Introduction to the Physics of Semiconductor Quantum Dots

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Outline: Measuring Energy Scales

- Coulomb blockade energy $U$ (Metal Single Electron Transistor)
- Energy level spacing $\Delta \varepsilon$ (Semiconductor SET)
- Coupling to the leads $\Gamma$ and $kT_K$
- Measuring charge instead of current
- Electron counting to determine $\Gamma$
Two Barriers with Small Island

source \hspace{1cm} \text{metal} \hspace{1cm} \text{metal} \hspace{1cm} \text{metal} \hspace{1cm} \text{drain}

Small Metal Island

$E_F$ \hspace{1cm} \text{U} = e^2/C \hspace{1cm} E_F - eV_{ds}$

No current at zero temperature
Schematic of Metal SET

source metal metal metal drain
(No tunneling) metal gate
Sequential Charging

At low $T$ and with very small $V_{ds}$ get one sharp peak for each electron added.
Charge Quantization
Current vs. Gate Voltage

Note: Because of superconductivity, below $T_C$, one gets a peak every time Cooper pair is added.
Condition for Charge Quantization

An extra electron stays on the island for time \( RC \). This time must be long enough that the uncertainty in its energy is less than \( U \).

\[ U > \frac{h}{RC}, \text{ but } U = \frac{e^2}{C} \]

\[ R > \frac{h}{e^2} \text{ or } G < \frac{e^2}{h} \]
Adding Charge by Source-Drain

\( V_{ds} \)
Coulomb Staircase

Peak in $dI/dV$
Coulomb Diamonds

First step in Coulomb staircase

Slopes of diamonds give capacitance ratios which converts voltage to energy.

SET made with nano-particle, Bolotin et al. APL 84, 3154 (2004)

Note: switching from nearby charges
Making Semiconductor SETs

GaAs Field Effect Transistor

Insulator (AlGaAs)
Gate (metal)
Semiconductor (GaAs)
Electrons
Drain
Source
Electron Beam Lithography

1. GaAs
2. Electron Beam
3. Electron sensitive layer
4. Develop
5. Evaporate Metal
6. Lift Off
Actual Process
GaAs SET

Schematic Potential in SET
Coulomb Charging Peaks

Data from Meirav et al. PRL 65, 771 (1990).

Note: Variation of peak height and spacing reflects individual levels.
Quantized Energy Levels

There is a peak in $dI/dV_{sd}$ for every energy level. Although these have been detected in metal SET’s it is hard because density of states is so large.
Excited State Spectroscopy

dI/dV_{sd} has peak when level crosses E_F

Very small dot ⇒ peaks no longer periodic along V_{sd} = 0

Electron interactions are more complicated than just U and involve exchange.

Kouwenhoven et al
Science 278, 1788, 1997
Probability of electron remaining in a level on the dot decays as $\exp(-t/\tau)$, so the level broadens into a Lorentzian with energy width $\Gamma = \hbar/\tau$
The chemical potential $\mu$ is proportional to the gate voltage. The full width at half maximum is $\Gamma$. $\tau = \hbar \Gamma^{-1}$ is the time for the electron to tunnel off.
For resonant tunneling near zero bias, i.e. $eV_{sd} < kT$, if $\Gamma$ is very small, $T(E) = \delta(E)$, $I = eV_{sd} \frac{df}{dE}$.
Fermi-Dirac Distribution

Thermal broadening gives width = 3.5kT
Height ~ 1/T

$df/dE$

Thermal Broadening

kT = 0.5
kT = 1
Thermal and Intrinsic Broadening


Dashed line from Fermi alone, solid includes Lorentzian
Absolute Thermometer

When $kT > \Delta \varepsilon$ the peak conductance becomes constant and the width changes slope slightly.

For thermometer application see Pekola et al. PRL 73, 2903 (1994)
Determining $\Gamma$ from peak width

Slope gives conversion of voltage to energy.

$T = 0$ intercept gives $\Gamma$

Note: in this case $\Gamma/h \sim 50\text{GHz}$

Condition for Charge Quantization is Condition for Level Separation

Above Coulomb gap, the current is \( I = N e/\tau \), \( \tau = h\Gamma^{-1} \) and \( N = eV/\Delta\varepsilon \)

\[ G = \frac{I}{V} = \frac{e^2}{h} \left( \frac{\Gamma}{\Delta\varepsilon} \right) \]

\[ G < \frac{e^2}{h} \Rightarrow \Gamma < \Delta\varepsilon \]
Ignore interactions among electrons on artificial atom.
States fill two at a time.
Actually more complicated, but it is a useful starting point.
Energy Scales in SET

Here $\Delta \epsilon > U$ for simplicity.
t is the hopping matrix element between dot and leads.

$E_{F}$

$\Gamma \sim |t|^2 g(E_{F})$

Filled states

$g(E)$
Paired Peaks

High conductance for odd N, low for even N.
Temperature Dependence
T Dependence at Fixed $V_G$

Note logarithmic decrease of conductance with $T$
Comparison with Scaling Theory

\[ T_K = \frac{\sqrt{4U}}{2} e^{\frac{\pi \epsilon_0 (\epsilon_0 + U)}{4U}} \]

Haldane
Charge Measurement

Presentations of Sami Amasha and Kenneth MacLean
Laterally Gated Quantum Dots

$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$

110 nm

GaAs

Si $\delta$-doping

2DEG

GaAs

$n = 2.2 \times 10^{11}$ cm$^{-2}$

$\mu = 6.4 \times 10^5$ cm$^2$/V$\cdot$s

Can trap just one electron

Ciorga et al. PRB 61, 16315 (2000)
Energy Scales

\[ k_B T = 10 \mu eV \]
\[ E_{orb} = 2 \text{ meV} \]
Energy Scales

$k_B T = 10 \, \mu\text{eV}$

$E_{\text{orb}} = 2 \, \text{meV}$

$U = 4 \, \text{meV}$
Measurement of Current

- $\Gamma$ tuned by gate
- $\Gamma \sim 0.01 - 100$ GHz
- $I \sim e \Gamma \sim 10$ fA – 10 nA
Charge Sensing

\[ I = 1 \text{ nA} \]

Conductance

\[ \text{Conductance} \]

\[ \text{Conductance} \]
Measuring Charge

Field et al PRL 70 1311 (1993)
Charge Sensing

For large $\Gamma$ current and charge can be measured simultaneously.
Real-time Charge Sensing


$I_0 = 1 \text{nA}
\text{R}_{qpc} \approx 100 \text{k}\Omega
\text{C}_{coax} \approx 500 \text{pF}
\tau_{\text{rise}} = 50 \mu\text{s}

\begin{align*}
\text{G}_{QPC} & \approx 100 \text{en}\text{a}
\Delta V_g (\text{mV})
\end{align*}

Measure at small $\Gamma$
Measuring Tunneling Rates
Single-Electron Counting

\[ \Gamma/h \text{ can be measured from } 1-1000 \text{ Hz, compared to } 10-50 \text{ GHz from peak shapes} \]
Dependence on Bias Voltage

Demonstrates: tunneling is elastic, exponential dependence on barrier height, ability to measure excited states.

Summary

• Measure energy scales of quantum dots
  – $U$, $\Delta\epsilon$ (or $E_{\text{orb}}$), $\Gamma$, $kT_K$

• Measure charge instead of current
  – Access much smaller $\Gamma$
  – From charge with dc bias we see evidence for dominance of elastic tunneling