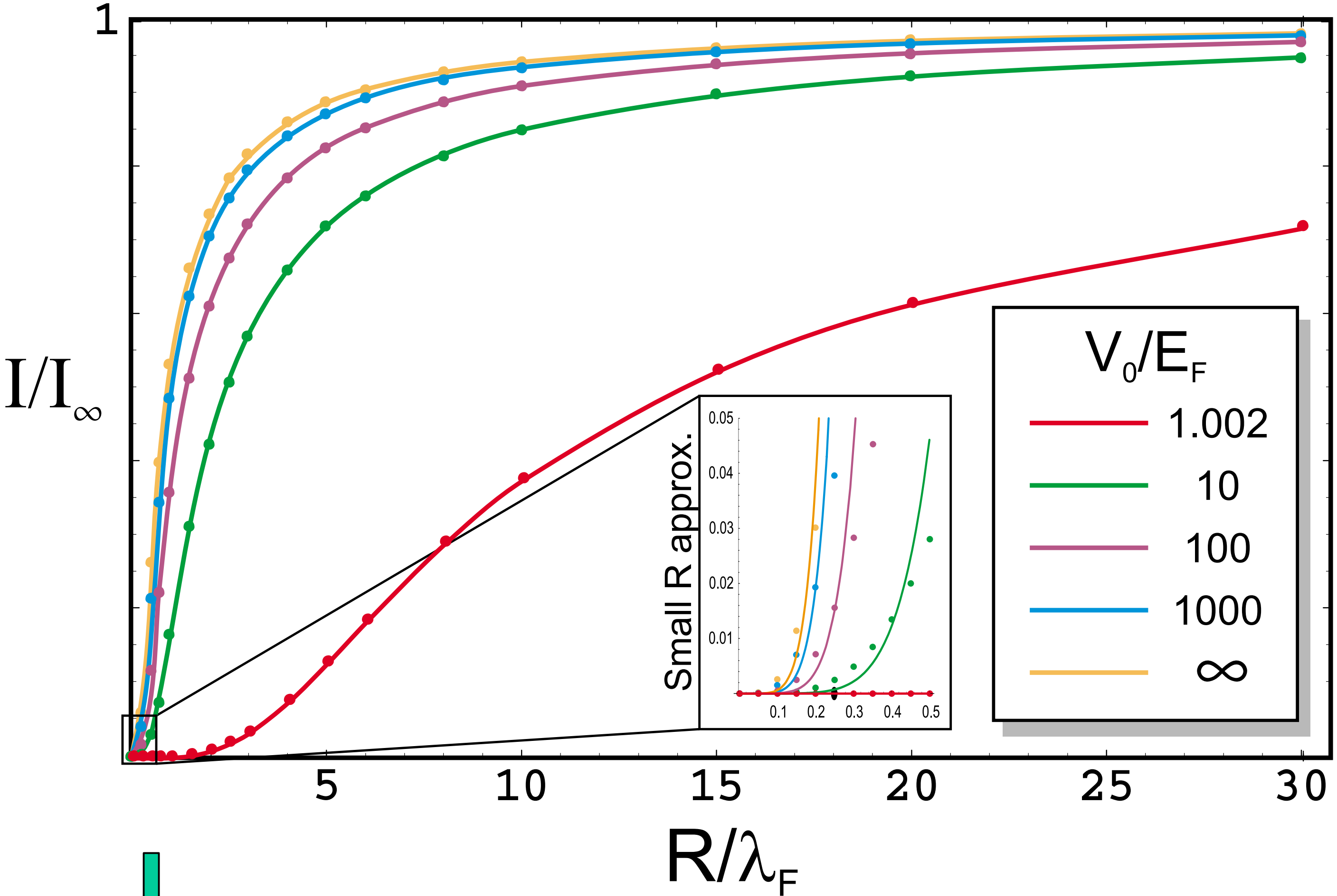
$$I_{SN} = \frac{I_0}{12 Z^4}$$

2-electron emission ≡ Andreev scattering of hole

Circular interface of arbitrary radius

- Six integrals with many oscillations to be evaluated numerically ...
- Necessary to introduce approximations
- Good for large R
- Small R limit is analytical

Circular interface



$\rightarrow$ $I_\infty \sim A \sim R^2$

(TD limit)

$\downarrow$

Small R: $$I \sim R^8 \sim A^4 \sim \left(\int_A dx_1 \int_A dx_2 \ldots \right)^2$$

Two-point interface

$$I = 2I(R \to 0) + \delta I(r)$$

Non-locally entangled current:

$$\delta I(r) \sim I(R \to 0)\left(\frac{\sin(k_F r)}{(k_F r)^3} - \frac{\cos(k_F r + \delta)}{(k_F r)^2}\right)^2 \exp\left(-\frac{2r}{\pi \xi_0}\right)$$

$$\delta \equiv \Delta / 2E_F$$

Note relatively fast decay for $r >> \lambda_F$

Failure of energy-independent hopping

$$T_{\mathbf{kq}} \approx \frac{\tau}{\Omega N(0)} \delta^2(\mathbf{k}_{||} - \mathbf{q}_{||}) \quad (\text{indep. of } k_z, q_z)$$

$$\Rightarrow \quad V \approx \text{cnst} \times \frac{\tau}{N(0)} \int_A d^2 r \, \psi_L^+(\mathbf{r}, z = 0^-) \psi_R(\mathbf{r}, z' = 0^+)$$

$$\lim_{R \to \infty} I(R)/R^2 = \infty$$

(divergent TD limit)

$$\delta I(r) \sim I(R \to 0) \left(\frac{\sin(k_F r)}{k_F r} \right)^2 \exp\left(-\frac{2r}{\pi \xi_0} \right)$$

(incorrect nonlocally entangled current)

CONCLUSIONS

- Equivalence between Andreev reflection and two-electron emission established
- Local 3D Hamiltonian valid for arbitrary interface shape
- NS current more focussed than NN' current
- Thermodynamic limit achieved for $R > 10\ \lambda_F$ due to fast spatial phase oscillations
- Role of barrier height investigated
- Non-locally entangled current decays fast for $R \gg \lambda_F$
- Failure of energy-independent hopping