Optomechanics - light and motion in the nanoworld

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[just moved from: Ludwig-Maximilians-Universität München]

University of Erlangen-Nuremberg and
Max-Planck Institute for the Physics of Light (Erlangen)

DIP project "Dynamics of Electrons and Collective Modes in Nanostructures"

SFB | TR12

SFB 631

nanosystems initiative munich

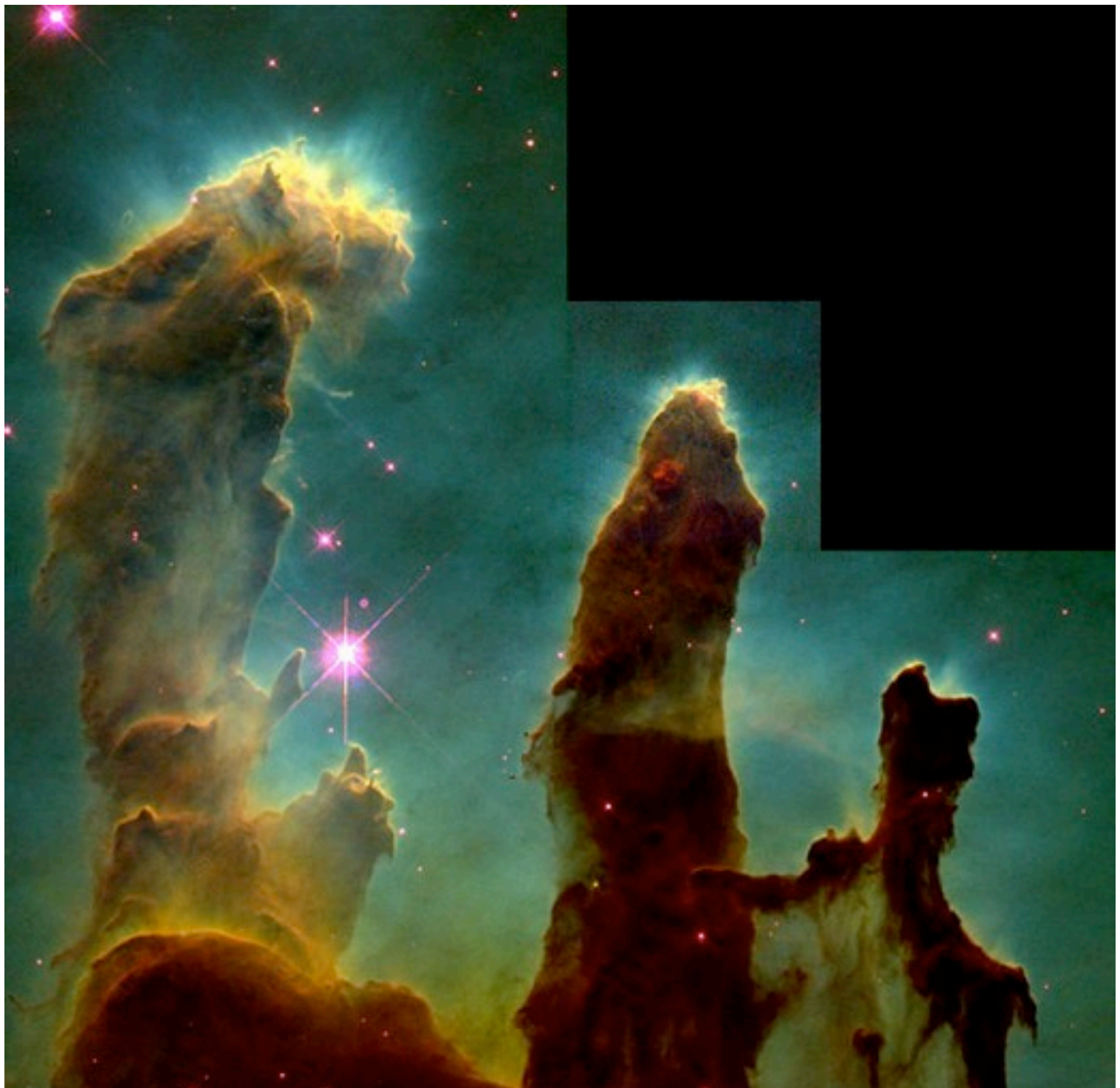
Emmy Noether-Programm
Deutsche Forschungsgemeinschaft
DFG

Radiation pressure

(Comet Hale-Bopp; by Robert Allevo)

Johannes Kepler
De Cometis, 1619

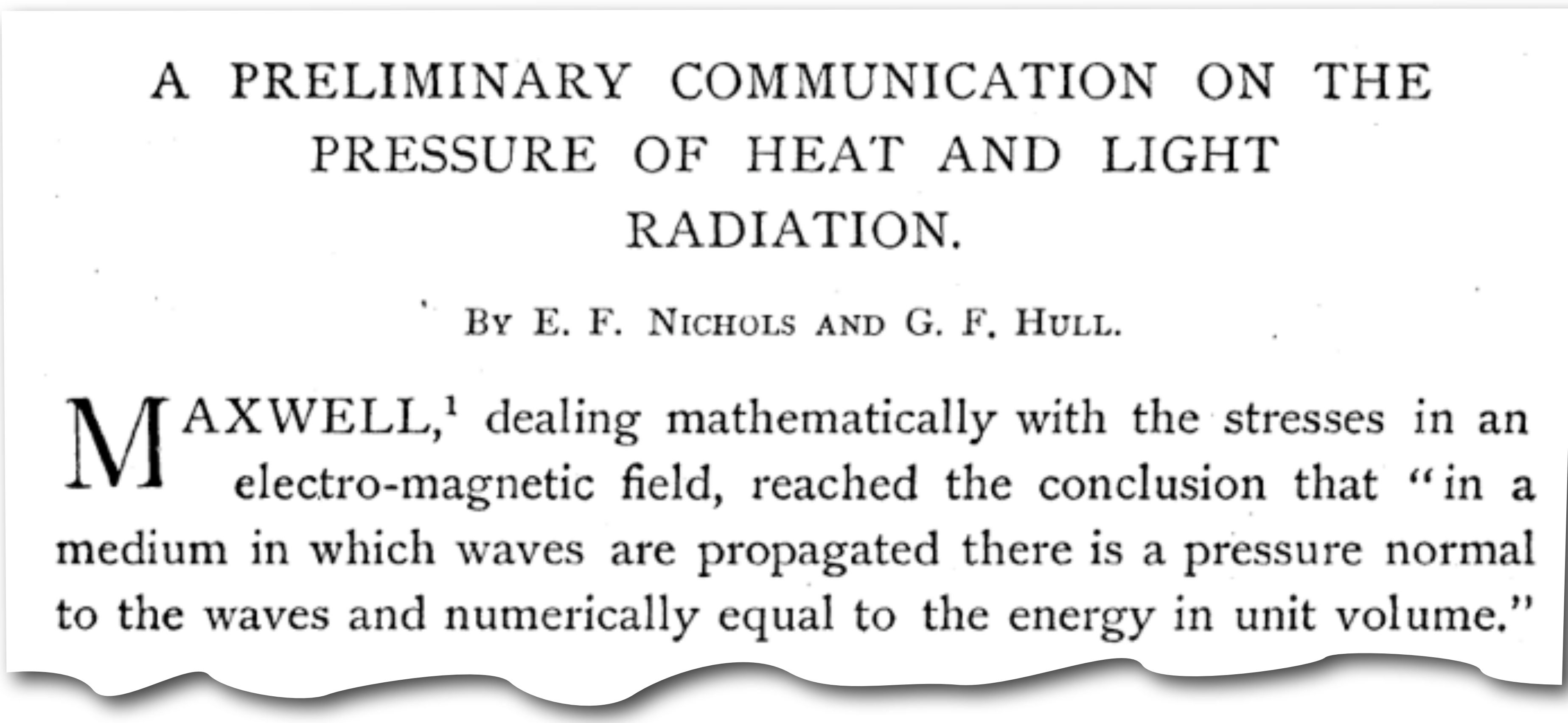
Radiation pressure



Johannes Kepler
De Cometis, 1619

Radiation pressure

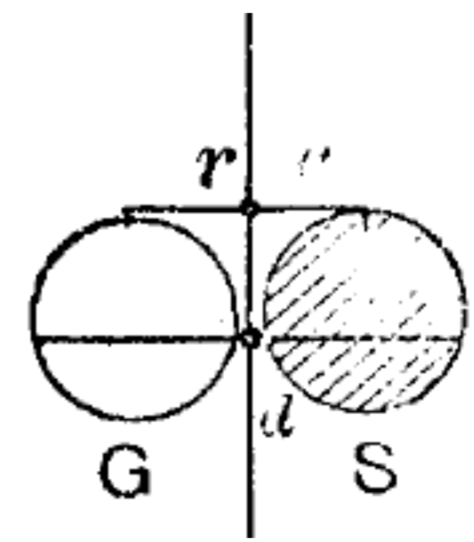
Nichols and Hull, 1901
Lebedev, 1901



A PRELIMINARY COMMUNICATION ON THE PRESSURE OF HEAT AND LIGHT RADIATION.

BY E. F. NICHOLS AND G. F. HULL.

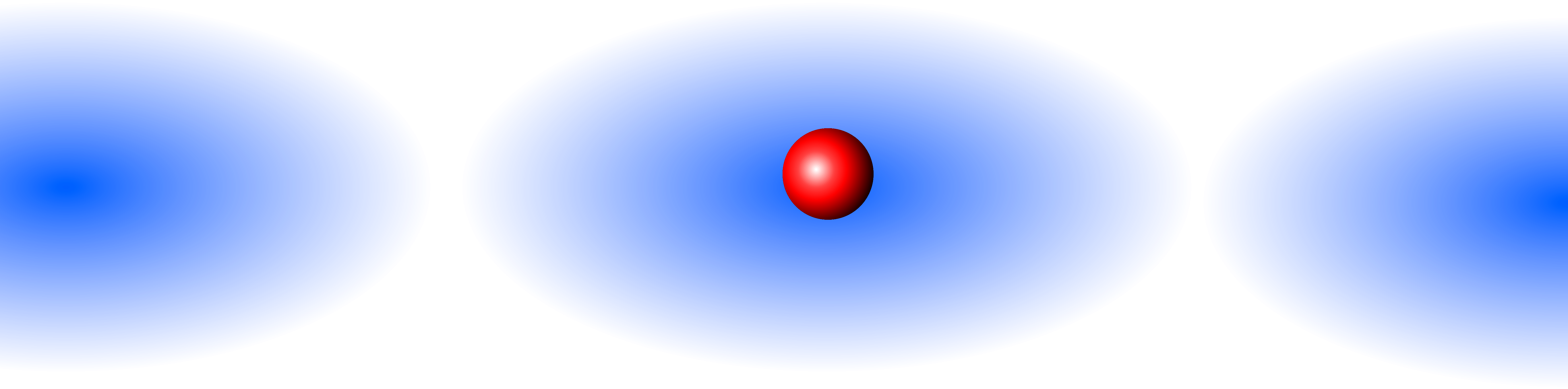
MAXWELL,[1] dealing mathematically with the stresses in an electro-magnetic field, reached the conclusion that "in a medium in which waves are propagated there is a pressure normal to the waves and numerically equal to the energy in unit volume."



$$F = \frac{2I}{c}$$

Nichols and Hull, Physical Review **13**, 307 (1901)

Radiation forces



Trapping and cooling
- Optical tweezers
- Optical lattices

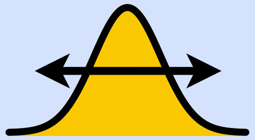
Optomechanics on different length scales

LIGO – Laser Interferometer Gravitational Wave Observatory

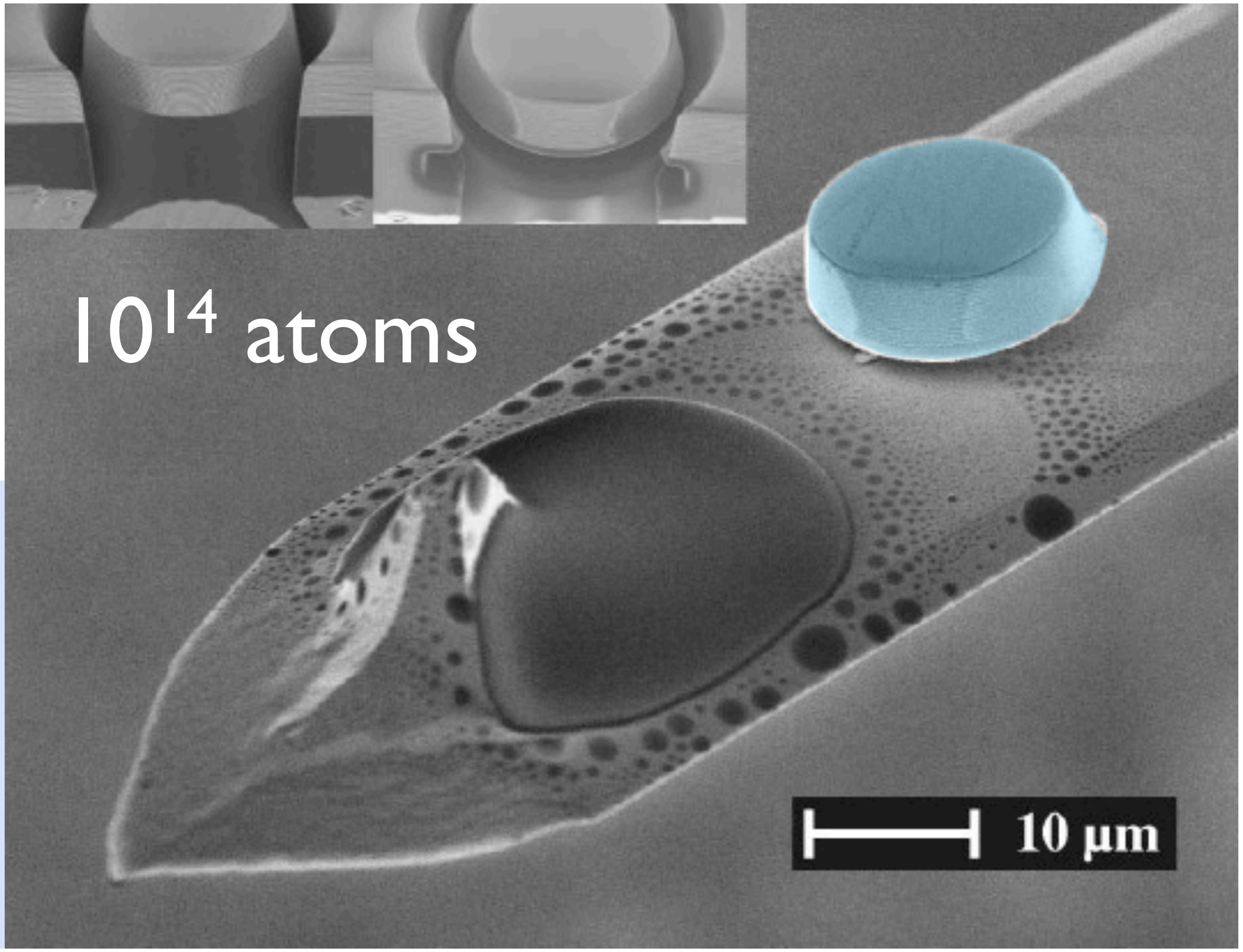
Optomechanics on different length scales

LIGO – Laser Interferometer Gravitational Wave Observatory

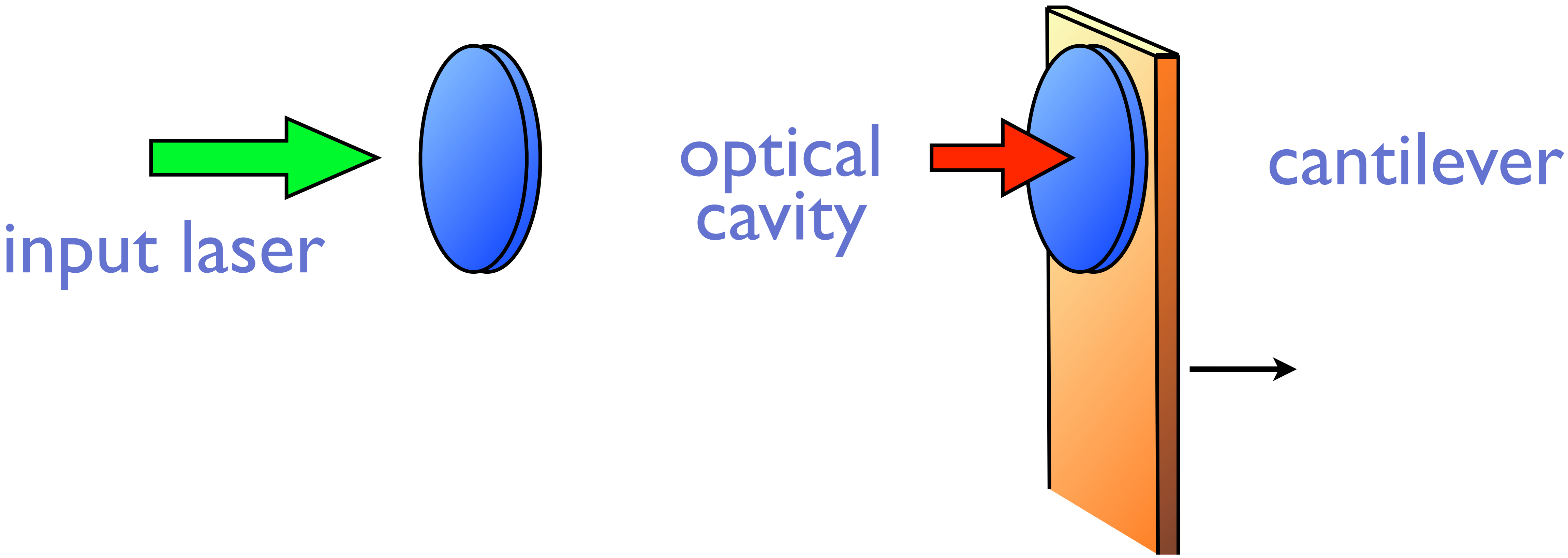
$$\omega_M \sim 1\text{kHz} - 100\text{MHz}$$
$$m \sim 10^{-12} - 10^{-10}\text{kg}$$
$$x_{\text{ZPF}} \sim 10^{-16} - 10^{-14}\text{m}$$
$$x_{\text{ZPF}} = \sqrt{\hbar/(2m\omega_M)}$$



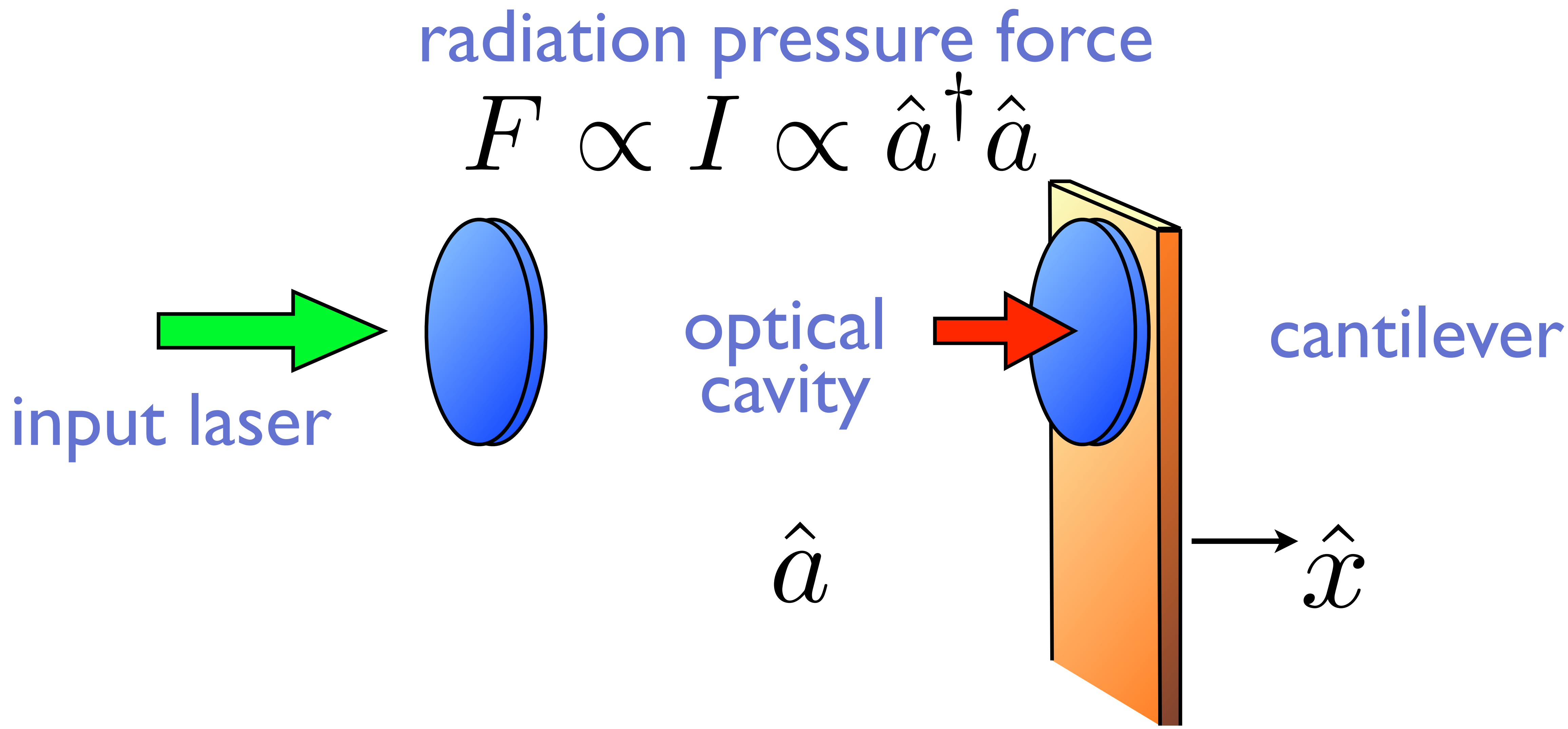
Mirror on cantilever – Bouwmeester lab, Santa Barbara



Optomechanical systems



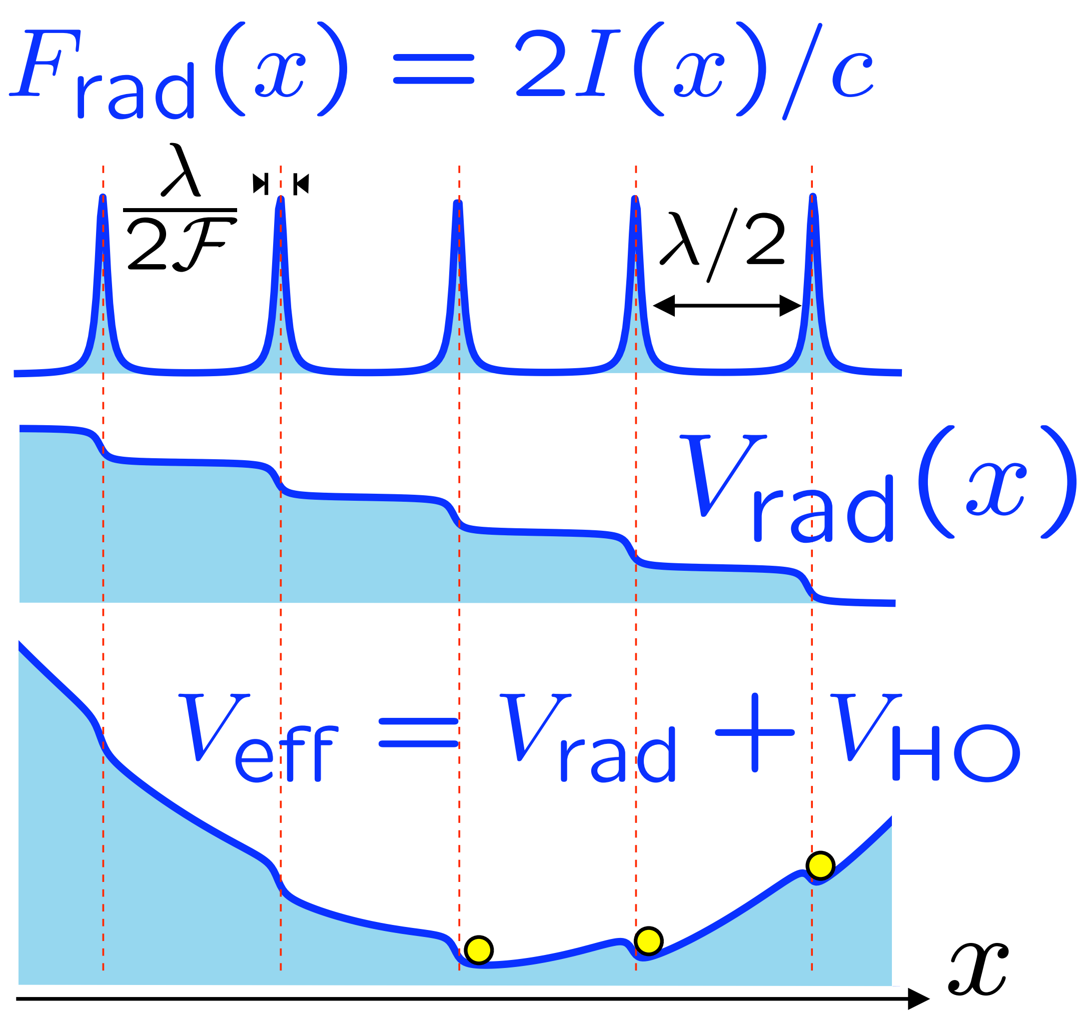
Optomechanical systems



$$\hat{H} = \hbar\omega_R(1 - \hat{x}/L)\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b} + \dots$$

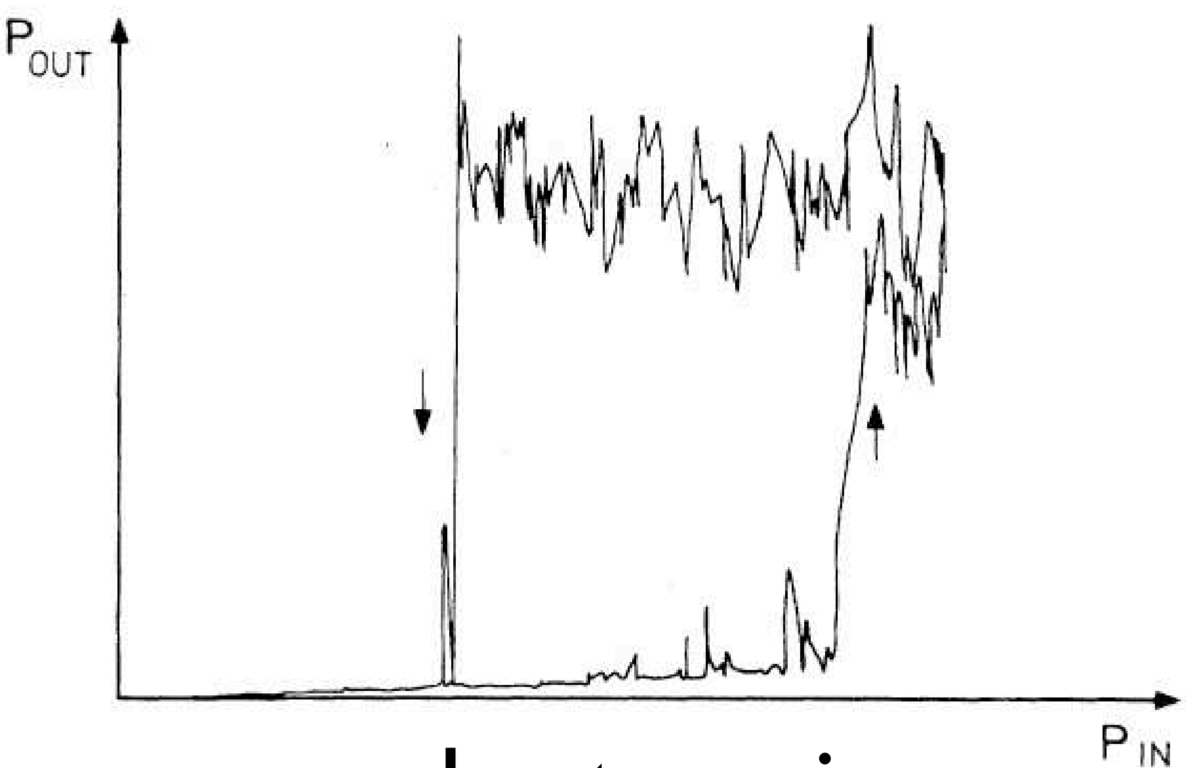
$$\hat{x} = x_{\mathrm{ZPF}}(\hat{b}^\dagger + \hat{b})$$

Basic physics: Statics



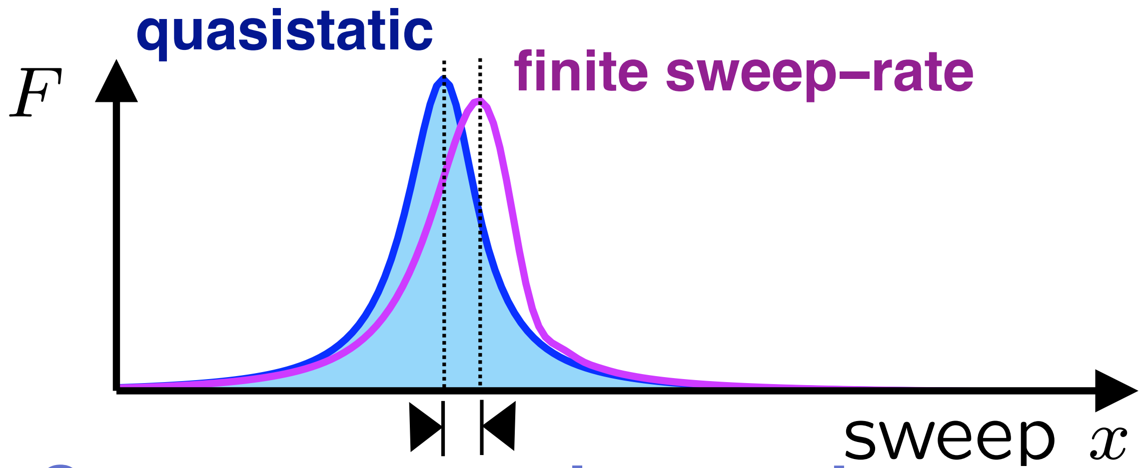
Experimental proof of static bistability:

A. Dorsel, J. D. McCullen, P. Meystre, E. Vignes and H. Walther: Phys. Rev. Lett. 51, 1550 (1983)

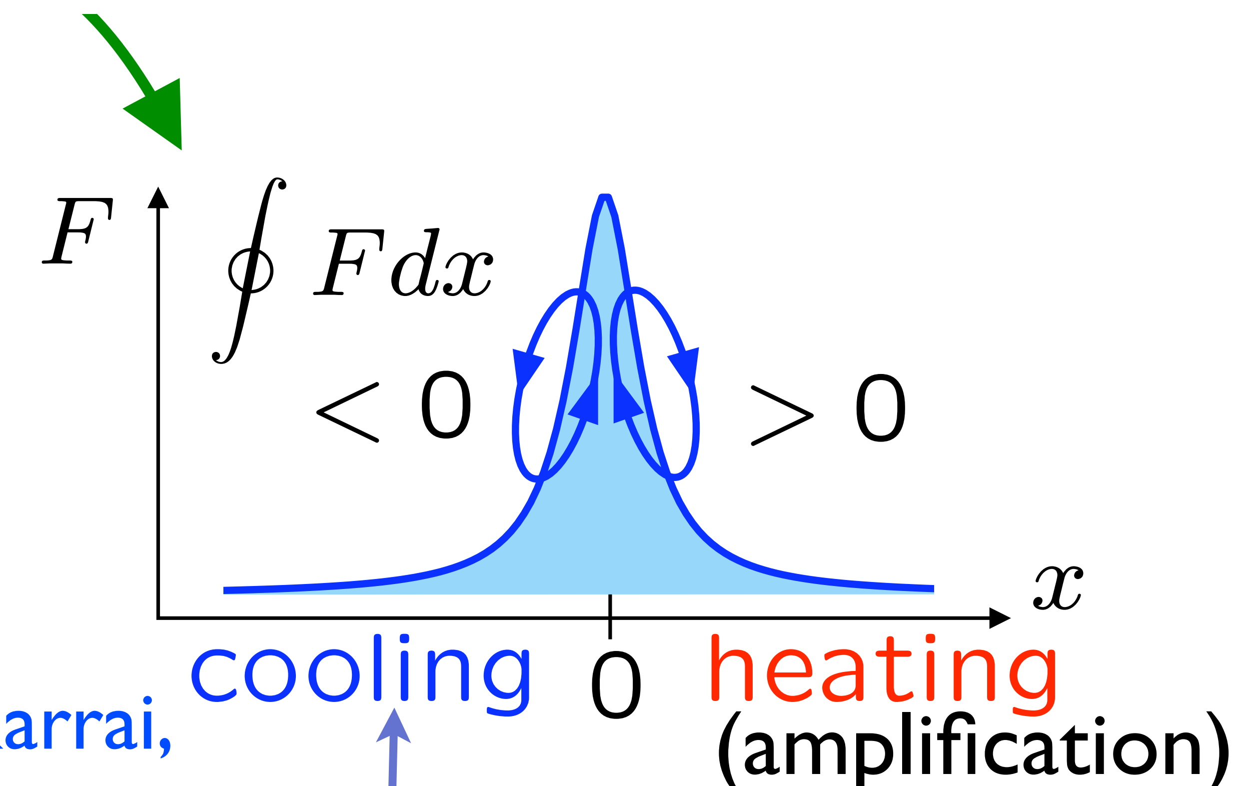


hysteresis

Basic physics: dynamics

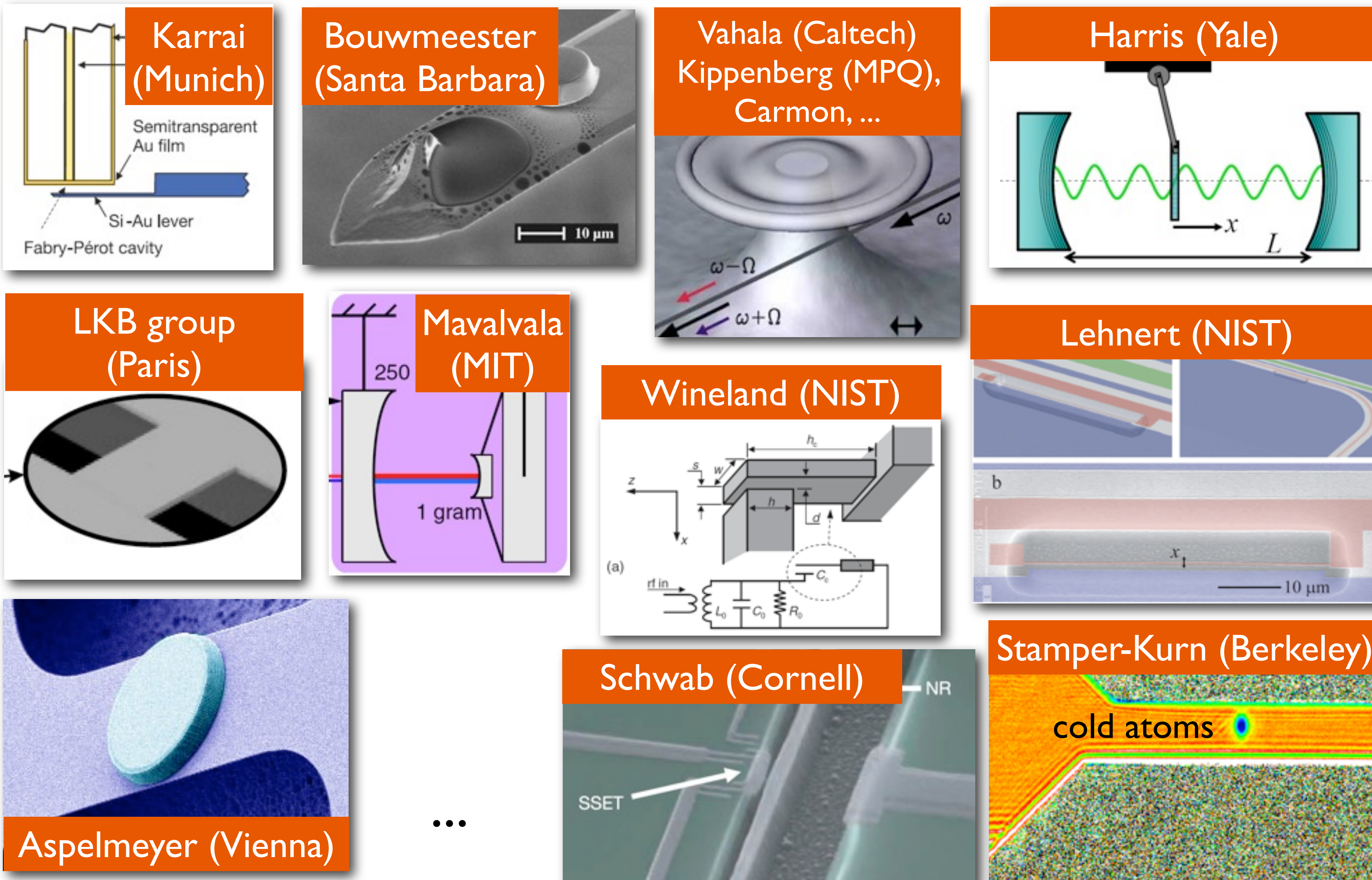


finite optical ringdown time κ^{-1} –
delayed response to cantilever motion

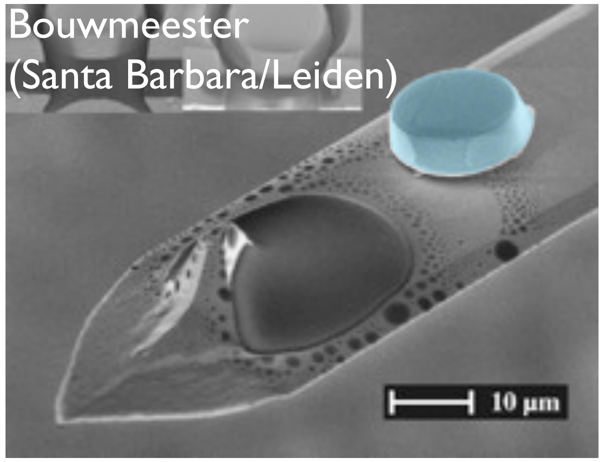


Höhberger-Metzger and Karrai,
Nature **432**, 1002 (2004):
300K to 17K [photothermal force]

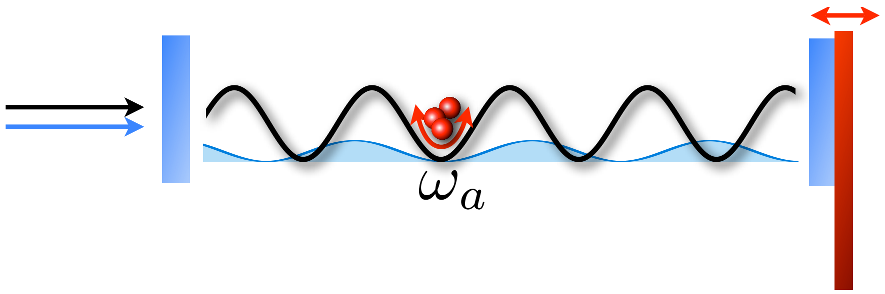
The zoo of optomechanical (and analogous) systems



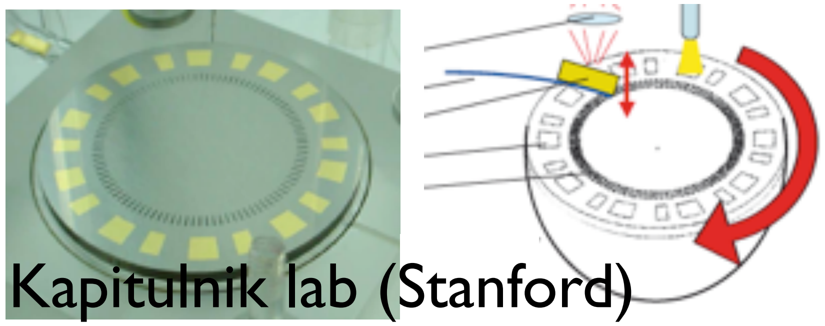
Optomechanics: general outlook



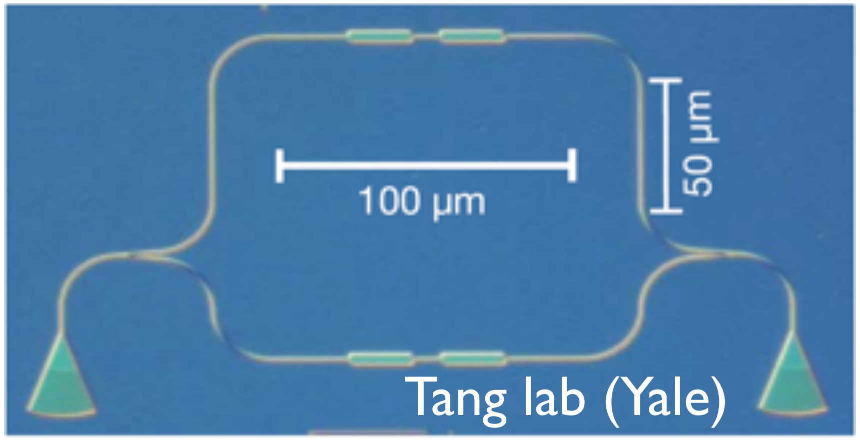
Fundamental tests of quantum mechanics in a new regime: entanglement with 'macroscopic' objects, unconventional decoherence? [e.g.: gravitationally induced?]



Mechanics as a 'bus' for connecting hybrid components: superconducting qubits, spins, photons, cold atoms, ...

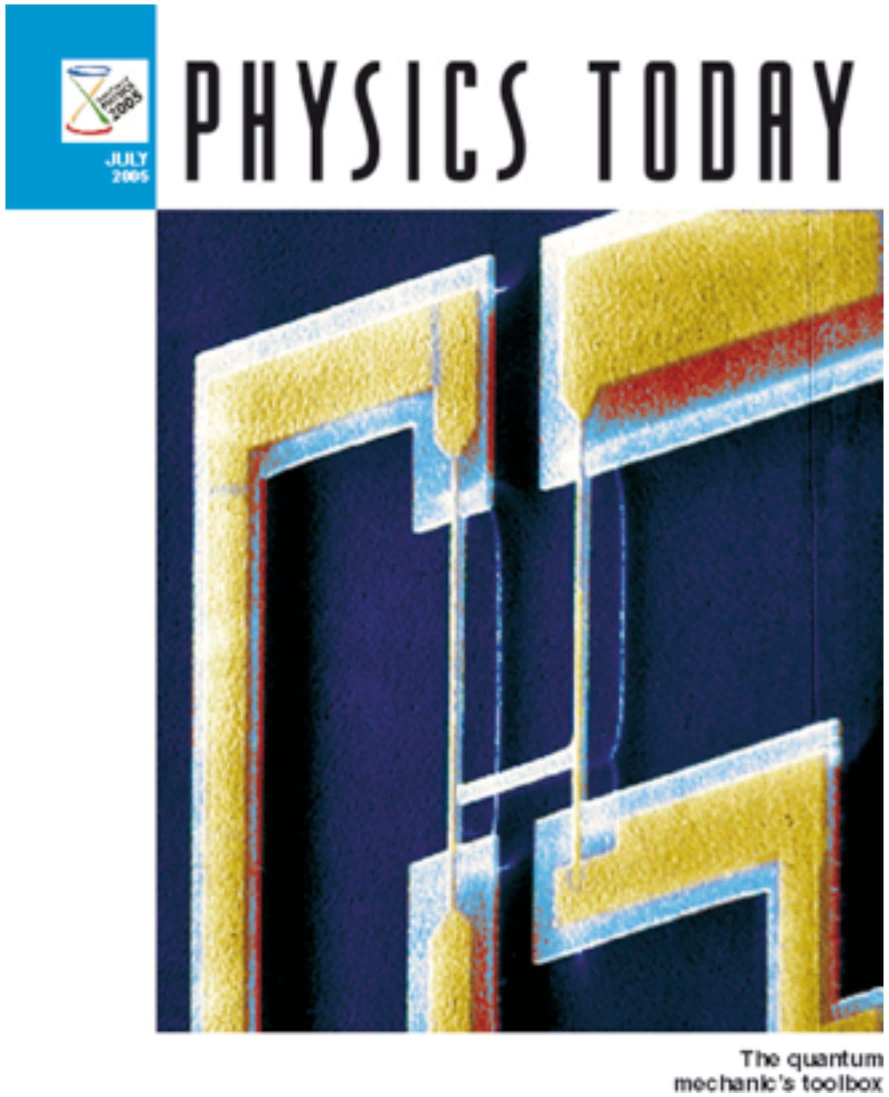


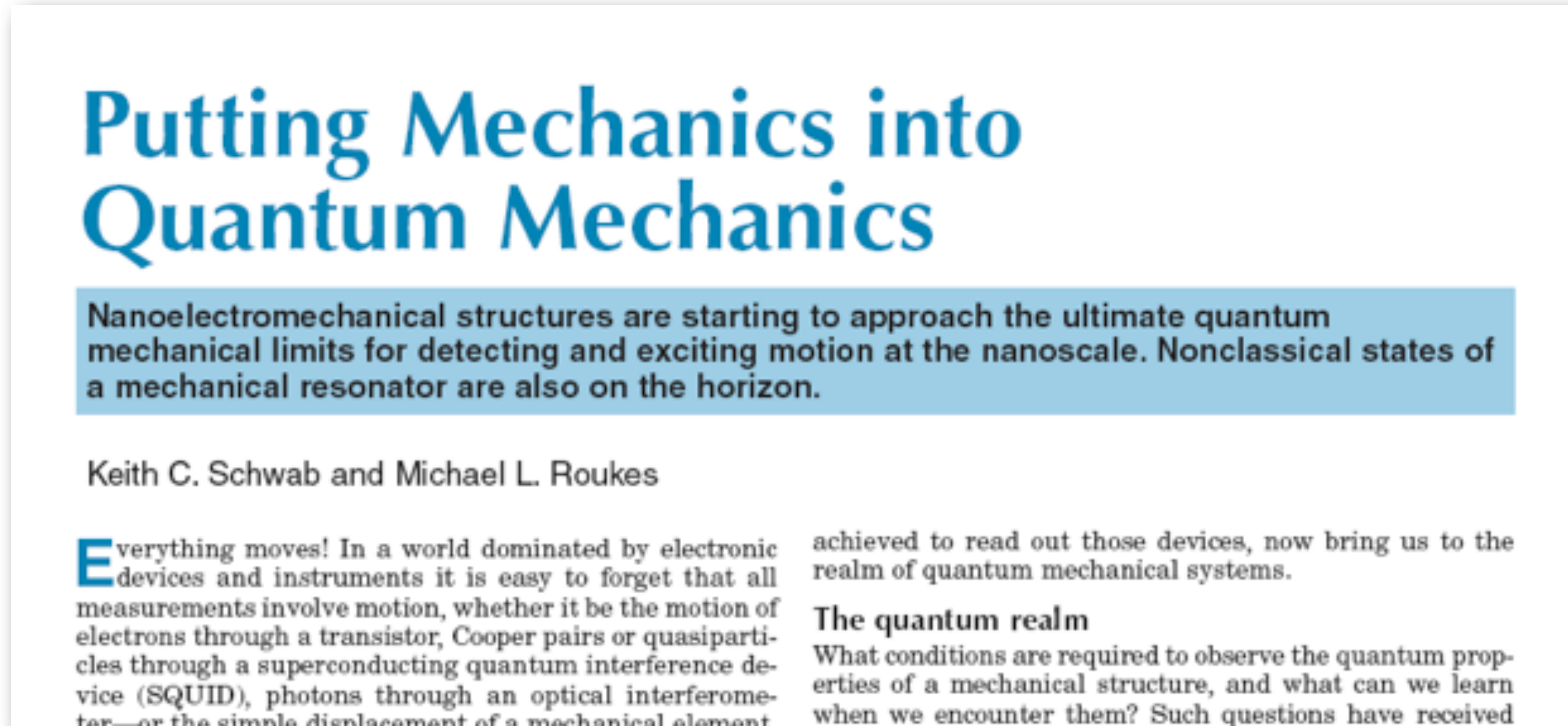
Precision measurements
[e.g. testing deviations from Newtonian gravity due to extra dimensions]



Optomechanical circuits & arrays
Exploit nonlinearities for classical and quantum information processing, storage, and amplification; study collective dynamics in arrays

Towards the quantum regime of mechanical motion





Putting Mechanics into Quantum Mechanics

Nanoelectromechanical structures are starting to approach the ultimate quantum mechanical limits for detecting and exciting motion at the nanoscale. Nonclassical states of a mechanical resonator are also on the horizon.

Keith C. Schwab and Michael L. Roukes

Everything moves! In a world dominated by electronic devices and instruments it is easy to forget that all measurements involve motion, whether it be the motion of electrons through a transistor, Cooper pairs or quasiparticles through a superconducting quantum interference device (SQUID), photons through an optical interferometer—or the simple displacement of a mechanical element. achieved to read out those devices, now bring us to the realm of quantum mechanical systems.

The quantum realm

What conditions are required to observe the quantum properties of a mechanical structure, and what can we learn when we encounter them? Such questions have received

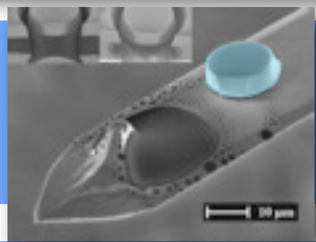
Schwab and Roukes, Physics Today 2005

- nano-electro-mechanical systems

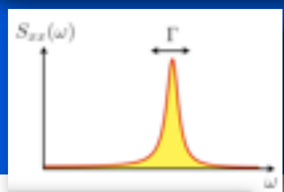
Superconducting qubit coupled to nanoresonator: Cleland & Martinis 2010

- optomechanical systems

Optomechanics (Outline)

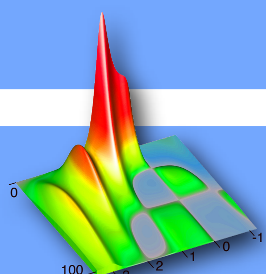


Introduction

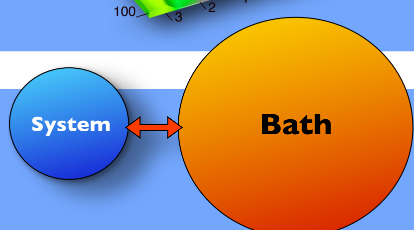


Displacement detection

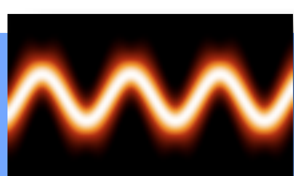
Linear optomechanics



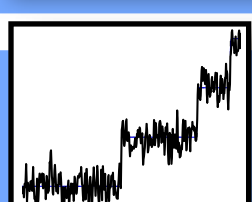
Nonlinear dynamics



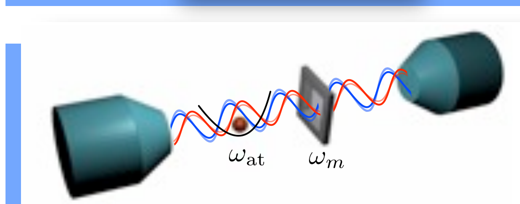
Quantum theory of cooling



Interesting quantum states

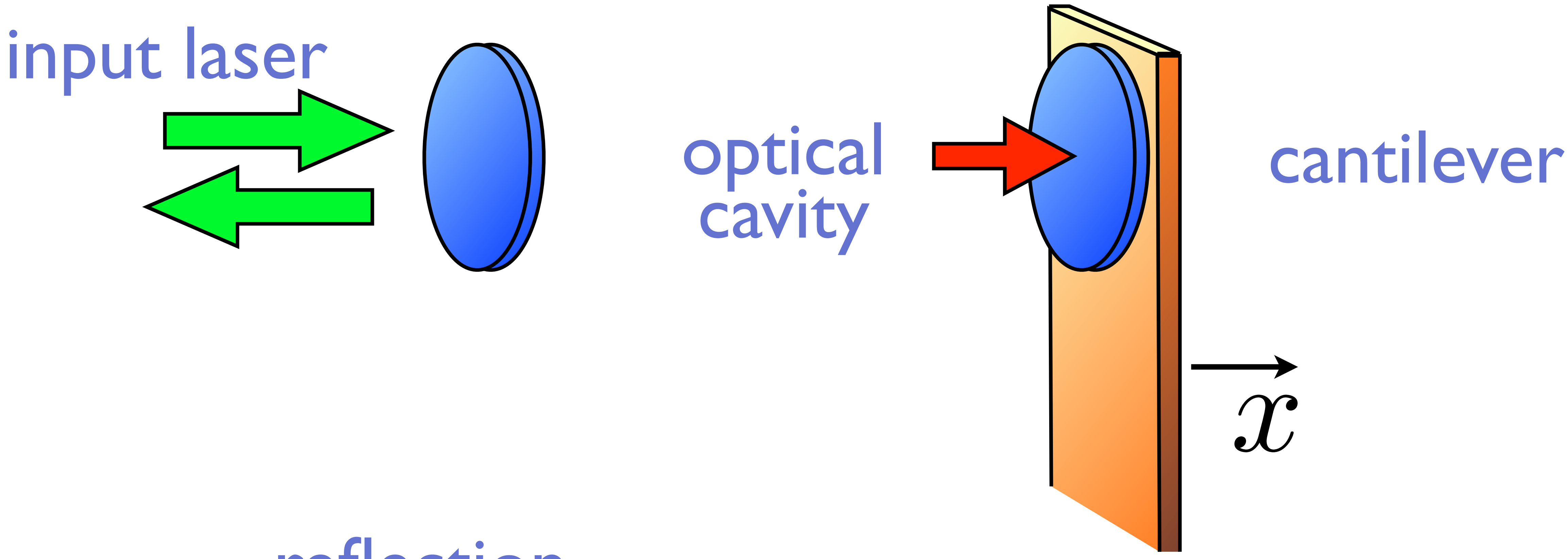


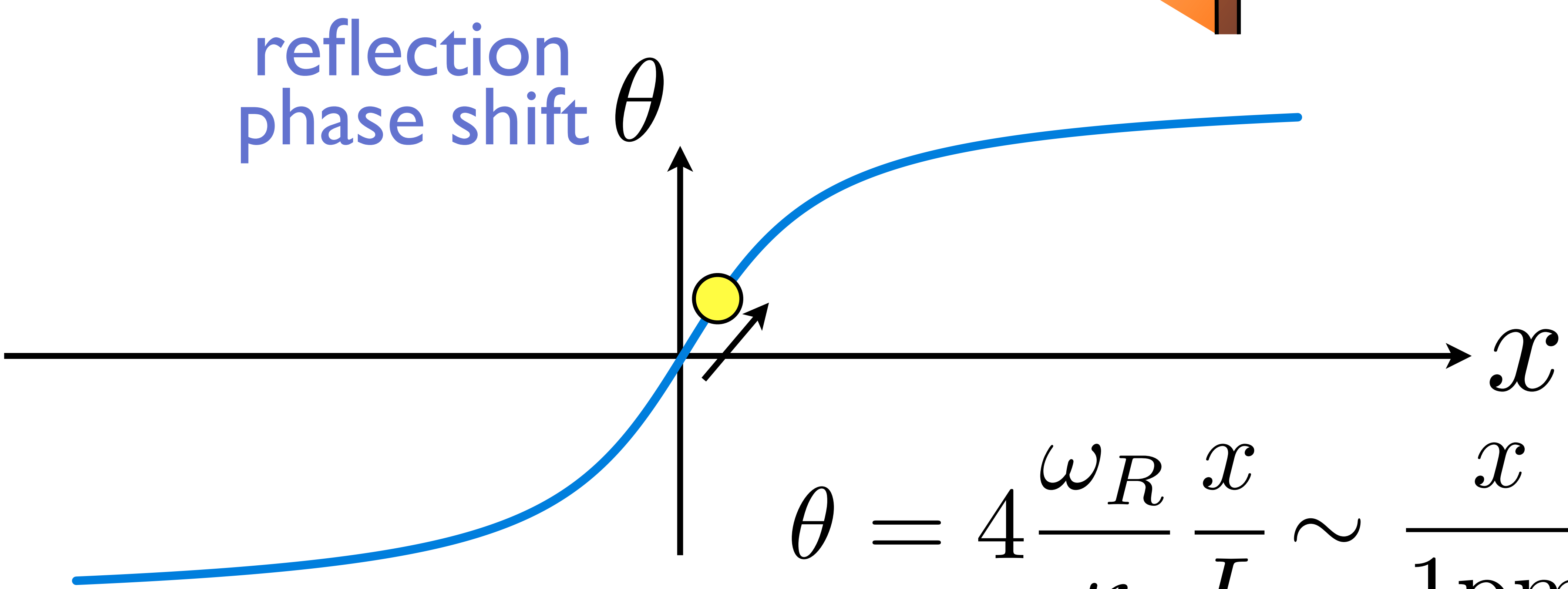
Towards Fock state detection



Coupling to the motion of a single atom

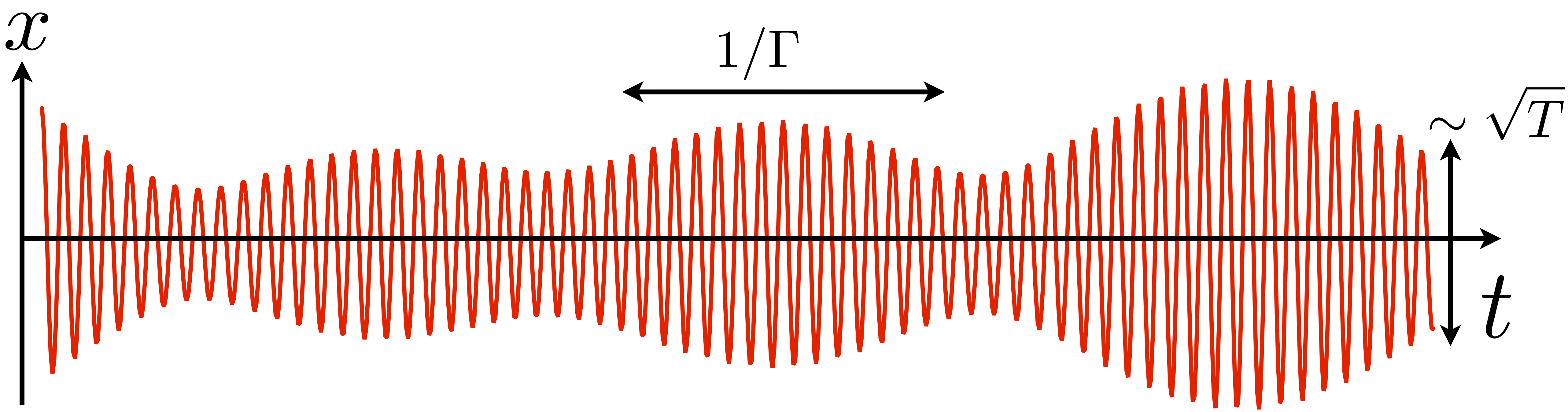
Optical displacement detection





$$\theta = 4\frac{\omega_R}{\kappa}\frac{x}{L} \sim \frac{x}{1\mathrm{pm}}$$

Thermal fluctuations of a harmonic oscillator



Classical equipartition theorem:

$$\frac{m\omega_M^2}{2}\left\langle x^2\right\rangle = \frac{k_B T}{2} \quad \Rightarrow \quad \left\langle x^2\right\rangle = \frac{k_B T}{m\omega_M^2}$$

extract temperature!

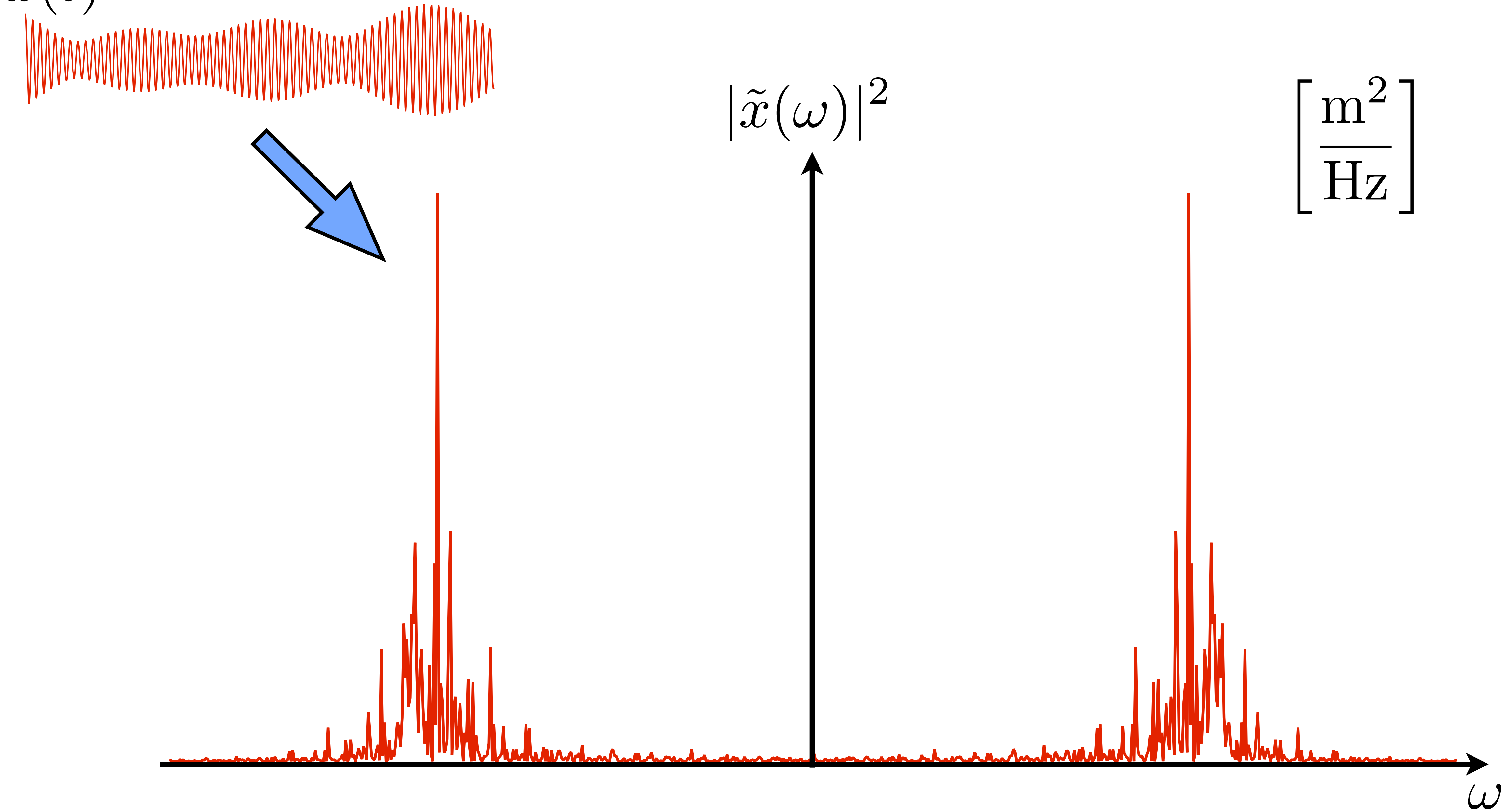
Possibilities:

- Direct time-resolved detection
- Analyze **fluctuation spectrum of x**

Fluctuation spectrum

$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^\tau dt e^{i\omega t} x(t)$$

$x(t)$



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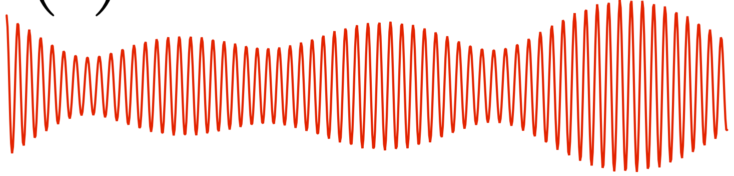
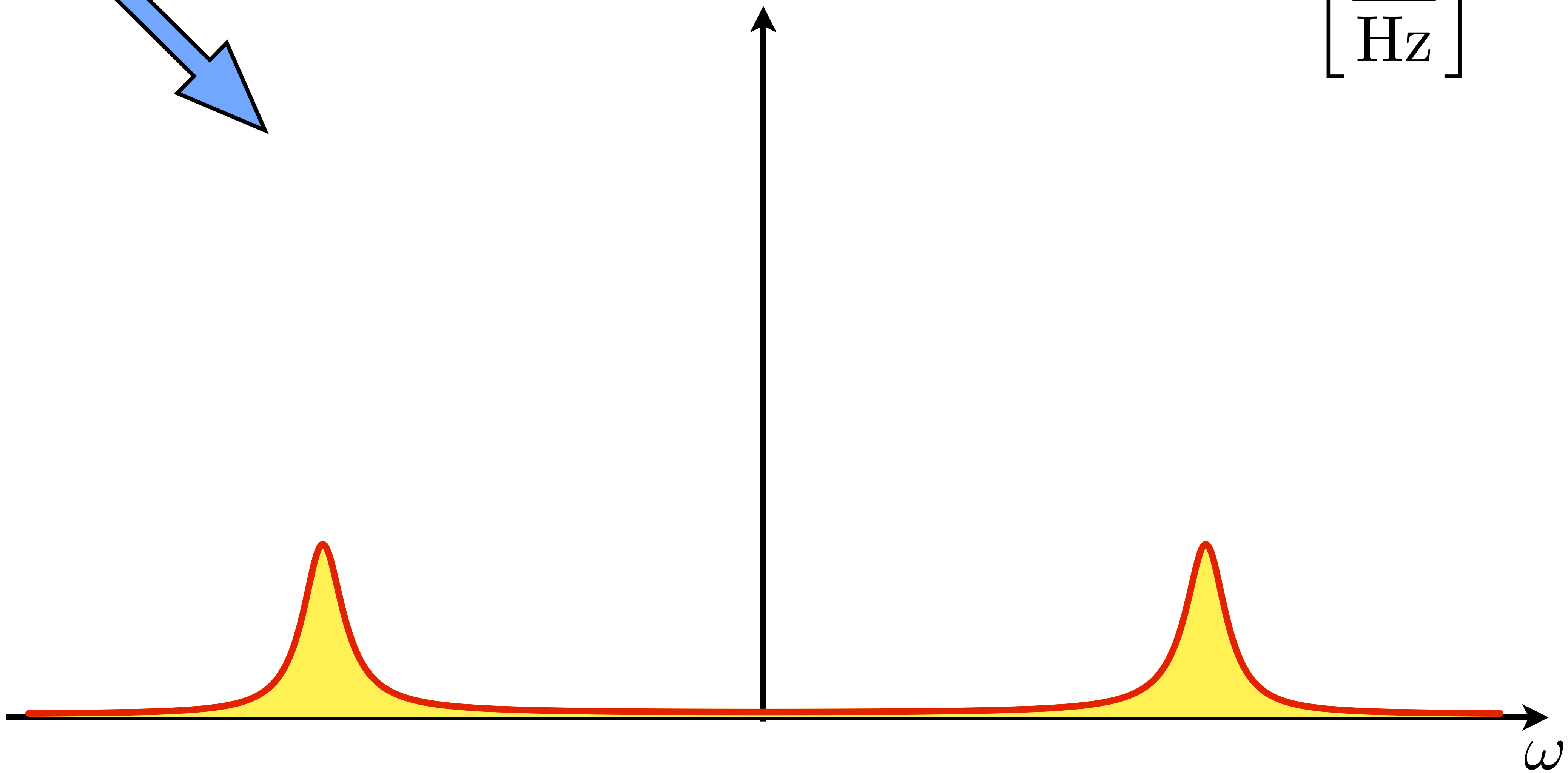


$$\langle |\tilde{x}(\omega)|^2 \rangle \equiv S_{xx}(\omega) \qquad \left[\frac{\mathrm{m}^2}{\mathrm{Hz}}\right]$$



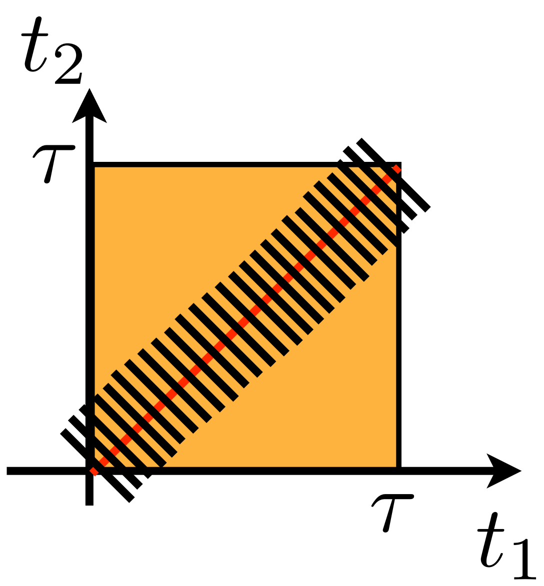
Fluctuation spectrum

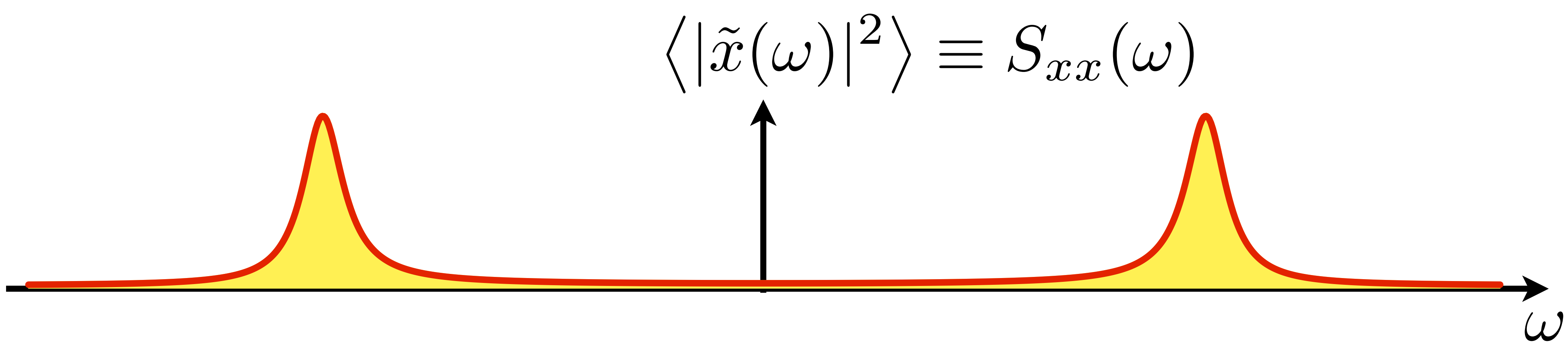
$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^\tau dt e^{i\omega t} x(t)$$

$$S_{xx}(\omega) \equiv \left\langle |\tilde{x}(\omega)|^2 \right\rangle =$$

$$\frac{1}{\tau} \int_0^\tau dt_1 \int_0^\tau dt_2 e^{i\omega(t_2 - t_1)} \left\langle x(t_2) x(t_1) \right\rangle$$

$$\approx \frac{1}{\tau} \tau \int_{-\infty}^{+\infty} dt e^{i\omega t} \left\langle x(t) x(0) \right\rangle$$





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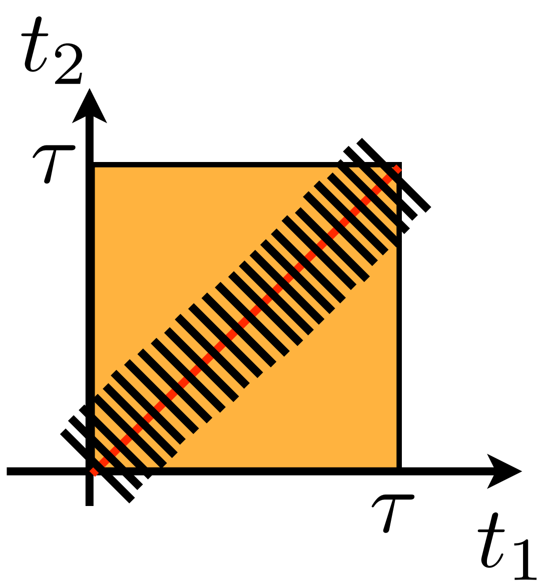
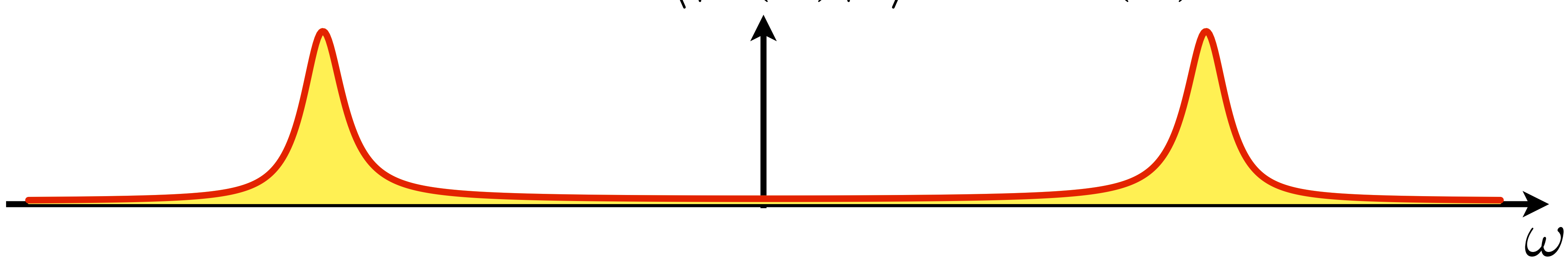


"Wiener-Khinchin theorem"

$$\left\langle |\tilde{x}(\omega)|^2 \right\rangle \equiv S_{xx}(\omega)$$



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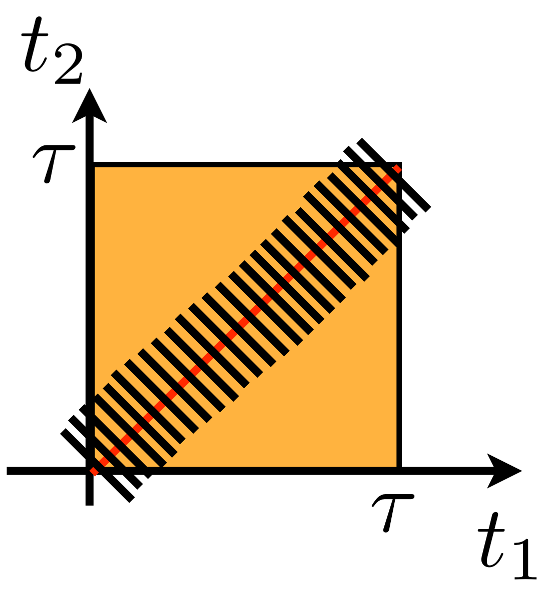

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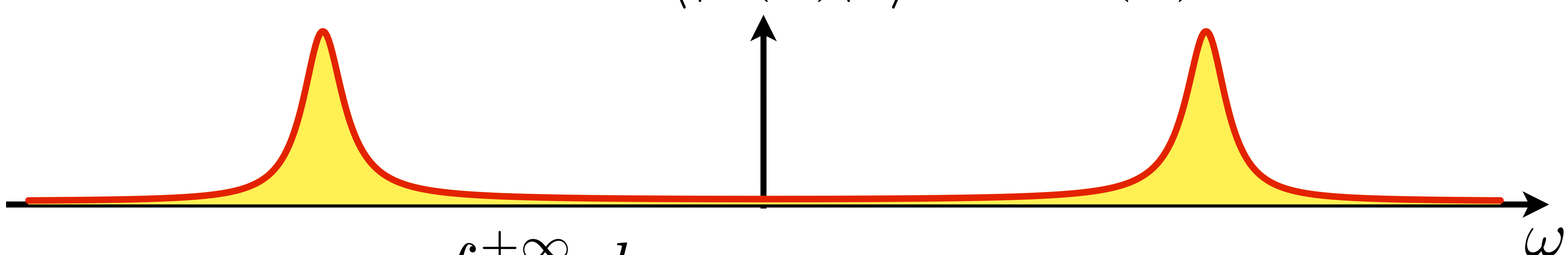

area yields variance of x:

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_{xx}(\omega) = \left\langle x^2 \right\rangle$$

Fluctuation-dissipation theorem

General relation between noise spectrum and linear response susceptibility

$$\langle \delta x \rangle (\omega) = \chi_{xx}(\omega) F(\omega)$$

susceptibility

$$S_{xx}(\omega) = \frac{2k_B T}{\omega} \mathrm{Im}\chi_{xx}(\omega)$$

(classical limit)

Fluctuation-dissipation theorem

General relation between noise spectrum and linear response susceptibility

$$\langle \delta x \rangle (\omega) = \chi_{xx}(\omega) F(\omega)$$

susceptibility

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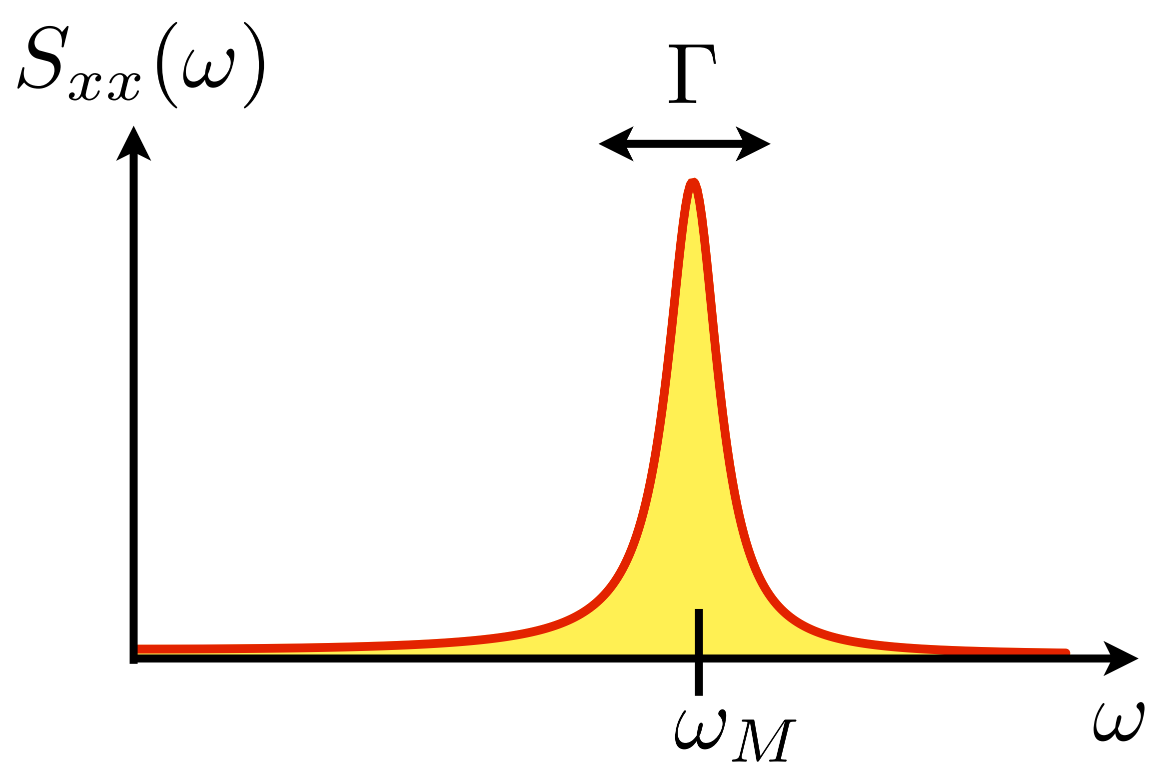
(classical limit)

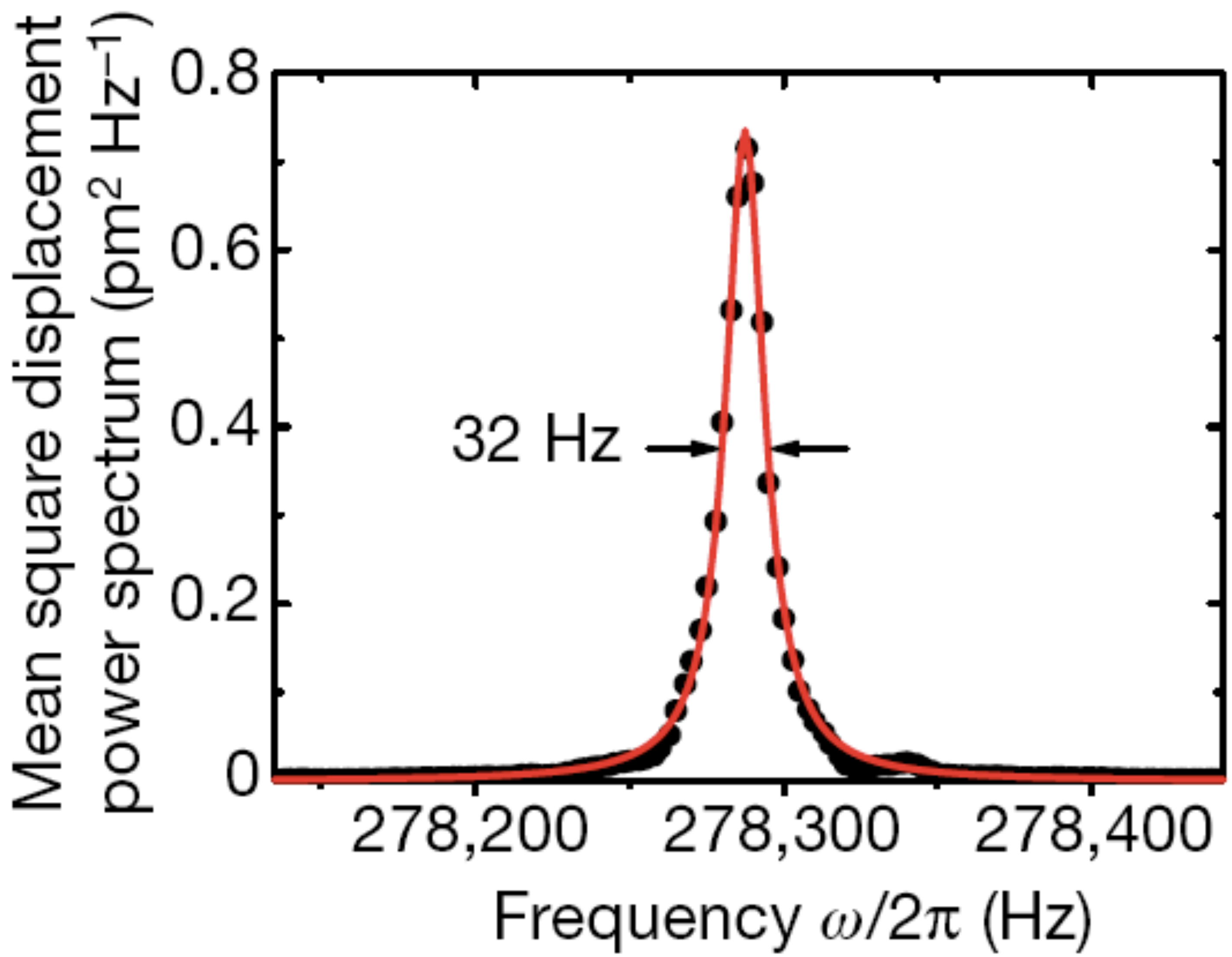
for the damped oscillator:

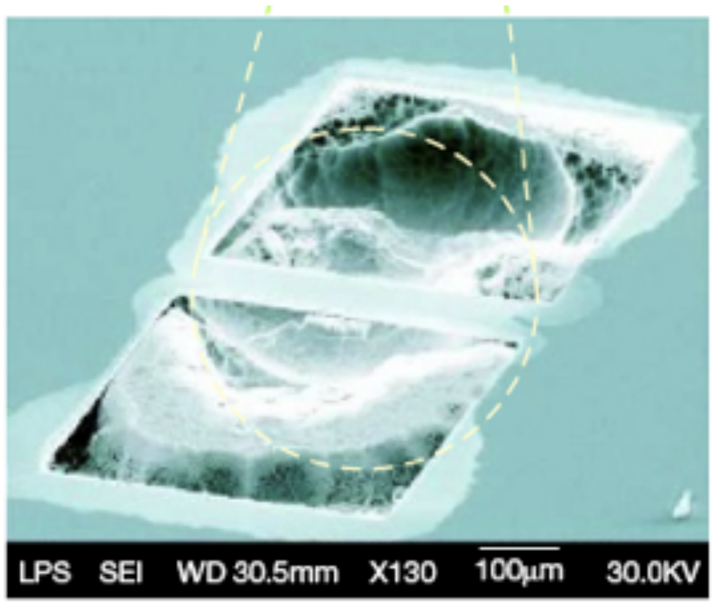
$$m\ddot{x} + m\omega_M^2 x + m\Gamma\dot{x} = F$$

$$x(\omega) = \underbrace{\frac{1}{m(\omega_M^2 - \omega^2) - im\Gamma\omega}}_{\chi_{xx}(\omega)} F(\omega)$$



Displacement spectrum

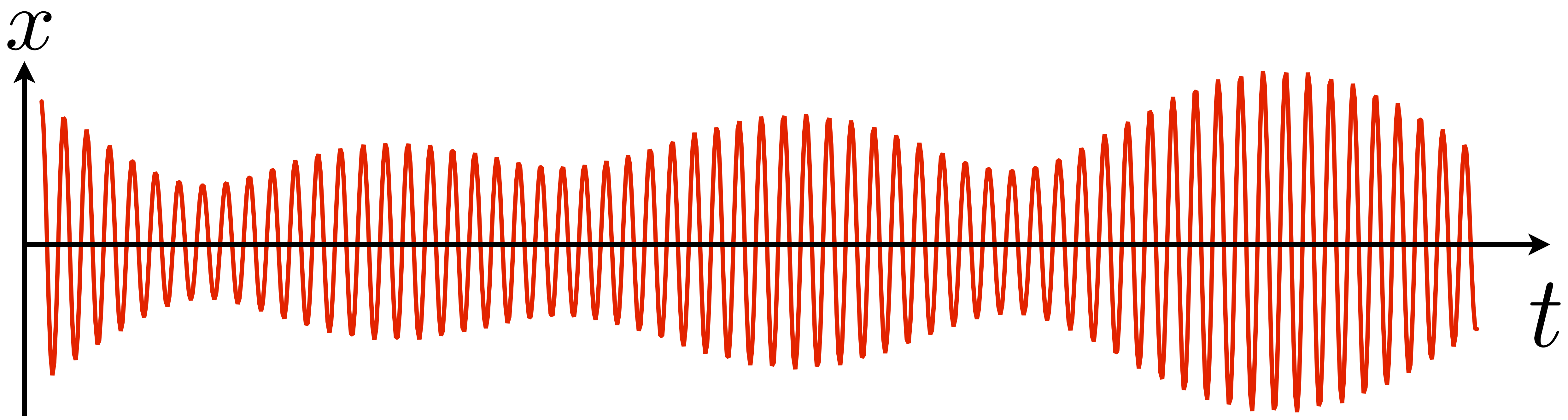




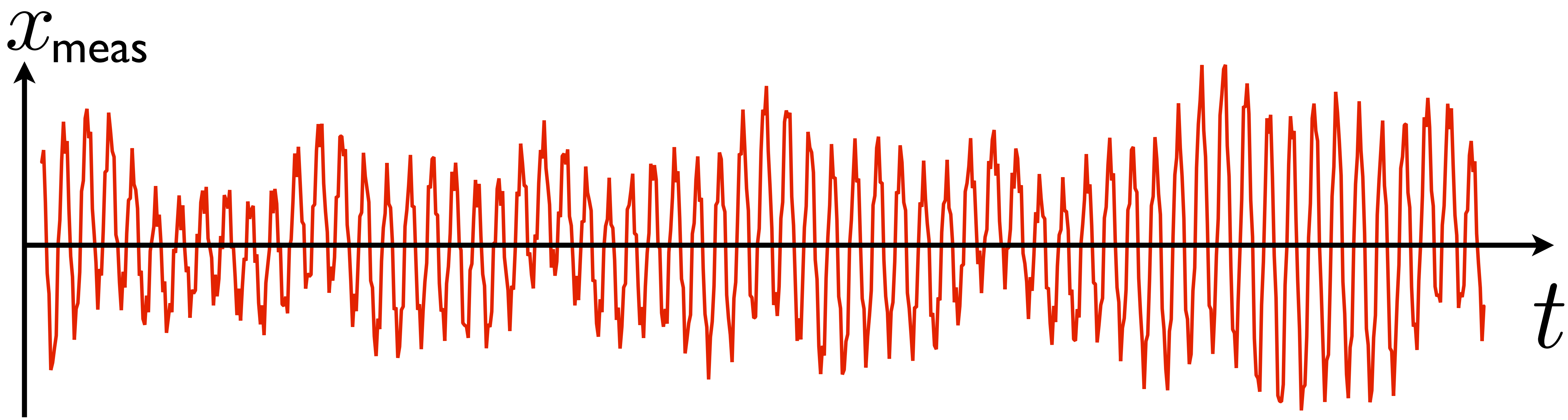
T=300 K

Experimental curve:
Gigan et al., Nature 2006

Measurement noise



Measurement noise

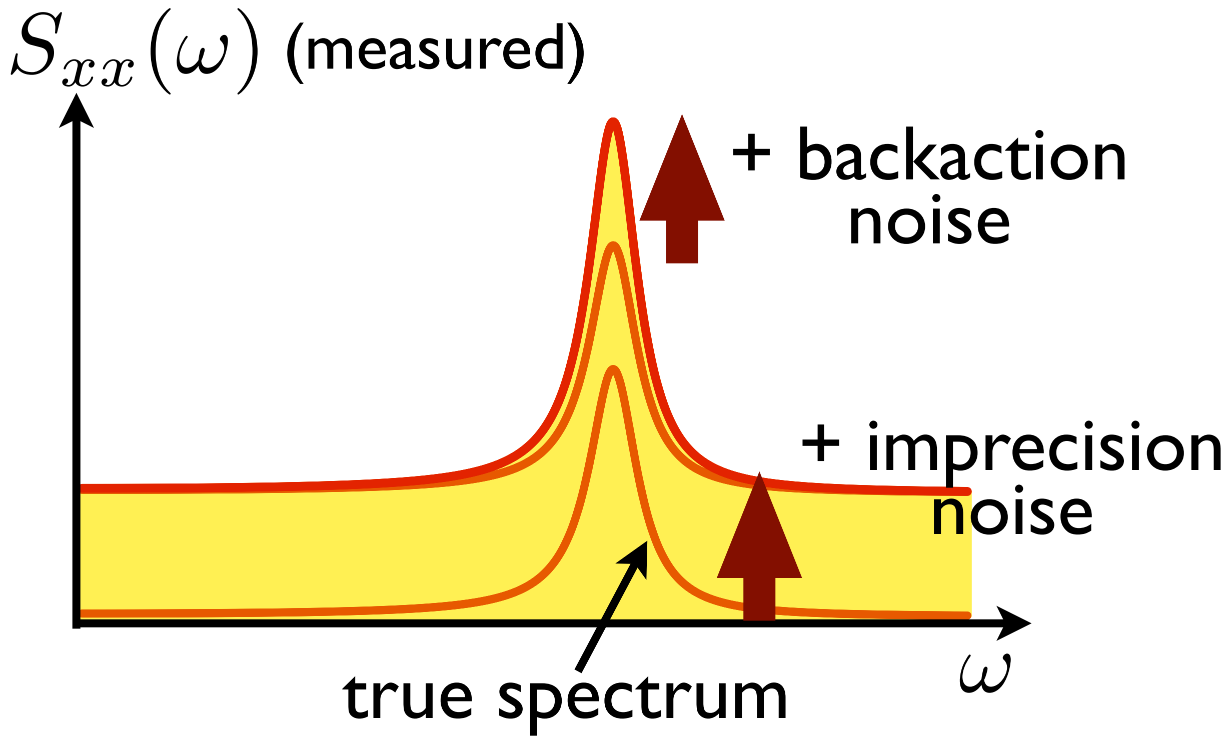


$$x_{\text{meas}}(t) = x(t) + x_{\text{noise}}(t)$$

Two contributions to $x_{\text{noise}}(t)$

1. measurement imprecision — phase noise of laser beam (shot noise limit!)
2. measurement back-action: fluctuating force on system — noisy radiation pressure force

“Standard Quantum Limit”



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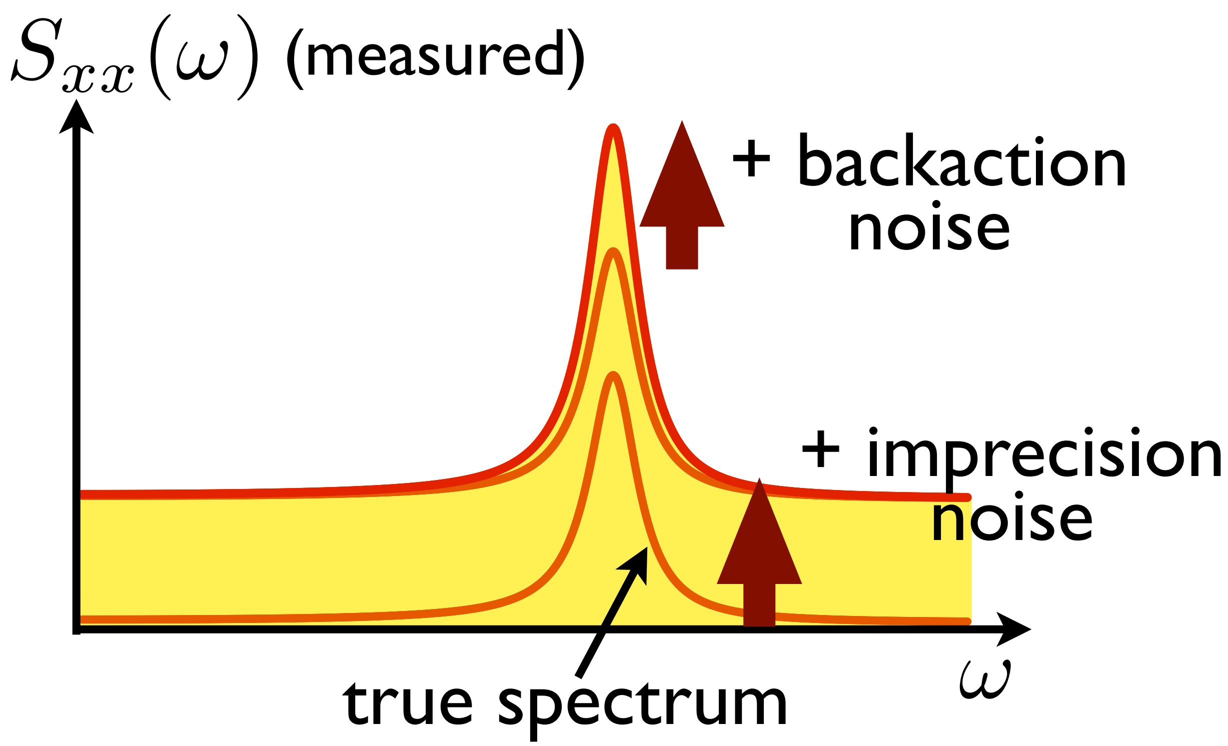

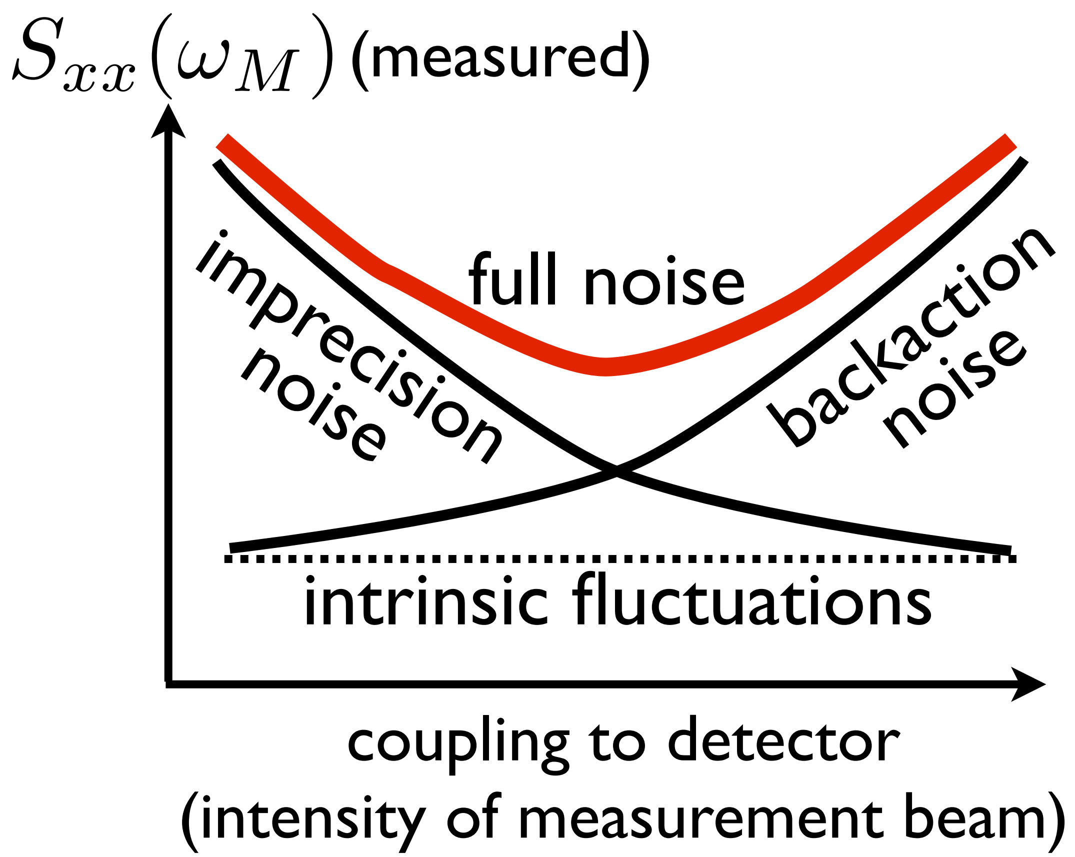

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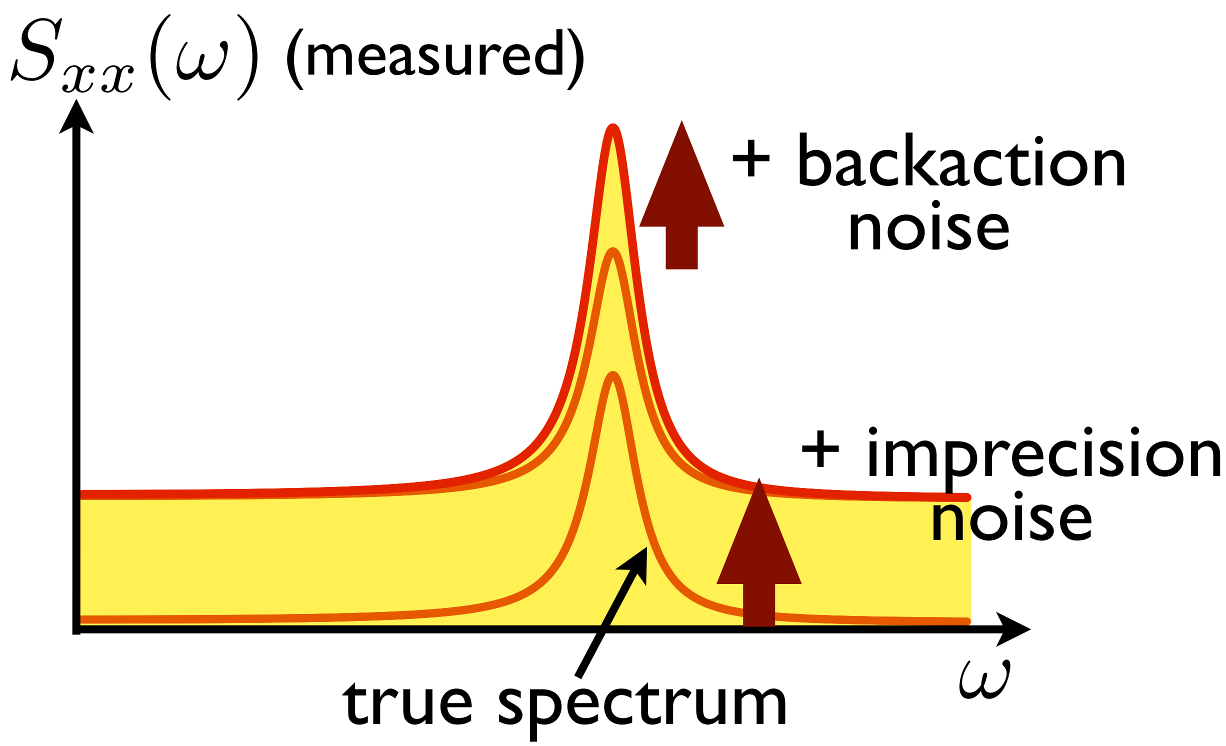
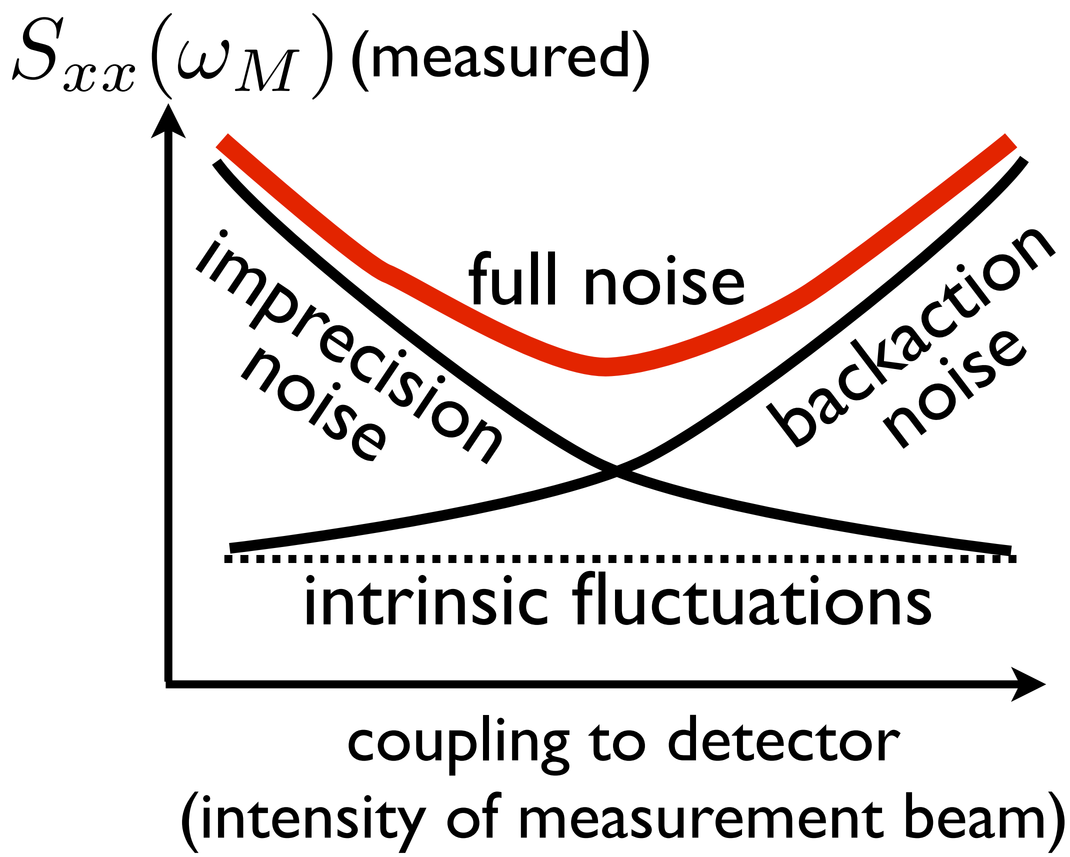




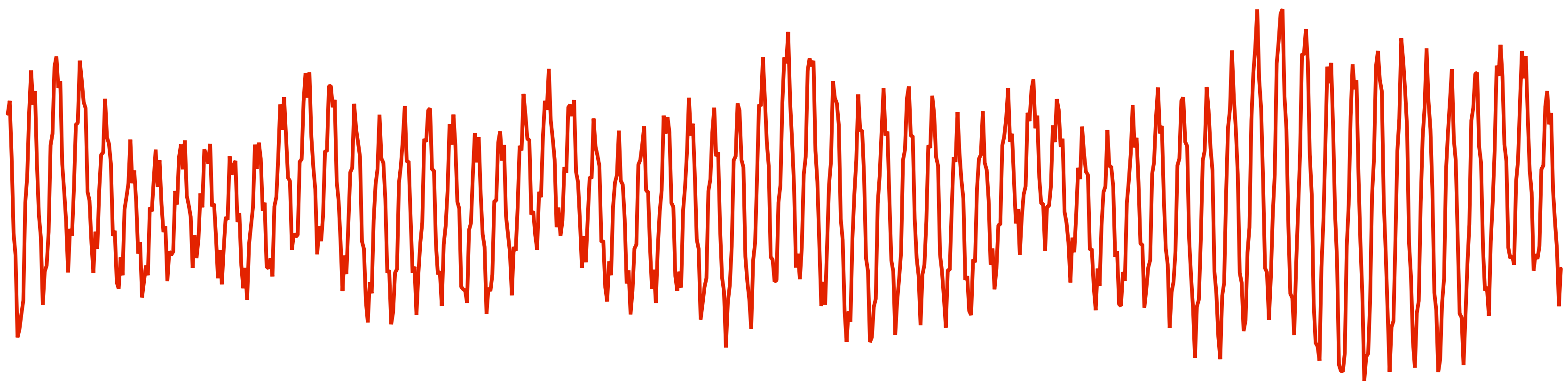
Best case allowed by quantum mechanics:

$$S_{xx}^{(\mathrm{meas})}(\omega) \geq 2 \cdot S_{xx}^{T=0}(\omega)$$

"Standard quantum limit (SQL) of displacement detection"

...as if adding the zero-point fluctuations a second time: "adding half a photon"

Notes on the SQL



- "weak measurement": integrating the signal over time to suppress the noise
- trying to detect slowly varying "quadratures of motion": $\hat{x}(t) = \hat{X}_1 \cos(\omega_M t) + \hat{X}_2 \sin(\omega_M t)$

$$\left[\hat{X}_1, \hat{X}_2\right] = 2x_{\mathrm{ZPF}}^2$$

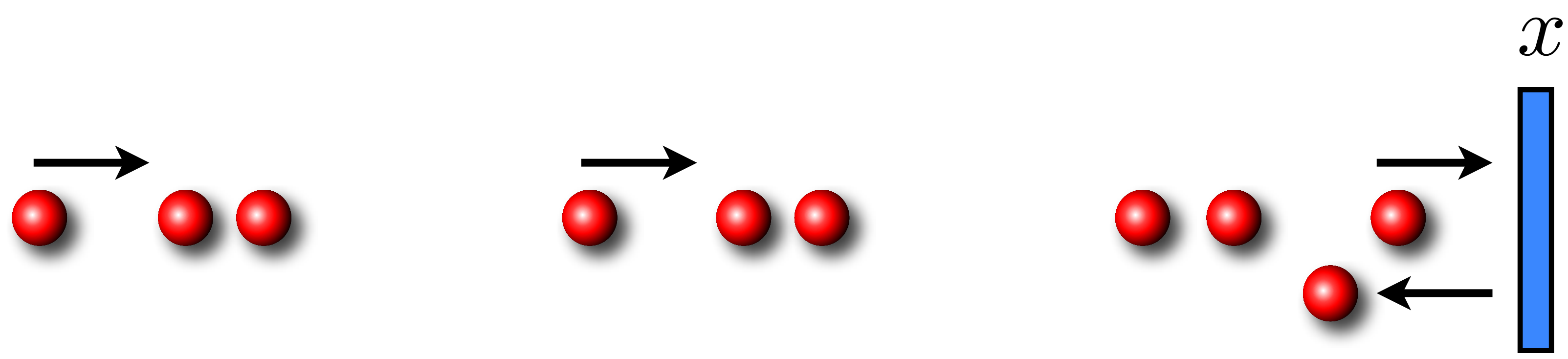
Heisenberg is the reason for SQL!

no limit for **instantaneous** measurement of x(t)!

- SQL means: detect $\hat{X}_{1,2}$ down to x_{ZPF} on a time scale $1/\Gamma$

Impressive: $x_{\mathrm{ZPF}} \sim 10^{-15} m$!

Enforcing the SQL (Heisenberg) in a weak optical measurement



reflection phase shift: $\theta = 2kx$
(here: free space)

N photons arrive in time t

fluctuations: $\delta N = \sqrt{\mathrm{Var} N} = \sqrt{\bar{N}}$

Poisson distribution for a coherent laser beam

1. Uncertainty in phase estimation:

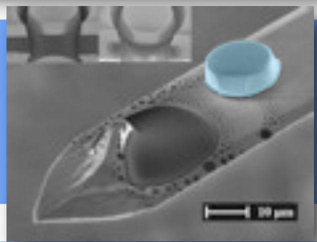
$$\delta N \cdot \delta\theta \geq \frac{1}{2} \quad \Rightarrow \quad \delta\theta \geq \frac{1}{2\sqrt{\bar{N}}} \quad \Rightarrow \quad \delta x = \frac{\delta\theta}{2k} \sim \frac{1}{2\sqrt{\bar{N}}2k}$$

2. Fluctuating force: momentum transfer $\Delta p = 2\hbar k \cdot N$

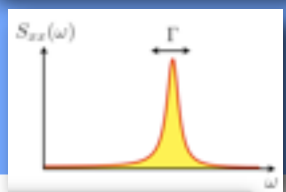
$$\delta p = \sqrt{\mathrm{Var}\Delta p} = 2\hbar k\sqrt{\bar{N}}$$

Uncertainty product: $\delta x \delta p \geq \frac{\hbar}{2}$ Heisenberg

Optomechanics (Outline)

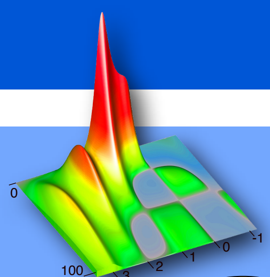


Introduction

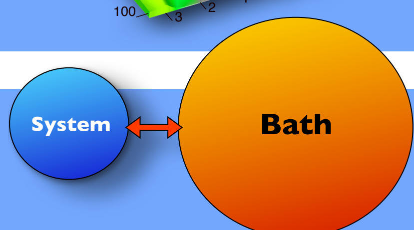


Displacement detection

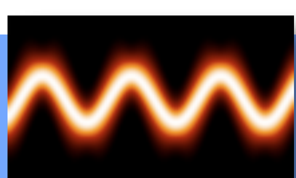
Linear optomechanics



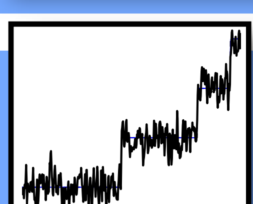
Nonlinear dynamics



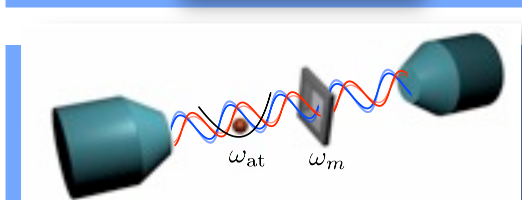
Quantum theory of cooling



Interesting quantum states

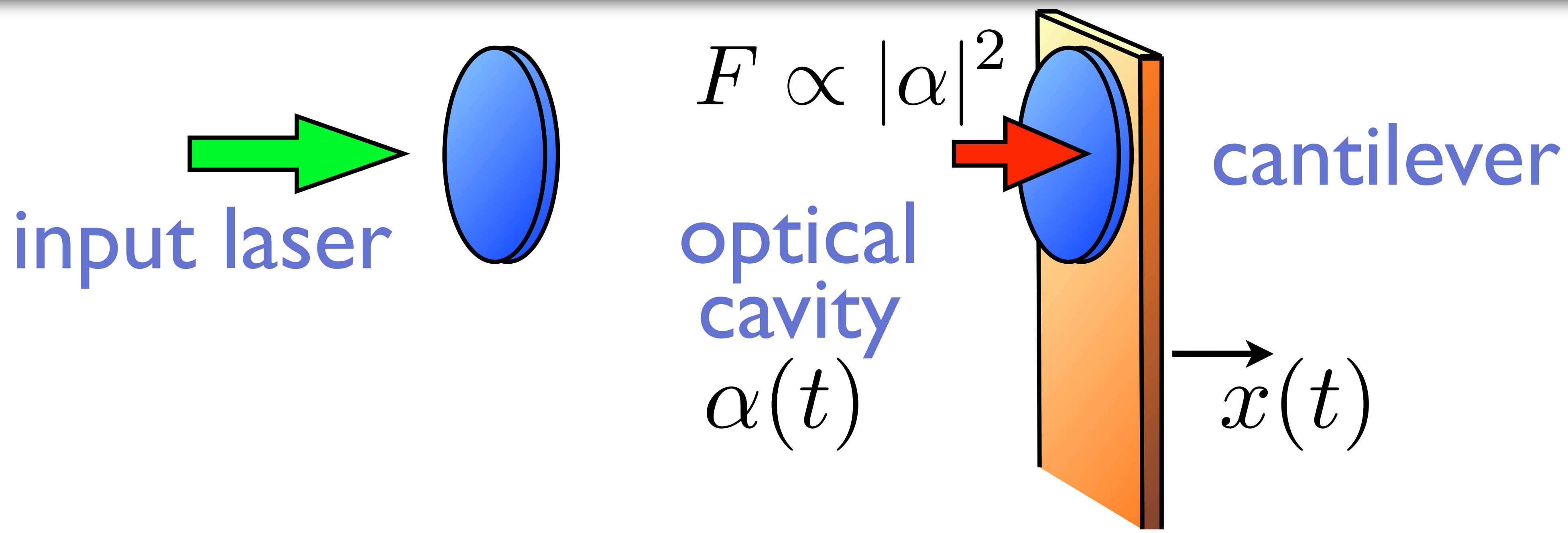


Towards Fock state detection

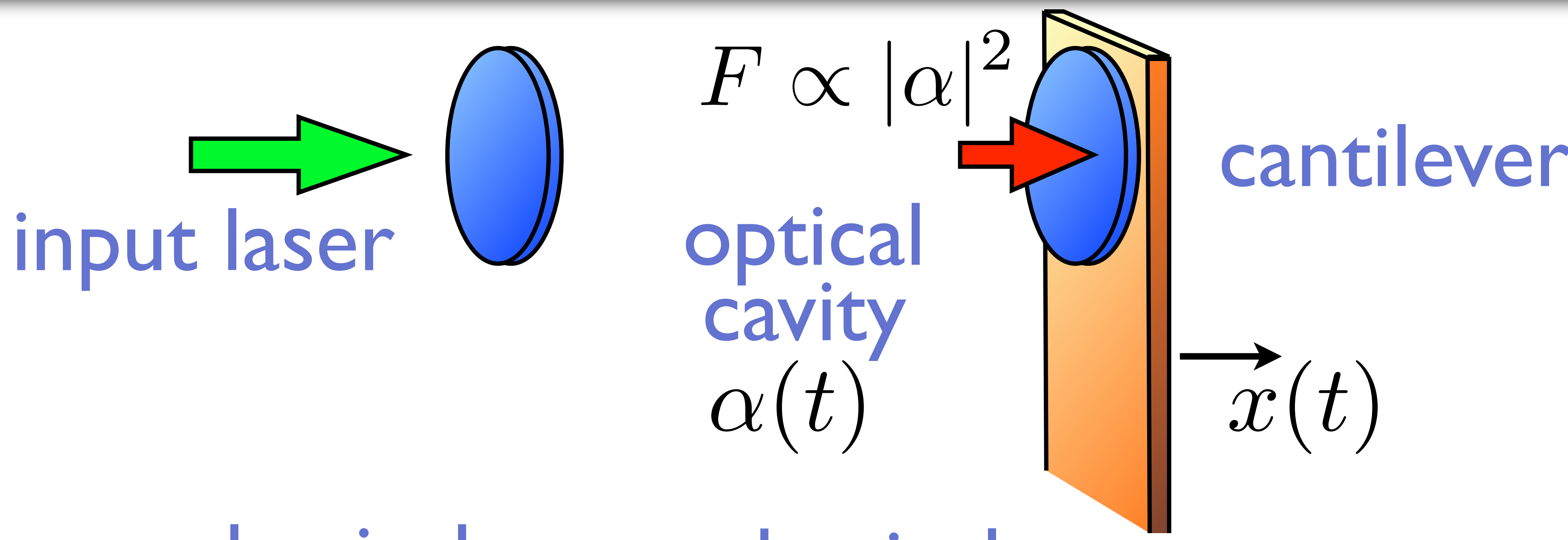


Coupling to the motion of a single atom

Equations of motion



Equations of motion



mechanical frequency — mechanical damping

$$\ddot{x} = -\omega_M^2 (x - x_0) - \Gamma \dot{x} + F/m$$

radiation pressure

equilibrium position

$$F = \frac{\hbar \omega_R}{L} |\alpha|^2$$

$$\dot{\alpha} = i\omega_R \frac{x}{L} \alpha - \frac{\kappa}{2}(\alpha - \alpha_{\text{in}})$$

detuning from resonance — cavity decay rate — laser amplitude

Linearized optomechanics

$$\alpha(t) = \bar{\alpha} + \delta\alpha(t)$$
$$x(t) = \bar{x} + \delta x(t)$$

$$\Rightarrow \ldots \Rightarrow$$

(solve for arbitrary $F_{\text{ext}}(\omega)$)

$$\delta x(\omega) = \underbrace{\frac{1}{m(\omega_M^2 - \omega^2) - im\omega\Gamma + \Sigma(\omega)}}_{\chi_{xx}^{\text{eff}}(\omega)} F_{\text{ext}}(\omega)$$

$$\delta\omega_M^2 = \frac{1}{m}\text{Re}\Sigma(\omega_M)$$

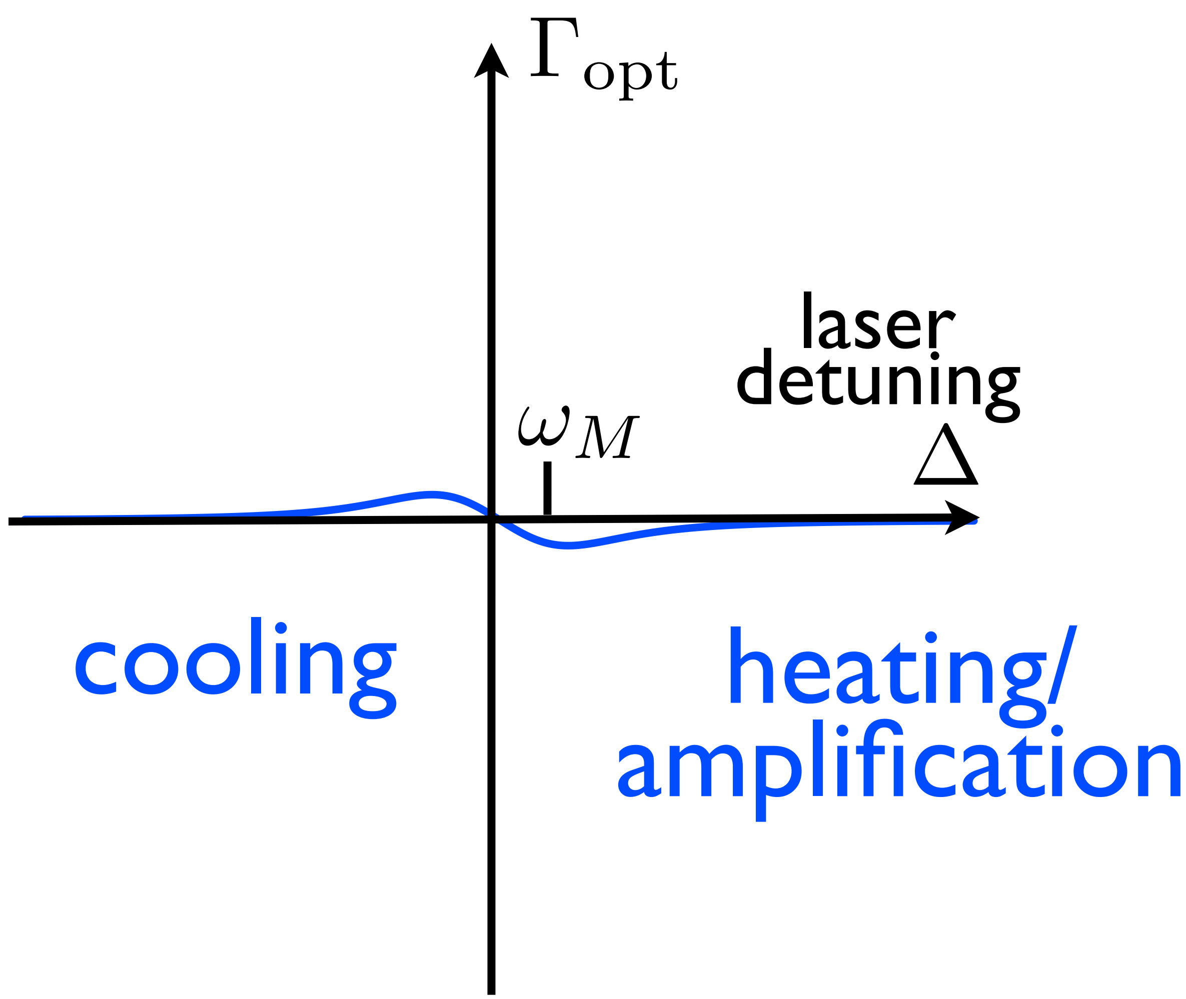
Optomechanical frequency shift ("optical spring")

$$\Gamma_{\text{opt}} = -\frac{1}{m\omega_M}\text{Im}\Sigma(\omega_M)$$

Effective optomechanical damping rate

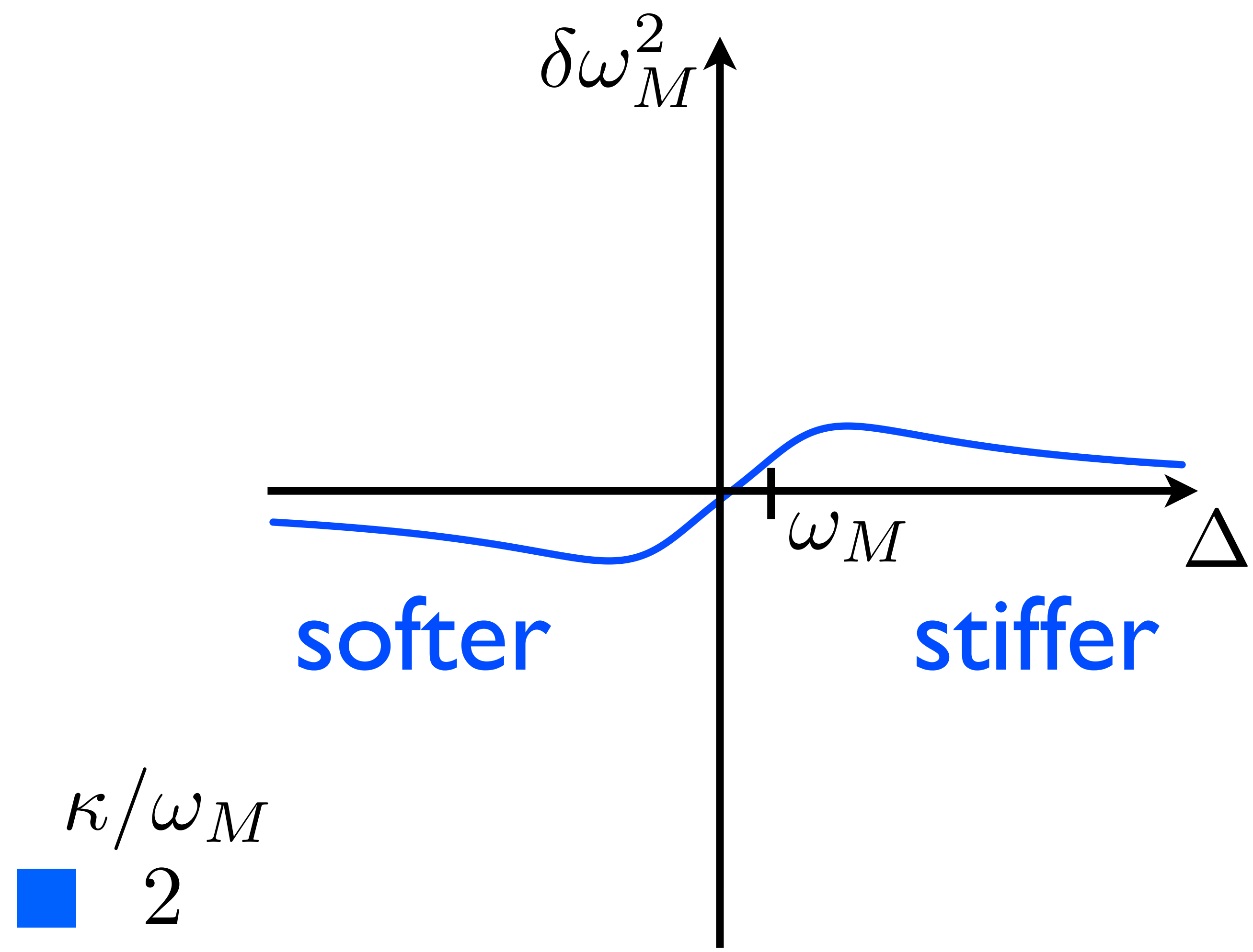
Linearized dynamics

Effective optomechanical damping rate



$(\Delta = \omega_R x_0 / L)$

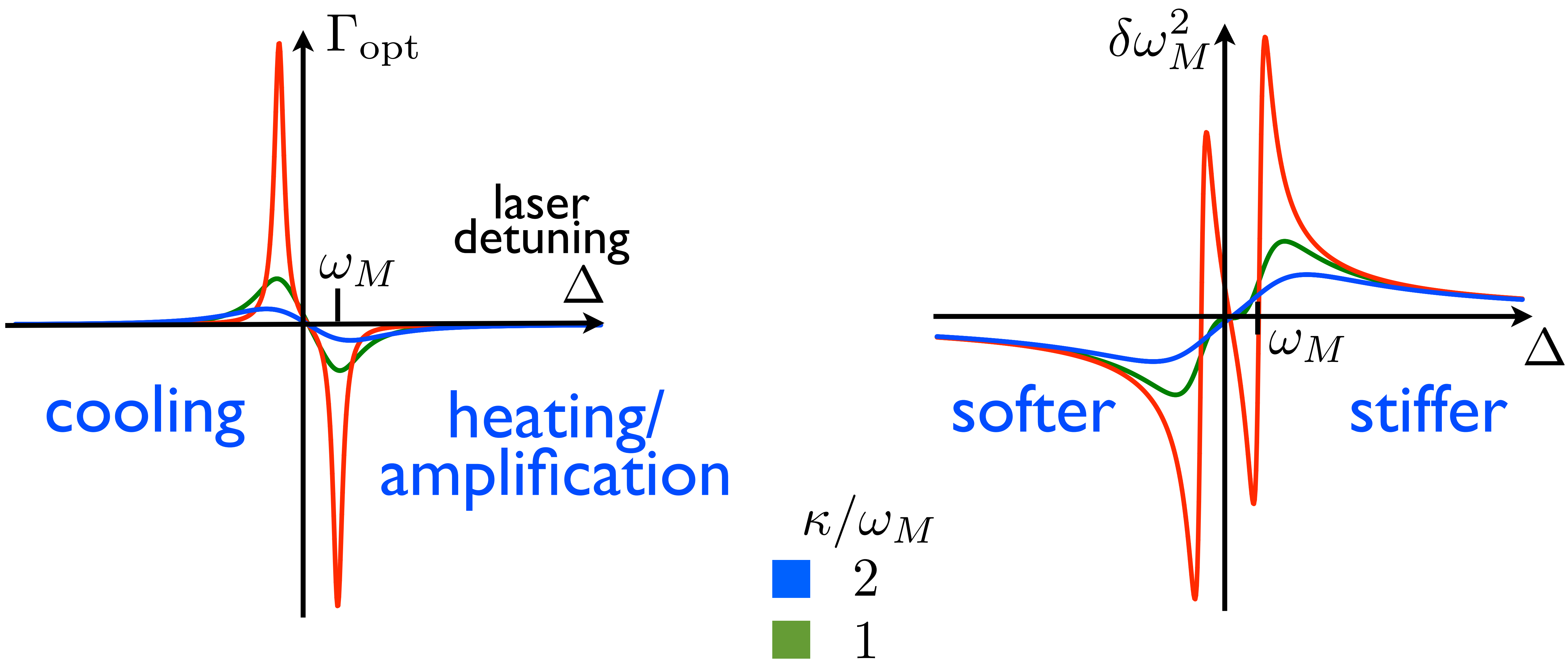
Optomechanical frequency shift ("optical spring")



Linearized dynamics

Effective optomechanical damping rate

Optomechanical frequency shift (“optical spring”)

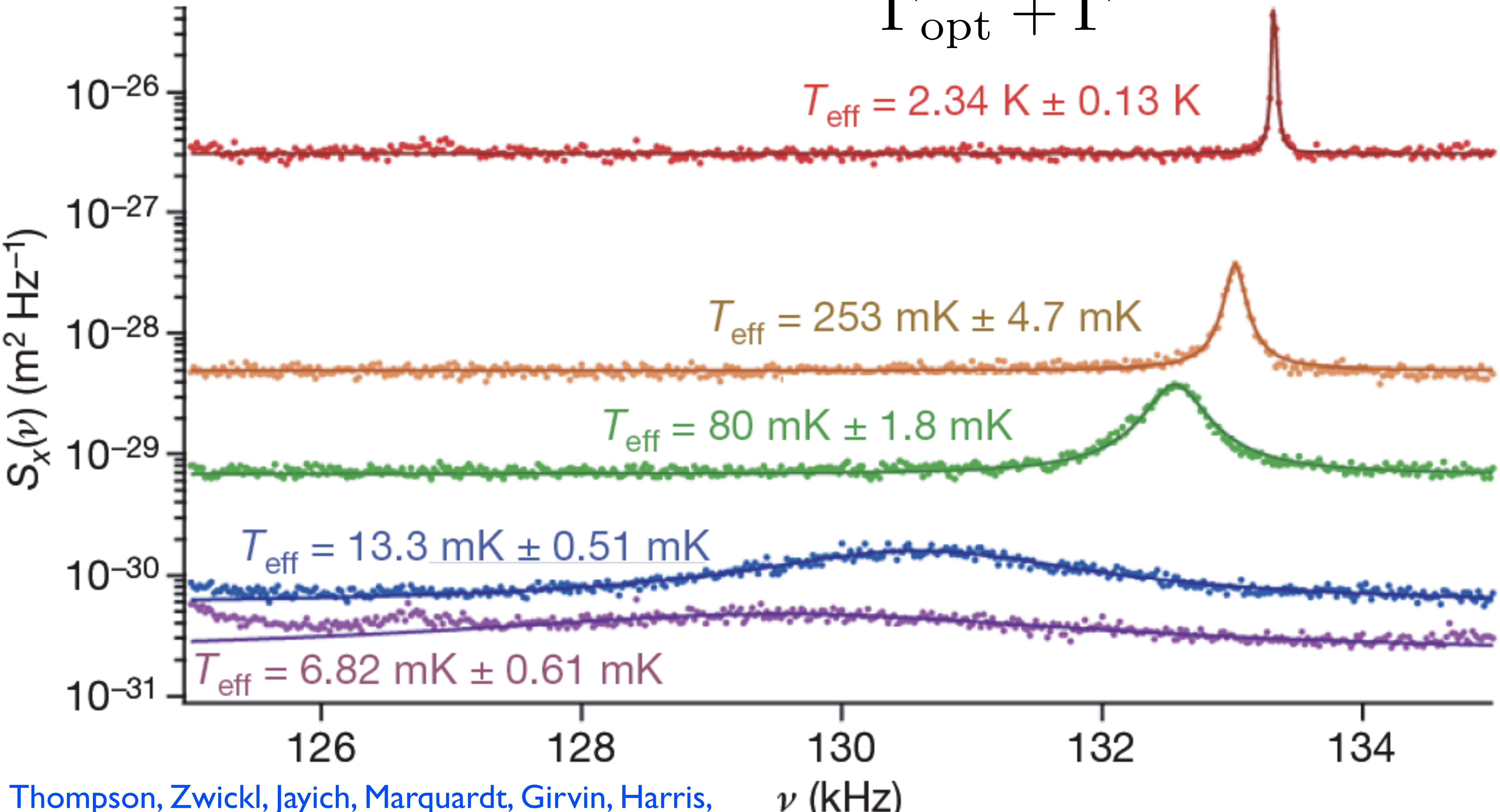


$(\Delta = \omega_R x_0 / L)$

Cooling by damping

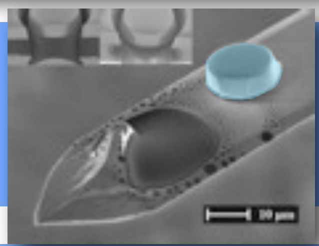
$$T_{\rm eff} = T \cdot \frac{\Gamma}{\Gamma_{\rm opt} + \Gamma}$$

T=300 K



Thompson, Zwickl, Jayich, Marquardt, Girvin, Harris, Nature 72, 452 (2008)

Optomechanics (Outline)



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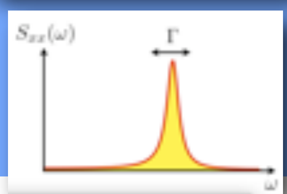

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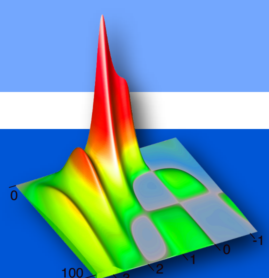
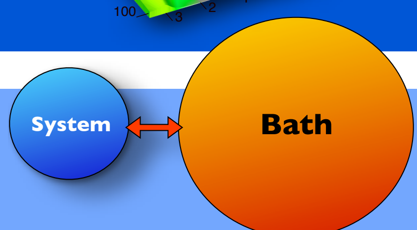


Nonlinear dynamics



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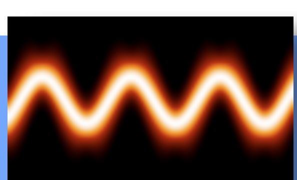

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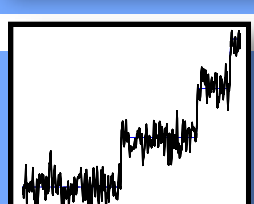

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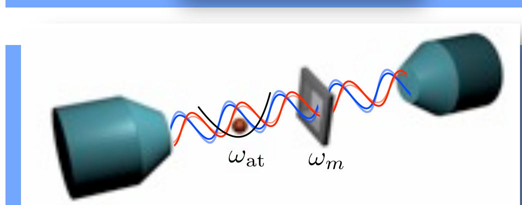
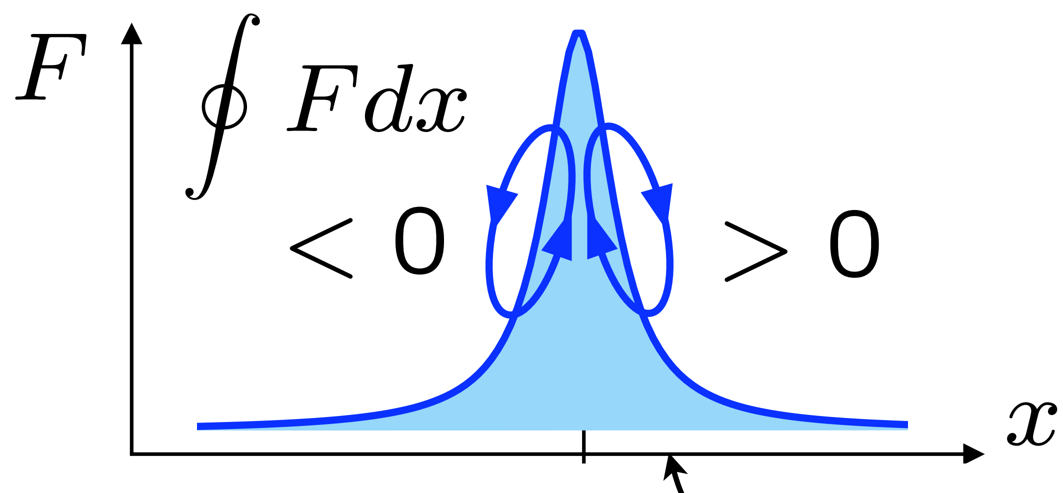


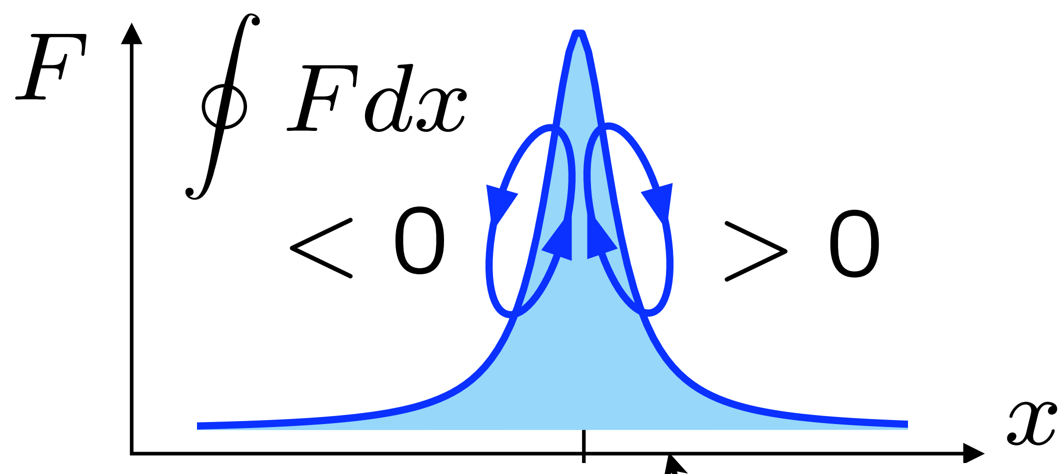
Coupling to the motion of a single atom

Self-induced oscillations



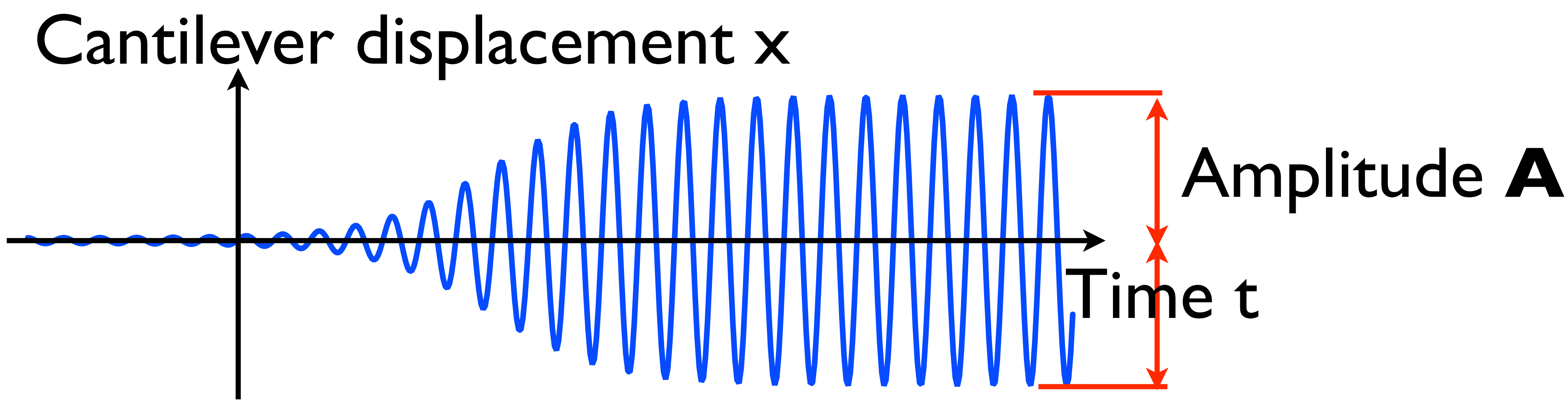
$$\Gamma_{\text{total}} = \Gamma_M + \underbrace{\Gamma_{\text{opt}}}_{< 0}$$

Self-induced oscillations



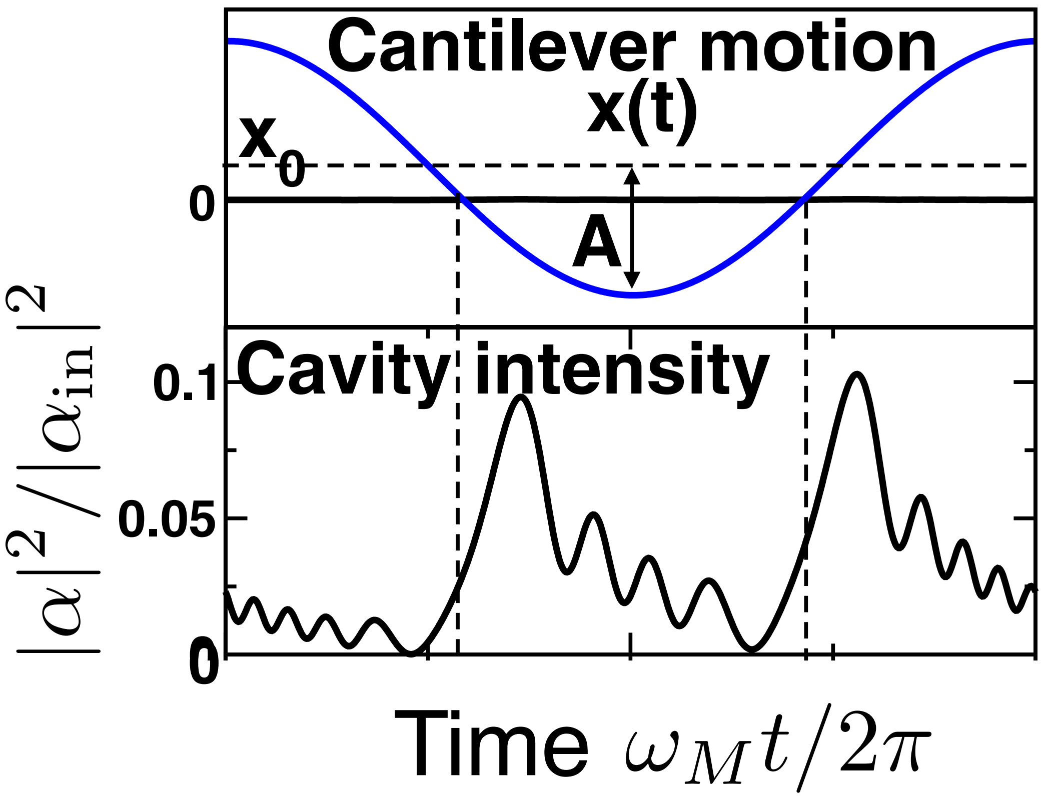
$$\Gamma_{\text{total}} = \Gamma_M + \Gamma_{\text{opt}} < 0$$

Beyond some laser input power threshold: instability



Self-induced oscillations

$$x(t) \approx \bar{x} + A\cos(\omega_M t)$$



FM, Harris, Girvin,
PRL 96, 103901 (2006)

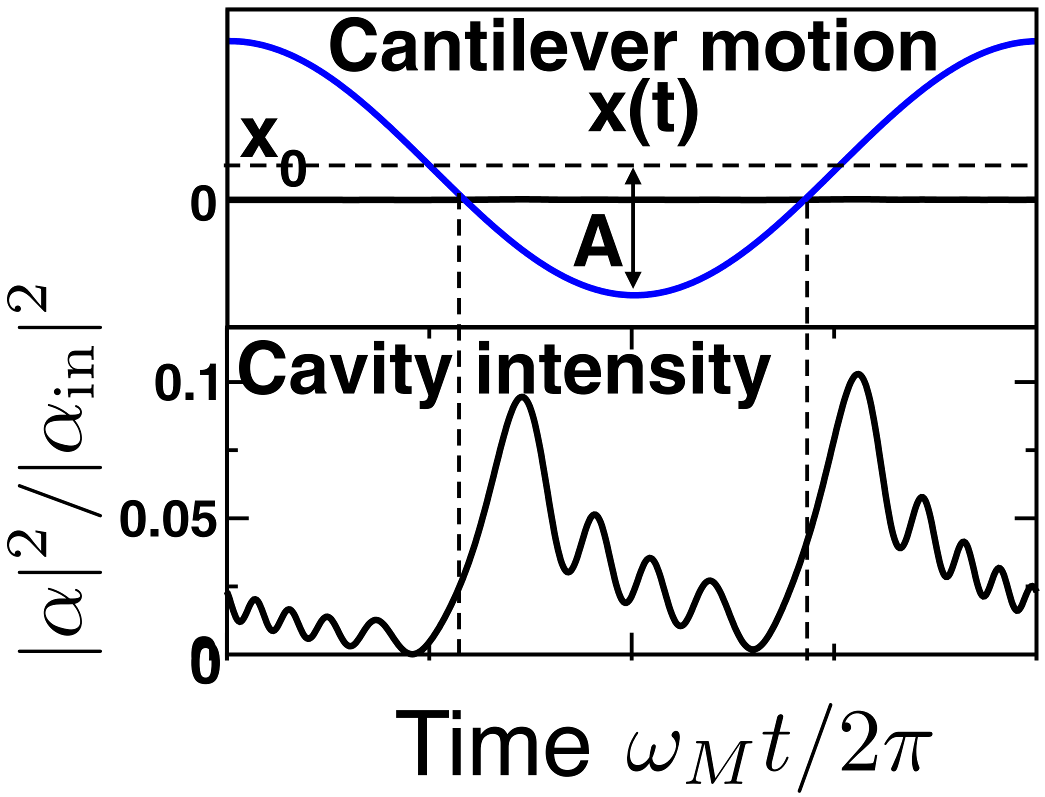
Self-induced oscillations

$$x(t) \approx \bar{x} + A\cos(\omega_M t)$$

cavity intensity oscillations:

$$\alpha(t) = e^{i\varphi(t)} \sum_n \alpha_n e^{in\omega_M t}$$

$$\alpha_n = \alpha_n(\bar{x}, A)$$ [analytical solution]



FM, Harris, Girvin,
PRL 96, 103901 (2006)

1. **force balance:** $\langle \ddot{x} \rangle = 0$

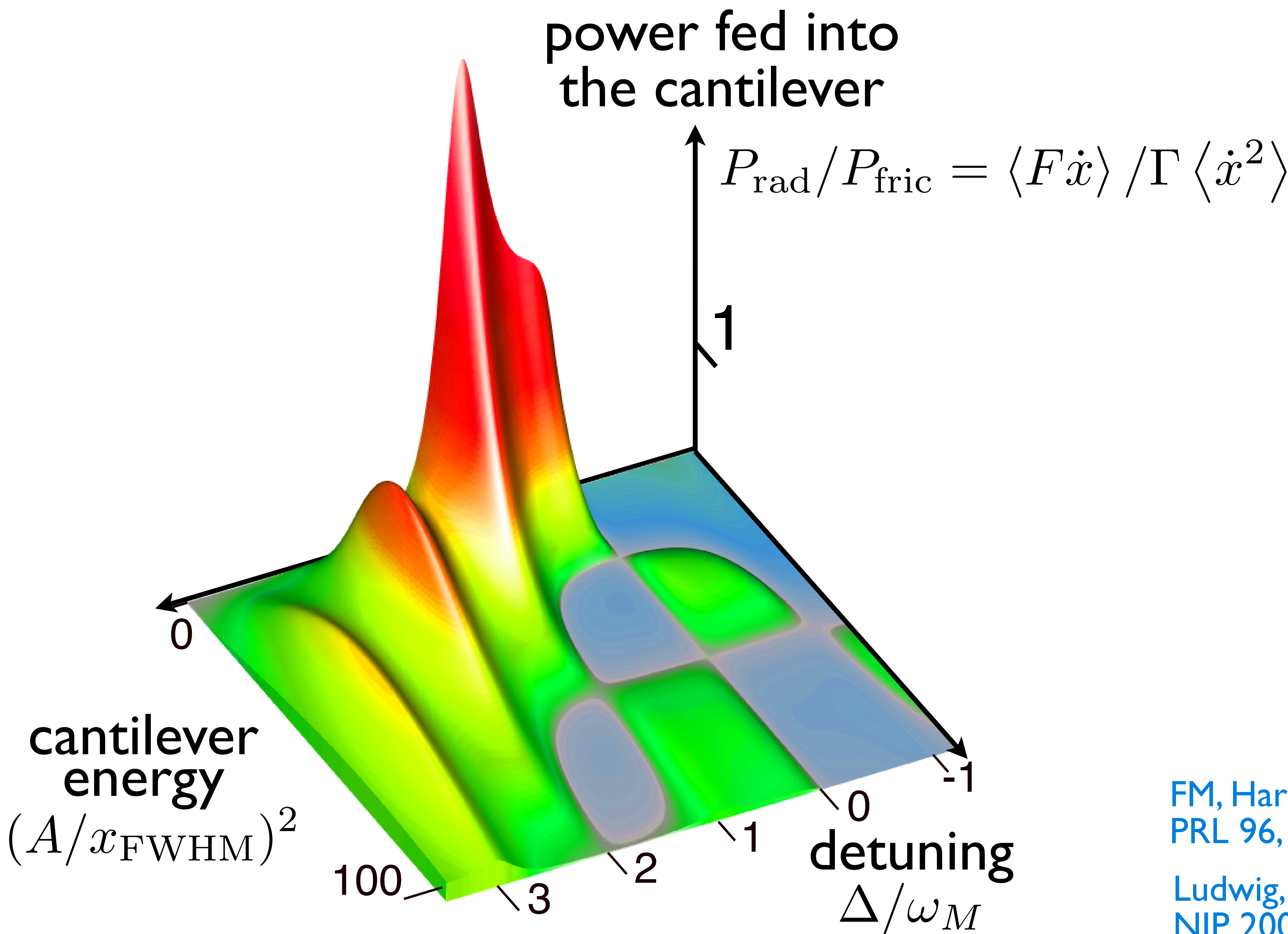
$$\langle F \rangle = m\omega_M^2(\bar{x} - x_0)$$

2. **power balance:** $\langle \dot{x}\ddot{x} \rangle = 0$

$$\langle F\dot{x} \rangle = \Gamma \langle \dot{x}^2 \rangle$$

Two equations for two unknowns: $\bar{x}, A$

Attractor diagram



FM, Harris, Girvin,
PRL 96, 103901 (2006)

Ludwig, Kubala, FM,
NJP 2008

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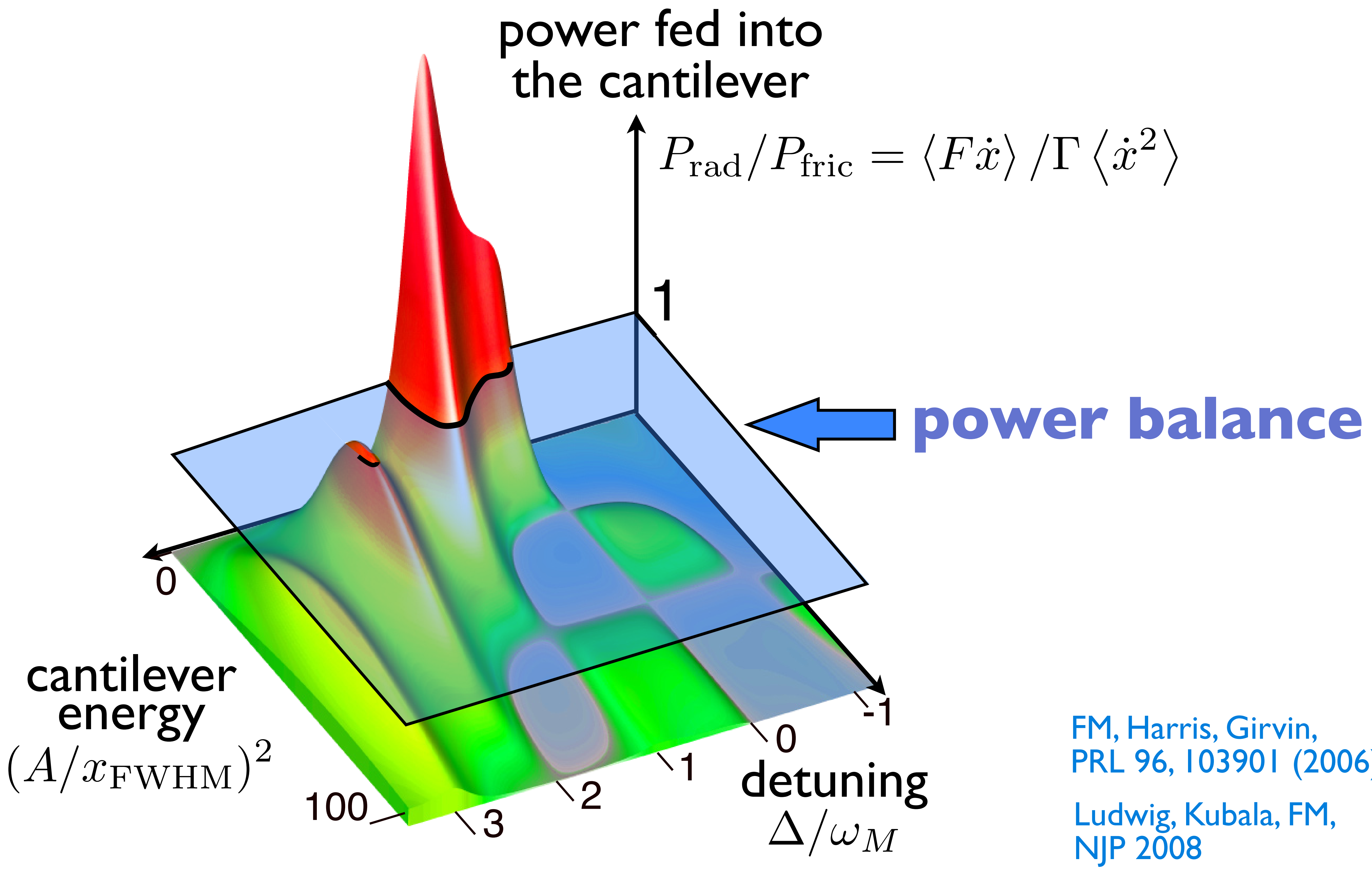

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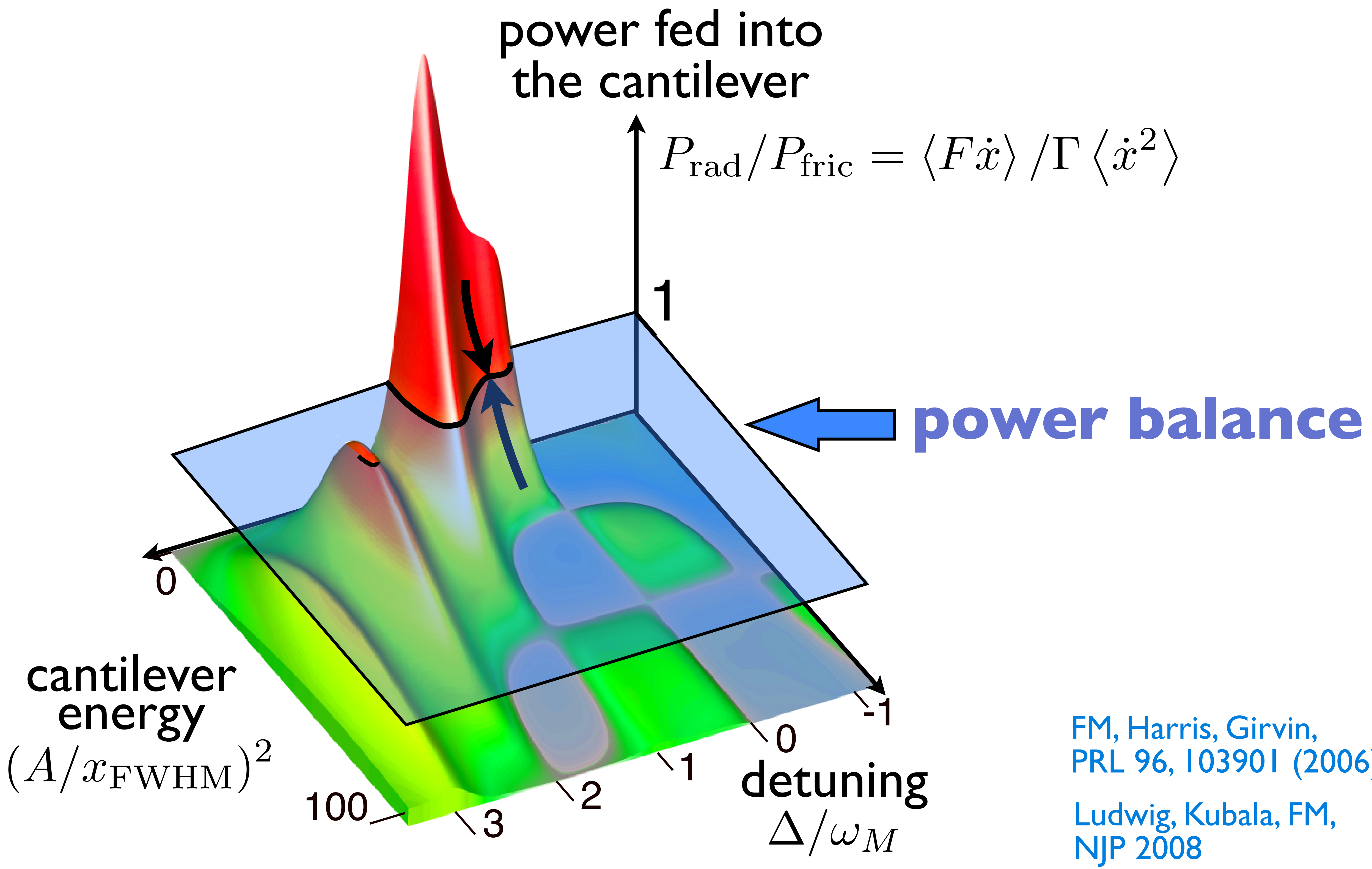

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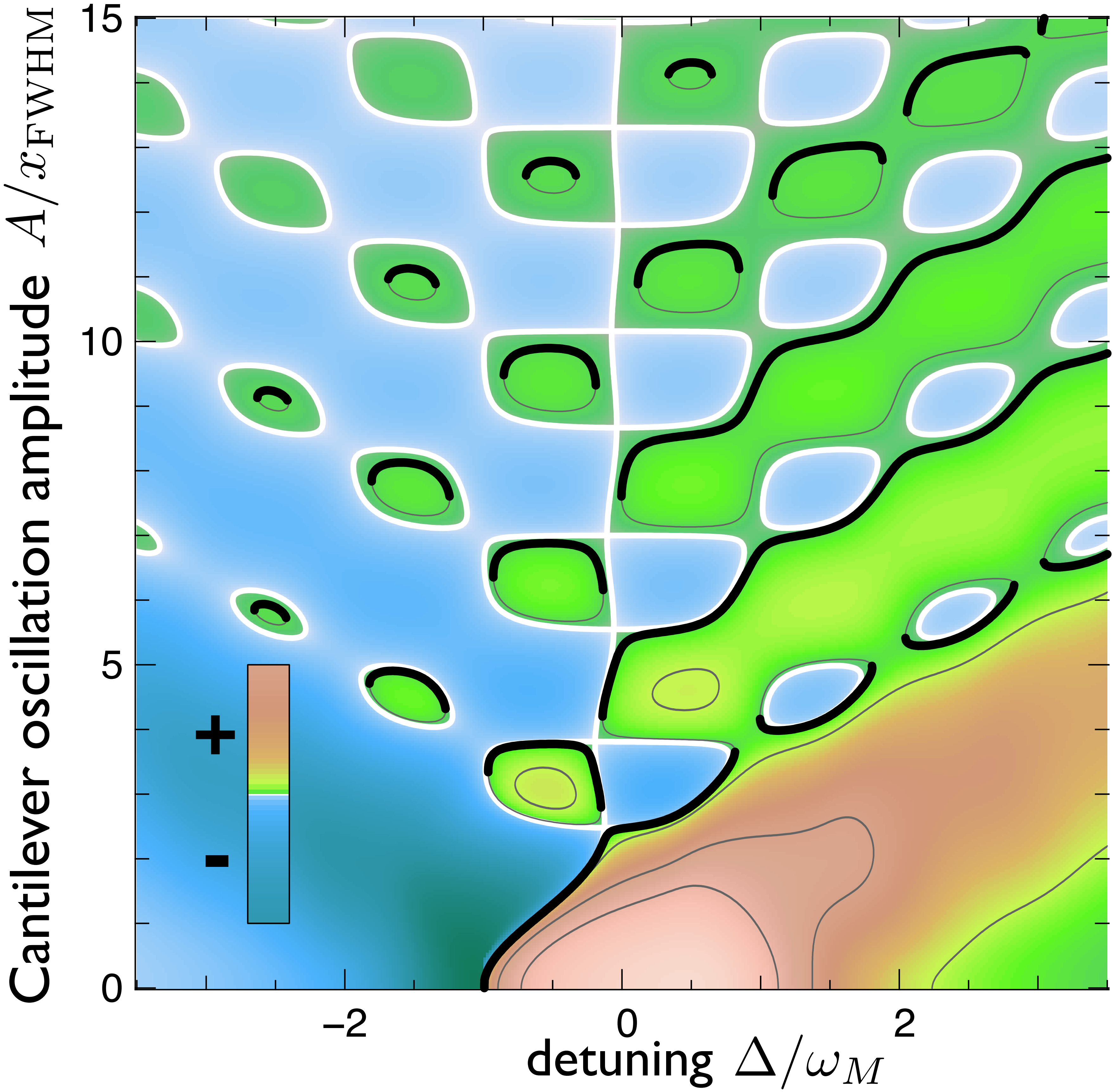


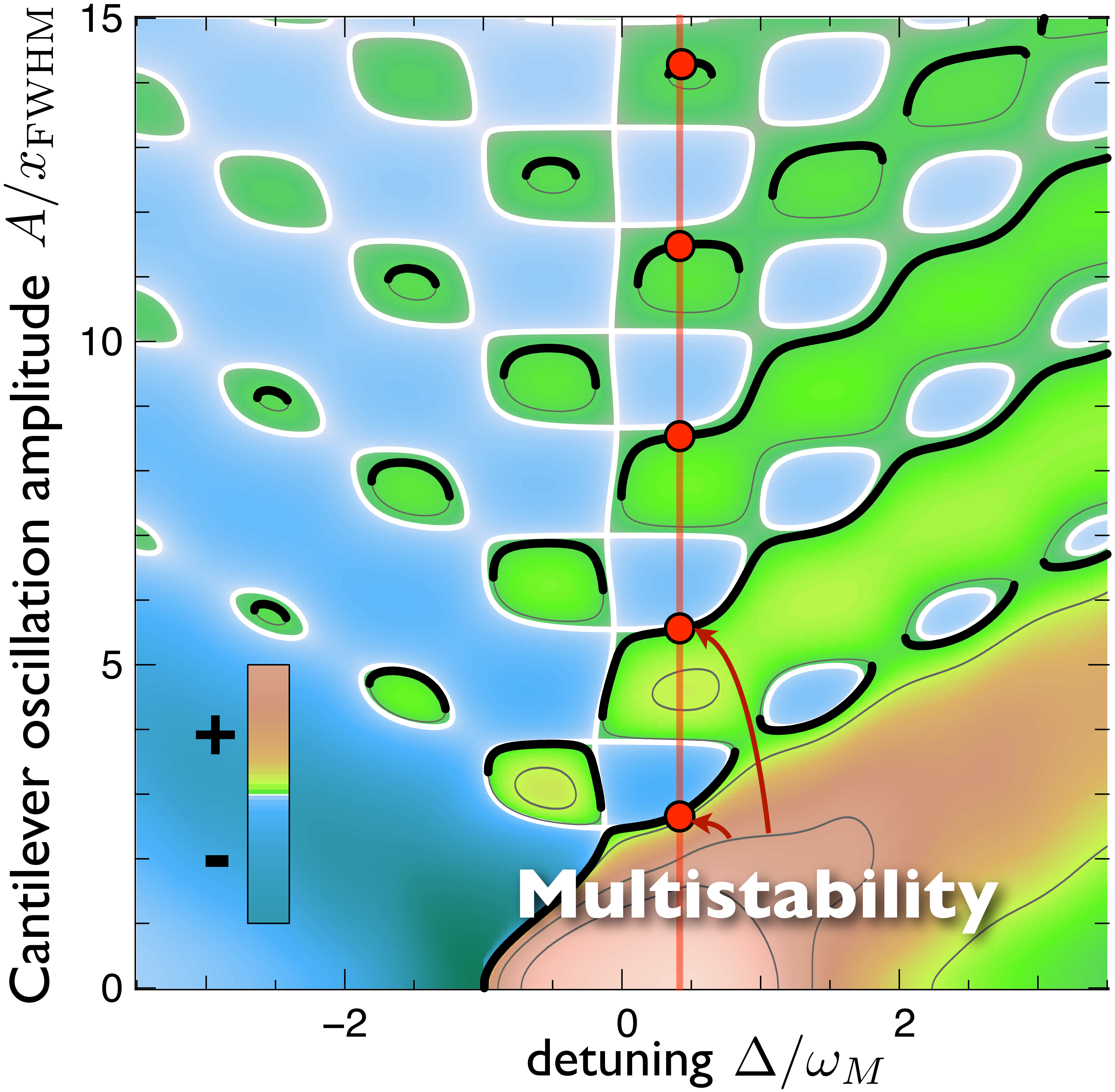
Color scale: power fed into cantilever

FM, Harris, Girvin, PRL **96**, 103901 (2006)

$\Gamma/\omega_M = 0.001$

$\omega_M/\kappa = 1$

Attractor diagram



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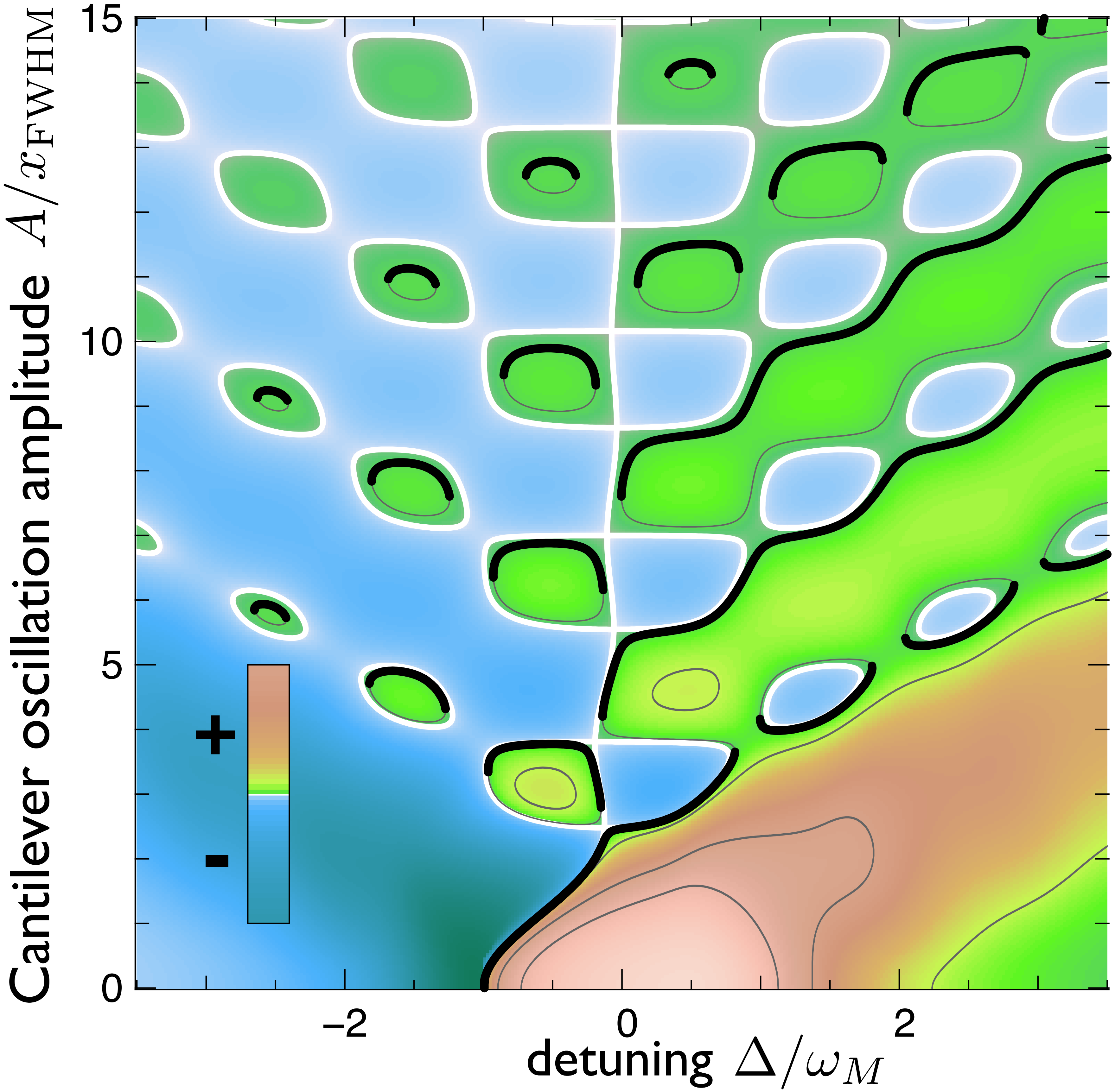

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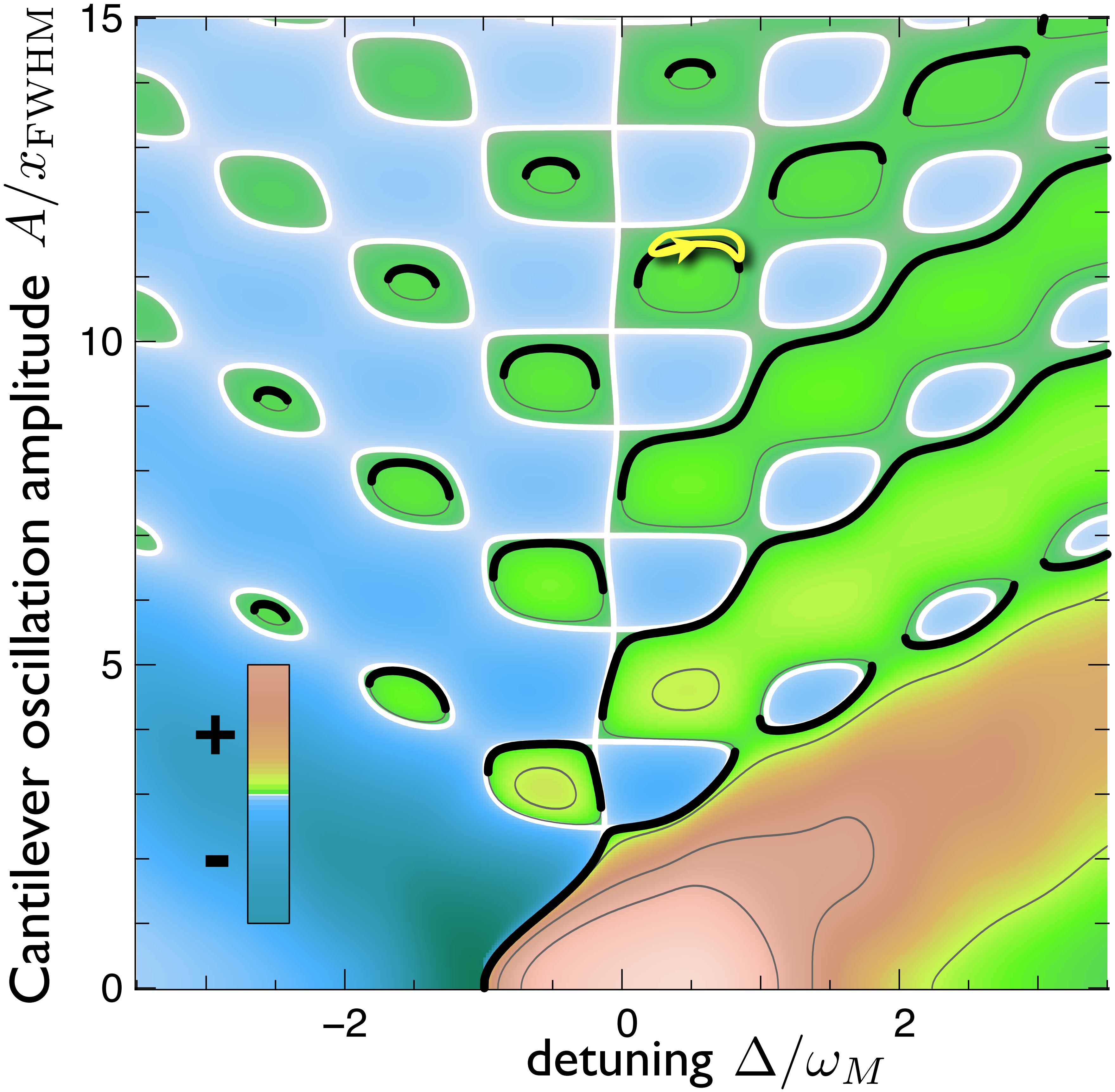

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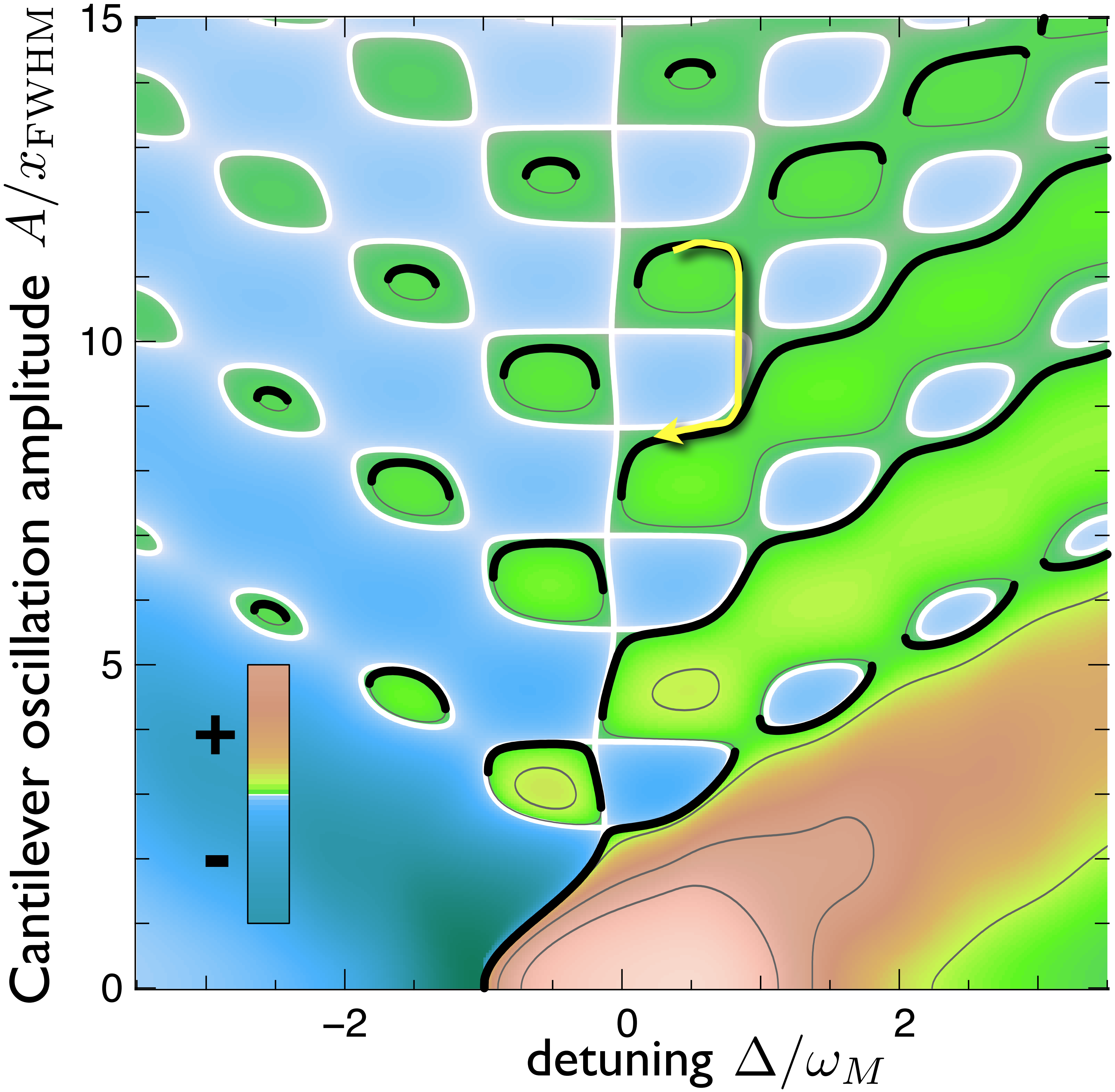

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FM, Harris, Girvin, PRL **96**, 103901 (2006)

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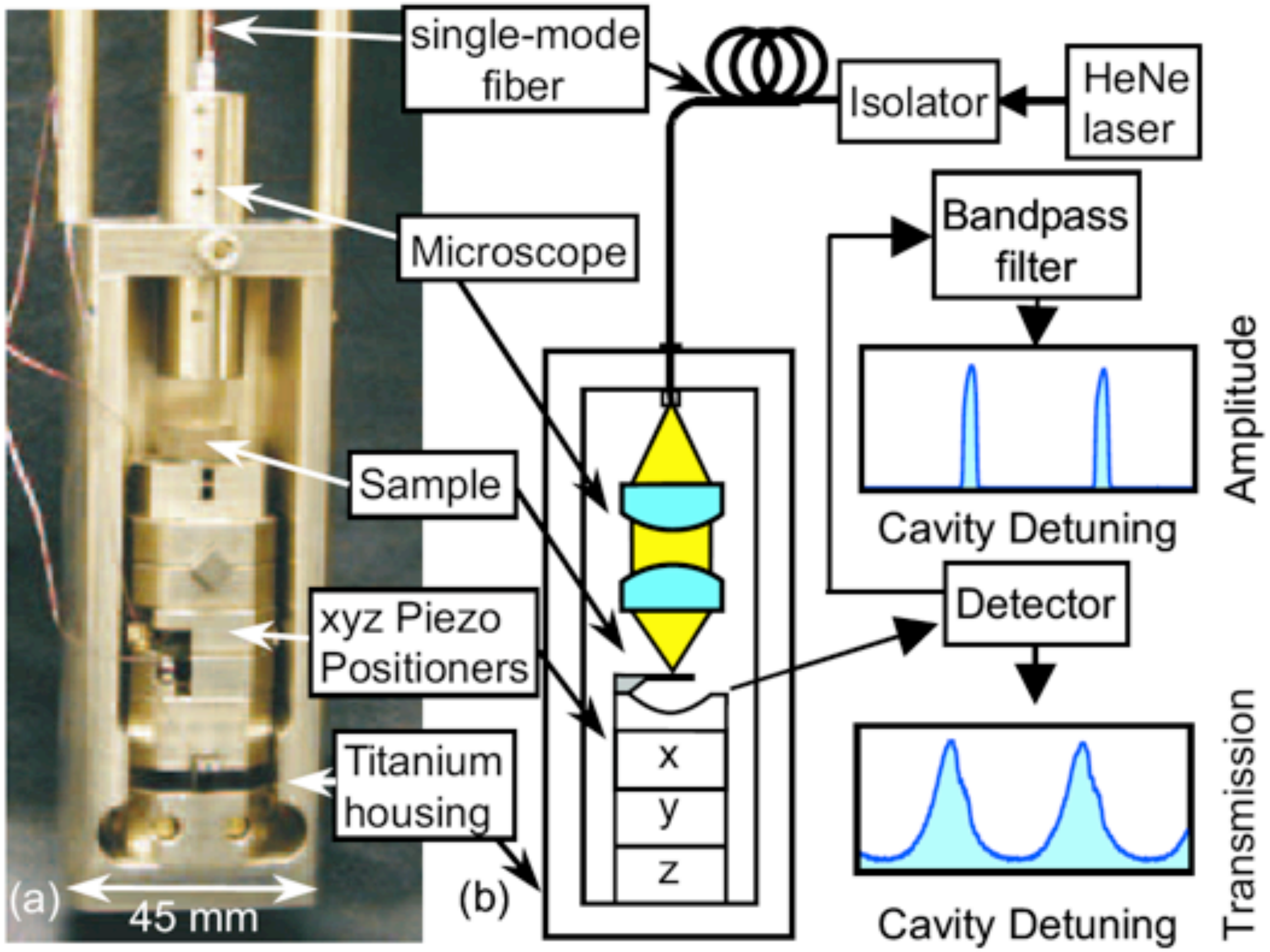
First experimental observation of attractor diagram



Time-delayed **bolometric** force:

$$F(t) \propto \int_{-\infty}^{t} e^{-(t-t')/\tau} I(t') \frac{dt'}{\tau}$$

Low optical finesse – light intensity follows instantaneously:

$$I(t) = I[x(t)]$$

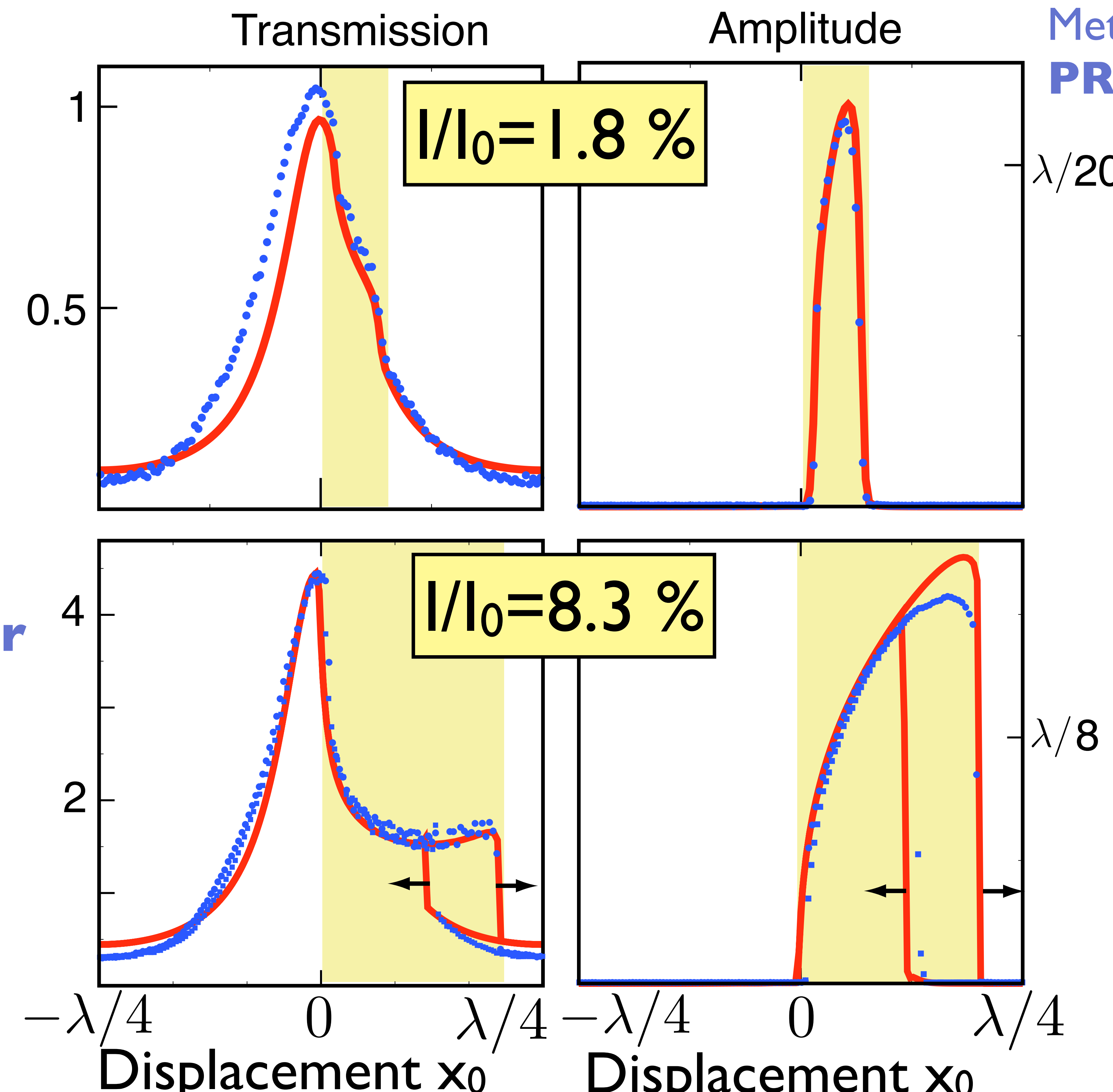
Ludwig, Neuenhahn, Metzger, Ortlieb, Favero, Karrai, FM,
Phys. Rev. Lett. **101**, 133903 (2008)

Comparison theory/experiment

Metzger et al., **PRL 2008**

low laser power

medium laser power

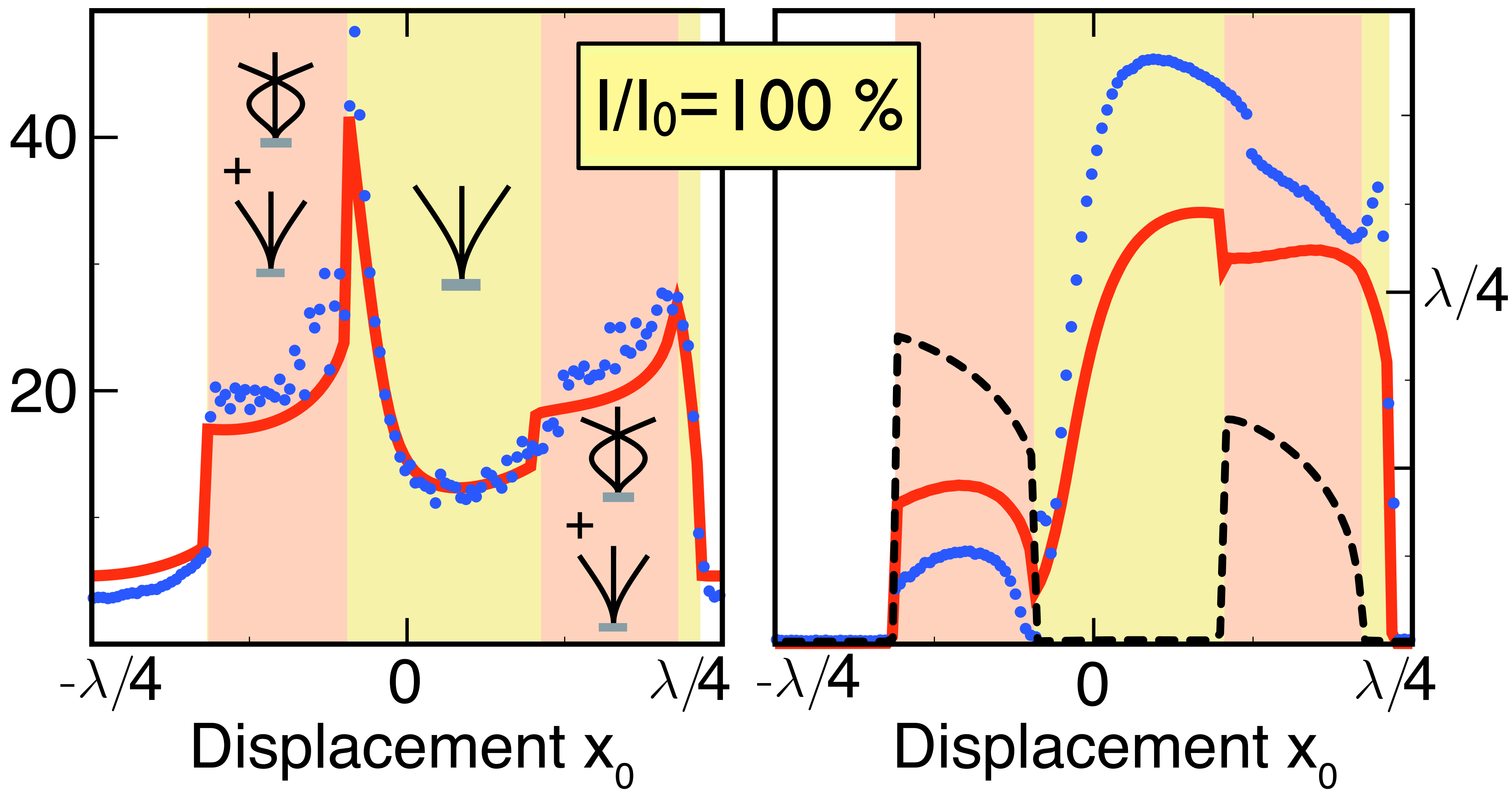


Comparison theory/experiment

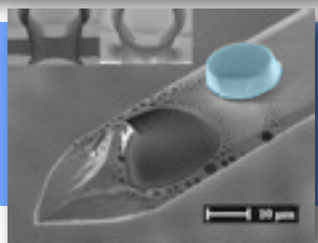
Metzger et al.,
PRL 2008

Coupled self-oscillations of two mechanical modes

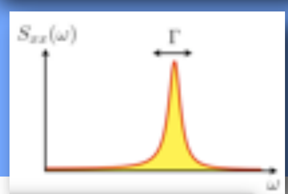
high laser power



Optomechanics (Outline)

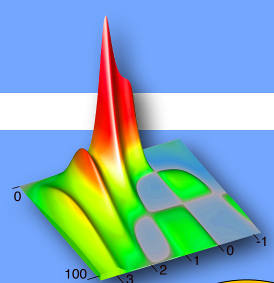


Introduction

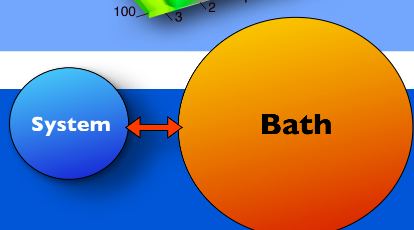


Displacement detection

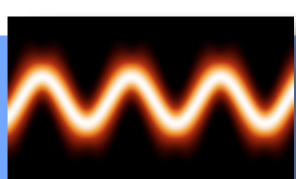
Linear optomechanics



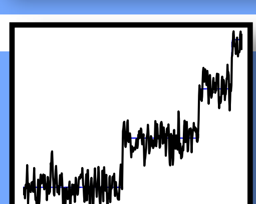
Nonlinear dynamics



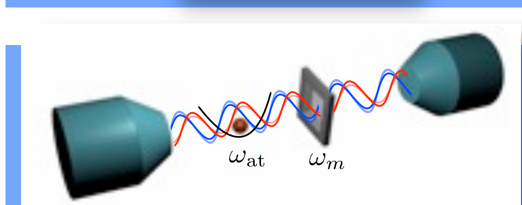
Quantum theory of cooling



Interesting quantum states

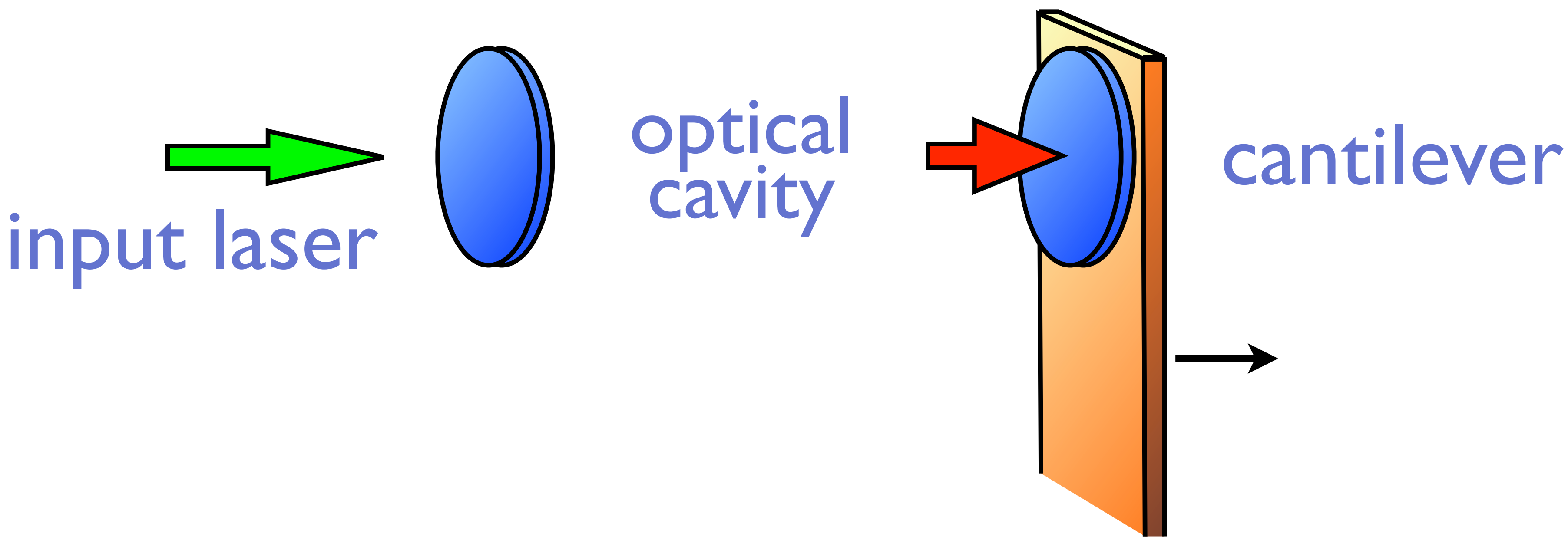


Towards Fock state detection



Coupling to the motion of a single atom

Cooling with light



Current goal in the field: *ground state* of mechanical motion of a macroscopic cantilever

$$k_B T_{\text{eff}} \ll \hbar \omega_M$$

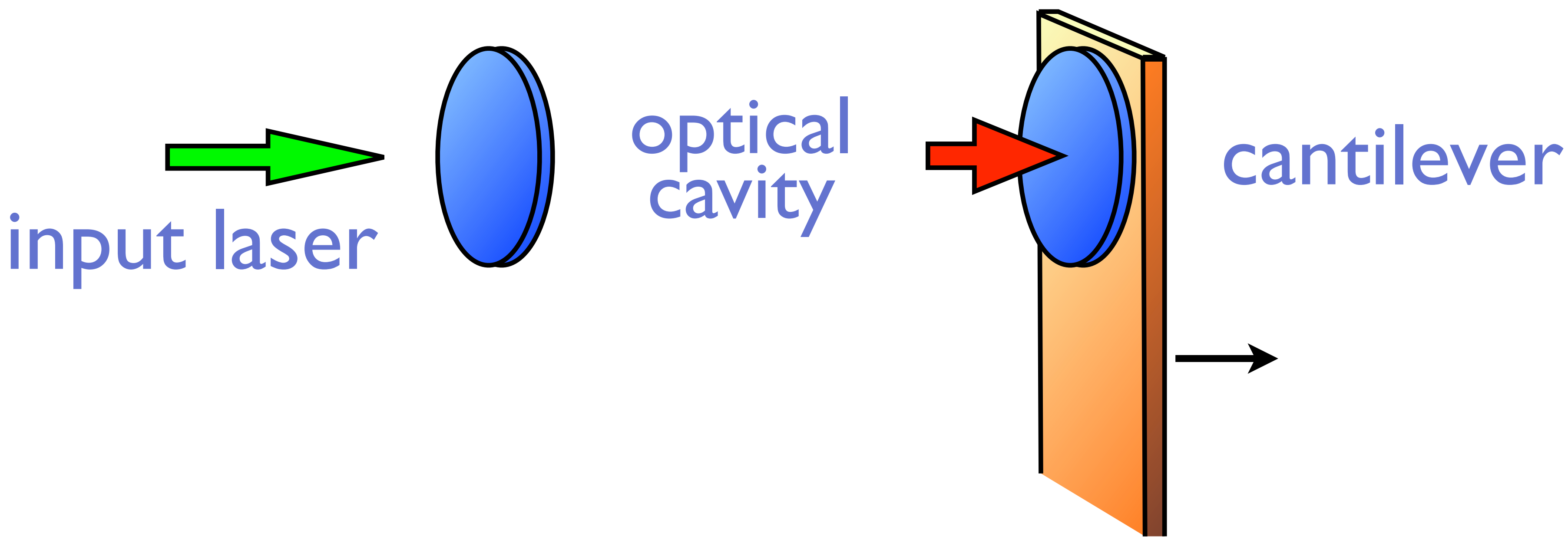
Classical theory:

$$T_{\text{eff}} = T \cdot \frac{\Gamma_M}{\Gamma_{\text{opt}} + \Gamma_M}$$

optomechanical damping rate

Pioneering theory and experiments: **Braginsky** (since 1960s)

Cooling with light



Current goal in the field: *ground state* of mechanical motion of a macroscopic cantilever

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Classical theory:

$$T_{\mathrm{eff}} = T \cdot \frac{\Gamma_M}{\Gamma_{\mathrm{opt}} + \Gamma_M} \rightarrow 0 \ ?$$

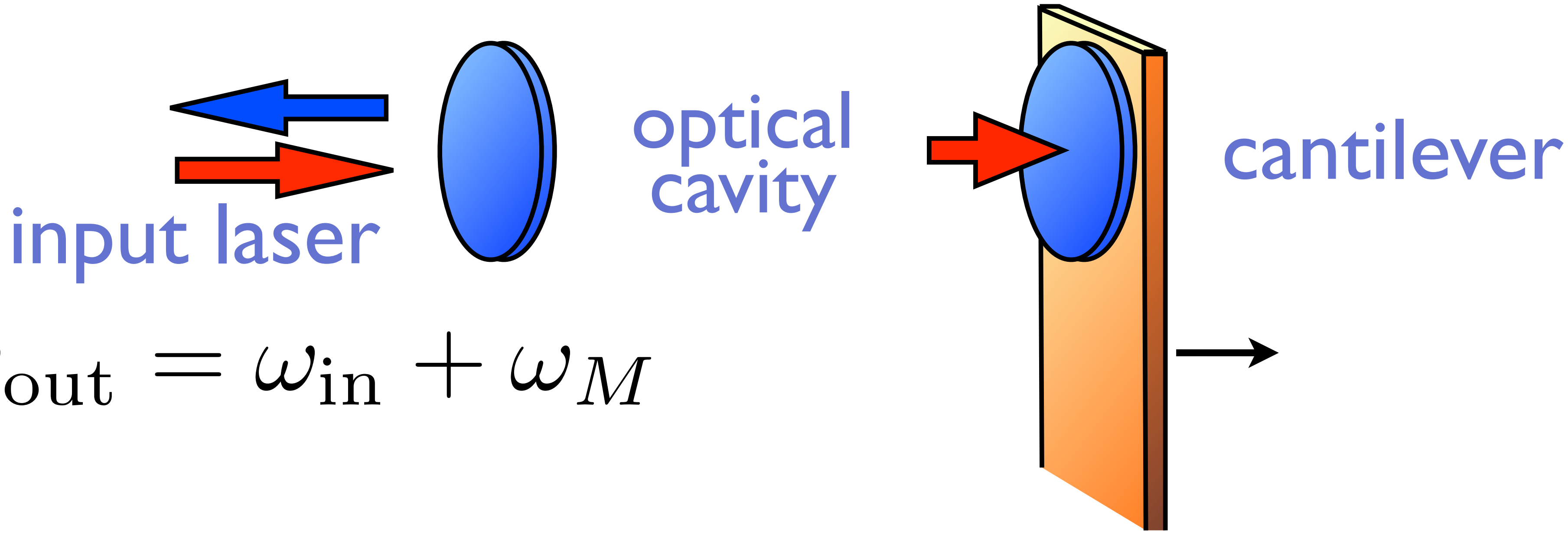
quantum limit?

shot noise!

optomechanical damping rate

Pioneering theory and experiments: **Braginsky** (since 1960s)

Cooling with light



$$\omega_{\rm out} = \omega_{\rm in} + \omega_M$$

Quantum picture: Raman scattering – sideband cooling

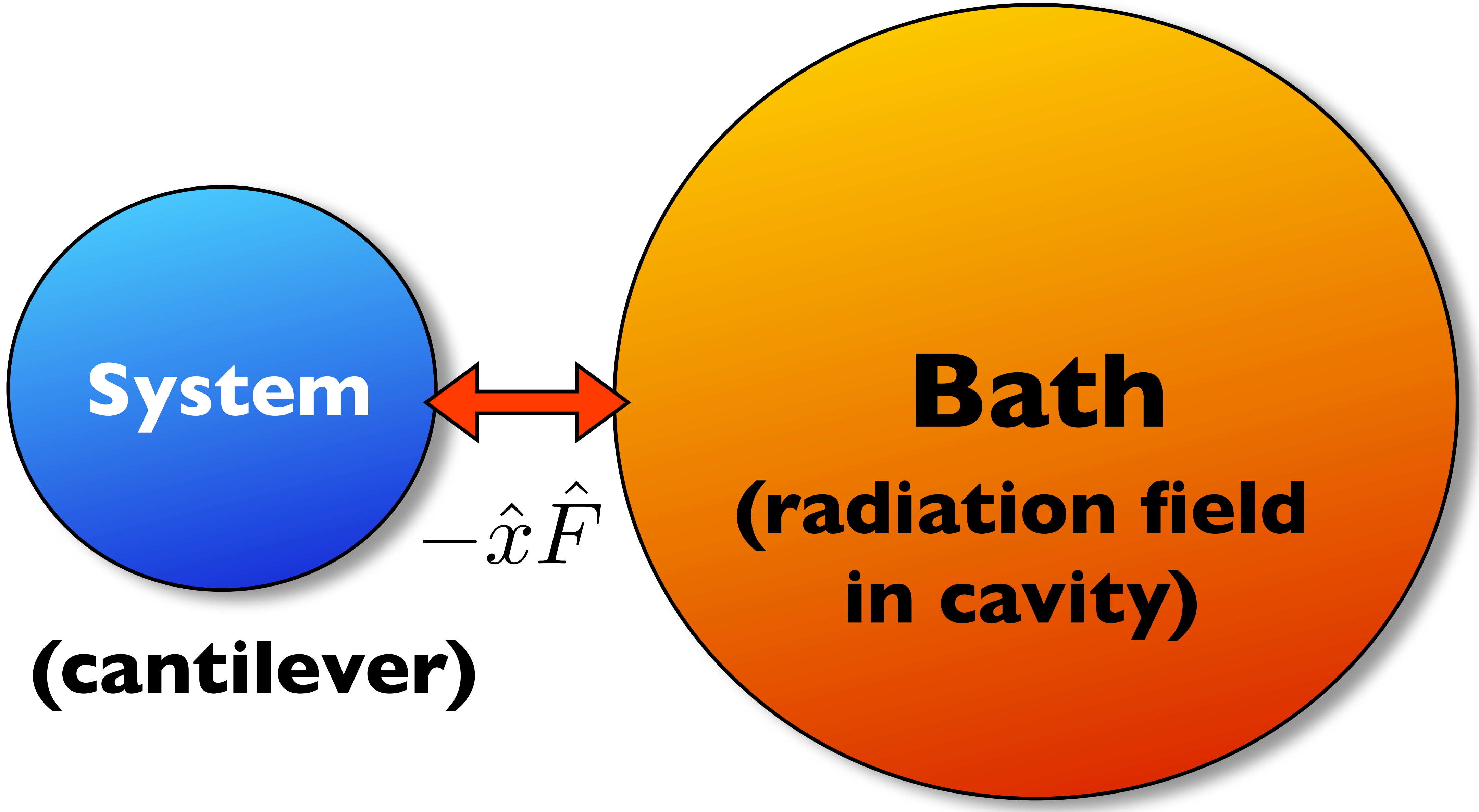
Original idea:

Sideband cooling in ion traps – Hänsch, Schawlow / Wineland, Dehmelt 1975

Similar ideas proposed for nanomechanics:

cantilever + quantum dot – Wilson-Rae, Zoller, Imamoglu 2004
cantilever + Cooper-pair box – Martin Shnirman, Tian, Zoller 2004
cantilever + ion – Tian, Zoller 2004
cantilever + supercond. SET – Clerk, Bennett / Blencowe, Imbers, Armour 2005,
Naik et al. (Schwab group) 2006

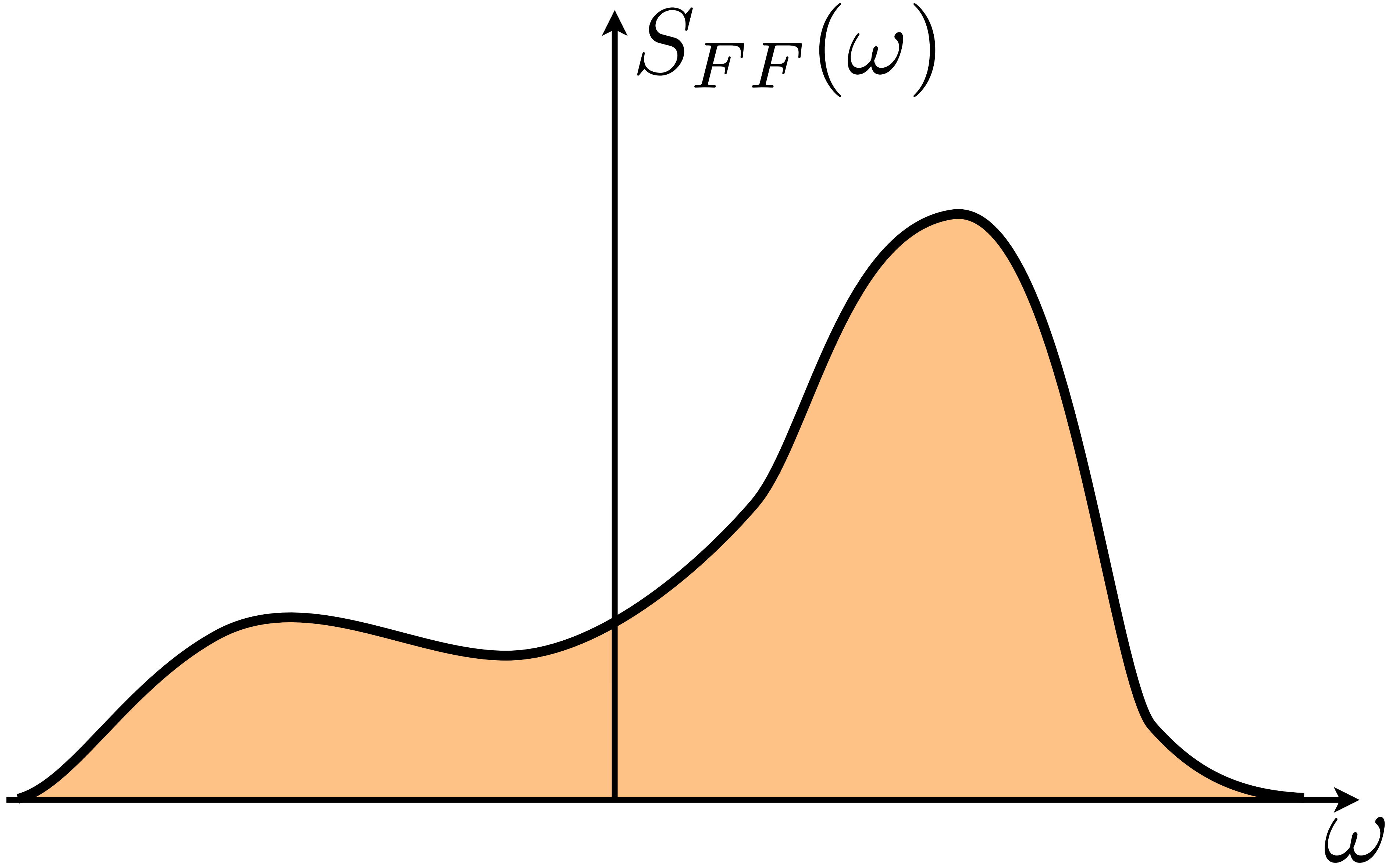
Quantum noise approach



(cantilever)

Quantum noise approach

spectrum $S_{FF}(\omega) = \int e^{i\omega t} \left\langle \hat{F}(t)\hat{F}(0) \right\rangle dt$



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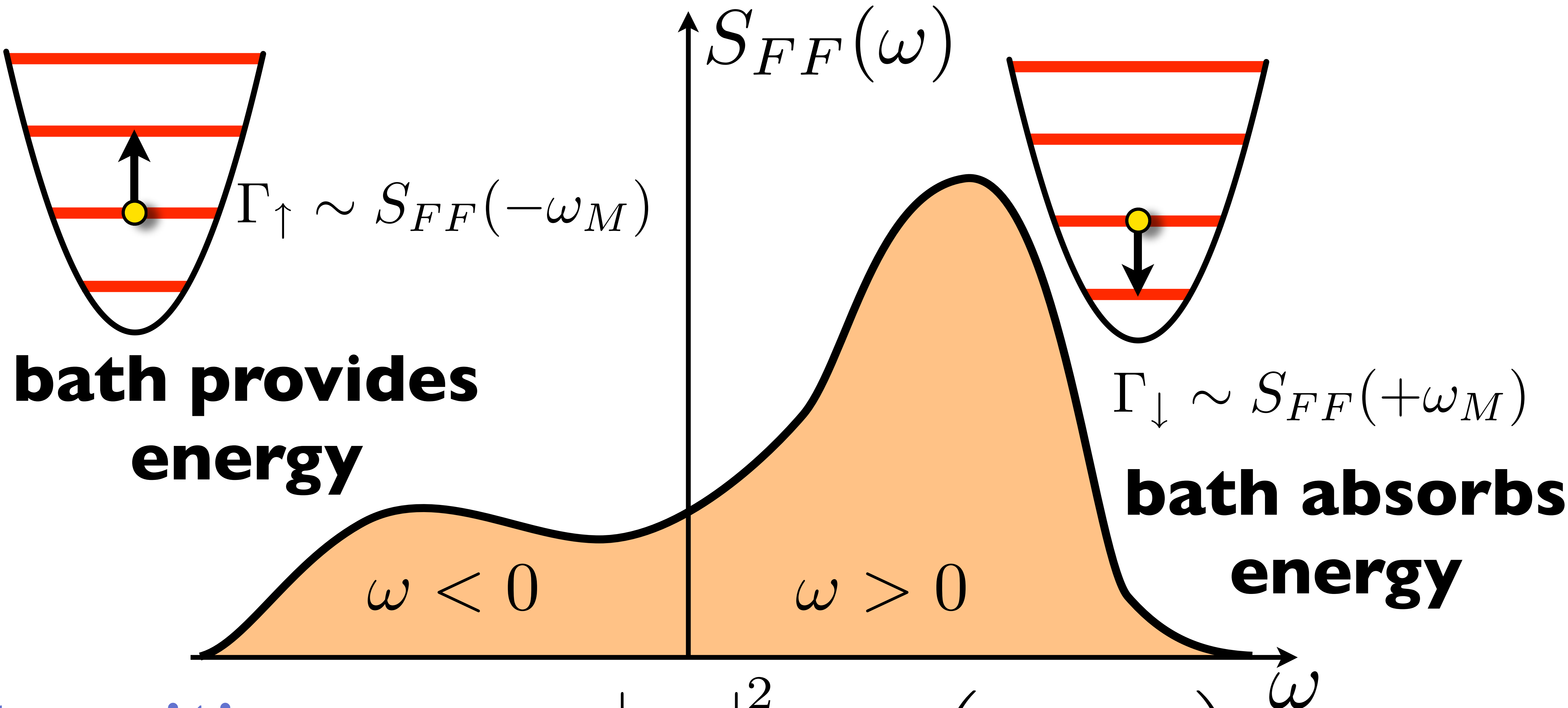


transition rate $\Gamma_{f \leftarrow i} = \frac{|x_{fi}|^2}{\hbar^2} S_{FF}\left(\frac{\epsilon_i - \epsilon_f}{\hbar}\right)$

Quantum theory of optomechanical cooling

Spectrum of radiation pressure fluctuations

$$S_{FF}(\omega) = \int e^{i\omega t} \left\langle \hat{F}(t)\hat{F}(0) \right\rangle dt$$

radiation pressure force

$$\hat{F} = \left(\frac{\hbar\omega_R}{L} \right) \hat{a}^\dagger \hat{a}$$

photon number

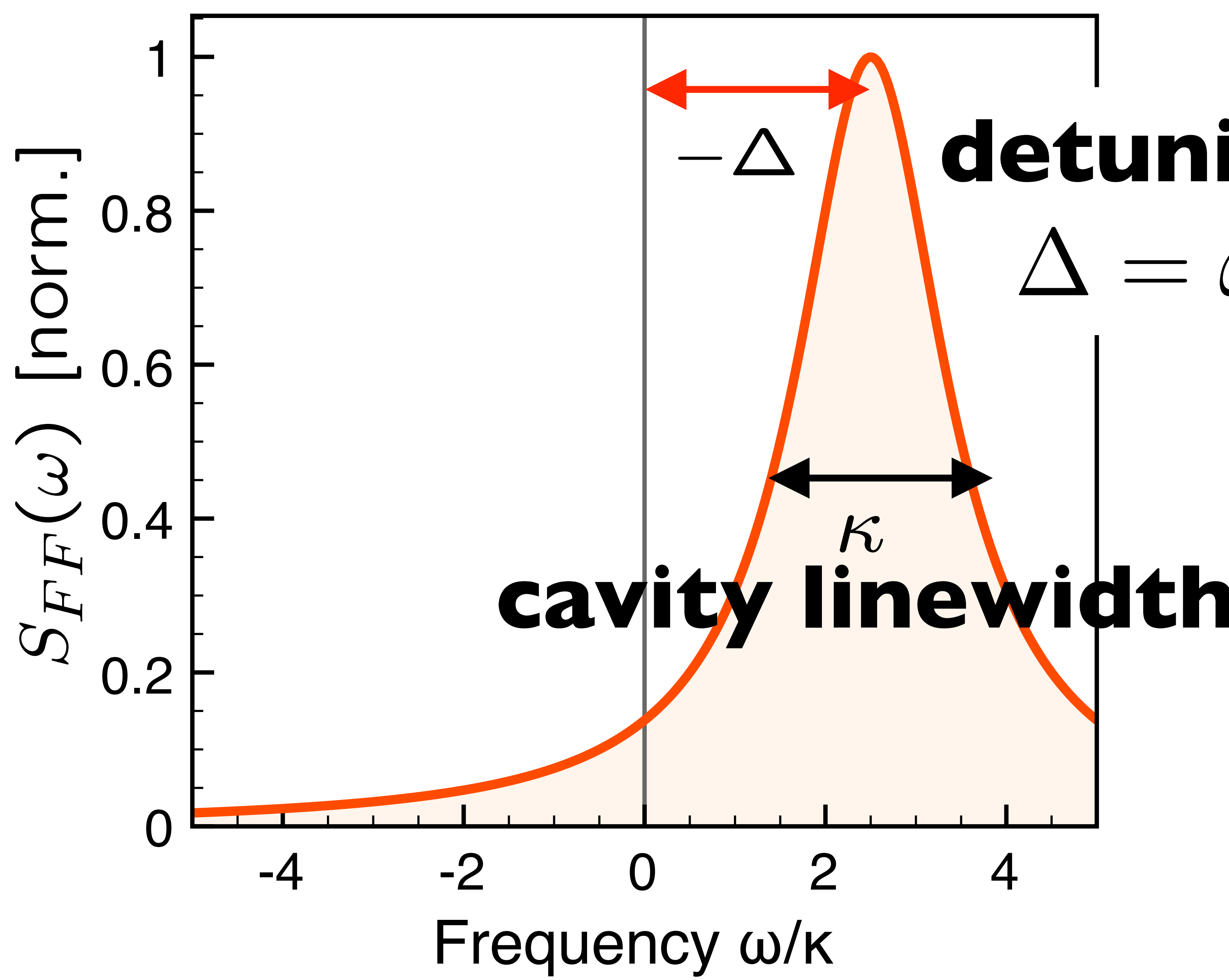
$$S_{FF}(\omega) = \left(\frac{\hbar\omega_R}{L} \right)^2 \bar{n}_p \frac{\kappa}{(\omega + \Delta)^2 + (\kappa/2)^2}$$

photon shot noise spectrum

Quantum theory of optomechanical cooling

Spectrum of radiation pressure fluctuations

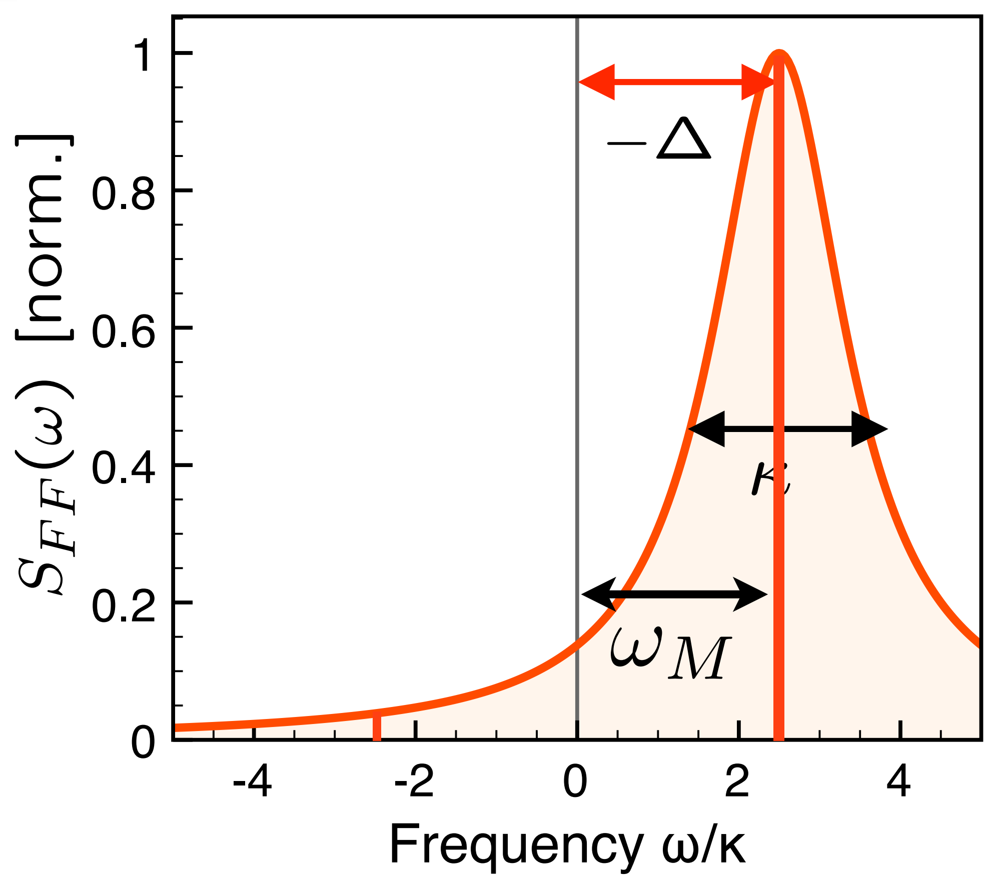
$$S_{FF}(\omega) = \int e^{i\omega t} \left\langle \hat{F}(t)\hat{F}(0) \right\rangle dt$$



detuning :
$$\Delta = \omega_L - \omega_R$$

cavity emits energy / absorbs energy

Quantum theory of optomechanical cooling



cavity emits energy / absorbs energy

Cooling rate

$$\Gamma_{\mathrm{opt}} = \frac{x_{\mathrm{ZPF}}{}^2}{\hbar^2}\left[S_{FF}(+\omega_M) - S_{FF}(-\omega_M)\right]$$

Quantum limit for cantilever phonon number

$$\frac{n_{\mathrm{opt}}+1}{n_{\mathrm{opt}}} = \frac{S_{FF}(+\omega_M)}{S_{FF}(-\omega_M)}$$

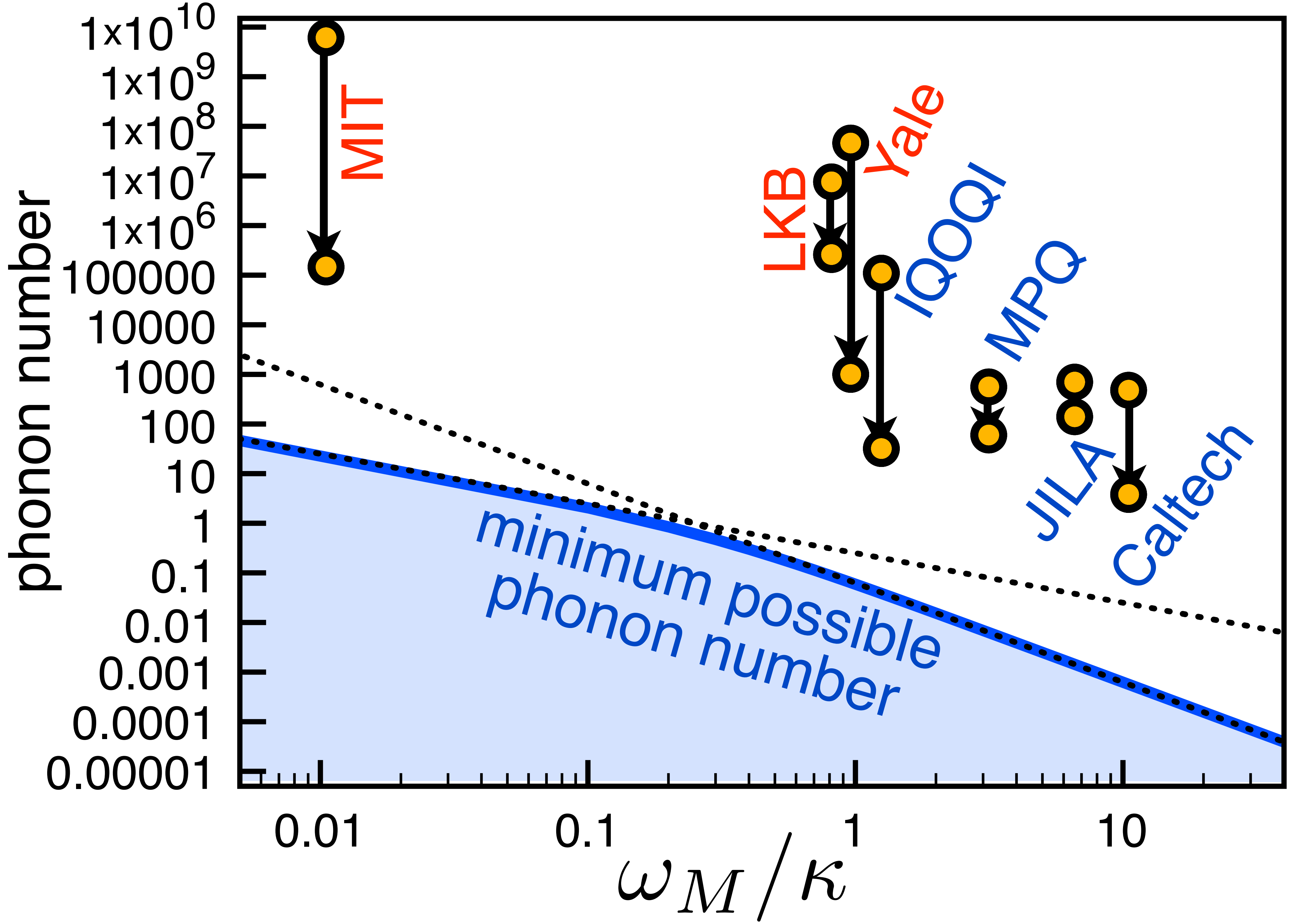
$$\Delta = -\omega_M \Rightarrow n_{\mathrm{opt}} = \left(\frac{\kappa}{4\omega_M}\right)^2$$

Ground-state cooling needs: high optical finesse / large mechanical frequency

FM, Chen, Clerk, Girvin, PRL **93**, 093902 (2007)
also: Wilson-Rae, Nooshi, Zwerger, Kippenberg, PRL **99**, 093901 (2007); Genes et al, PRA 2008

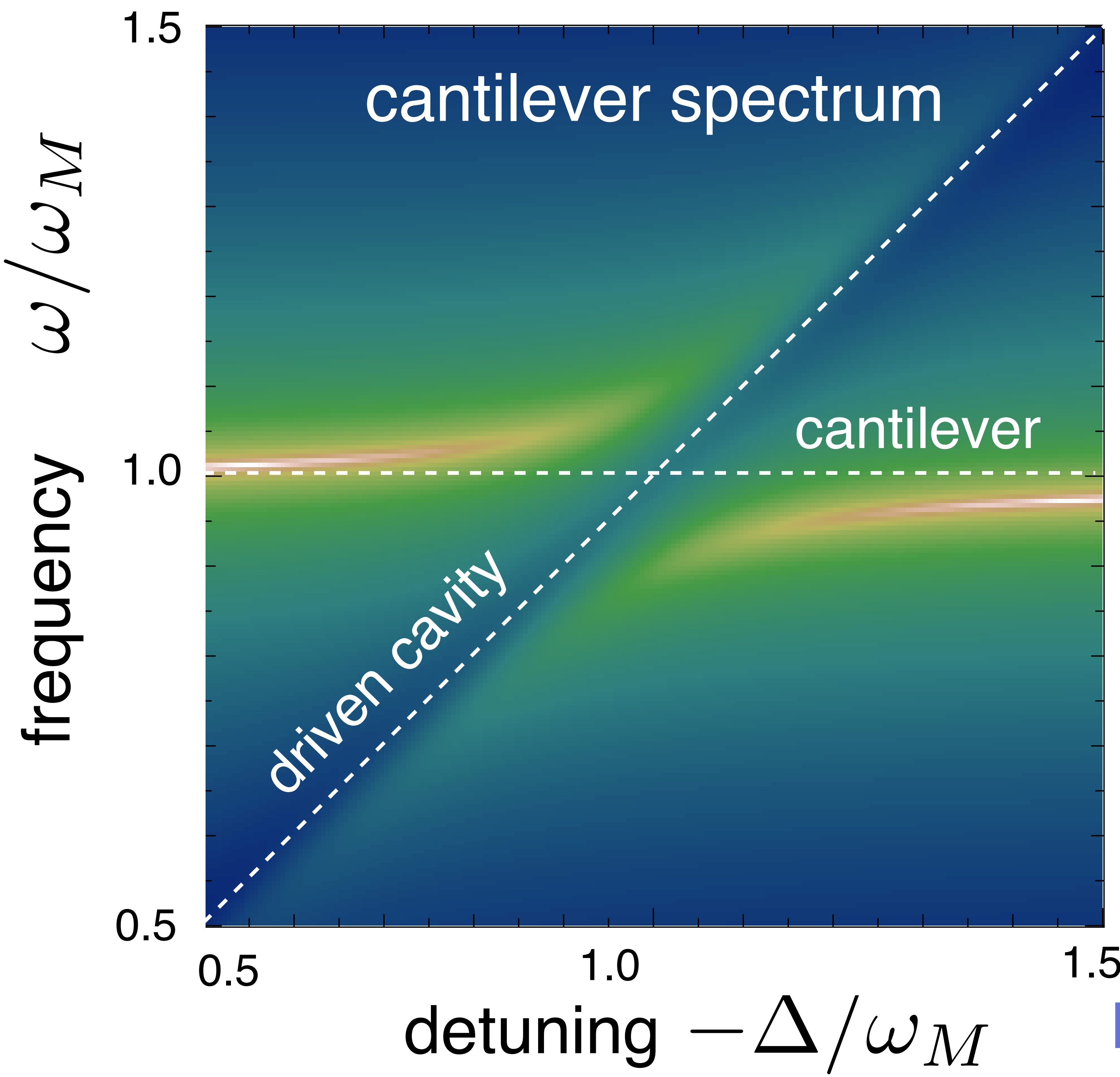
experiment with $\kappa/\omega_M \approx 1/20$
Kippenberg group 2007

Towards the ground state



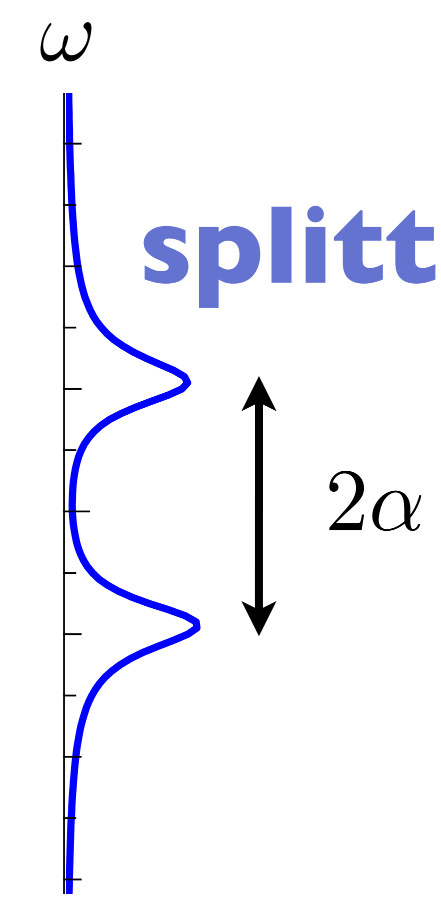
Results from groups at MIT (Mavalvala), LKB (Pinard, Heidmann, Cohadon), Yale (Harris), IQOQI Wien (Aspelmeyer), MPQ Munich (Kippenberg), JILA Boulder (Lehnert), Caltech (Schwab)

Strong coupling: resonances of light and mechanics hybridize



strong coupling

$\Gamma_{\mathrm{opt}} > \kappa/2$

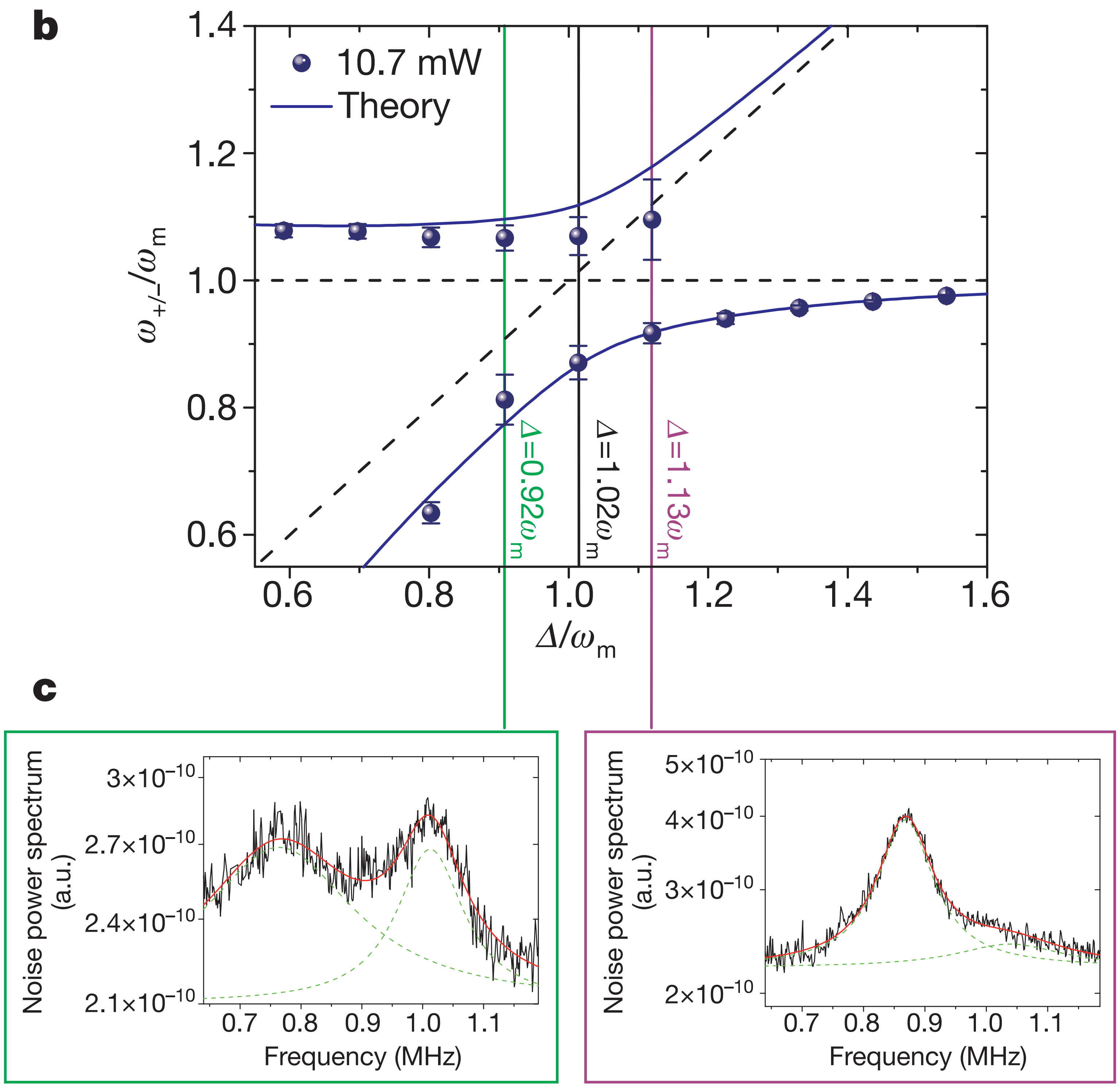


$$\alpha = \omega_R \sqrt{\bar{n}_p} \frac{x_{\mathrm{ZPF}}}{L}$$

FM, Chen, Clerk, Girvin, PRL **93**, 093902 (2007)

Experiment: Optomechanical hybridization

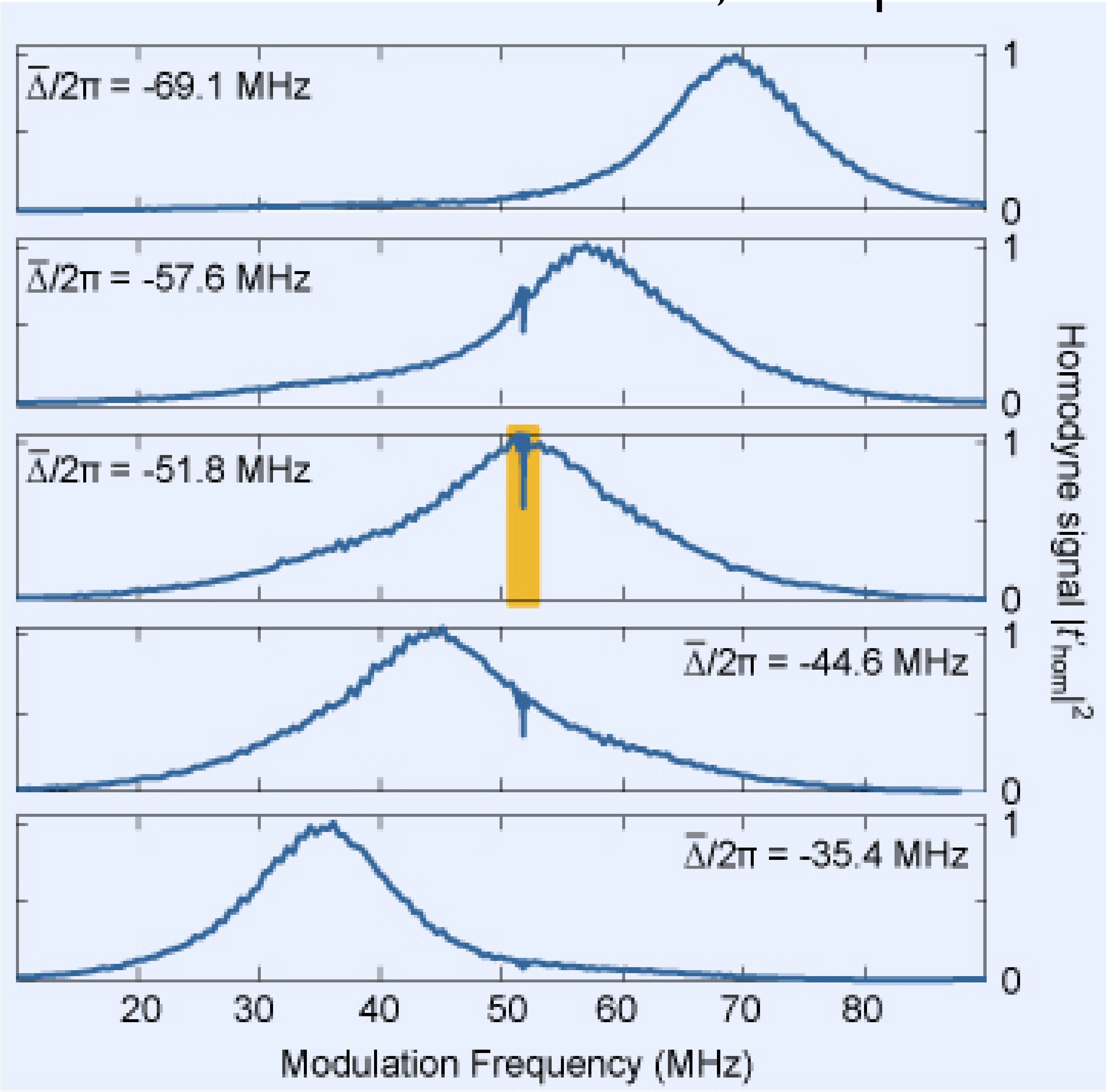
(Mechanical spectrum under strong illumination)



Aspelmeyer group 2009

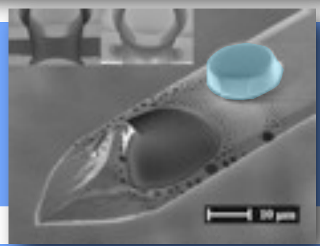
Experiment: Optomechanically induced transparency

(Light field transmission of a second, weak probe beam)



Kippenberg group 2010

Optomechanics (Outline)



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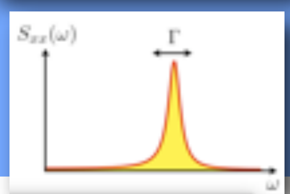
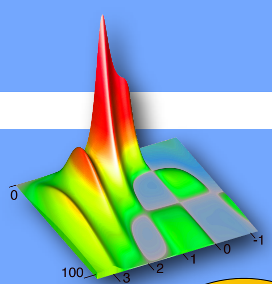
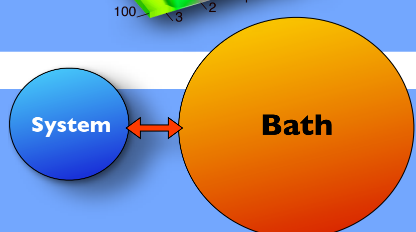


Displacement detection

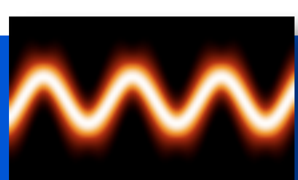
Linear optomechanics



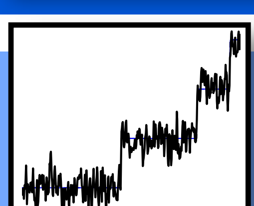
Nonlinear dynamics



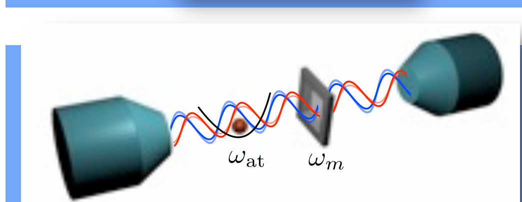
Quantum theory of cooling



Interesting quantum states



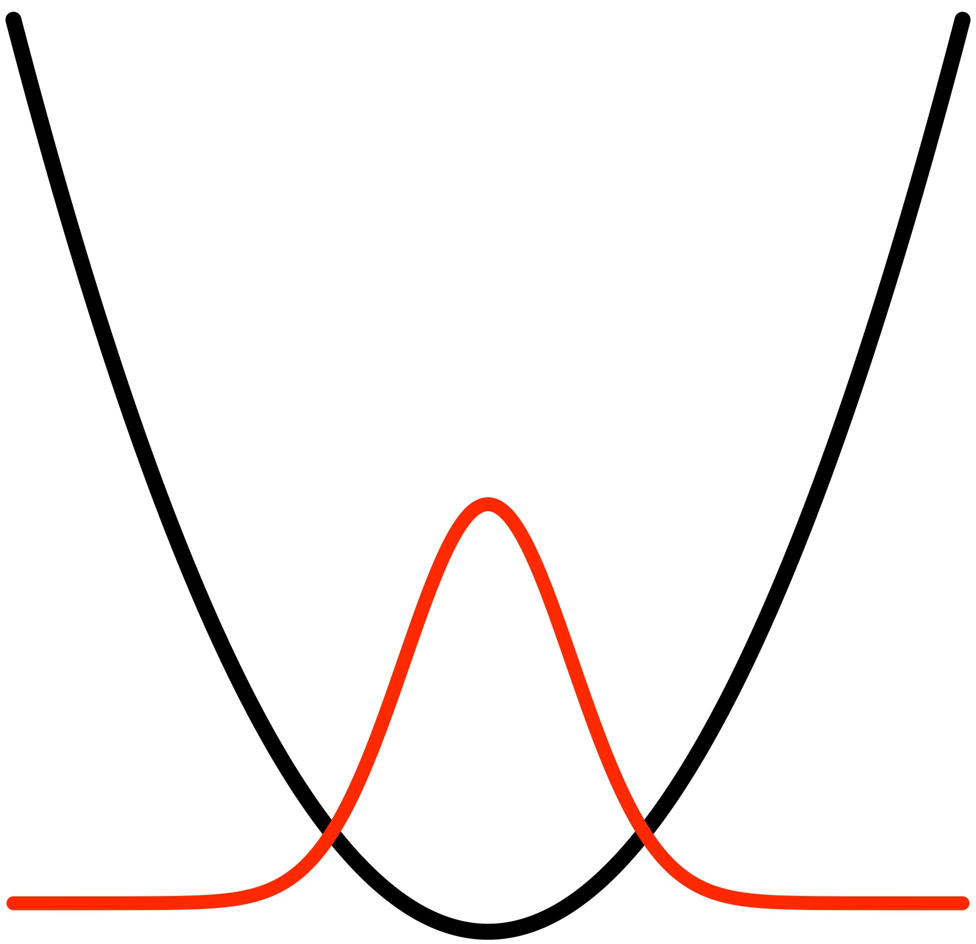
Towards Fock state detection



Coupling to the motion of a single atom

Squeezed states

Squeezing the mechanical oscillator state



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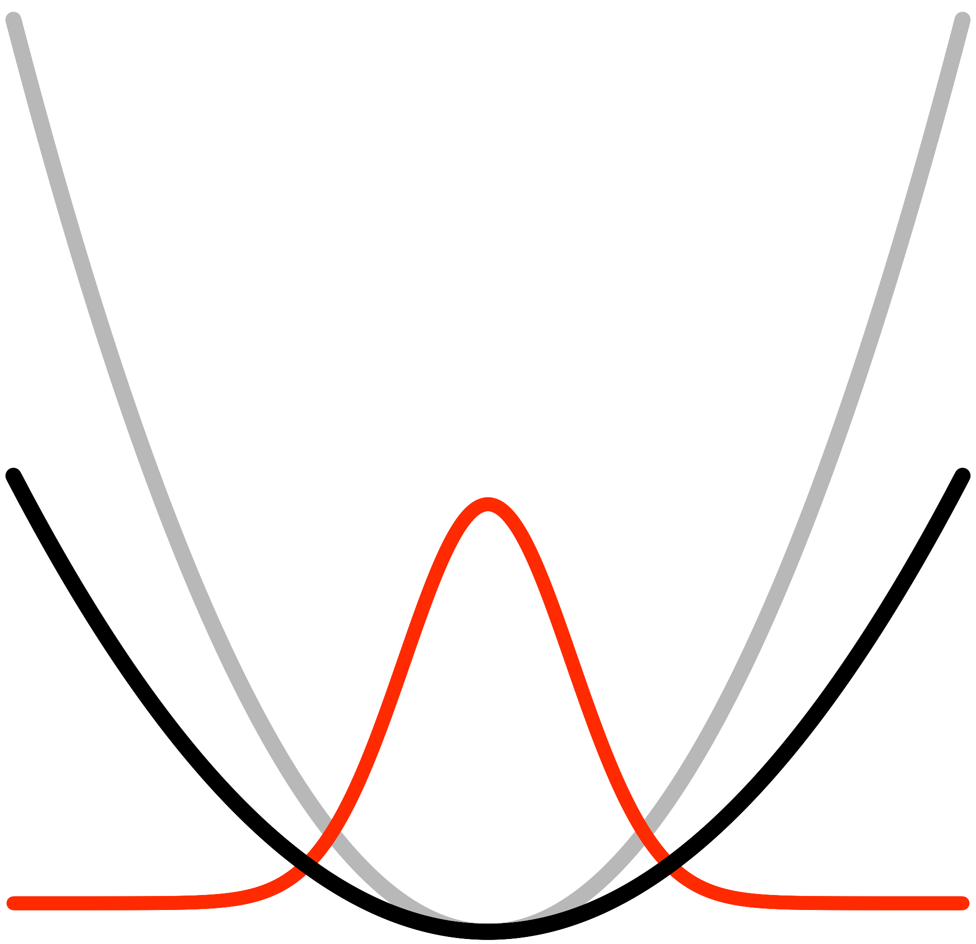

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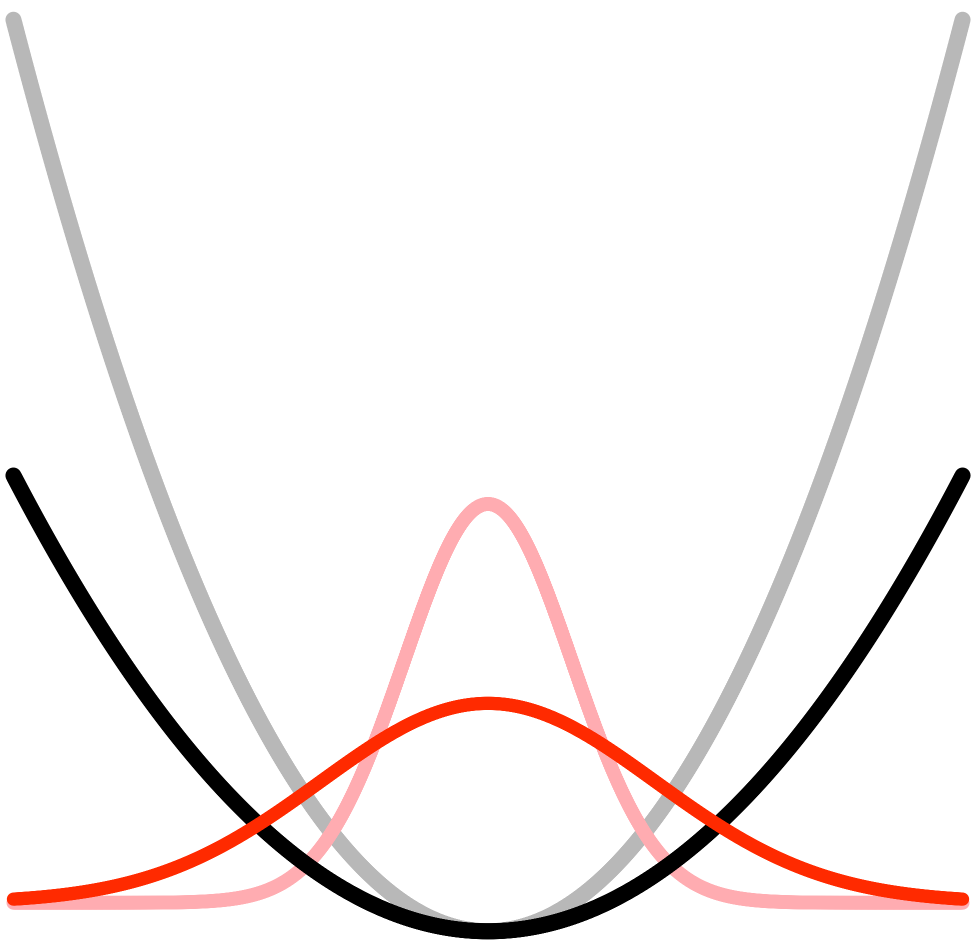

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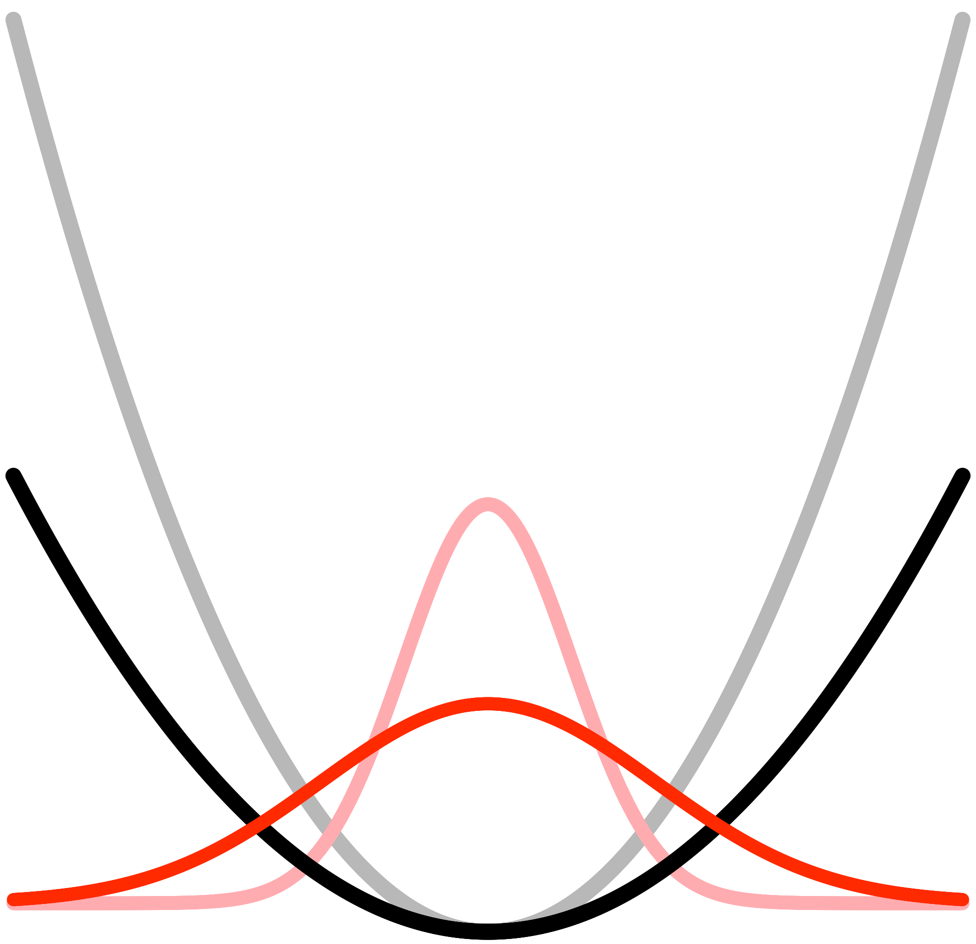

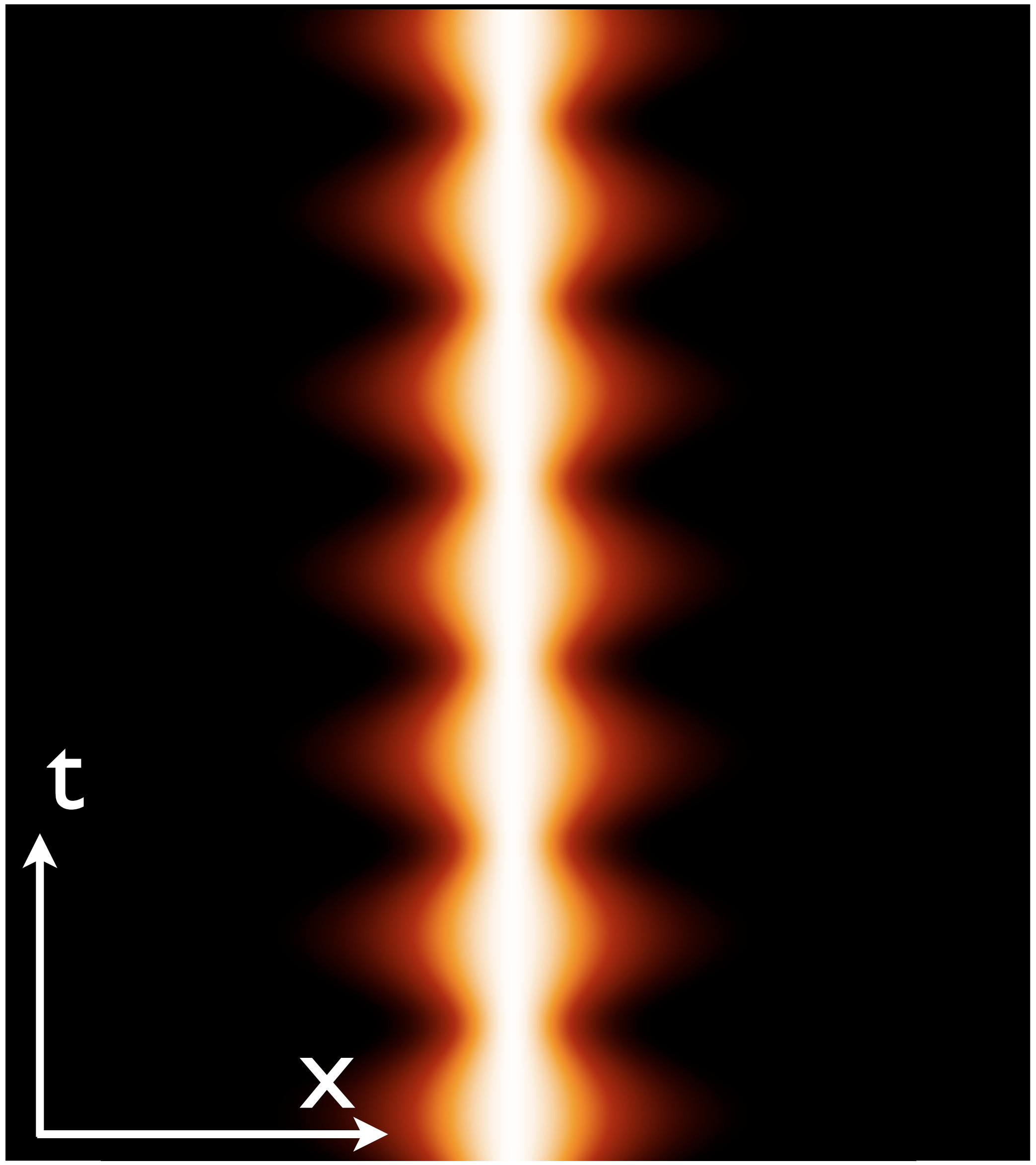

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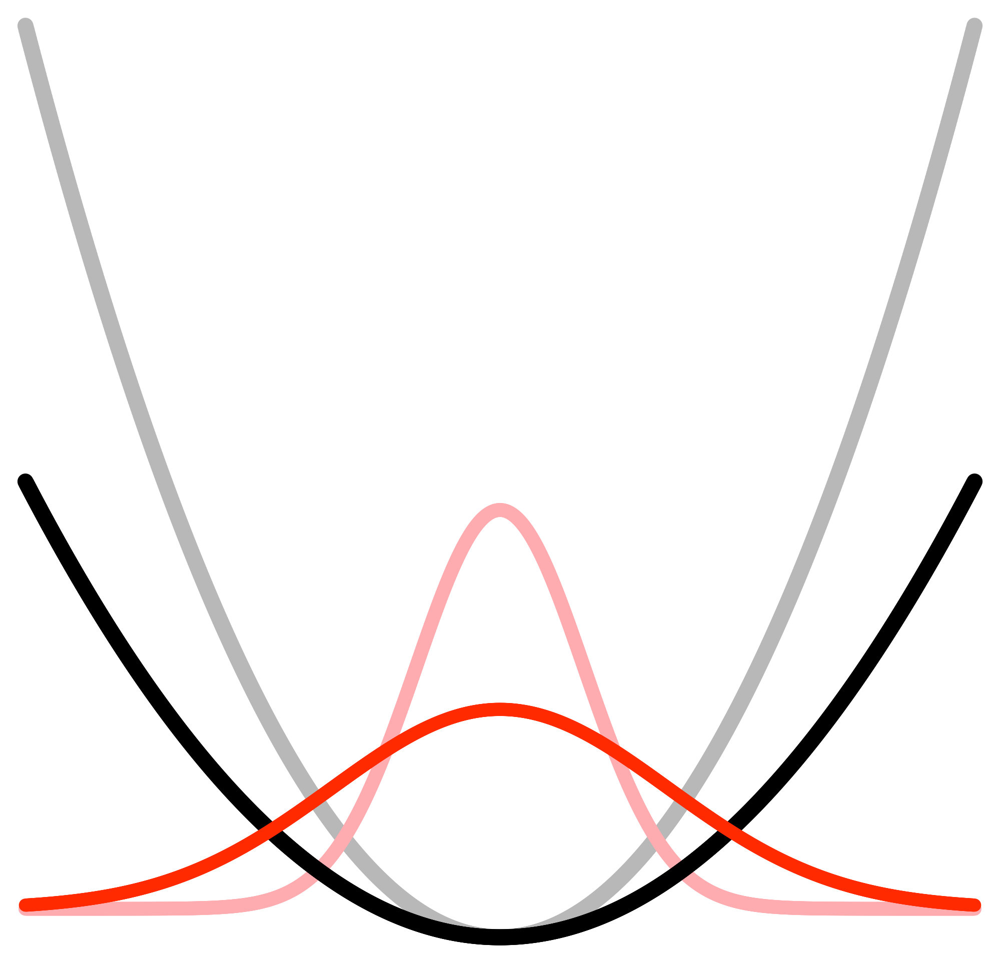
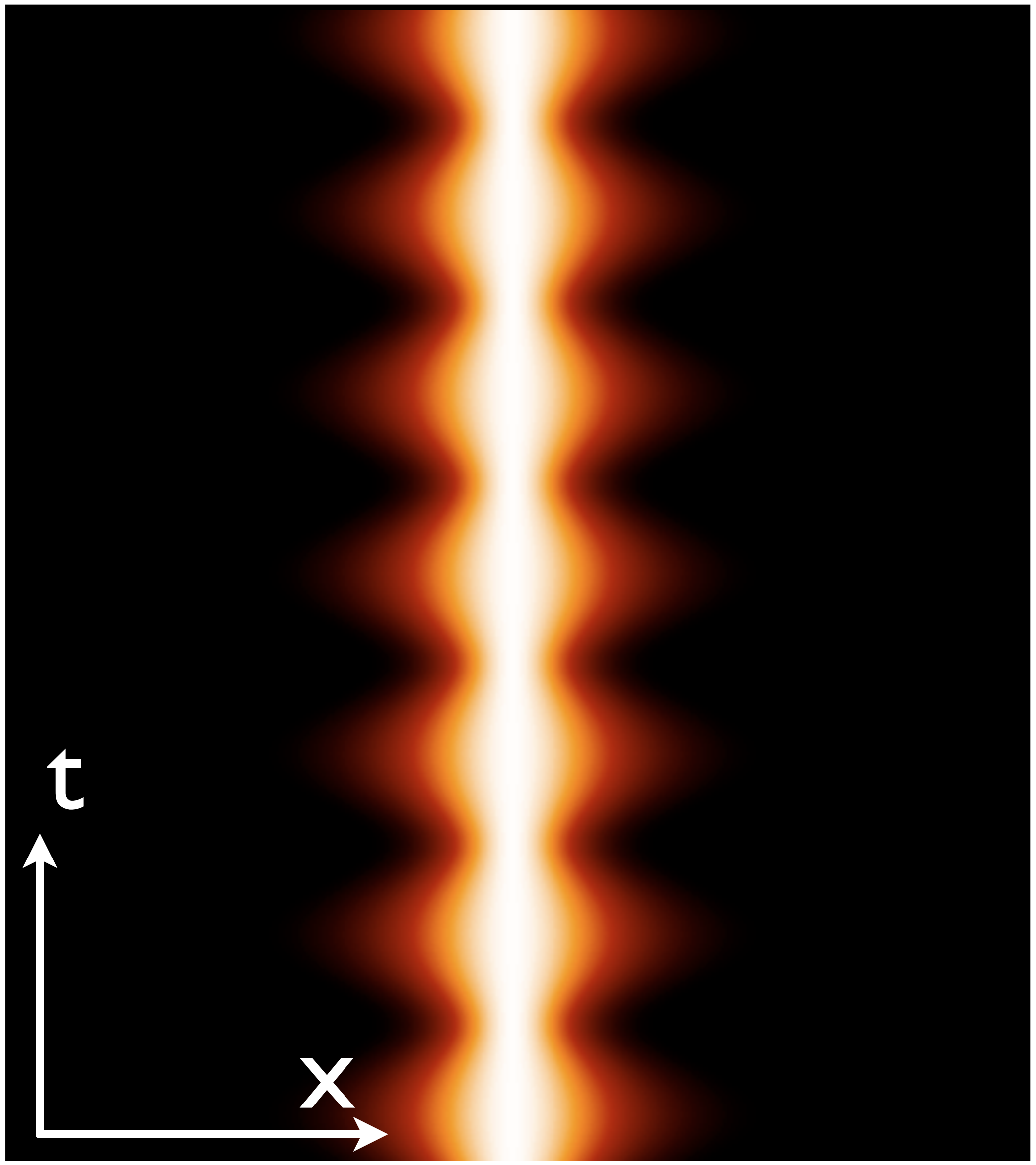
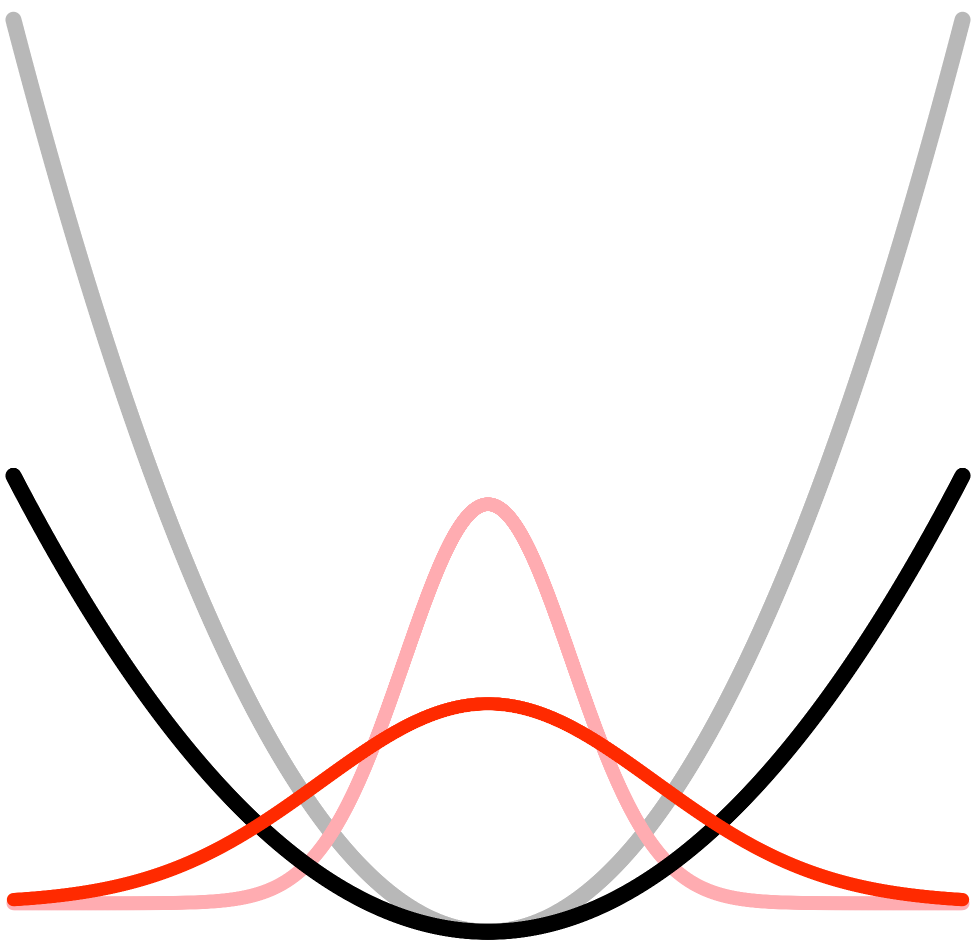


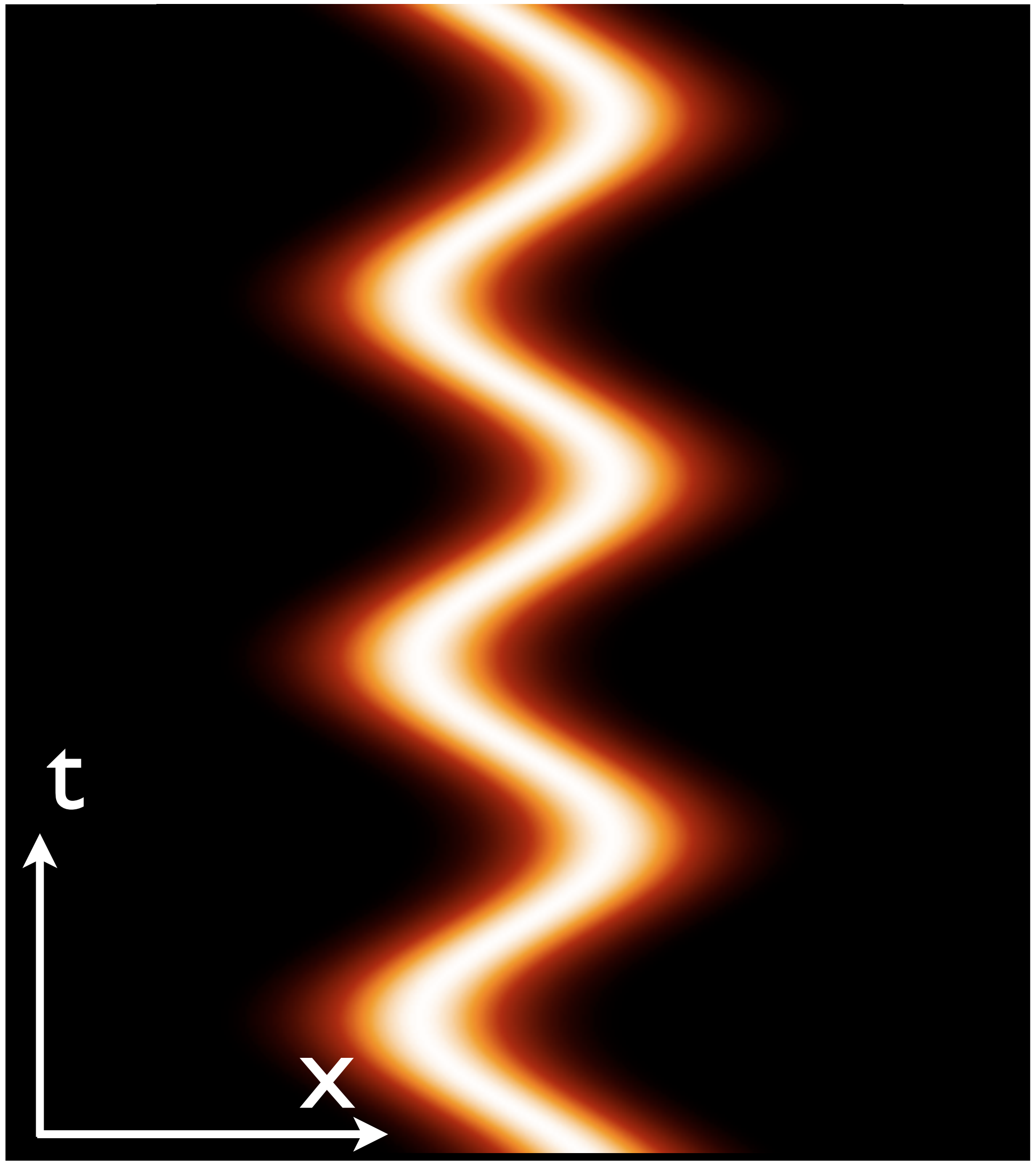


Periodic modulation of spring constant: $\delta\omega_M^2(t) \propto \cos(2\omega_M t)$
Parametric amplification

Squeezed states

Squeezing the mechanical oscillator state





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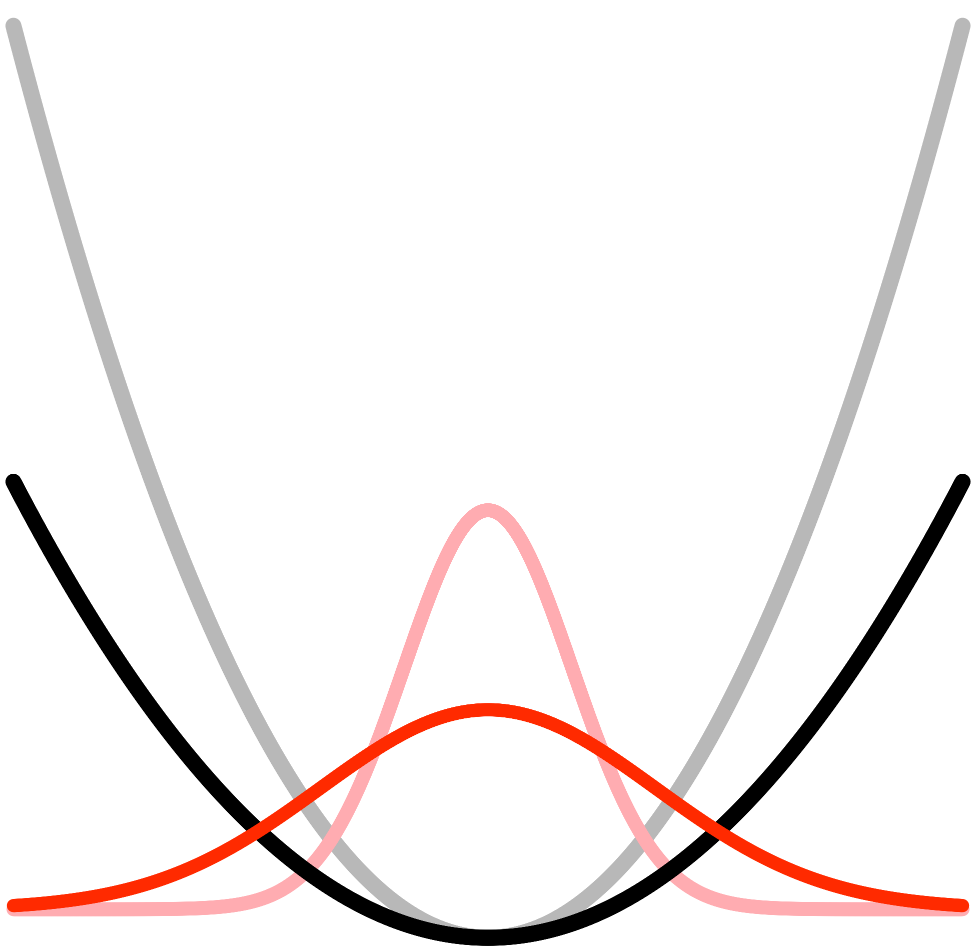

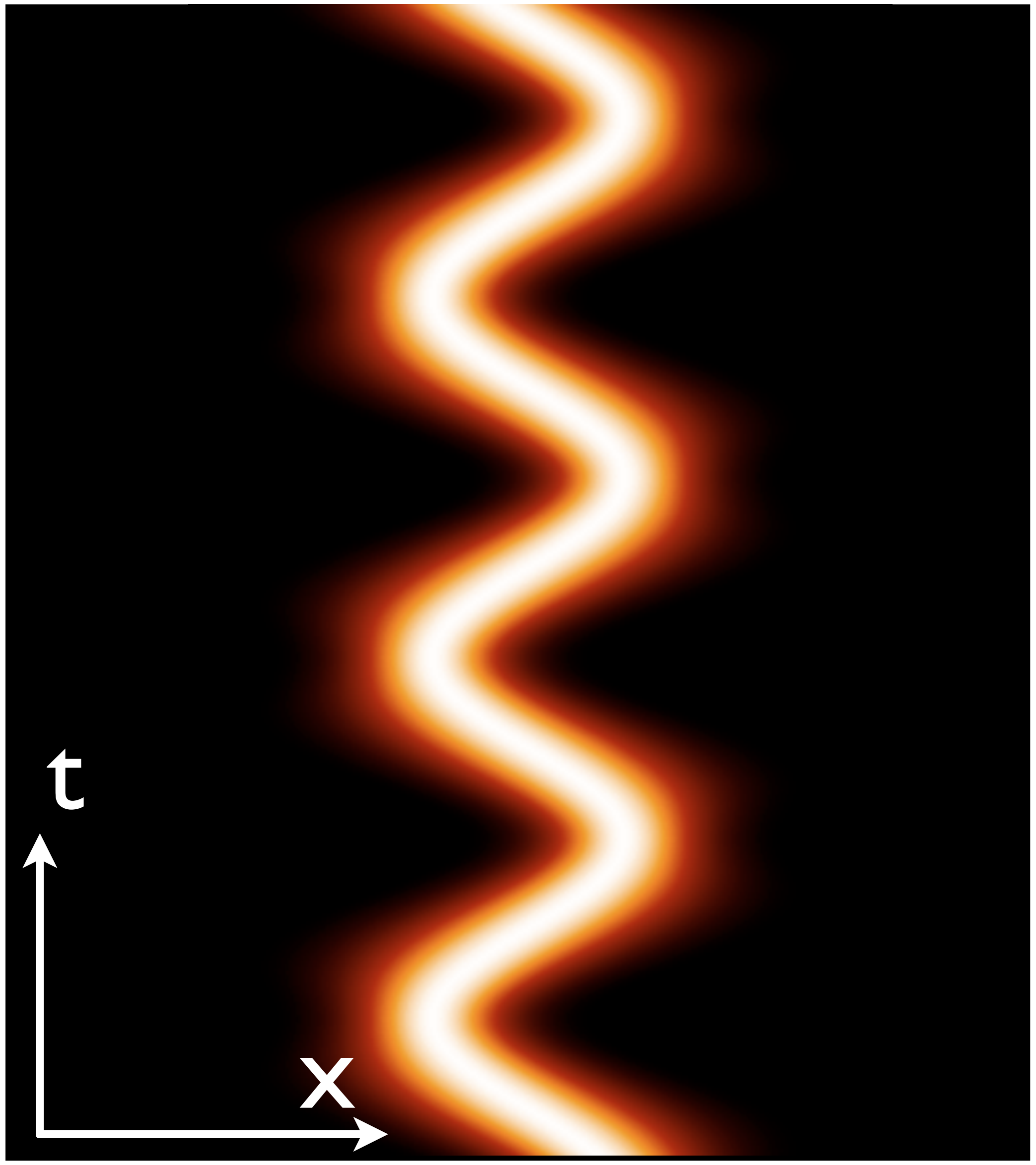

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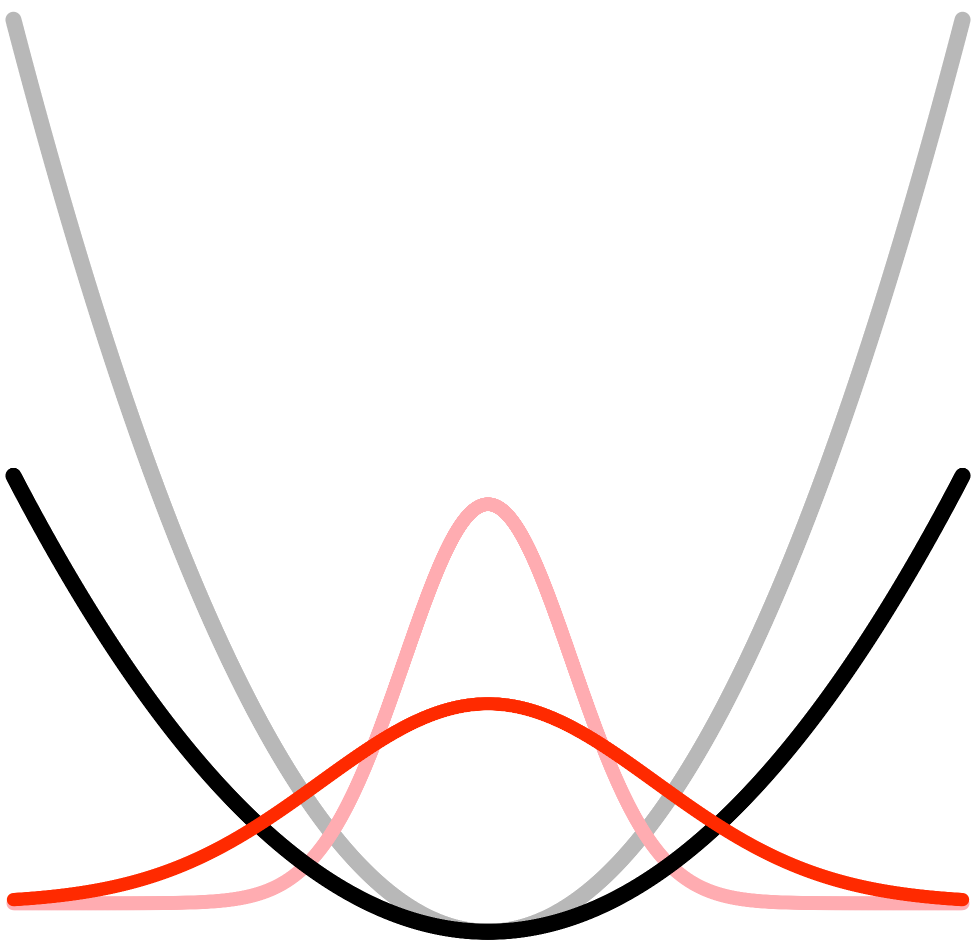

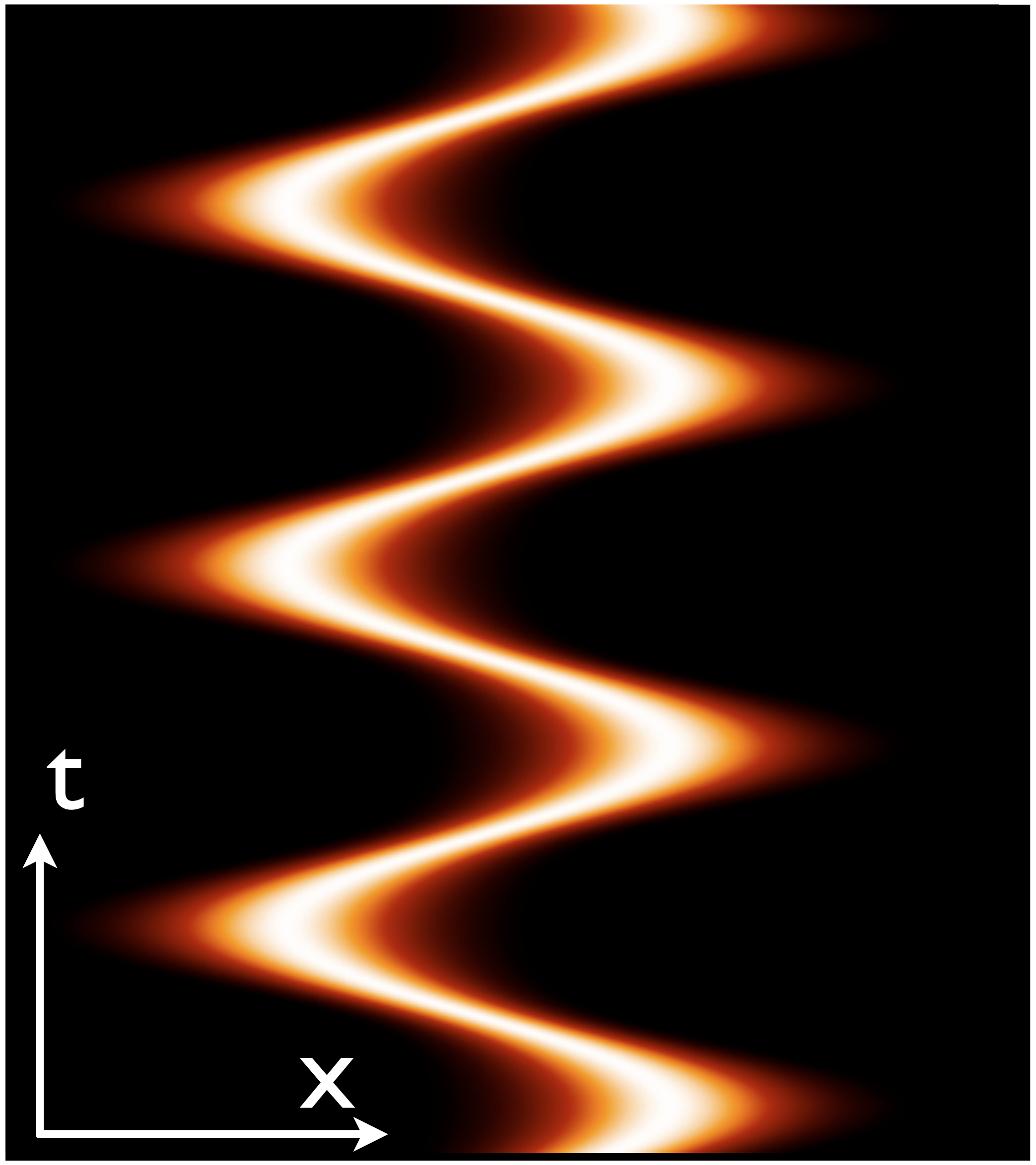

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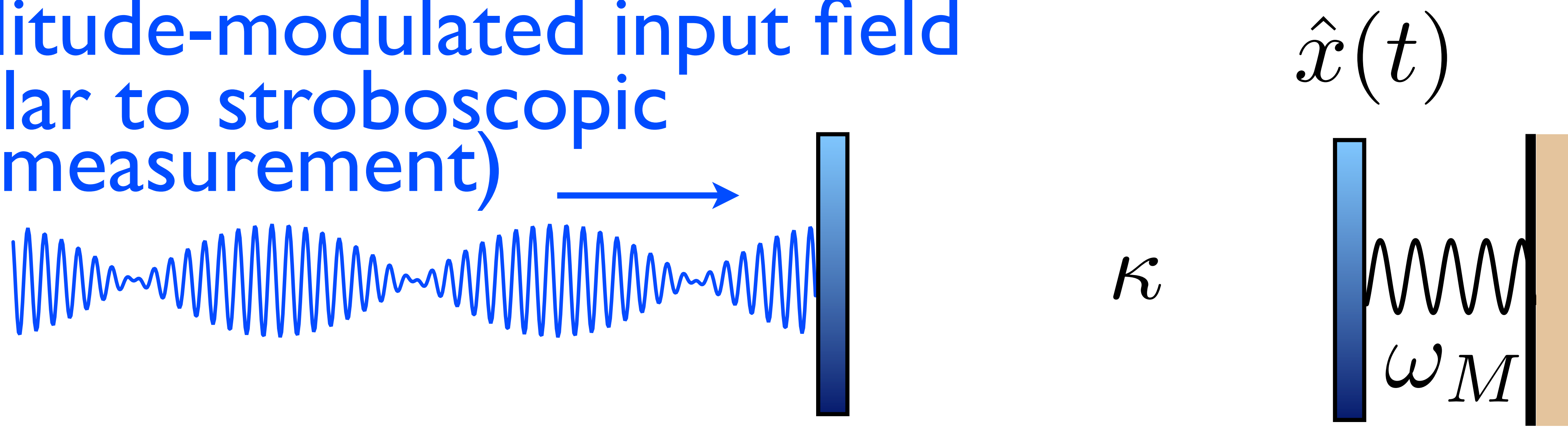
Parametric amplification

Measuring quadratures ("beating the SQL")

Clerk, Marquardt, Jacobs; NJP **10**, 095010 (2008)

amplitude-modulated input field (similar to stroboscopic measurement)



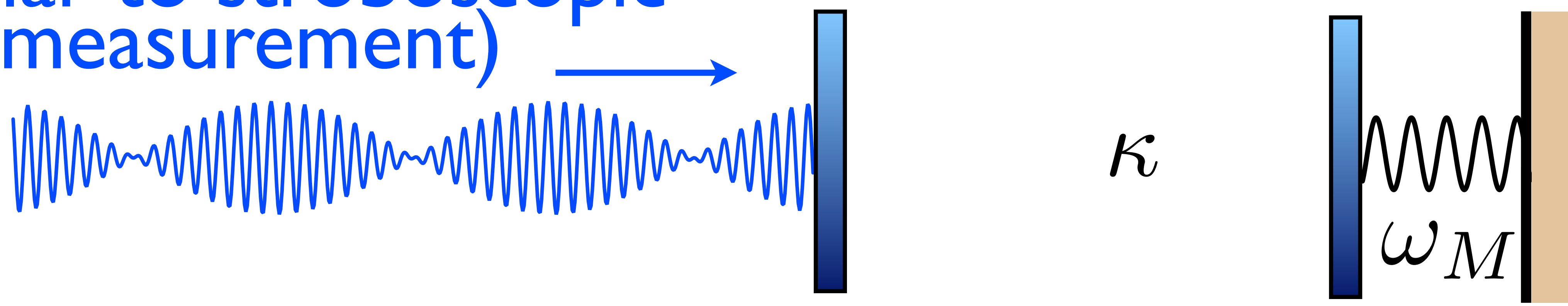
measure only one quadrature, back-action noise affects only the other one....need: $\kappa \ll \omega_M$

Measuring quadratures ("beating the SQL")

Clerk, Marquardt, Jacobs; NJP **10**, 095010 (2008)

amplitude-modulated input field (similar to stroboscopic measurement)

$\hat{x}(t)$



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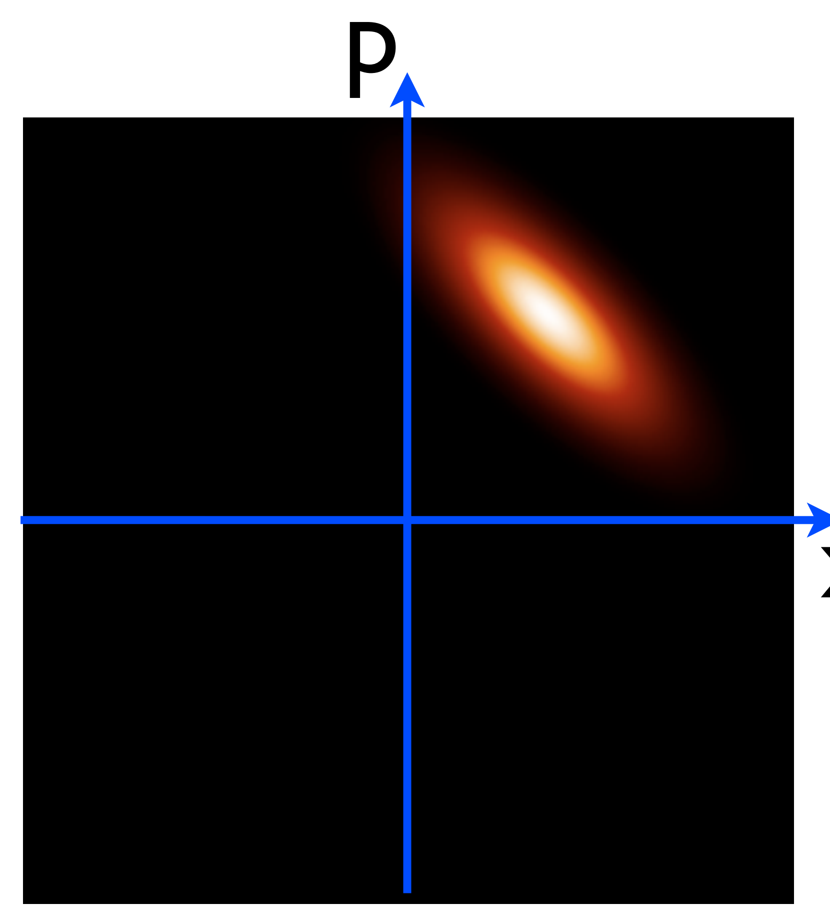

reconstruct mechanical Wigner density

(quantum state tomography)

$$W(x,p) \propto \int dy e^{ipy/\hbar} \rho(x - y/2, x + y/2)$$

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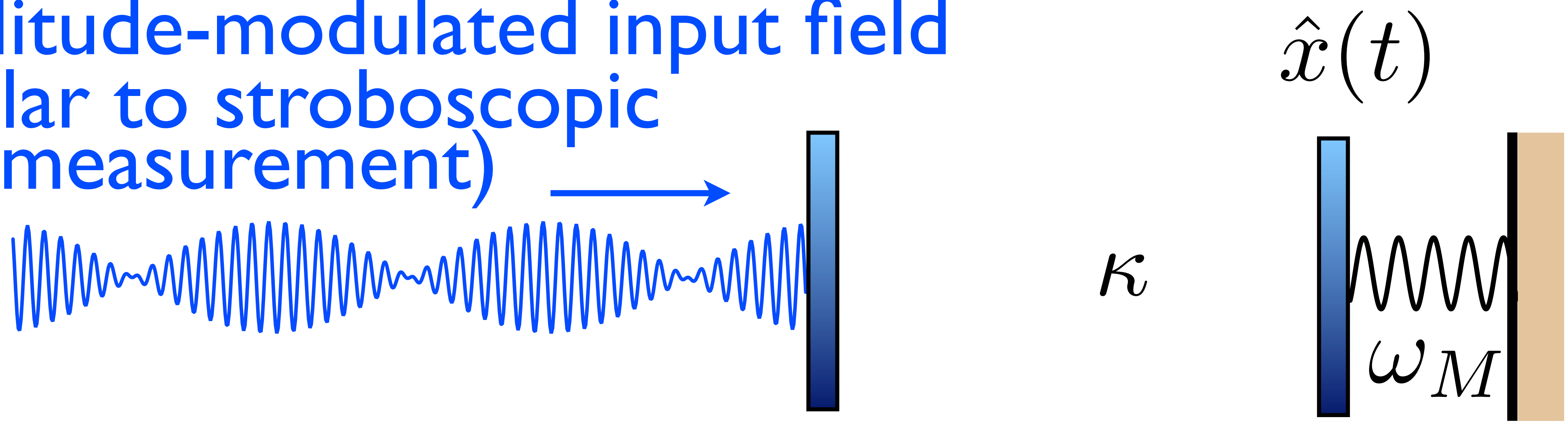

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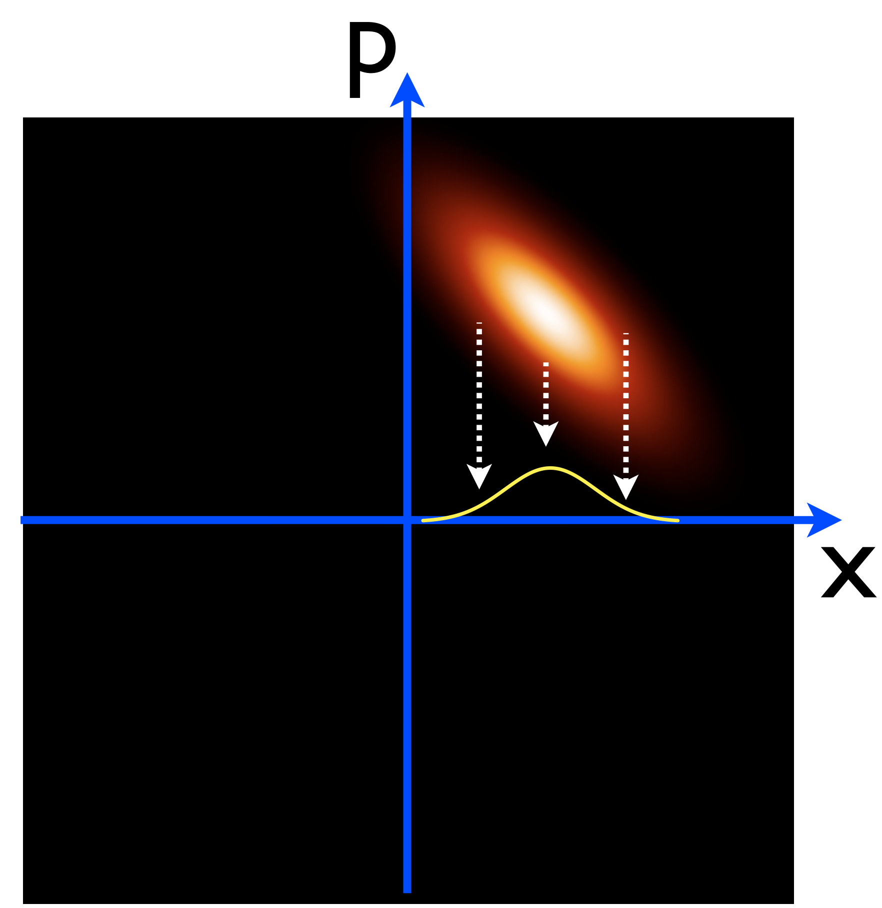

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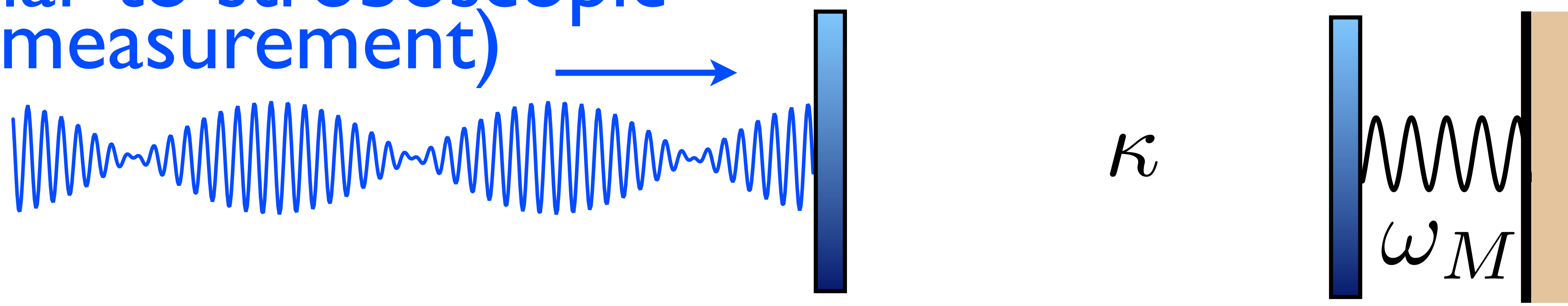

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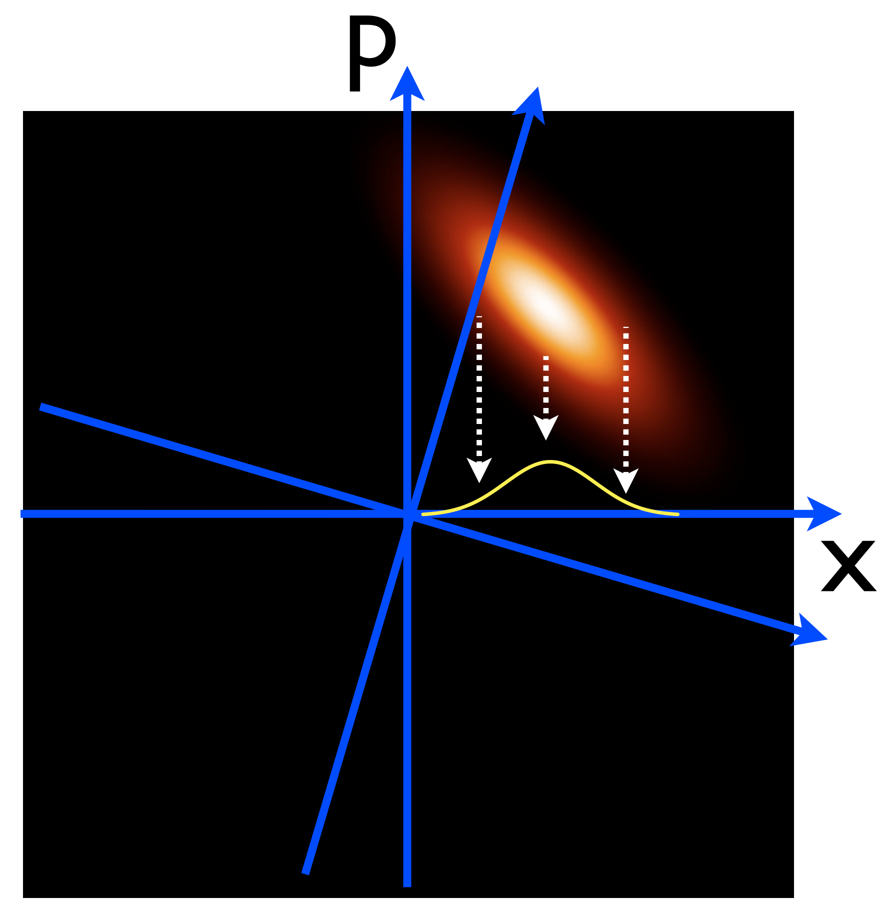
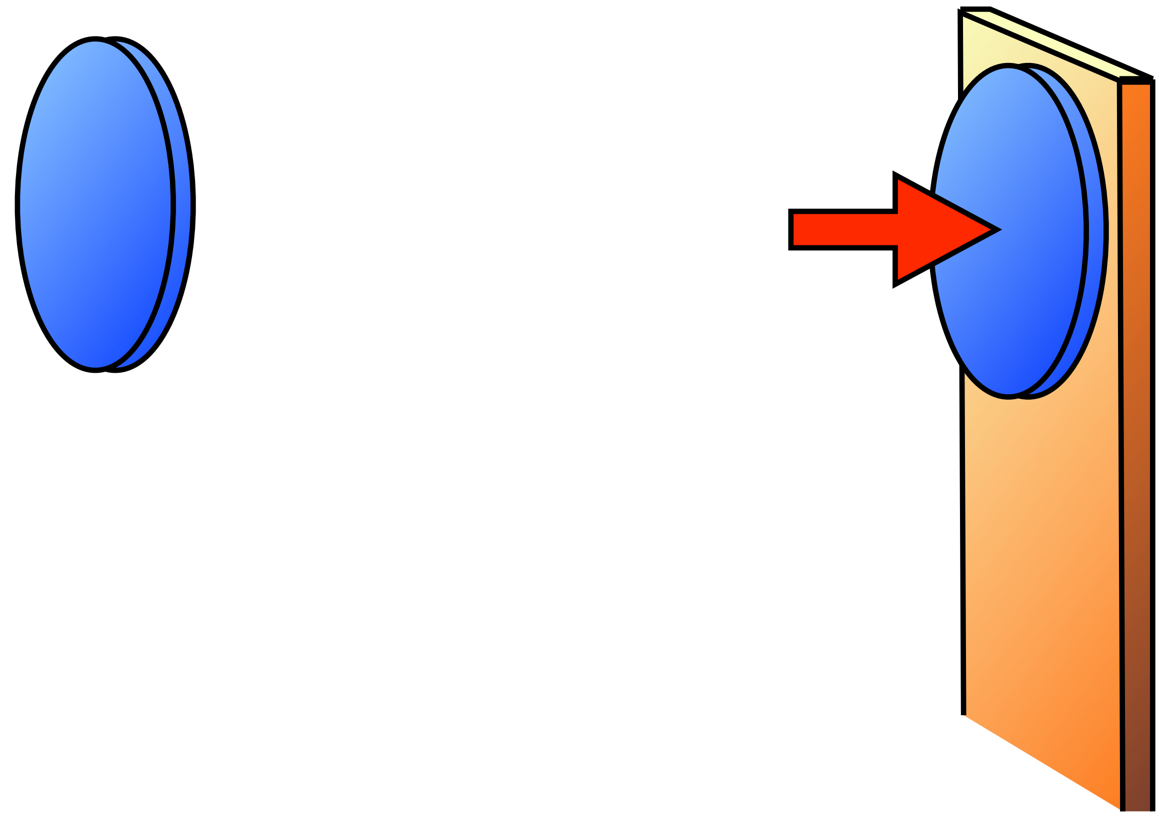
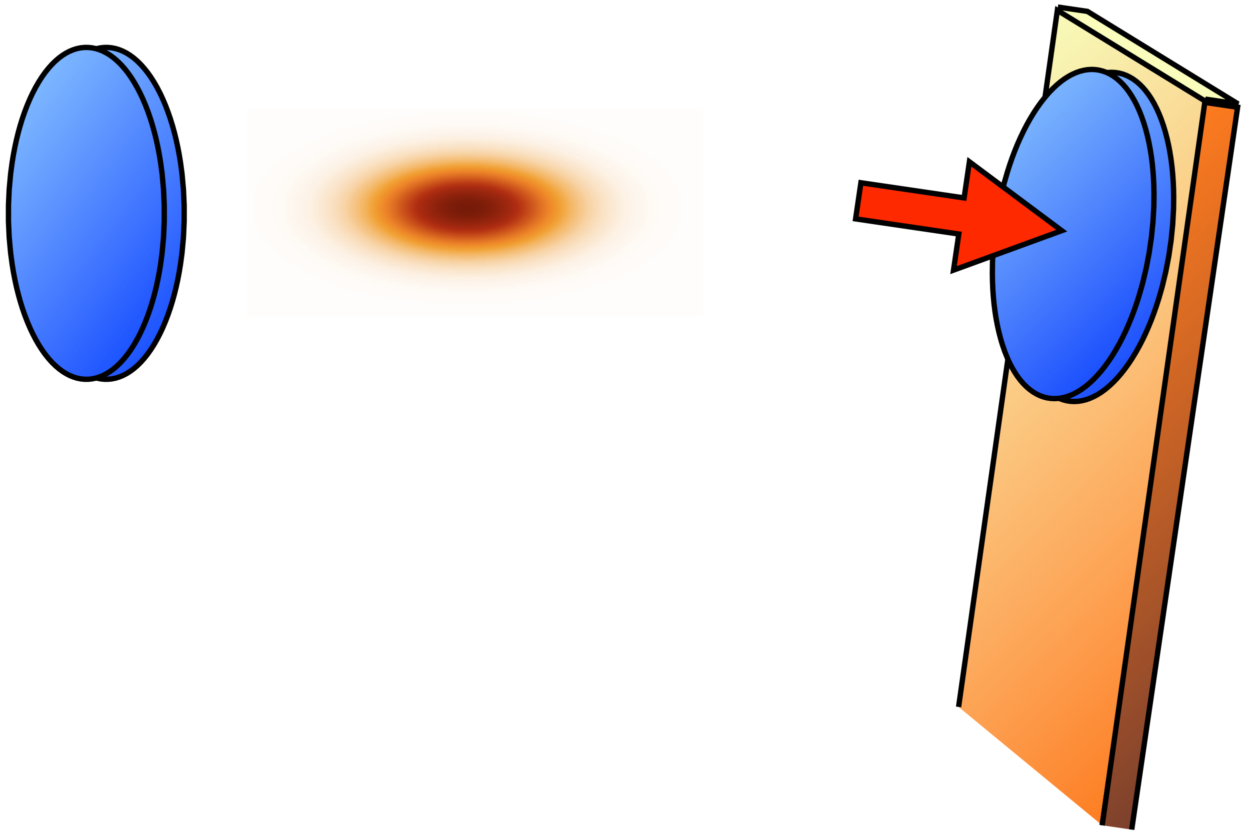


Optomechanical entanglement



Optomechanical entanglement



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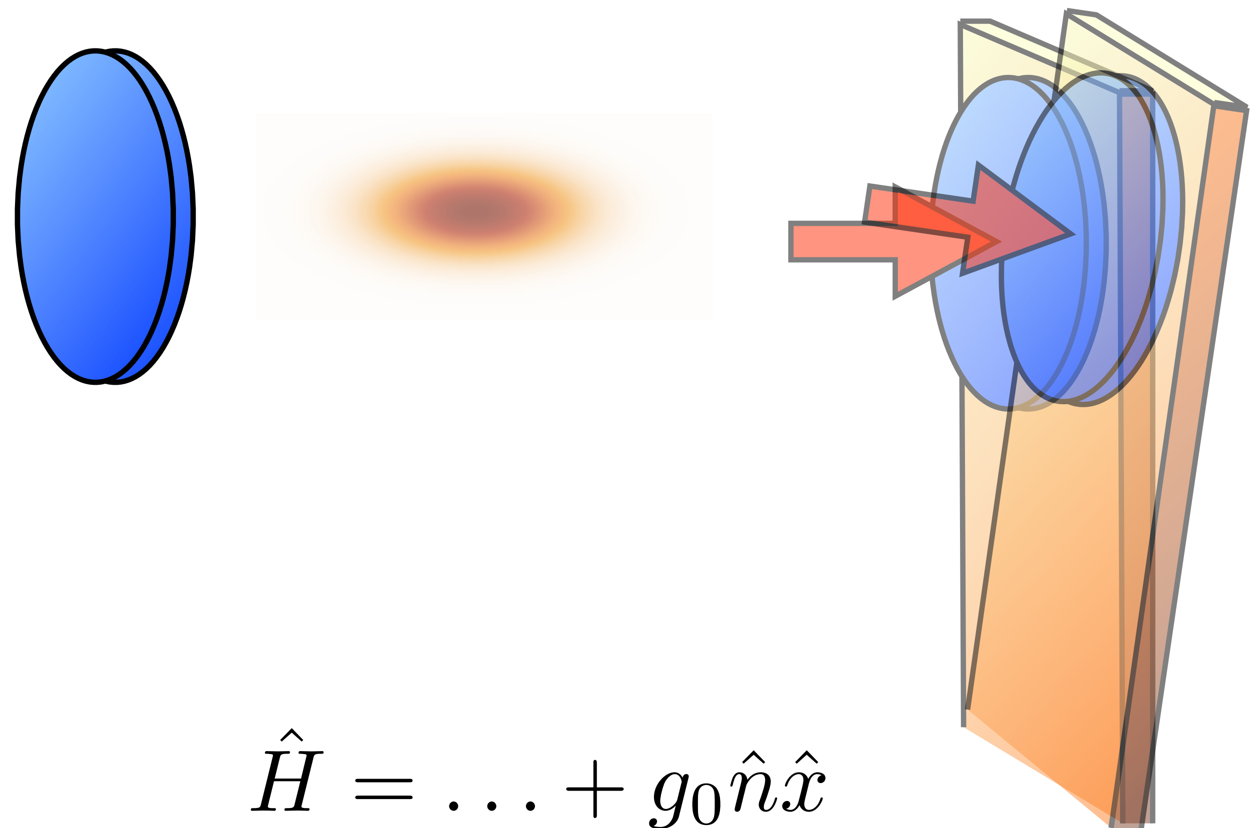
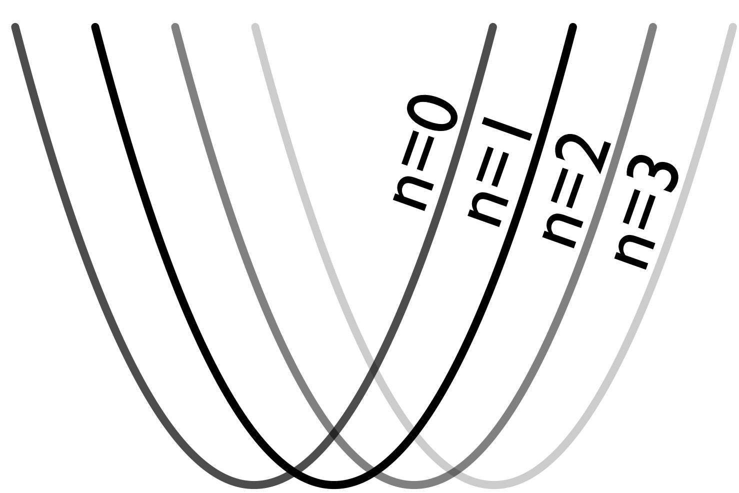


$$\hat{H} = \ldots + g_0 \hat{n} \hat{x}$$

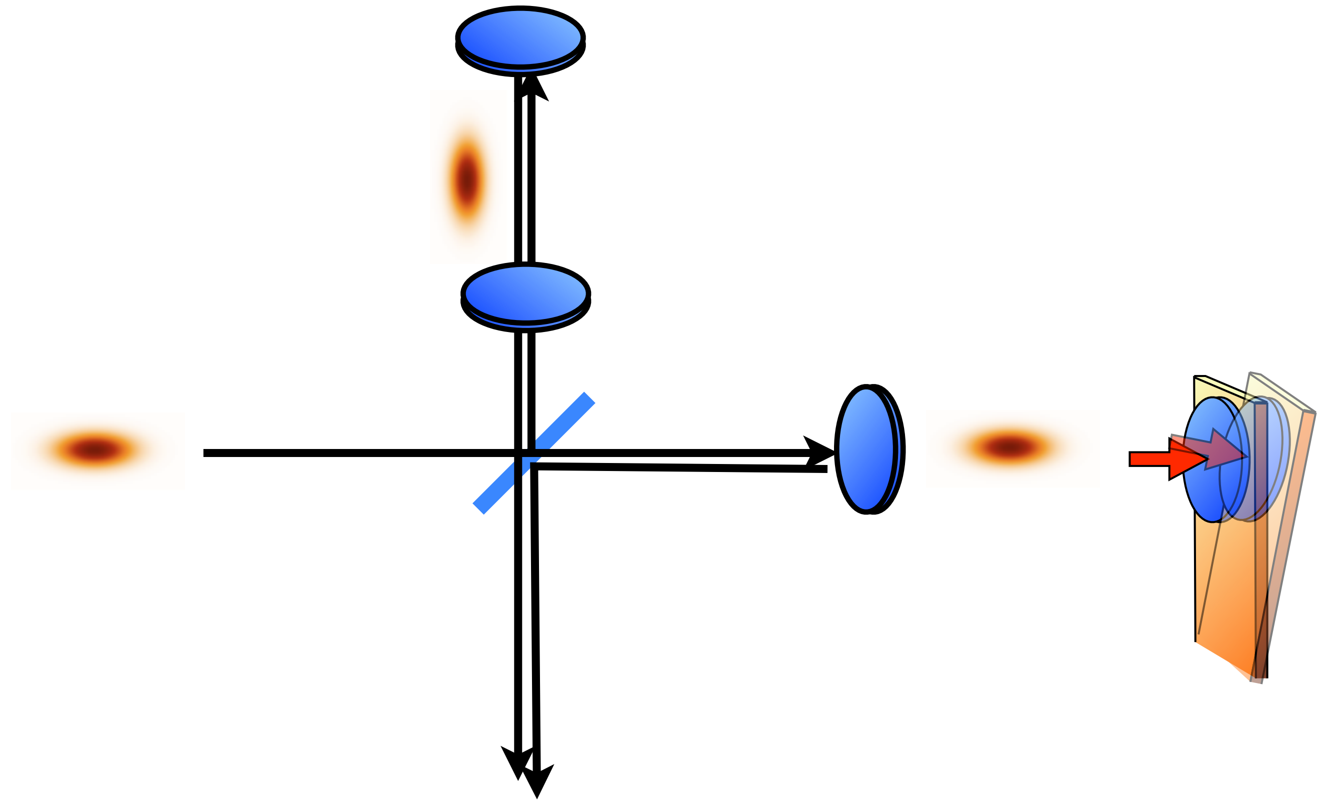
$$|\Psi\rangle = \sum_{n=0}^{\infty} c_n e^{-i\varphi_n(t)} |n\rangle \otimes |\alpha = \alpha_n(t)\rangle$$

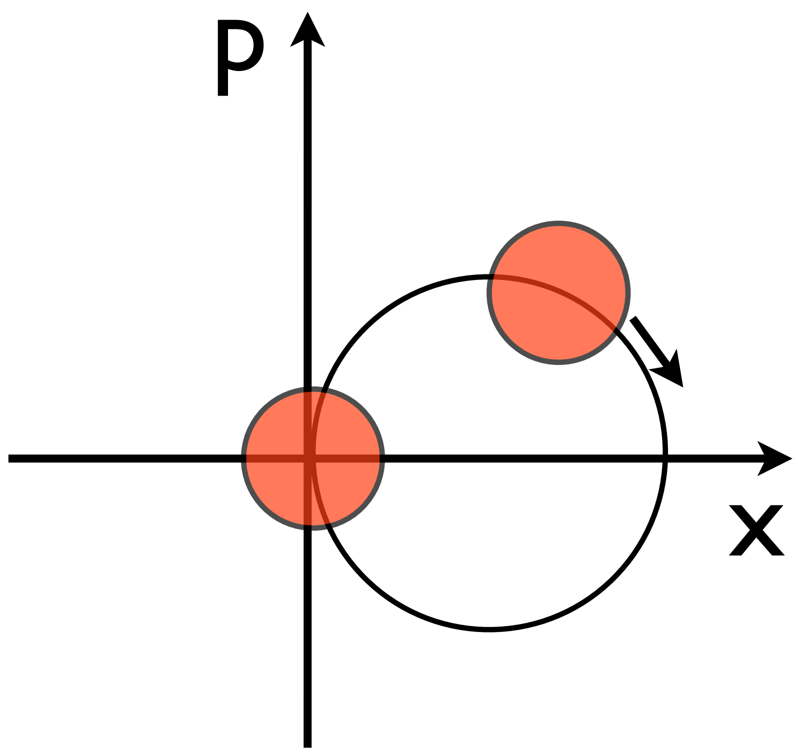
coherent mechanical state



entangled state
(light field/mechanics)

Proposed optomechanical which-path experiment and quantum eraser



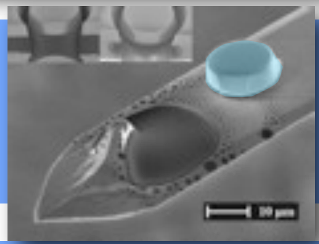


Recover photon coherence if interaction time equals a multiple of the mechanical period!

cf. Haroche experiments in 90s

Marshall, Simon, Penrose, Bouwmeester, PRL **91**, 130401 (2003)

Optomechanics (Outline)



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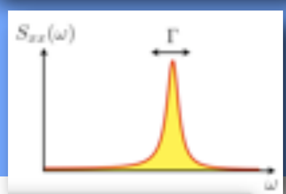
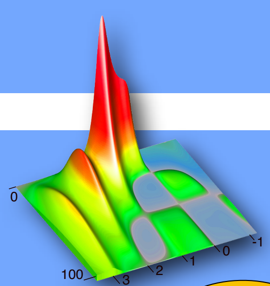
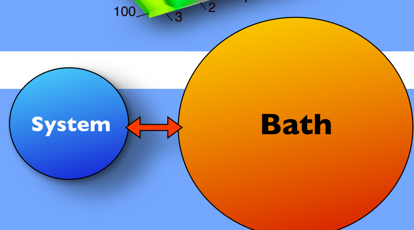


Displacement detection

Linear optomechanics



Nonlinear dynamics



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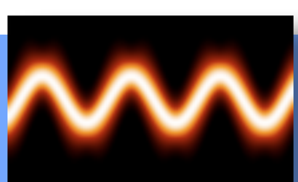
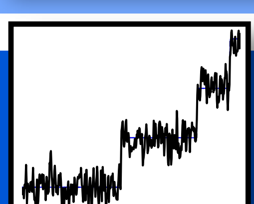
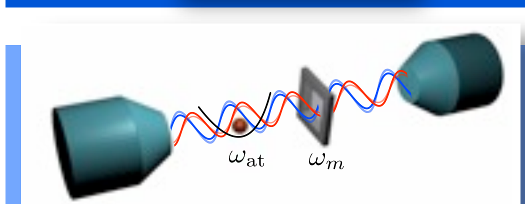


Interesting quantum states

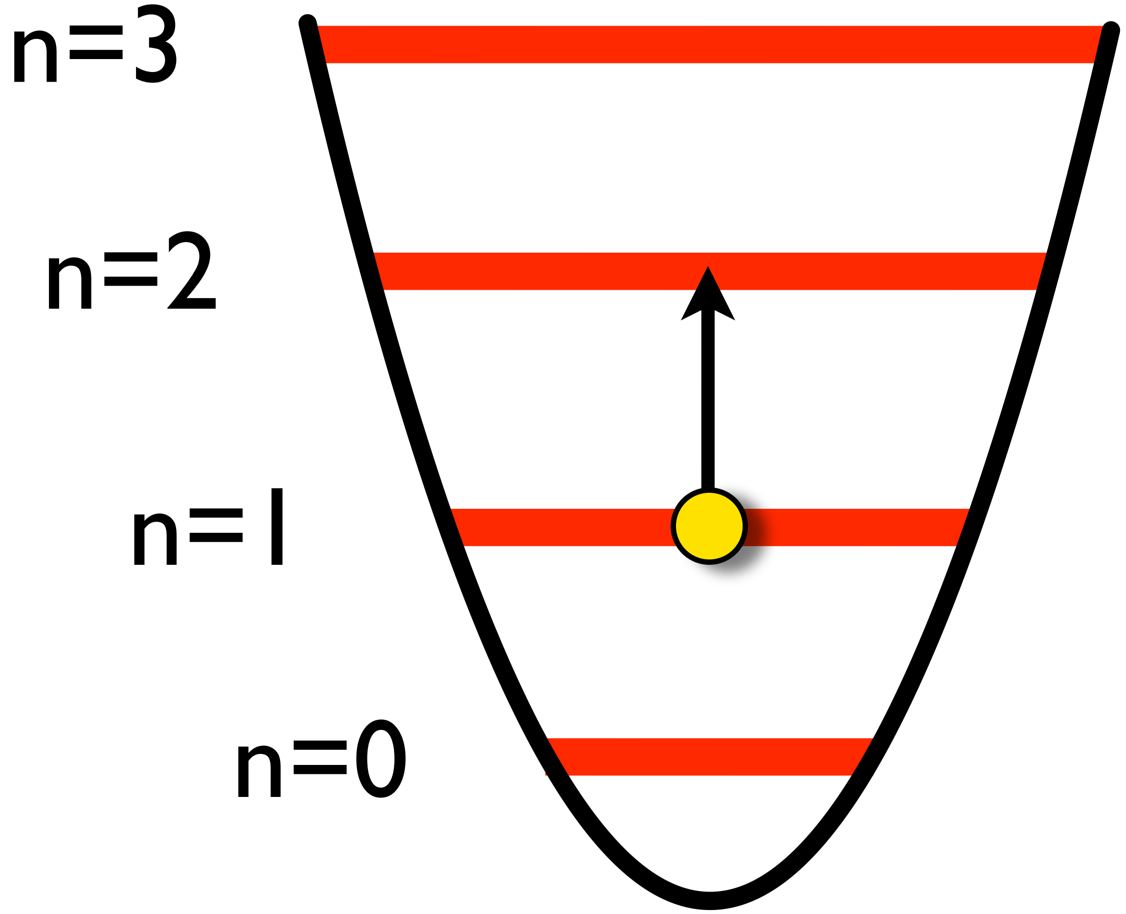


Towards Fock state detection



Coupling to the motion of a single atom

Towards Fock state detection of a macroscopic object



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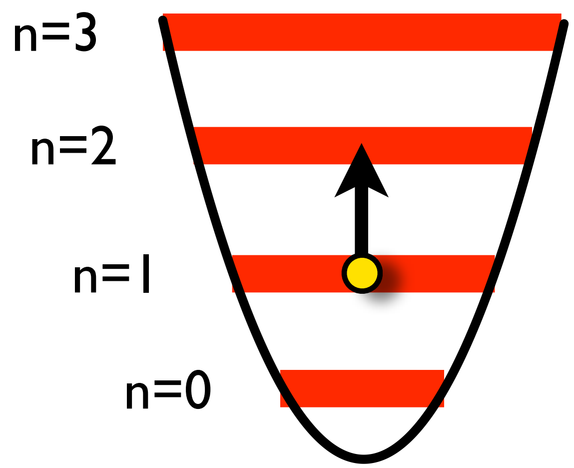
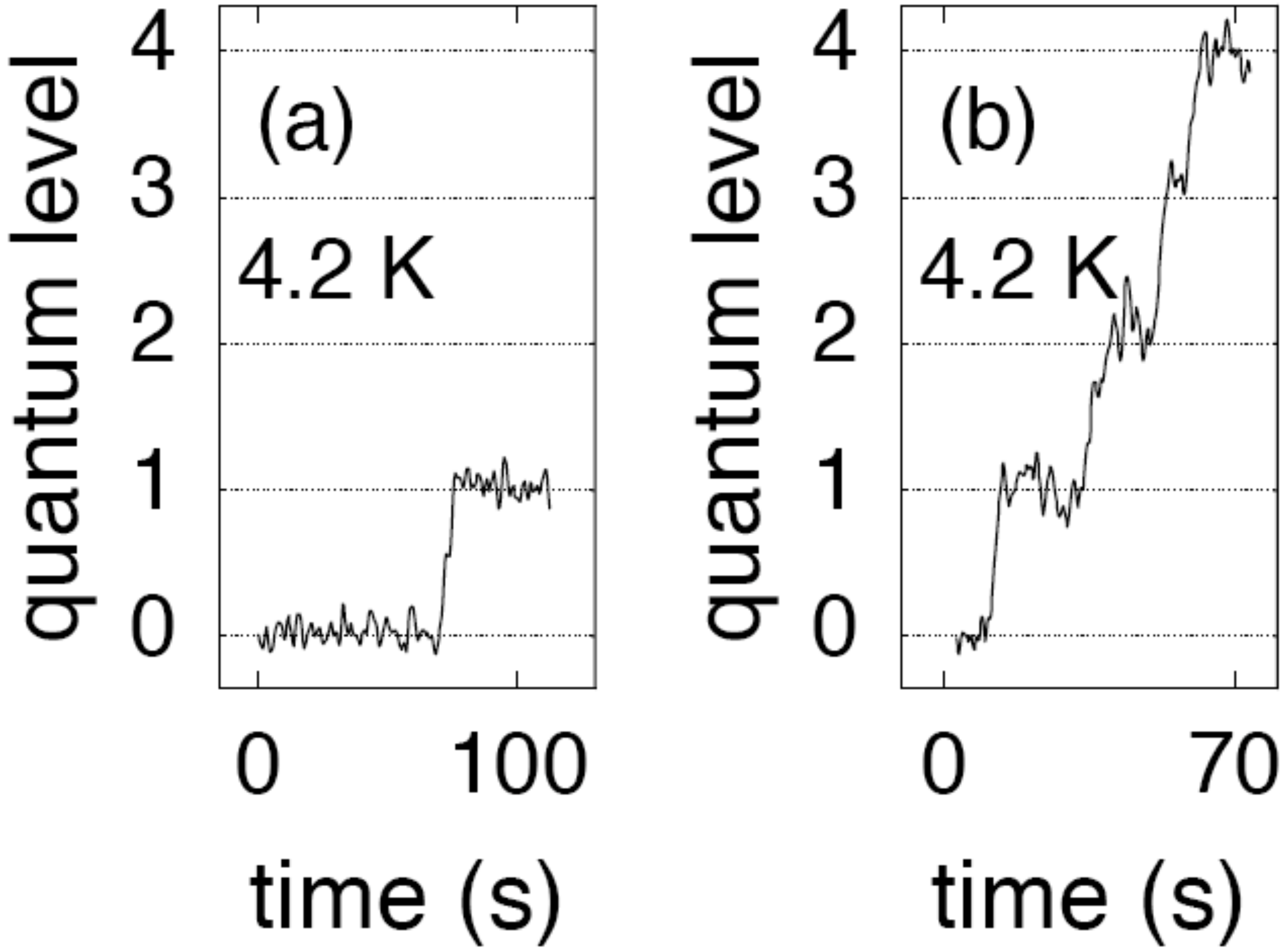
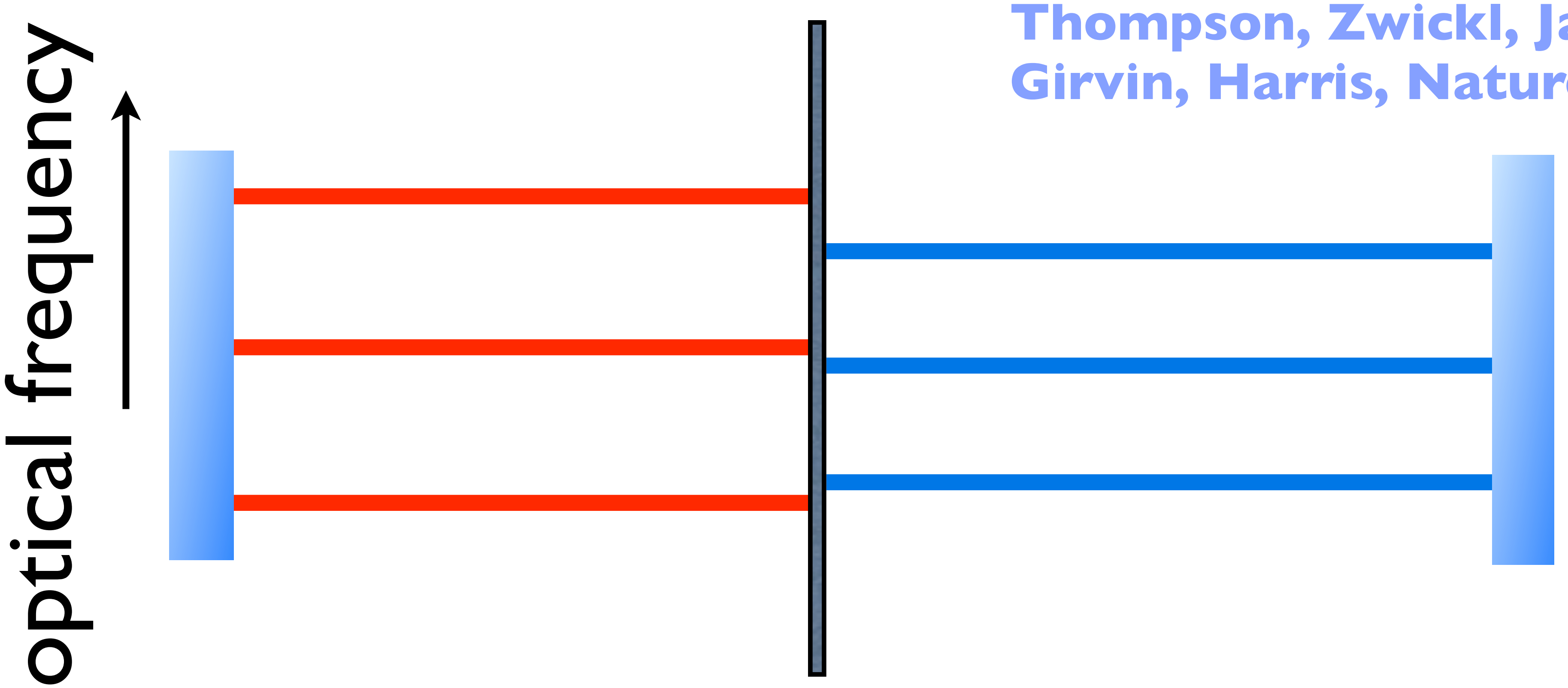


Electron oscillations in Penning trap:
Peil and Gabrielse, 1999



“Membrane in the middle” setup

Thompson, Zwickl, Jayich, Marquardt, Girvin, Harris, Nature 72, 452 (2008)



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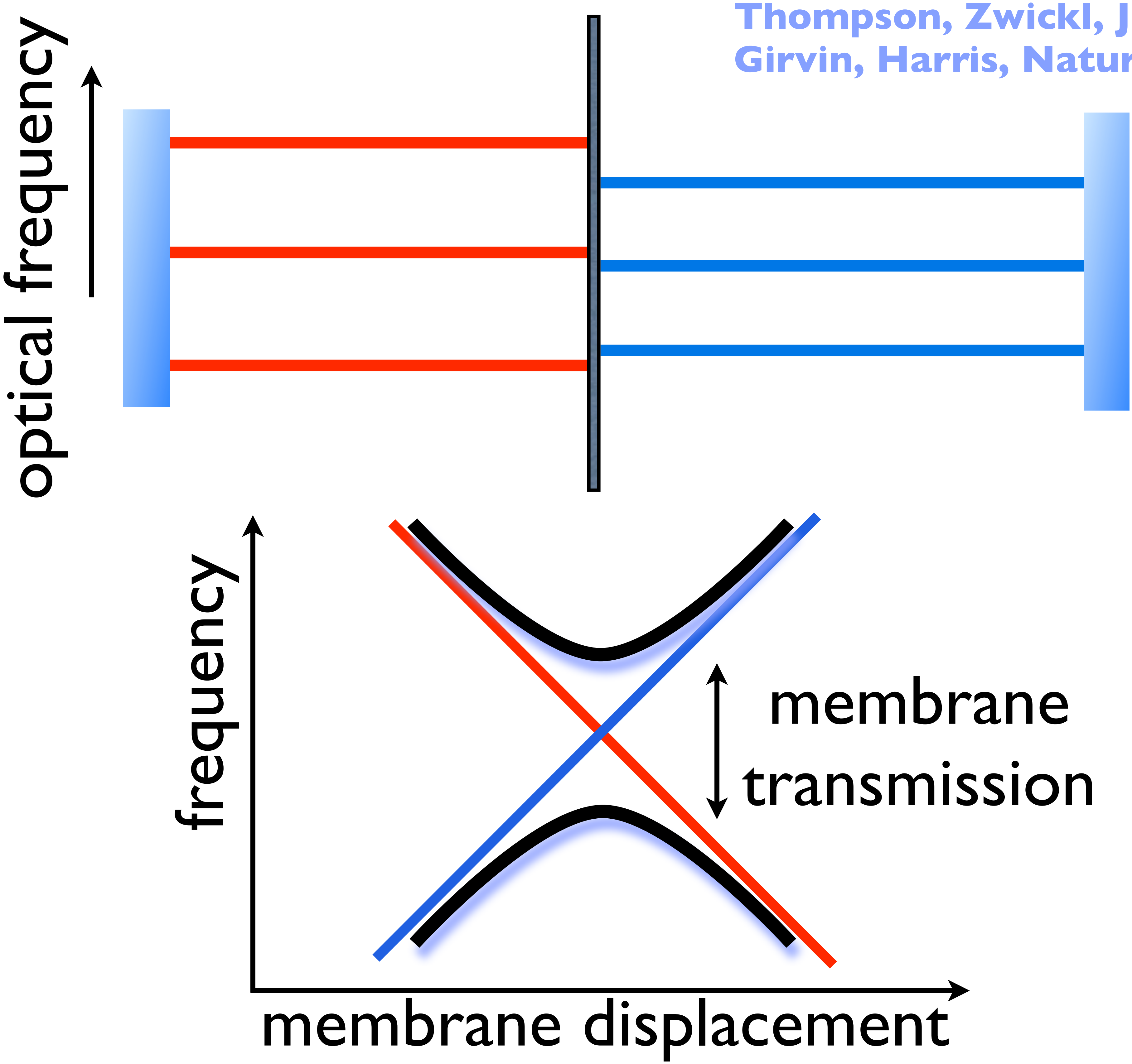
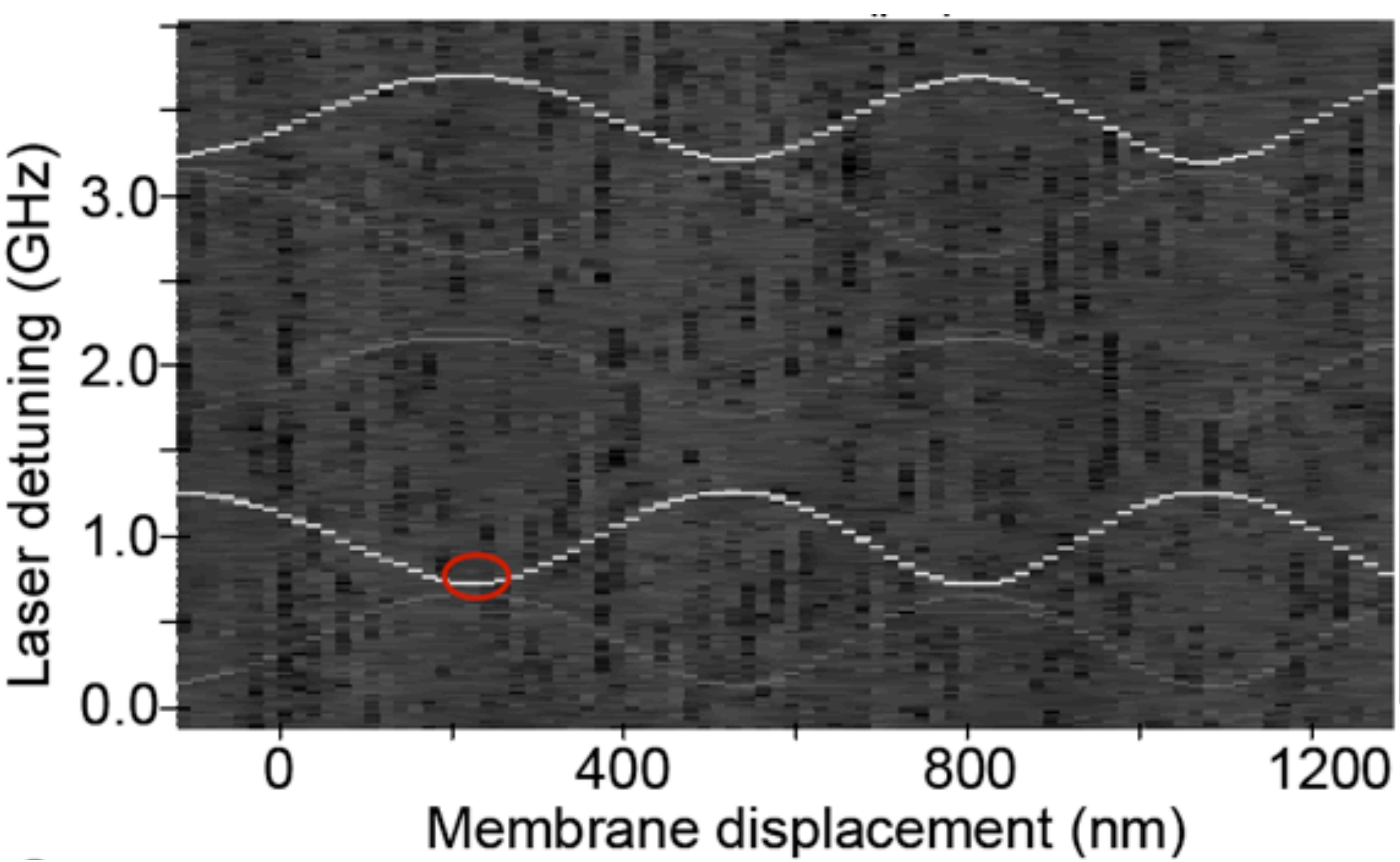


Experiment (Harris group, Yale)



Mechanical frequency:

$\omega_M = 2\pi \cdot 134$ kHz

Mechanical quality factor:

$Q = 10^6 \div 10^7$

Current optical finesse:

$7000 \div 15000$ $(5 \cdot 10^5)$

[almost sideband regime]

Optomechanical cooling from **300K** to **7mK**

Thompson, Zwickl, Jayich, Marquardt, Girvin, Harris, Nature 72, 452 (2008)

Towards Fock state detection of a macroscopic object

Detection of displacement x: *not* what we need!

Towards Fock state detection of a macroscopic object

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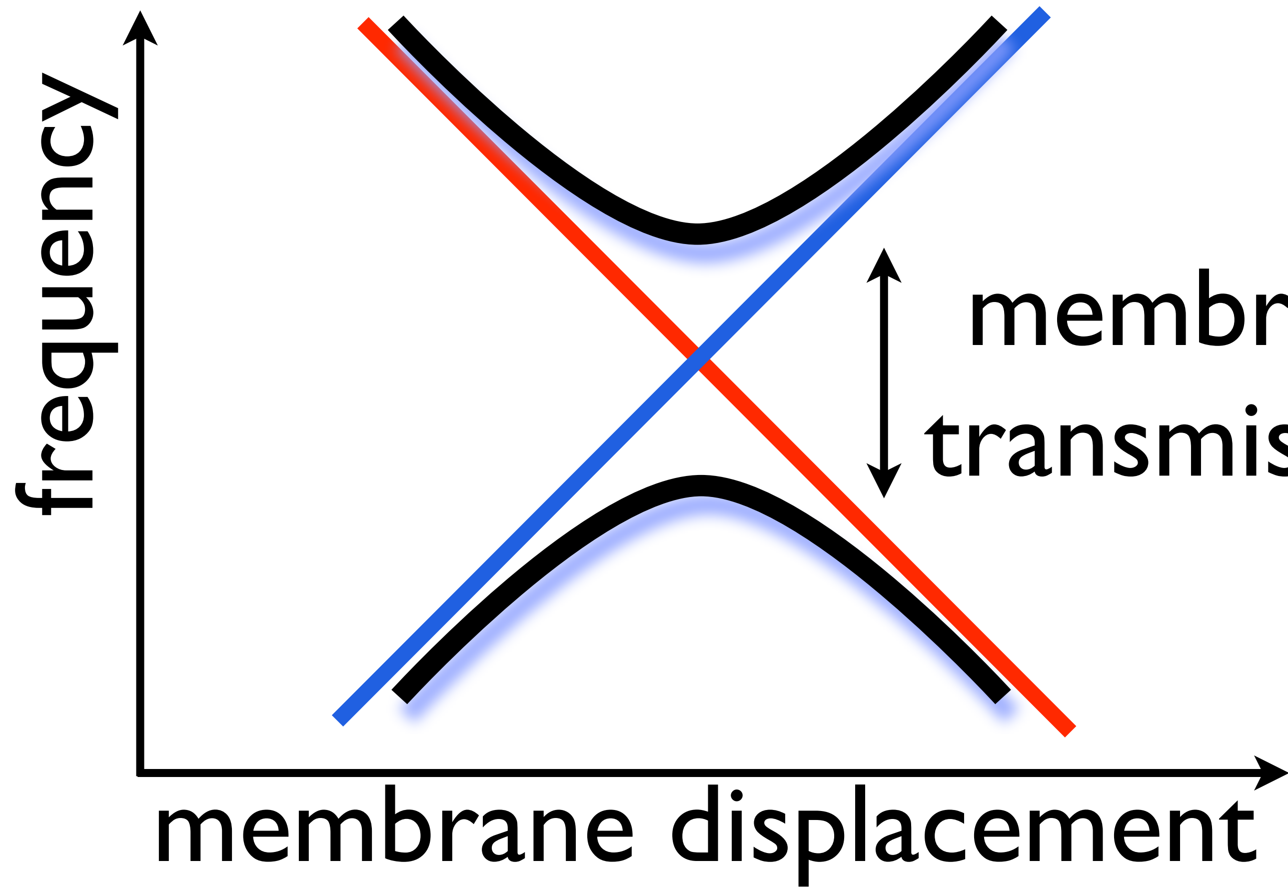

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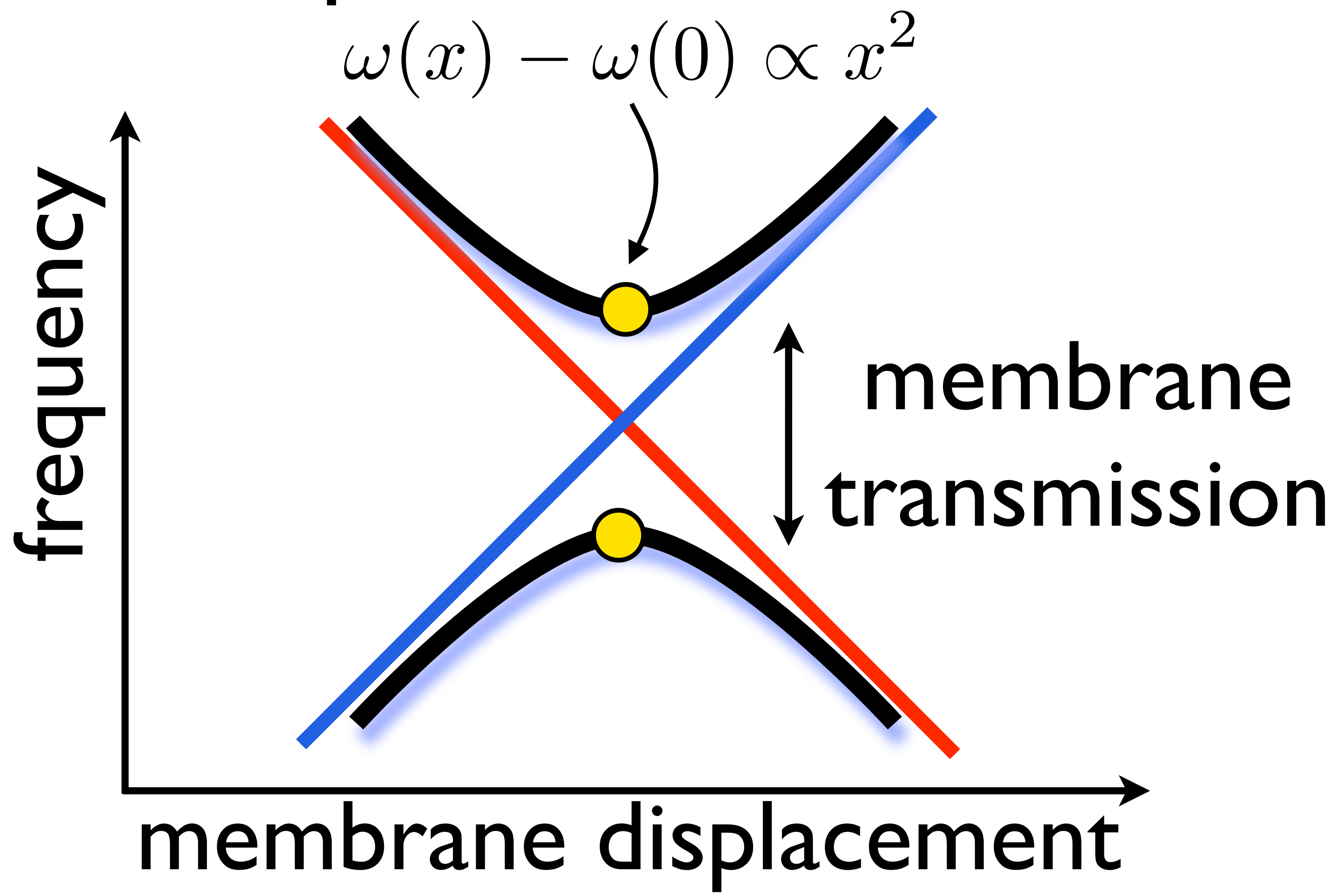

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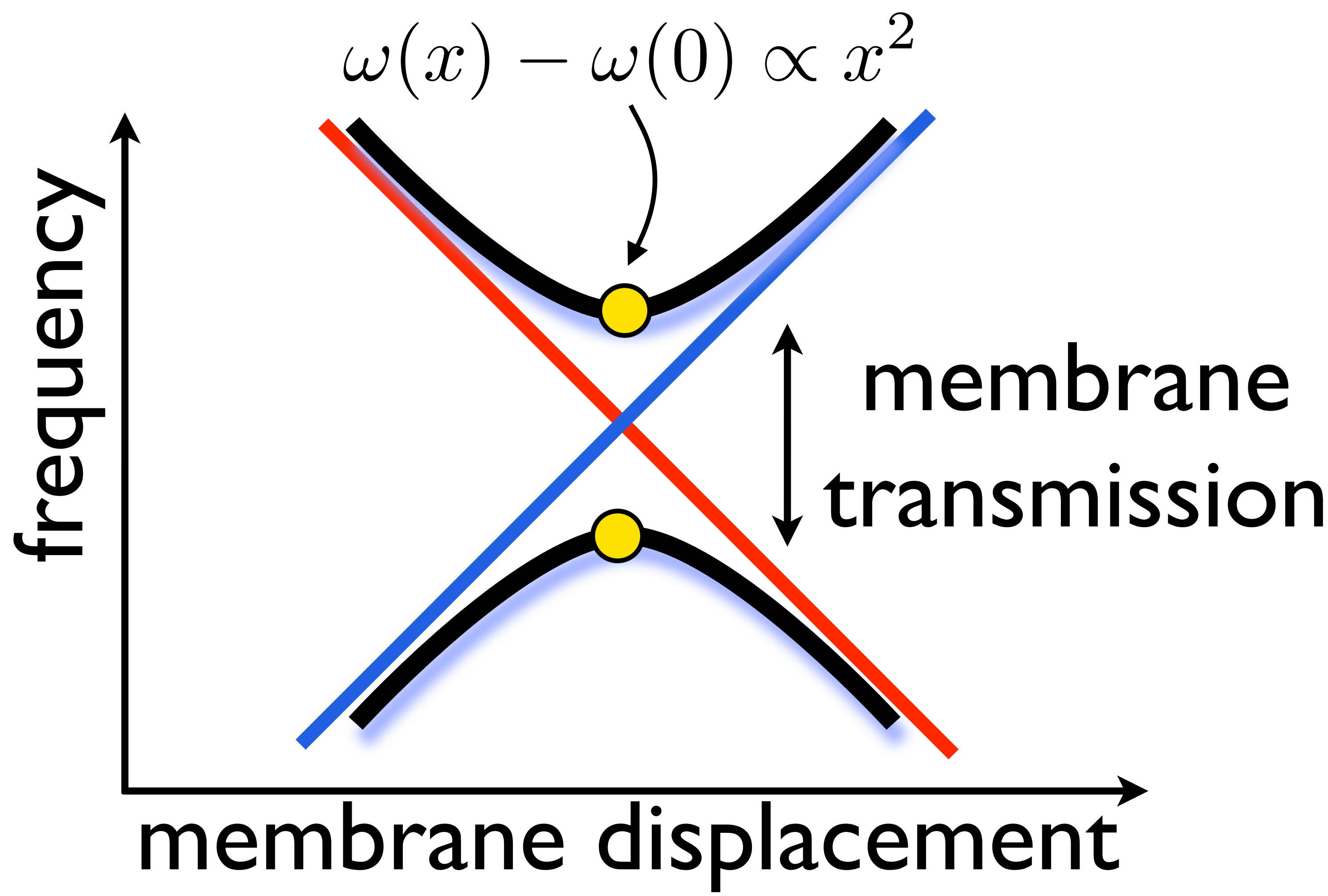
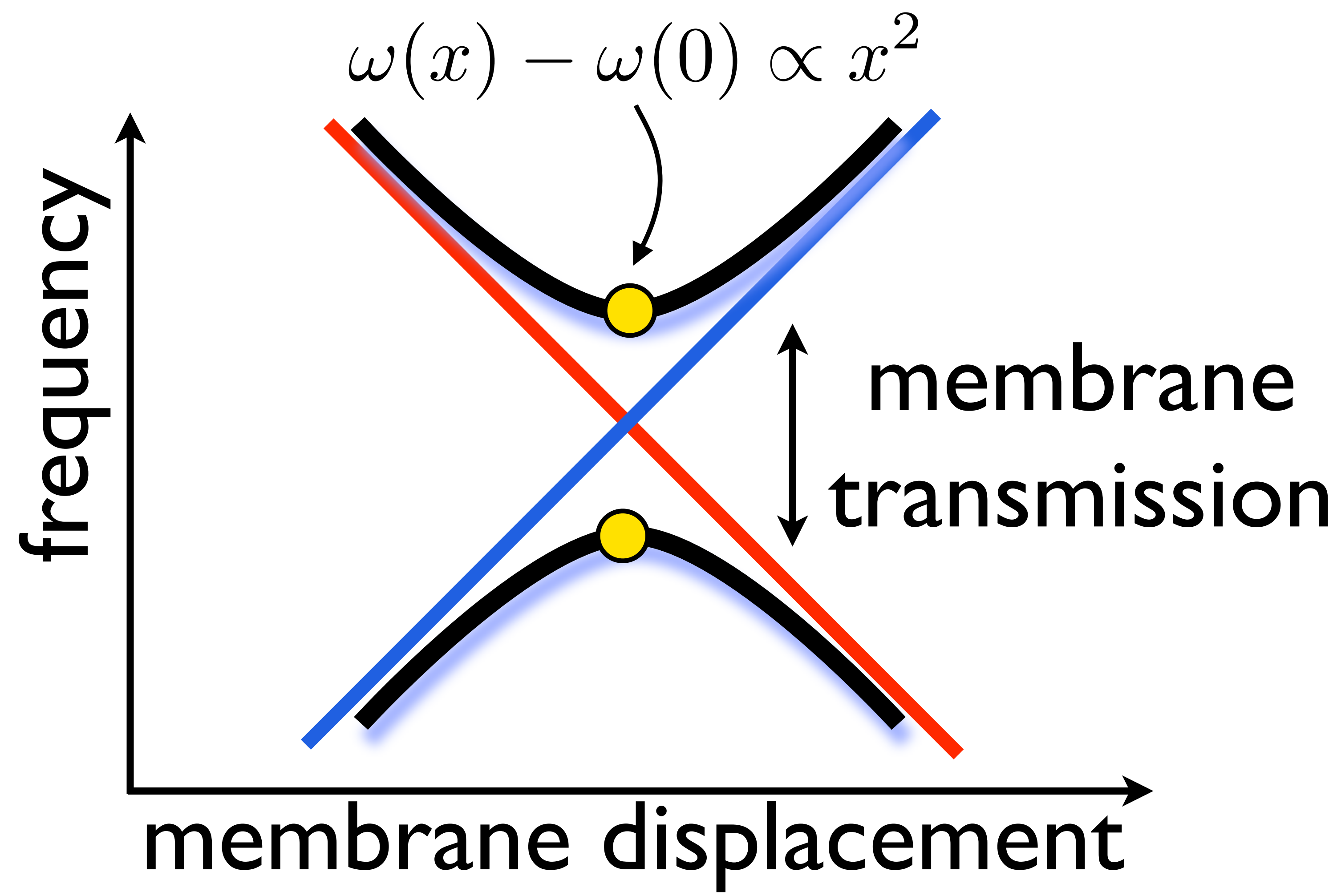


phase shift of measurement beam:

$$\hat{\theta} \propto \hat{x}(t)^2 \propto (\hat{b}(t) + \hat{b}^\dagger(t))^2 = \hat{b}^2 e^{-i2\omega_M t} + \hat{b}^{\dagger 2} e^{+i2\omega_M t} + \hat{b}^\dagger \hat{b} + \hat{b}\hat{b}^\dagger$$

Towards Fock state detection of a macroscopic object



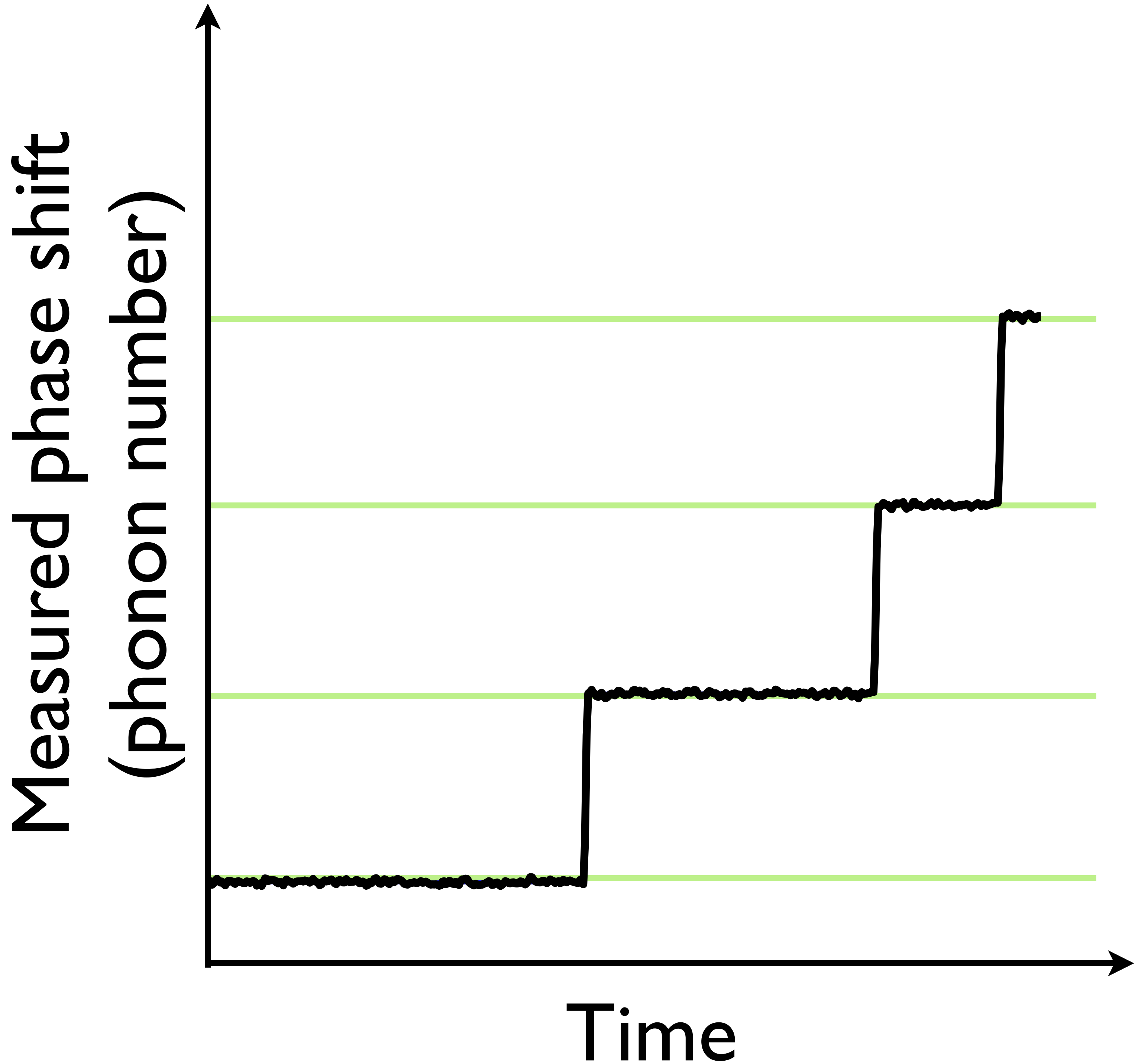
phase shift of measurement beam:

$$\overline{\hat{\theta}} \propto \overline{\hat{x}(t)^2} \propto \overline{(\hat{b}(t) + \hat{b}^\dagger(t))^2} \approx 2\hat{b}^\dagger\hat{b} + 1$$

(Time-average over cavity ring-down time)

QND measurement of phonon number!

Towards Fock state detection of a macroscopic object



Thompson, Zwickl, Jayich, FM,
Girvin, Harris, Nature 72, 452 (2008)

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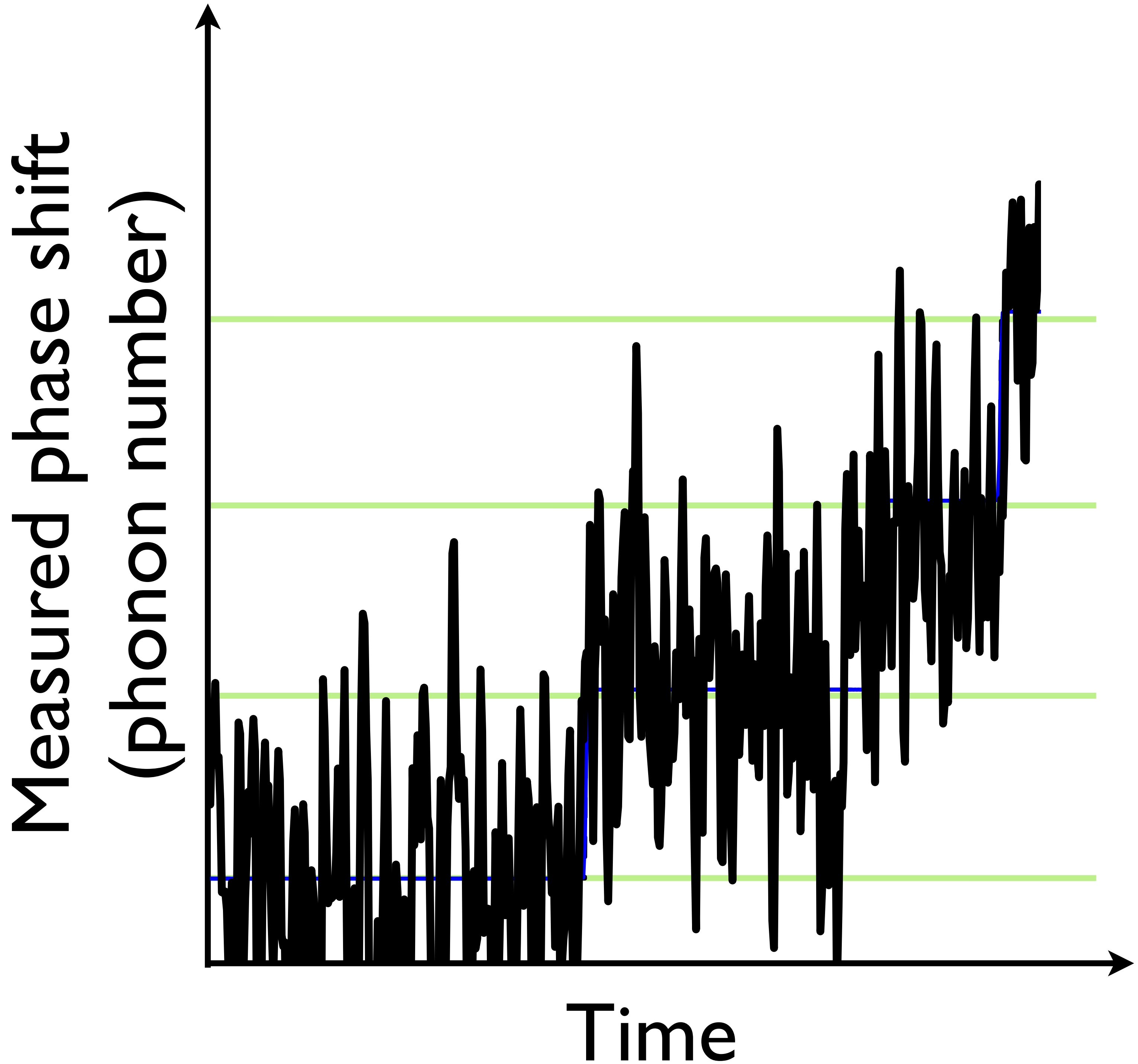

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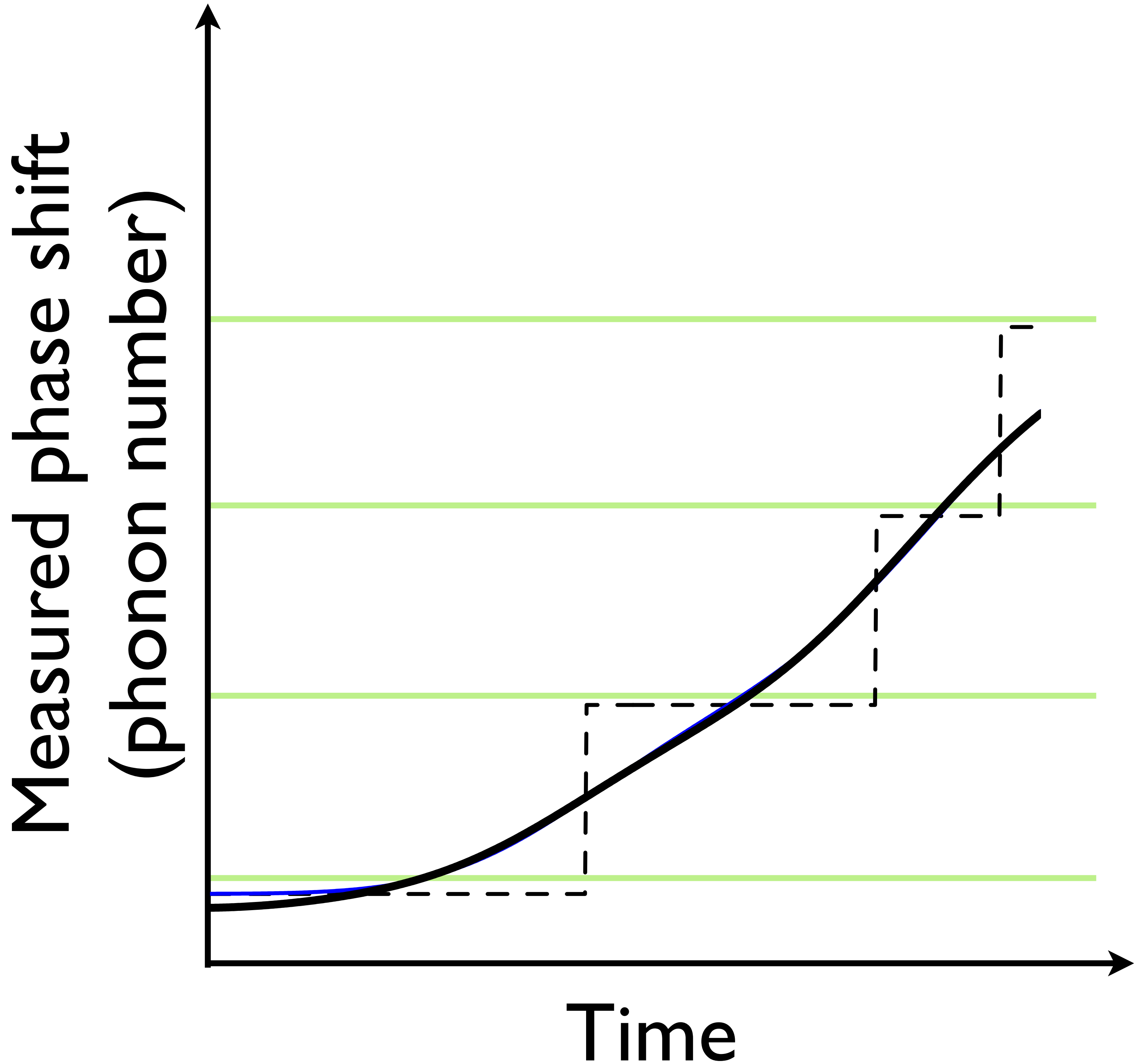

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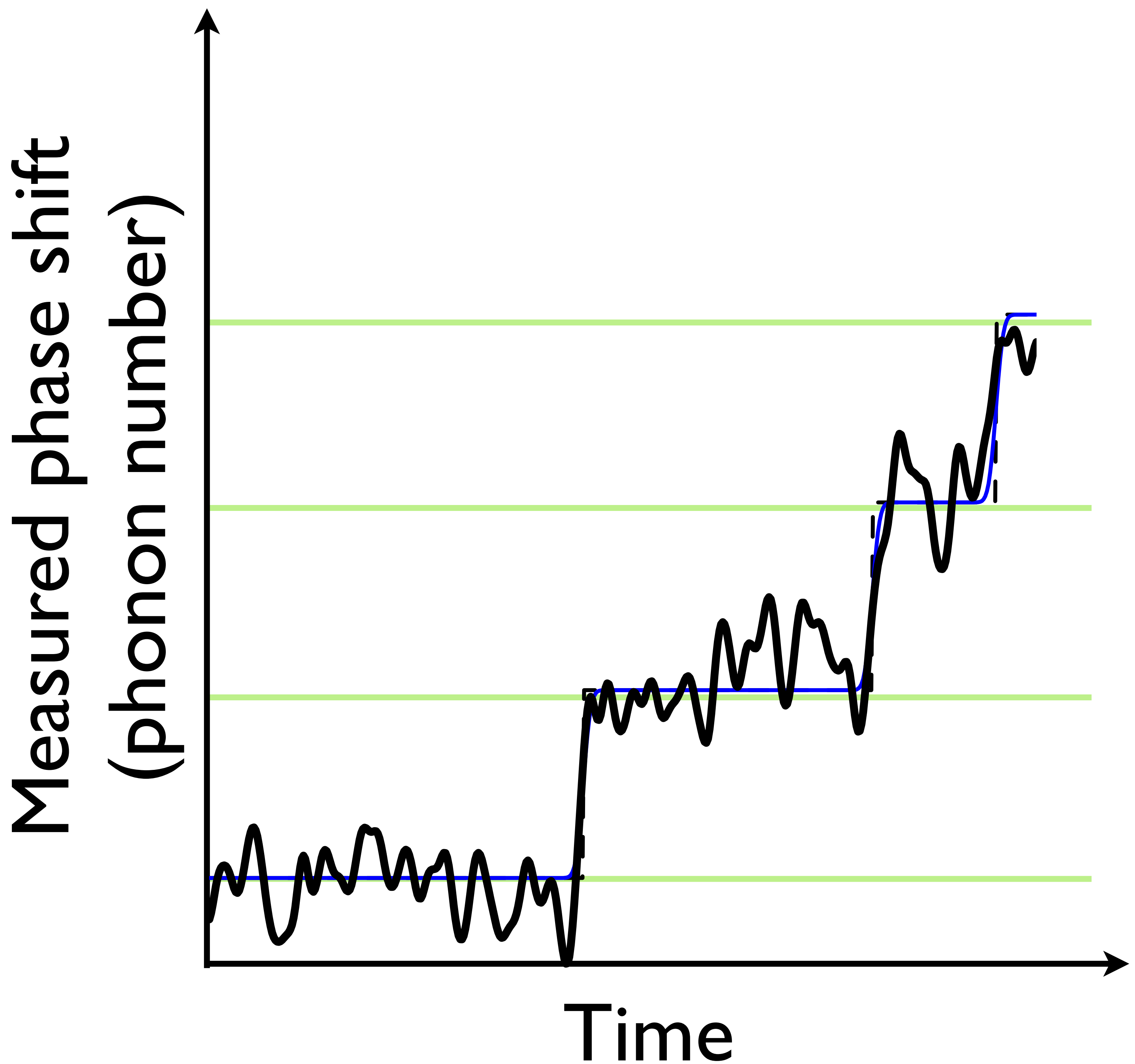

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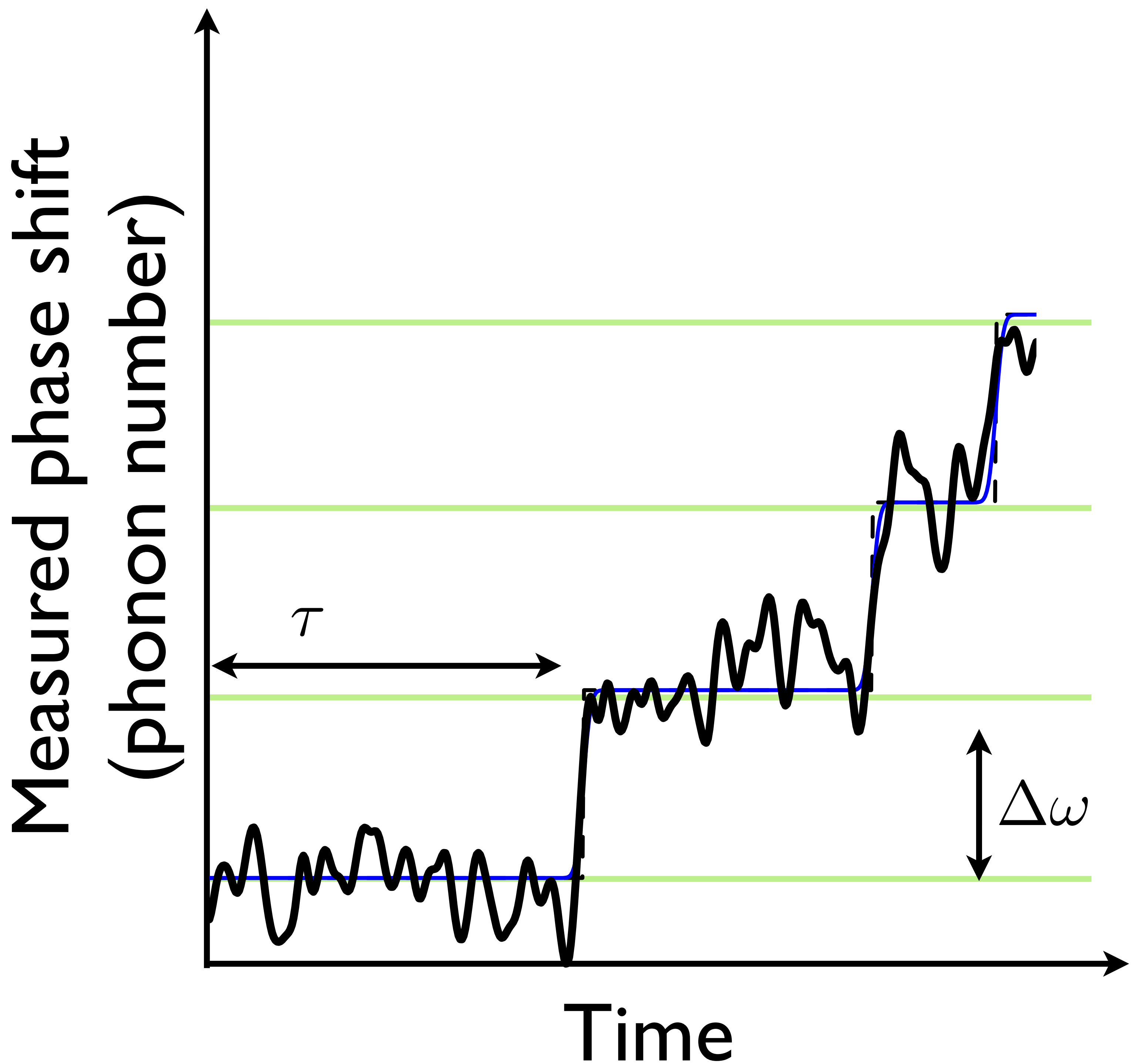
Girvin, Harris, Nature 72, 452 (2008)

Towards Fock state detection of a macroscopic object



Signal-to-noise ratio: $\frac{\tau \Delta\omega^2}{S_\omega}$

Optical freq. shift per phonon:

$$\Delta\omega = x_{\mathrm{ZPF}}^2 \omega''$$

Noise power of freq. measurement:

$$S_\omega = \frac{\kappa}{16 \bar{n}_{\mathrm{cavity}}}$$

Ground state lifetime:

$$\frac{1}{\tau} = \Gamma \bar{n}_{\mathrm{thermal}}$$

Thompson, Zwickl, Jayich, FM, Girvin, Harris, Nature 72, 452 (2008)

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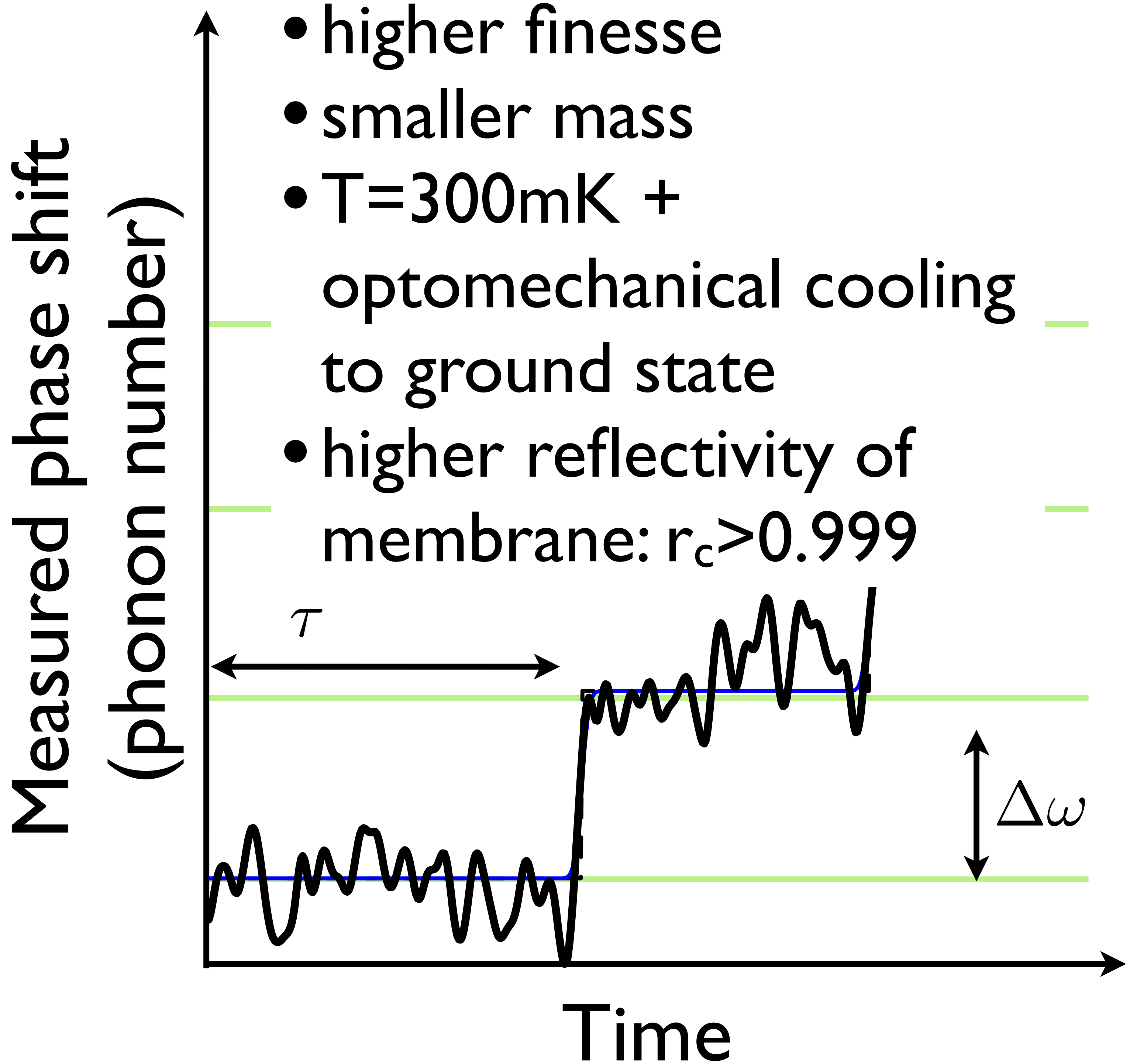
need:
- higher finesse
- smaller mass
- T=300mK + optomechanical cooling to ground state
- higher reflectivity of membrane: r_c>0.999



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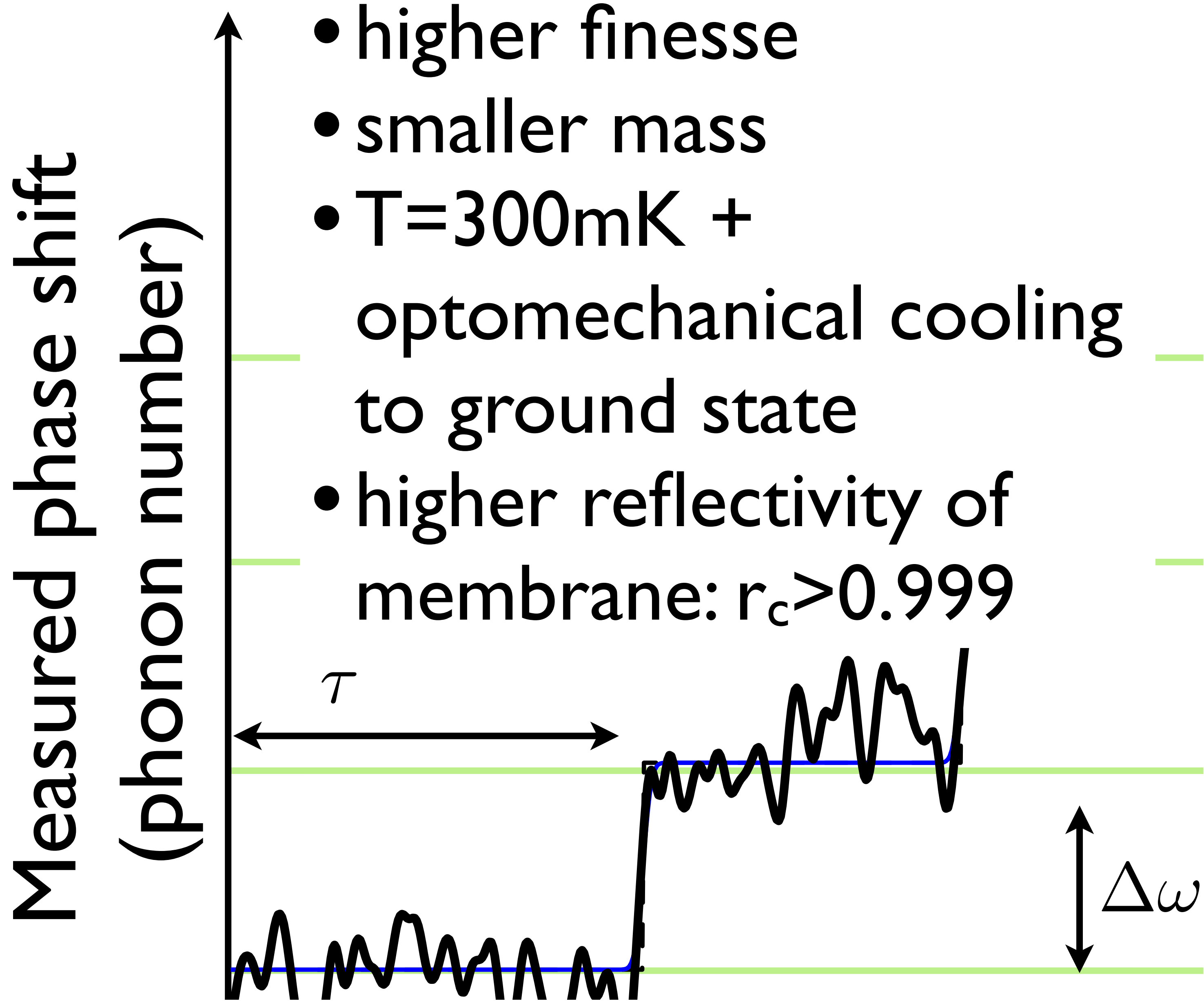

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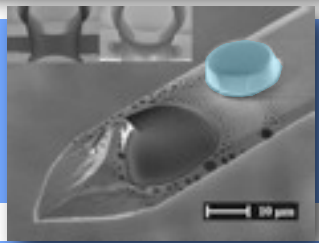
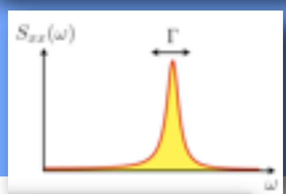
Alternative:

Phonon shot noise measurement (Clerk, FM, Harris 2010)

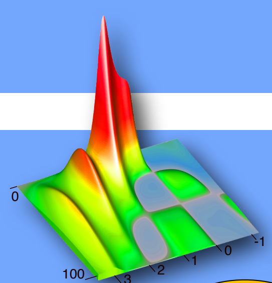
Optomechanics (Outline)



Introduction

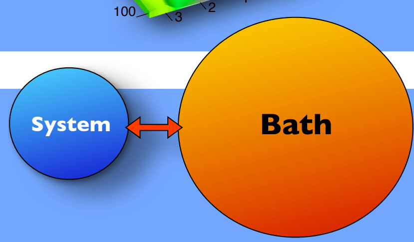


Displacement detection

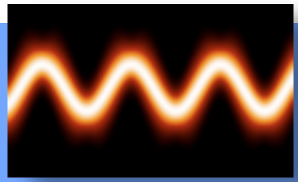


Linear optomechanics

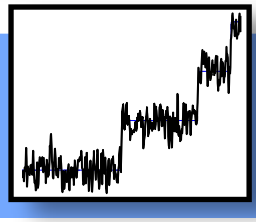
Nonlinear dynamics



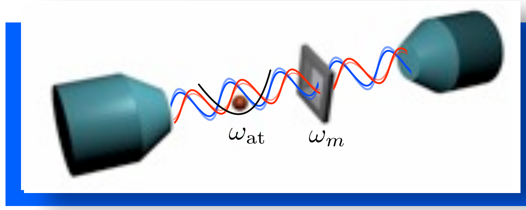
Quantum theory of cooling



Interesting quantum states



Towards Fock state detection



Coupling to the motion of a single atom

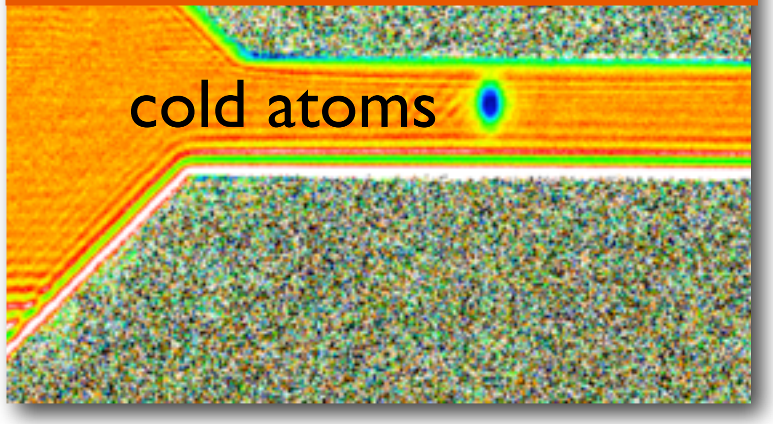
Atom-membrane coupling

Note: Existing works simulate optomechanical effects using cold atoms

K. W. Murch, K. L. Moore, S. Gupta, and D. M. Stamper-Kurn, Nature Phys. **4**, 561 (2008).

F. Brennecke, S. Ritter, T. Donner, and T. Esslinger, Science **322**, 235 (2008).

Stamper-Kurn (Berkeley)

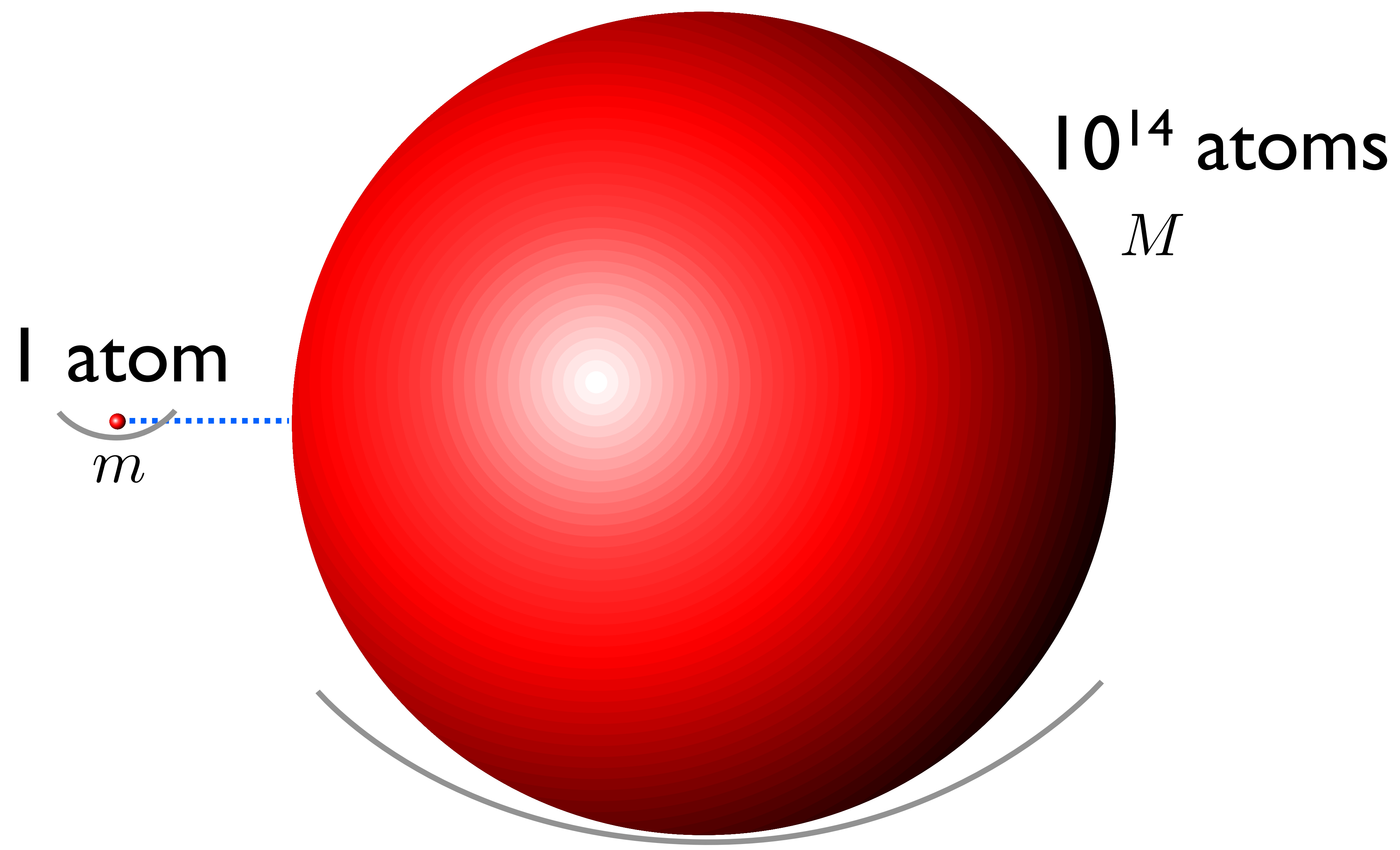


...profit from small mass of atomic cloud

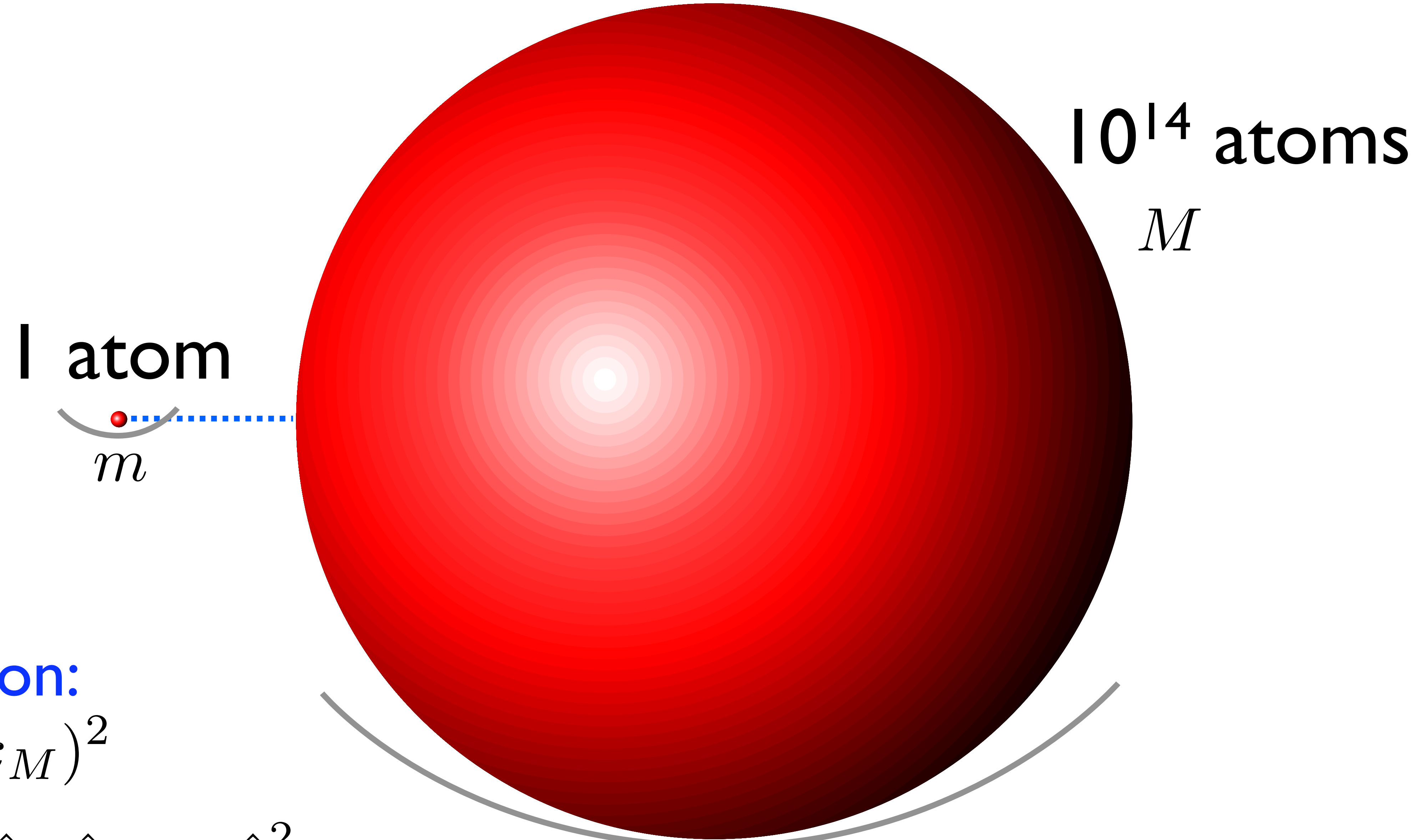
Here: Coupling a single atom to a macroscopic mechanical object

Challenge: huge mass ratio

Coupling a single atom to a heavy object: Why it is hard



Coupling a single atom to a heavy object: Why it is hard



interaction:

$$(\hat{x}_m - \hat{x}_M)^2$$

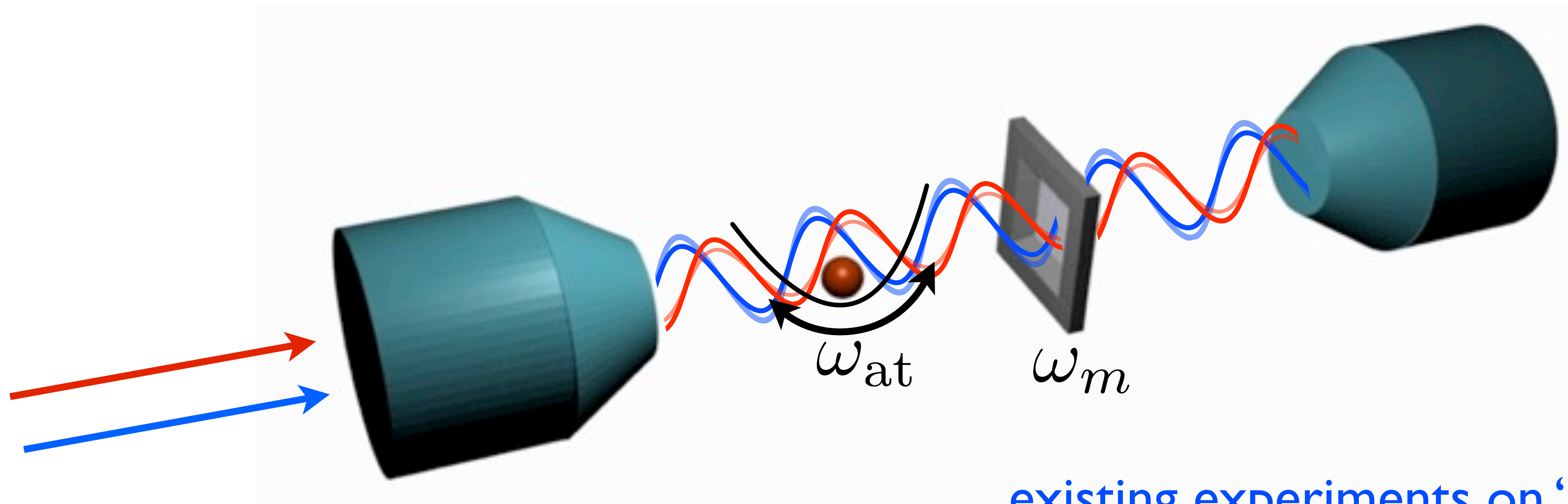
$$= \hat{x}_m^2 - 2\hat{x}_m\hat{x}_M + \hat{x}_M^2$$

$$= \frac{\hbar}{2m\omega}\left[(\hat{a}+\hat{a}^\dagger)^2 - 2\sqrt{\frac{m}{M}}(\hat{a}+\hat{a}^\dagger)(\hat{b}+\hat{b}^\dagger) + \frac{m}{M}(\hat{b}+\hat{b}^\dagger)^2\right]$$

freq. shift — coupling — freq. shift

$m/M \sim 10^{-14} - 10^{-8} \Rightarrow$ coupling term small!

Strong atom-membrane coupling via the light field



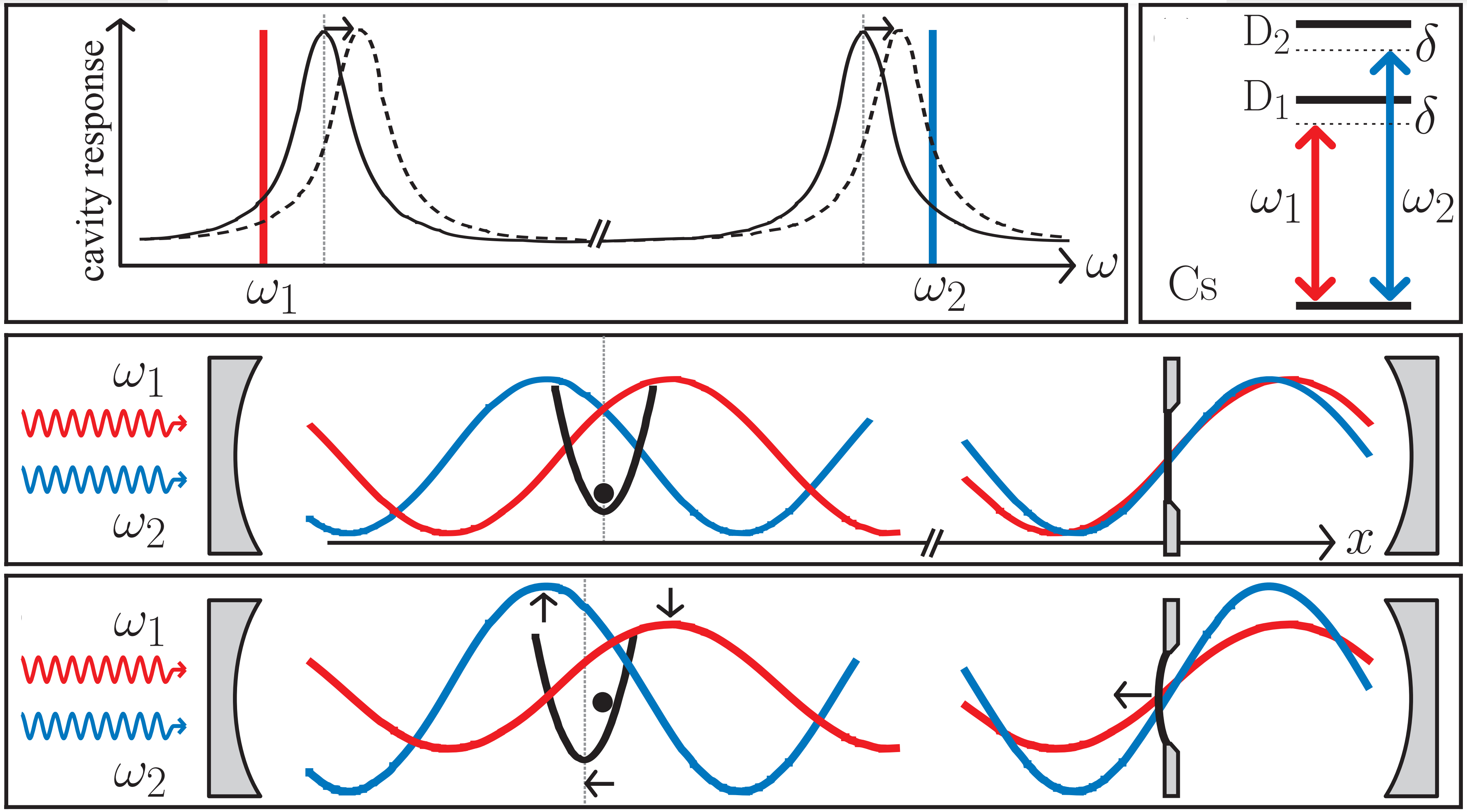
existing experiments on "optomechanics with cold atoms": labs of Dan-Stamper Kurn (Berkeley) and Tilman Esslinger (ETH)

collaboration:
LMU (M. Ludwig, FM, P. Treutlein),
Innsbruck (K. Hammerer, C. Genes, M. Wallquist, P. Zoller),
Boulder (J. Ye), Caltech (H. J. Kimble)
Hammerer et al., PRL 2009

Goal:

$$\hat{H} = \underbrace{\hbar\omega_{\rm at}\hat{a}^\dagger\hat{a}}_{\text{atom}} + \underbrace{\hbar\omega_m\hat{b}^\dagger\hat{b}}_{\text{membrane}} + \underbrace{\hbar G_{\rm eff}(\hat{a}^\dagger+\hat{a})(\hat{b}^\dagger+\hat{b})}_{\text{atom-membrane coupling}}$$

Cavity-mediated coupling



for each cavity mode:

$$\hat{H}_{\text{atom}-\text{cavity}} = U_0 \sin^2(k\hat{x}_{\text{at}})\hat{C}^\dagger\hat{C} = \ldots + g_{\text{at}}(\hat{c}^\dagger + \hat{c})(\hat{a}^\dagger + \hat{a})$$

$$(\hat{C} = \bar{C} + \hat{c})$$ (likewise for membrane)

atom-membrane coupling via virtual transitions:

$$G_{\text{eff}} = 4\frac{g_{\text{at}}g_{\text{m}}}{\Delta} \sim 100\text{kHz}$$

detuning laser-cavity

$\Delta \gg \omega_m, \kappa$

Decoherence and decay

Thermal ground-state decay rate:

$$\Gamma_{\rm th} = \bar{n}_{\rm th}\Gamma_m = \frac{k_B T}{\hbar Q}$$

Note: T limited by light absorption, T~1K

Relaxation of atom/membrane motion via driven cavity modes:

$$\Gamma_c \sim \frac{\kappa g^2}{\Delta^2}$$

Choose $\Delta \gg \omega_m, \kappa$

Atomic momentum diffusion from photon scattering:

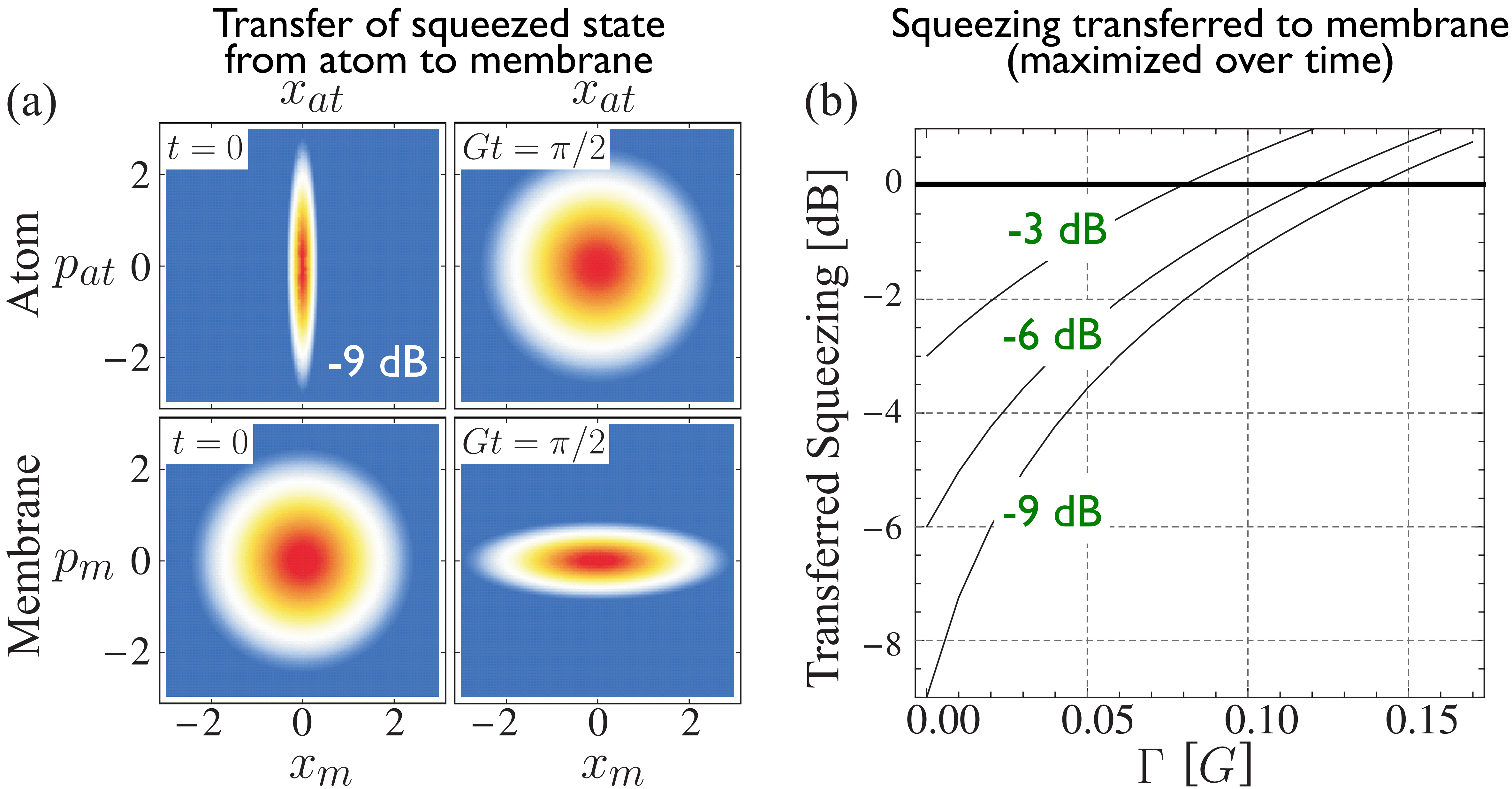
$$\Gamma_{\rm at} \sim \gamma \frac{g_{\rm at}^2}{\Omega^2}$$

Parameter optimization for currently achievable setups:

$$\Gamma_{\rm at}, \Gamma_c, \Gamma_{\rm at} \sim 0.1 \times G_{\rm eff}$$

can reach strong coupling regime!

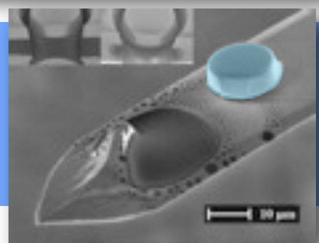
Example: State transfer



Exploit toolbox for single-atom manipulation

- Creation of arbitrary atom states and transfer to membrane
- Transfer of membrane states to atom and measurement

Optomechanics (Outline)



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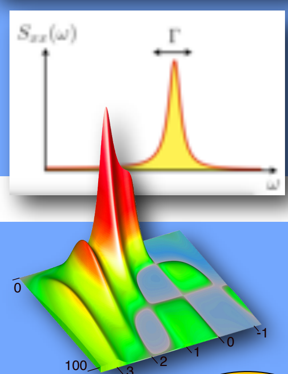
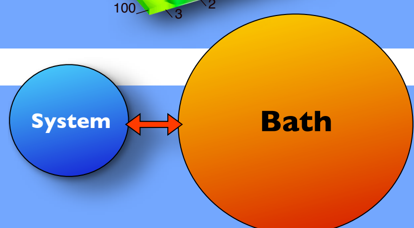


Displacement detection

Nonlinear dynamics



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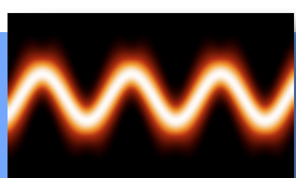
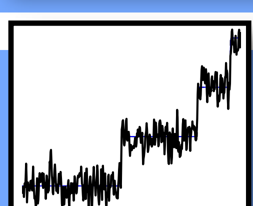
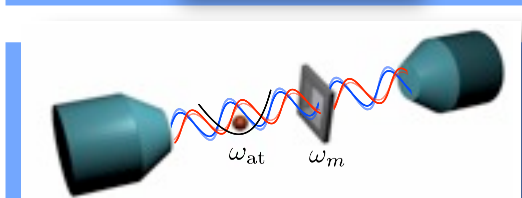


Interesting quantum states



Towards Fock state detection



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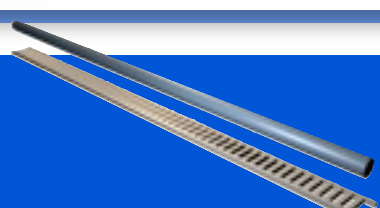
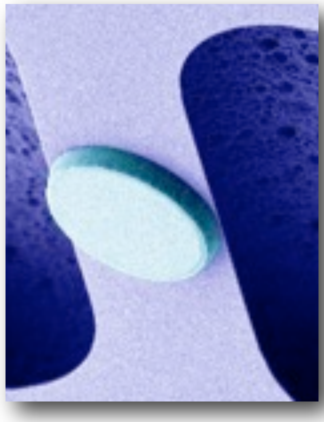


Optomechanical arrays

Scaling down

cm

usual optical cavities



10 μm

LKB, Aspelmeyer, Harris, Bouwmeester, ...

Scaling down

cm

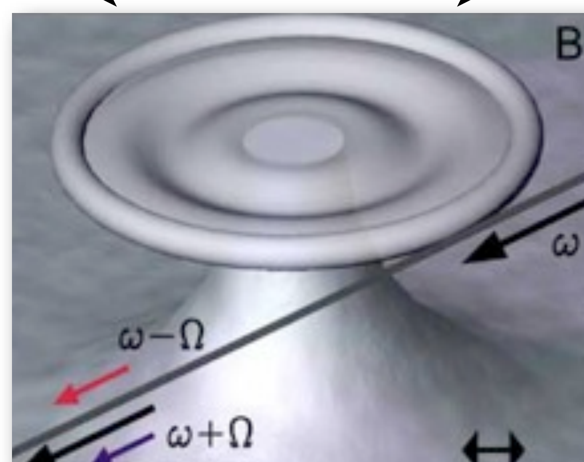
usual optical cavities

10 µm

LKB, Aspelmeyer, Harris, Bouwmeester, ...

50 µm

microtoroids



Vahala, Kippenberg, Carmon, ...

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cm

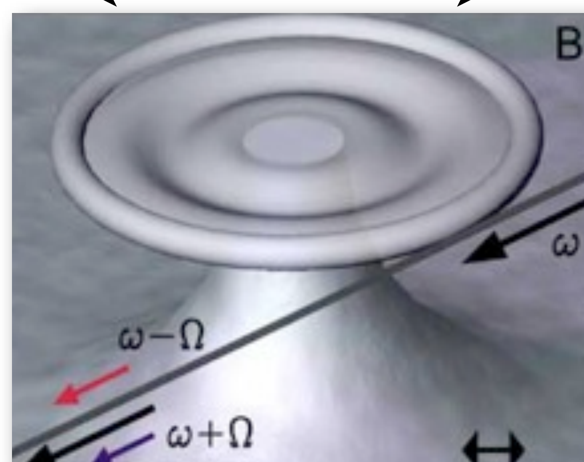
usual optical cavities

10 µm

LKB, Aspelmeyer, Harris, Bouwmeester, ...

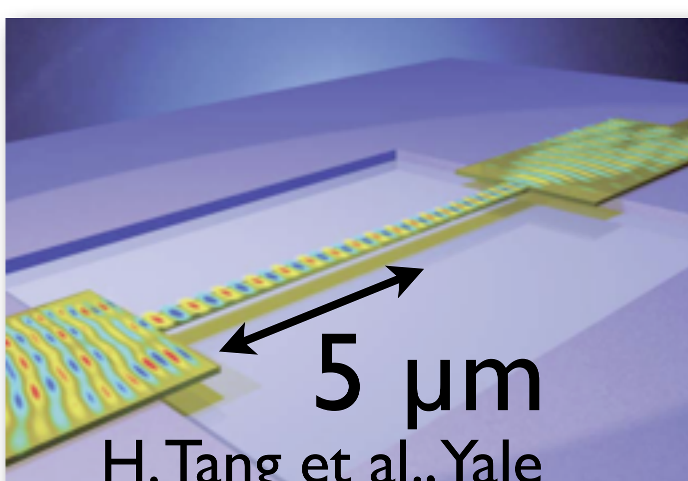
50 µm

microtoroids

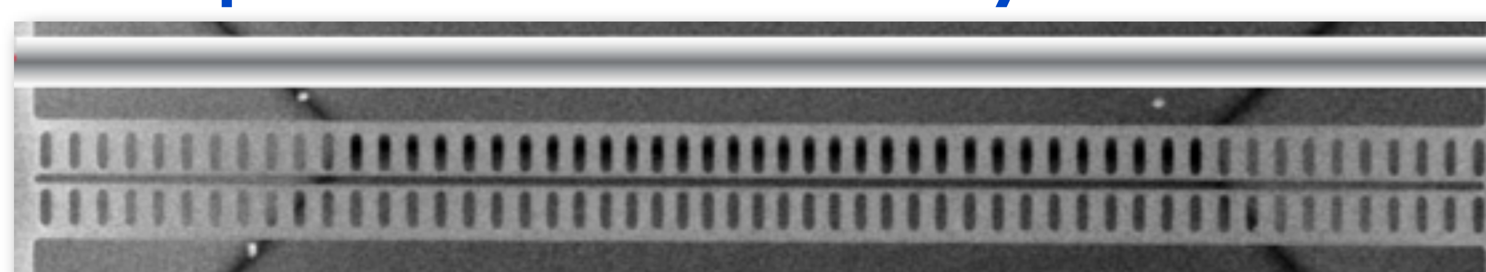


Vahala, Kippenberg, Carmon, ...

optomechanics in photonic circuits



optomechanical crystals



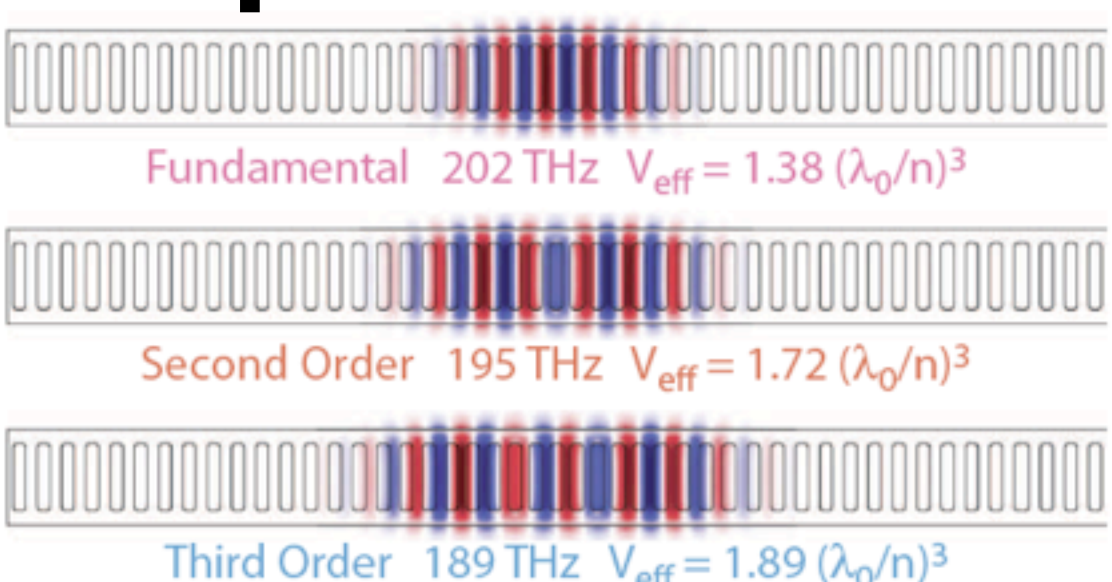
O. Painter et al., Caltech

10 µm

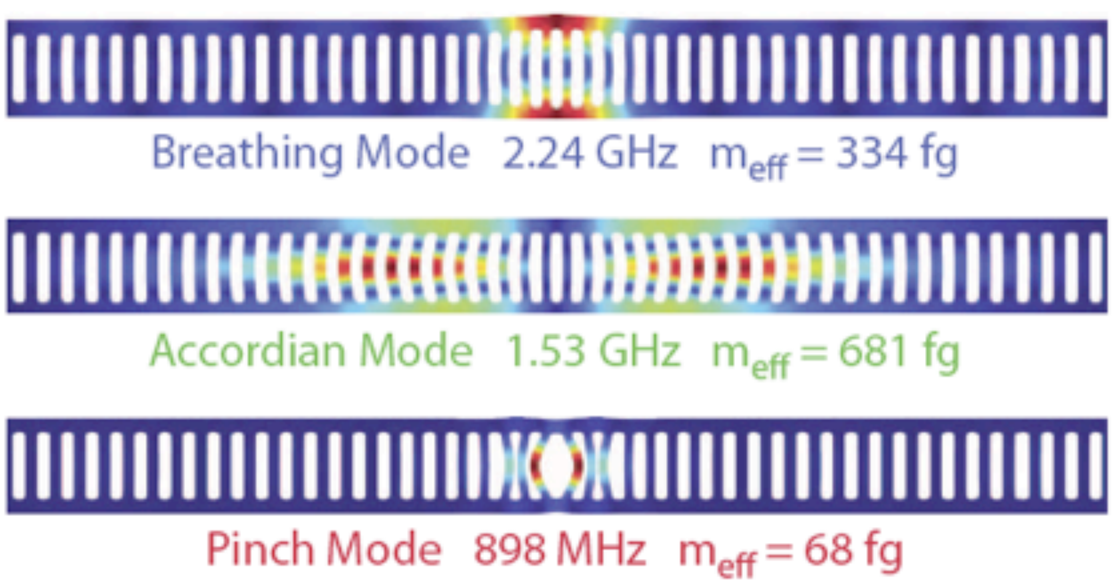
Optomechanical crystals

free-standing photonic crystal structures

optical modes



vibrational modes



advantages:

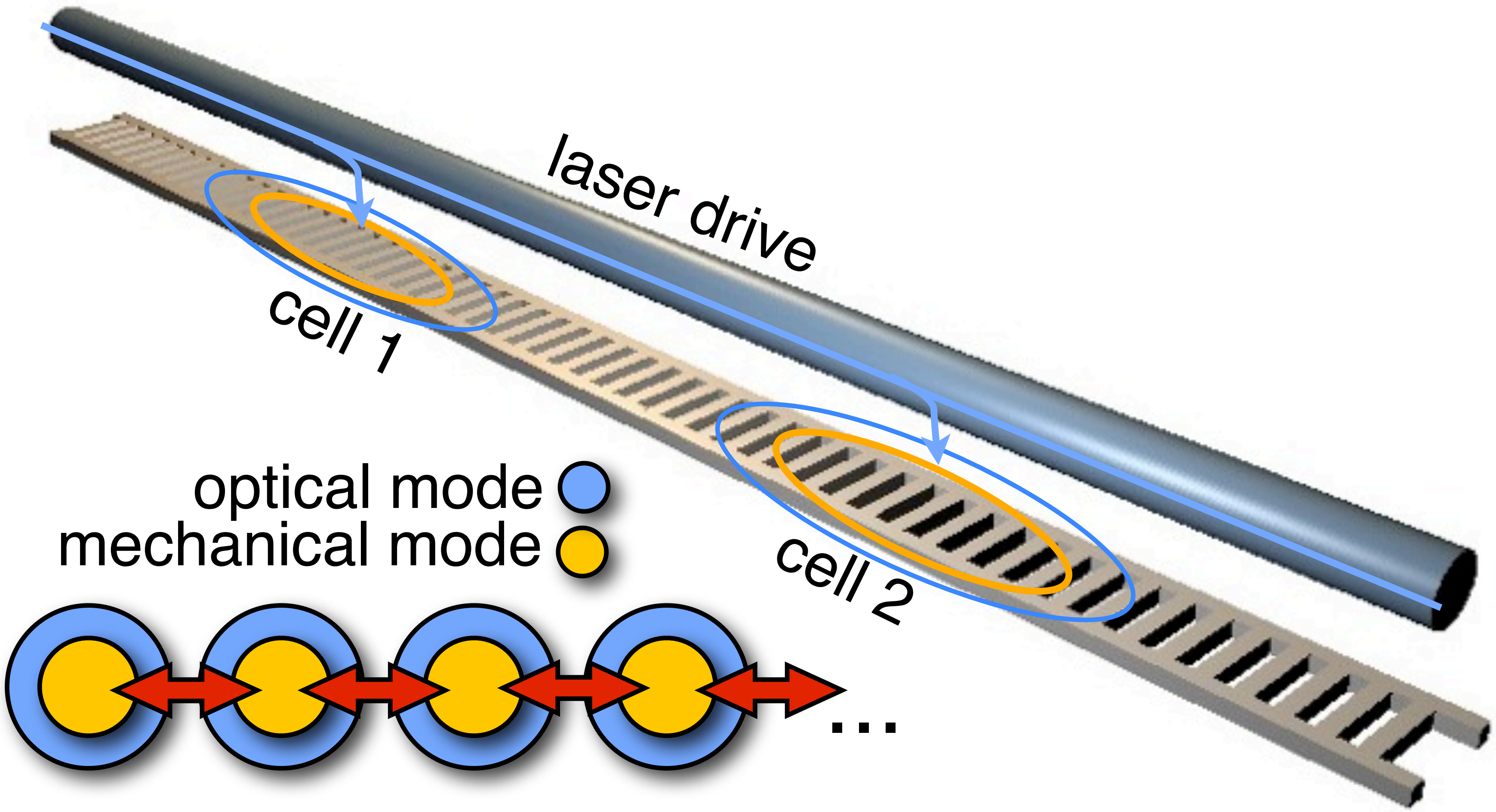
tight vibrational confinement:
high frequencies, small mass
(stronger quantum effects)

tight optical confinement:
large optomechanical coupling
(100 GHz/nm)

integrated on a chip

from: M. Eichenfield et al., Optics Express 17, 20078 (2009), Painter group, Caltech

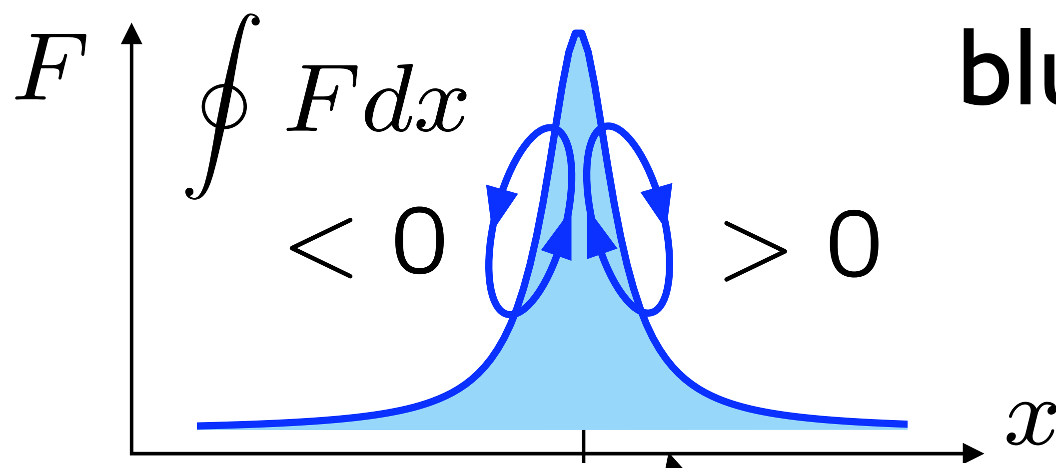
Optomechanical arrays



collective nonlinear dynamics:
classical / quantum

cf. Josephson arrays

Nonlinear dynamics of a single optomechanical cell: Self-induced oscillations



blue-detuned laser:
anti-damping

< 0

red-detuned laser:
damping (cooling)

$$\Gamma = \Gamma_M + \Gamma_{\mathrm{opt}}$$

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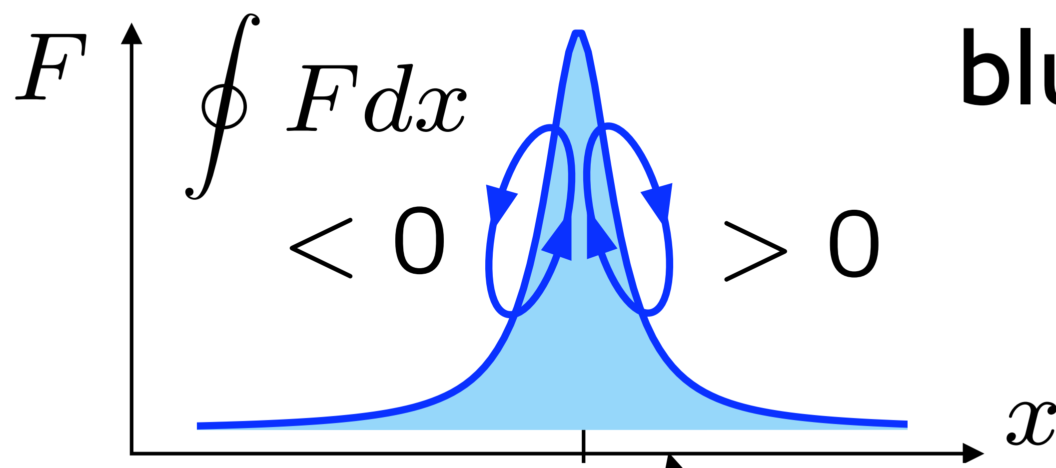

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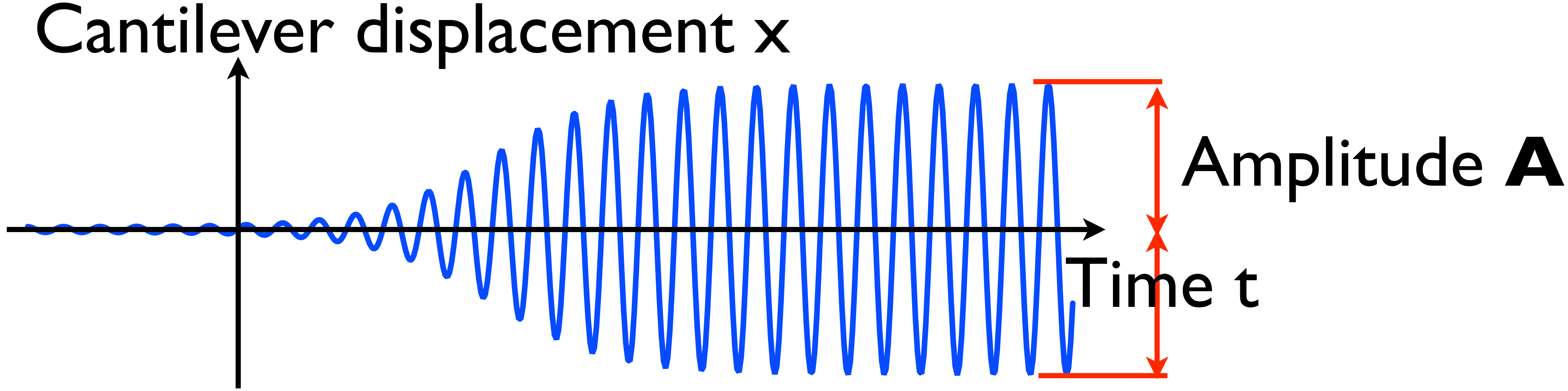
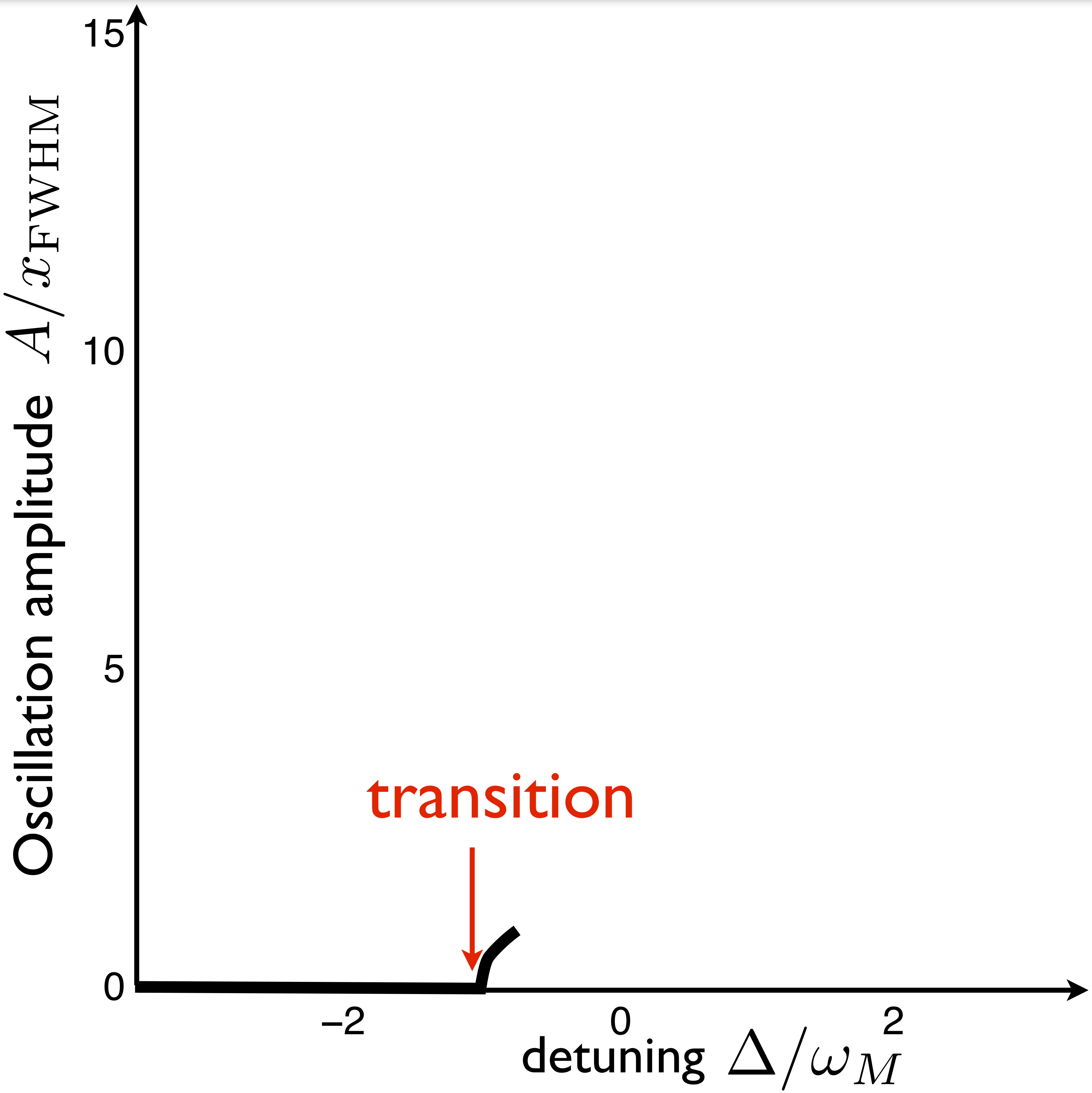
Beyond some laser input power threshold: instability



Attractor diagram



Höhberger, Karrai, IEEE proceedings 2004

Carmon, Rokhsari, Yang, Kippenberg, Vahala, PRL 2005

FM, Harris, Girvin, PRL 2006

Metzger et al., PRL 2008

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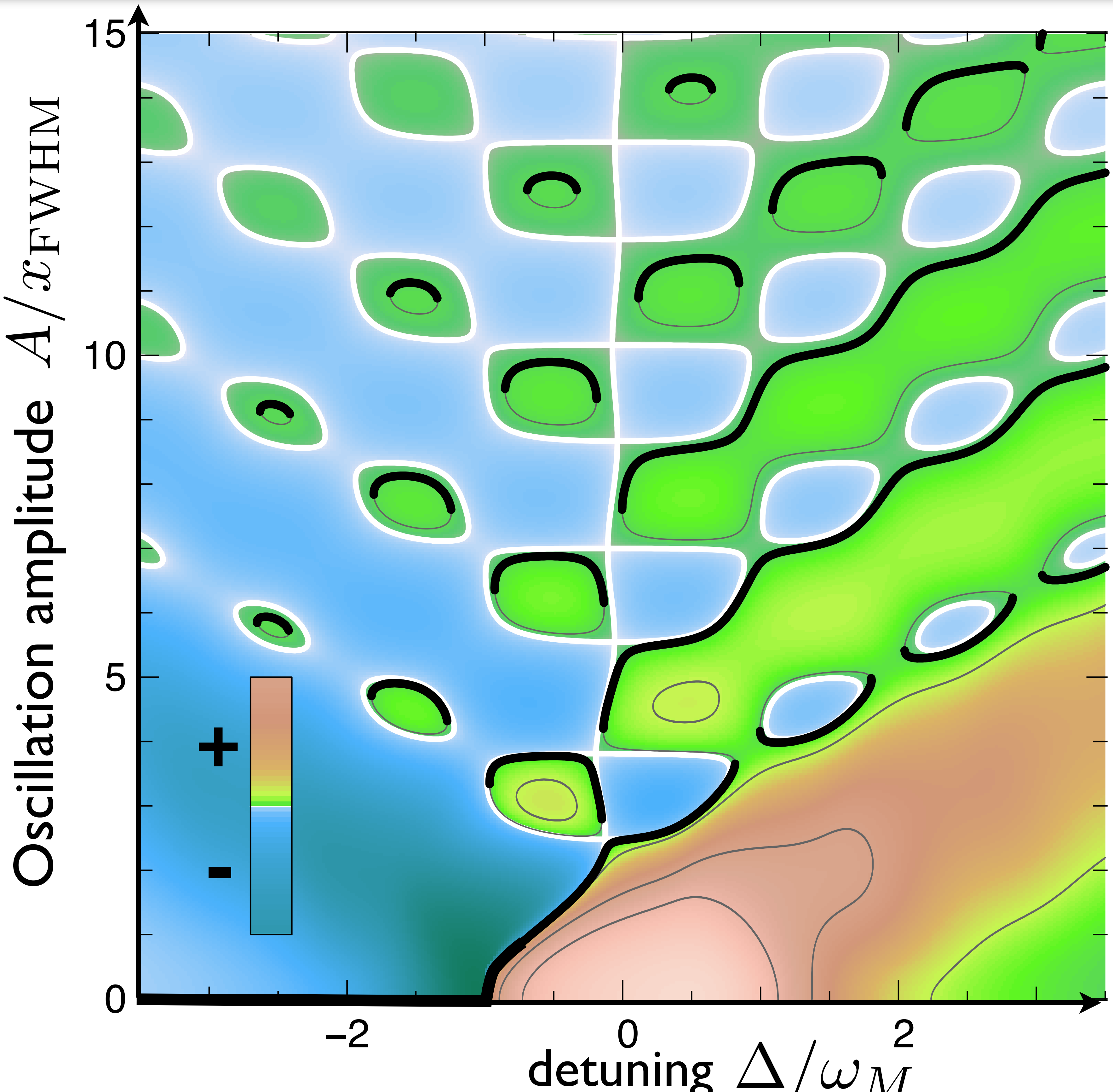

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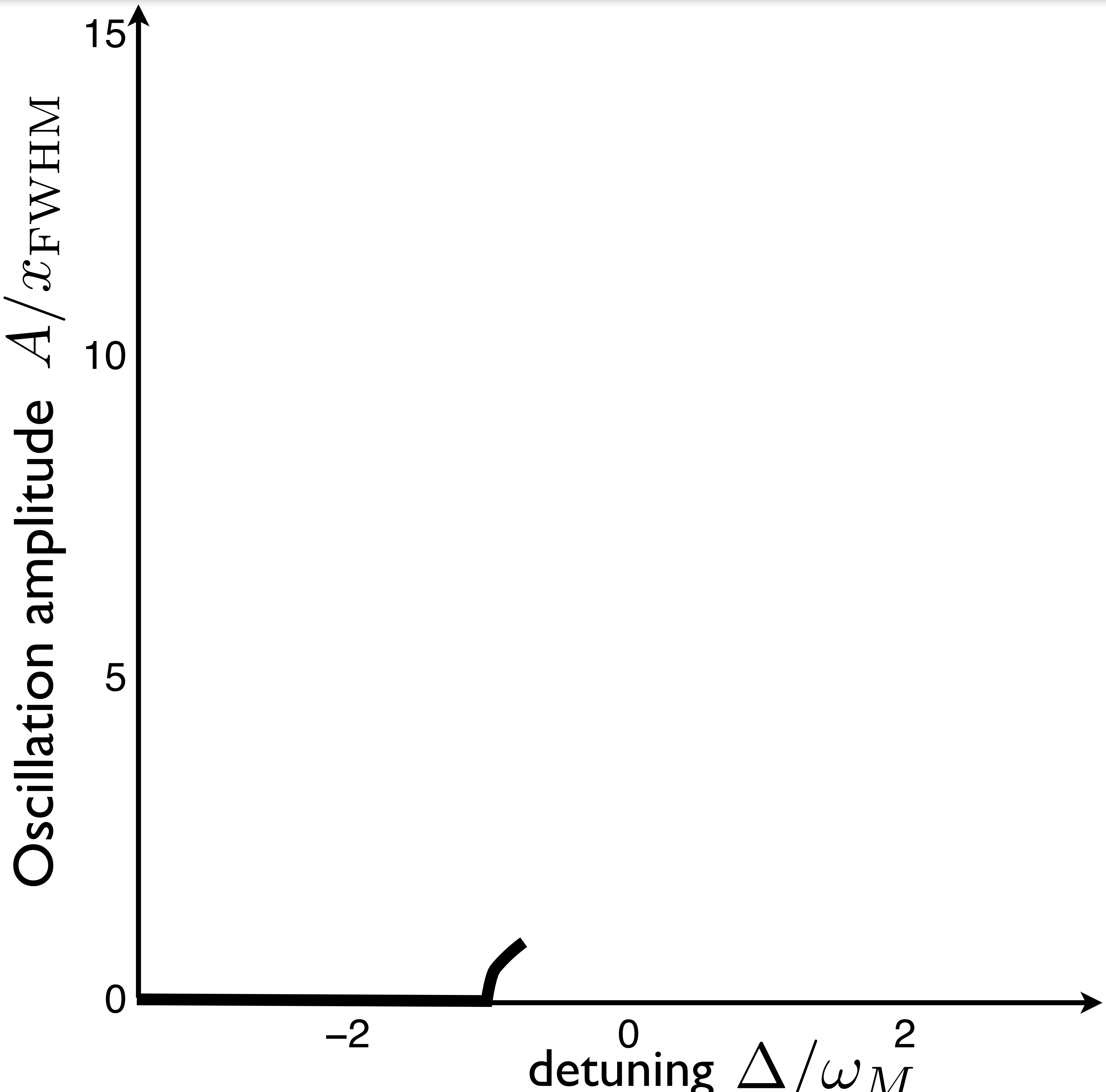

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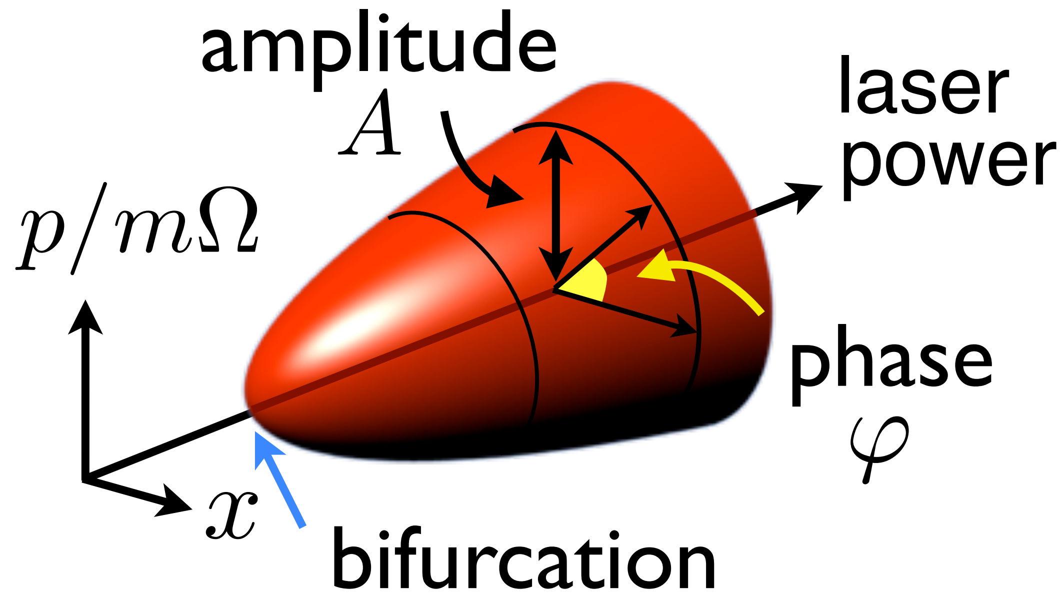
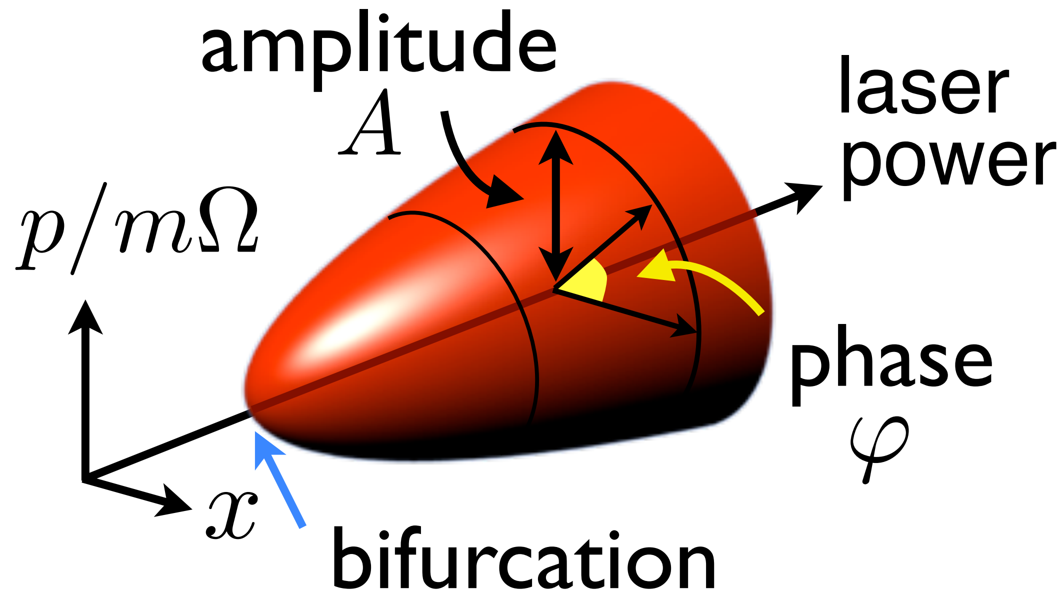
An optomechanical cell as a Hopf oscillator



Amplitude fixed, phase undetermined!

An optomechanical cell as a Hopf oscillator



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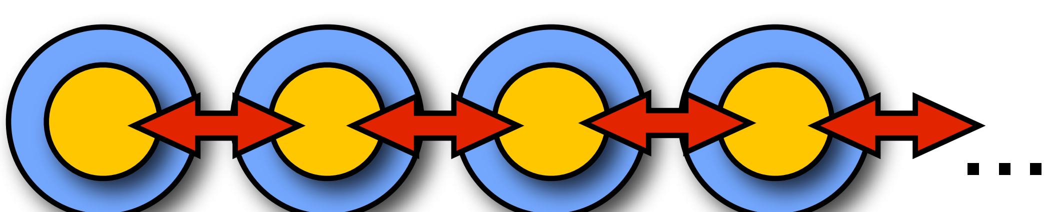
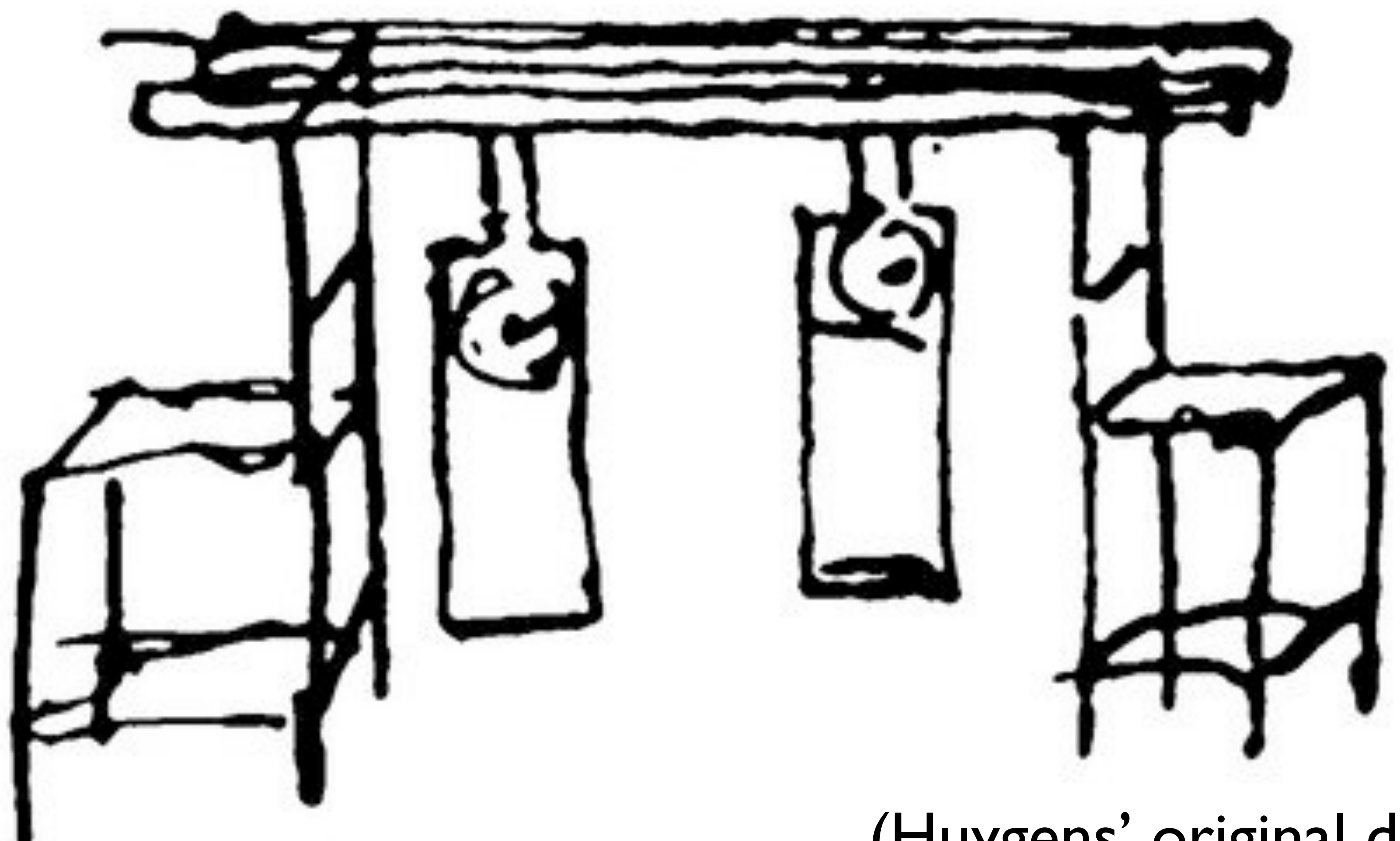


Collective dynamics in an array of coupled cells?

Phase-locking: synchronization!

Synchronization: Huygens' observation



(Huygens' original drawing!)

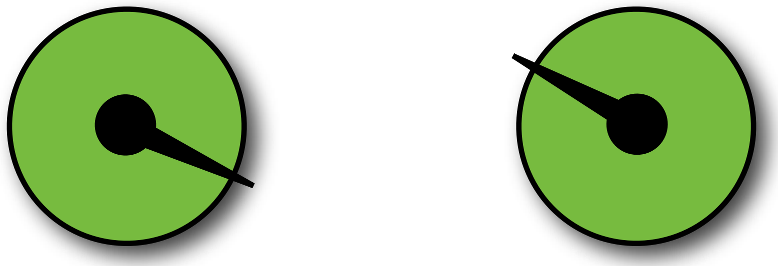
Coupled pendula synchronize...

...even though frequencies slightly different
...due to nonlinear effects

Fireflies synchronizing (Source: YouTube)

Coupled phase oscillators

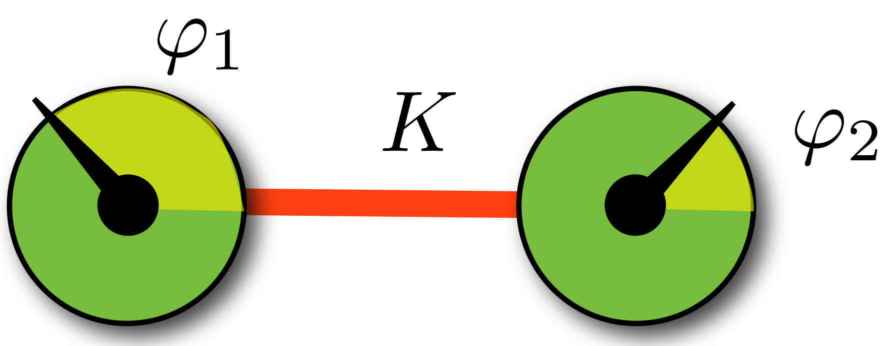
Coupled phase oscillators



Coupled phase oscillators

Coupled phase oscillators

The Kuramoto model



Kuramoto model:

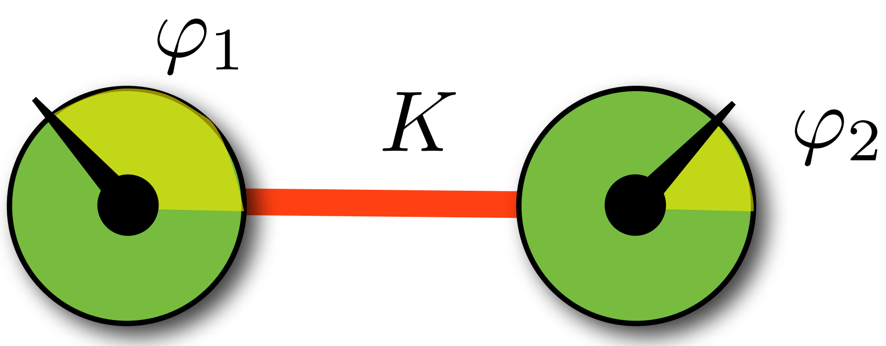
$$\dot{\varphi}_1 = \Omega_1 + K \sin(\varphi_2 - \varphi_1)$$
$$\dot{\varphi}_2 = \Omega_2 + K \sin(\varphi_1 - \varphi_2)$$

- captures essential features
- often found as limiting model

Kuramoto 1975, 1984

Acebron et al. , Rev. Mod. Phys. 77, 137 (2005)

The Kuramoto model



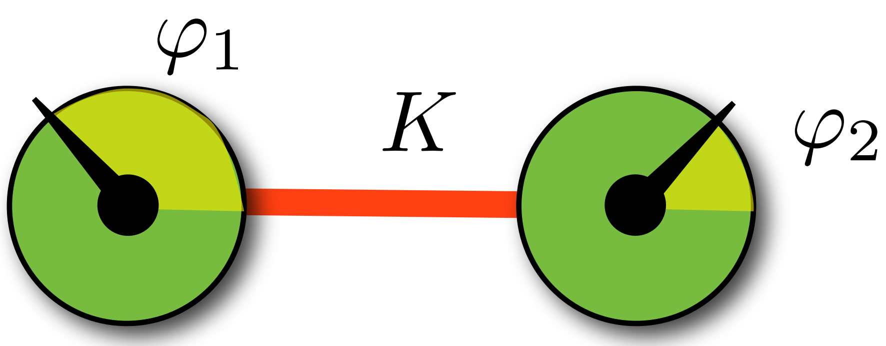
$$\dot{\varphi}_1 = \Omega_1 + K\sin(\varphi_2 - \varphi_1)$$
$$\dot{\varphi}_2 = \Omega_2 + K\sin(\varphi_1 - \varphi_2)$$

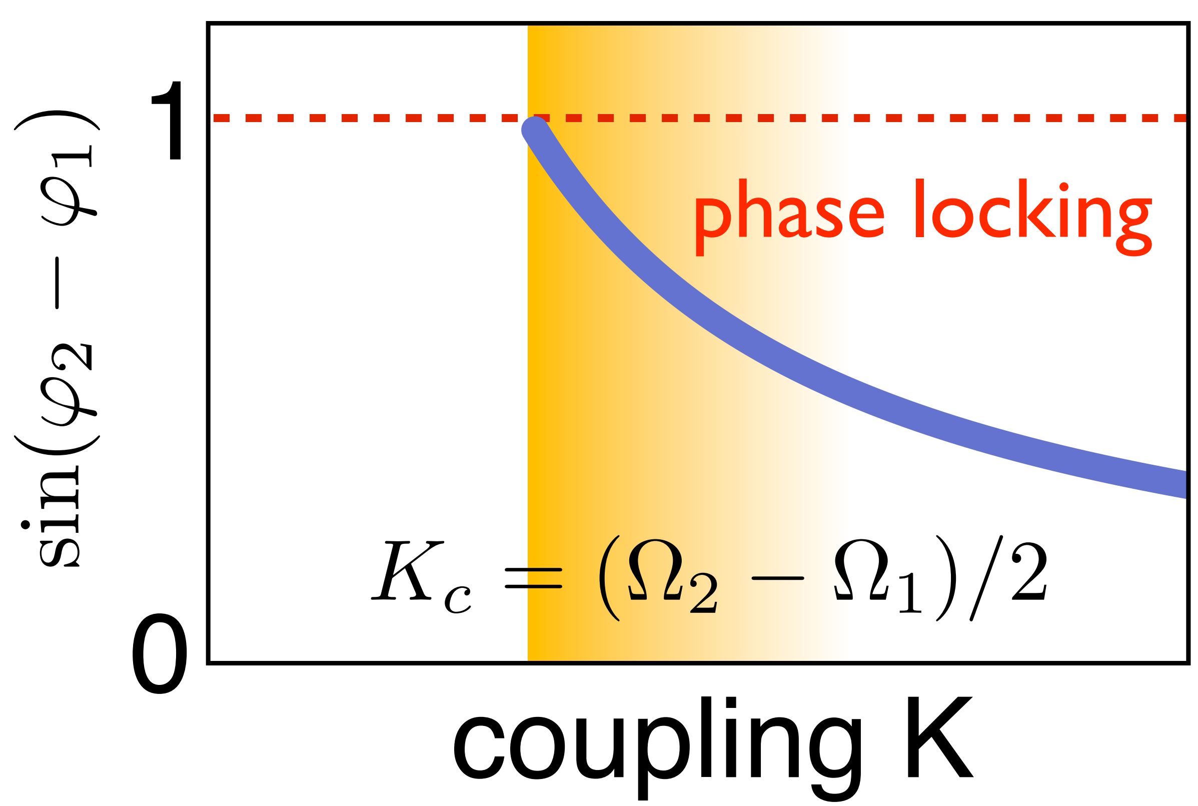
Synchronization:

$$\dot{\varphi}_1 = \dot{\varphi}_2 \quad \Rightarrow \quad \sin(\varphi_2 - \varphi_1) = \frac{\Omega_2 - \Omega_1}{2K}$$

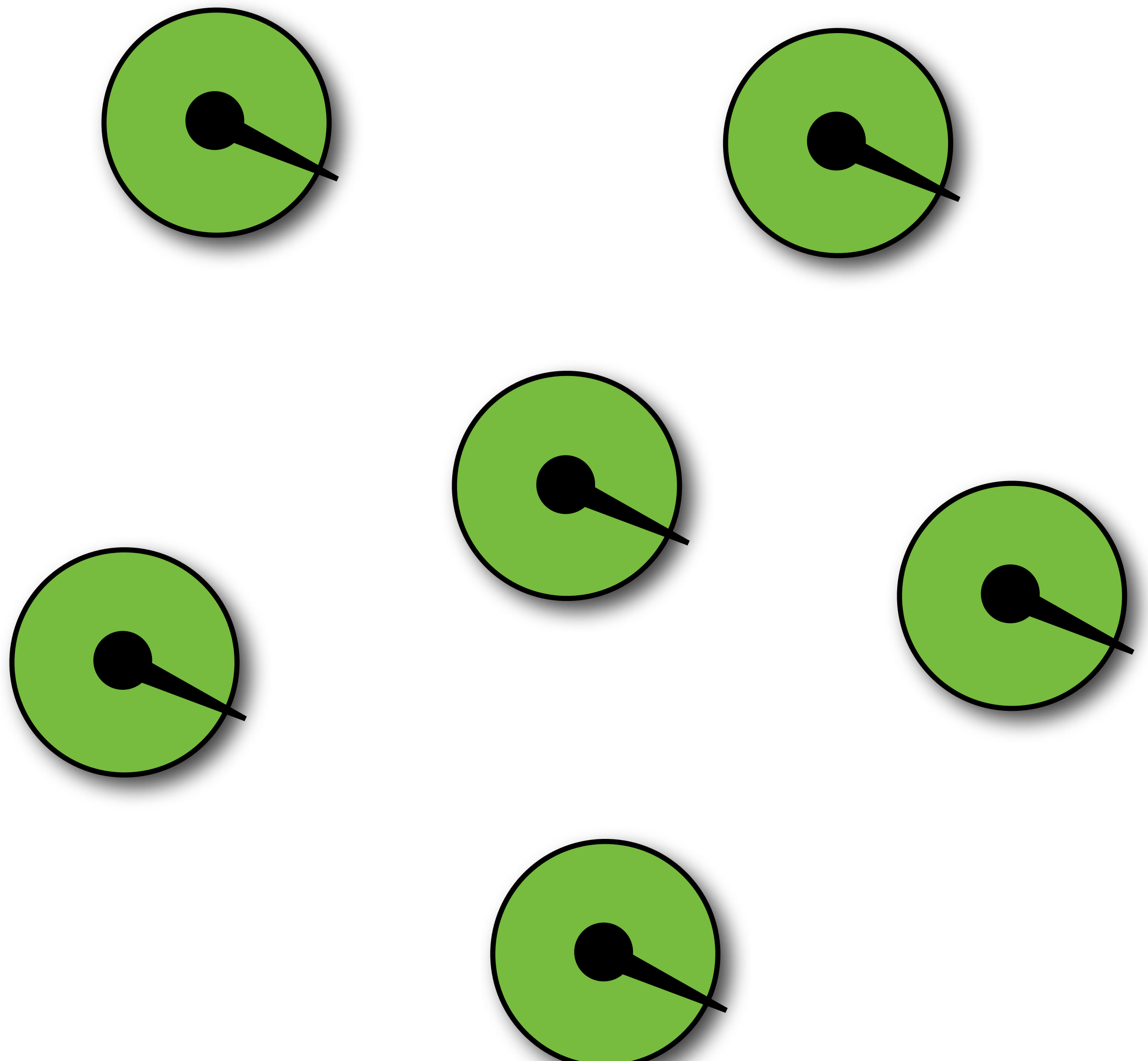
phase lag

The Kuramoto model





Many phase oscillators



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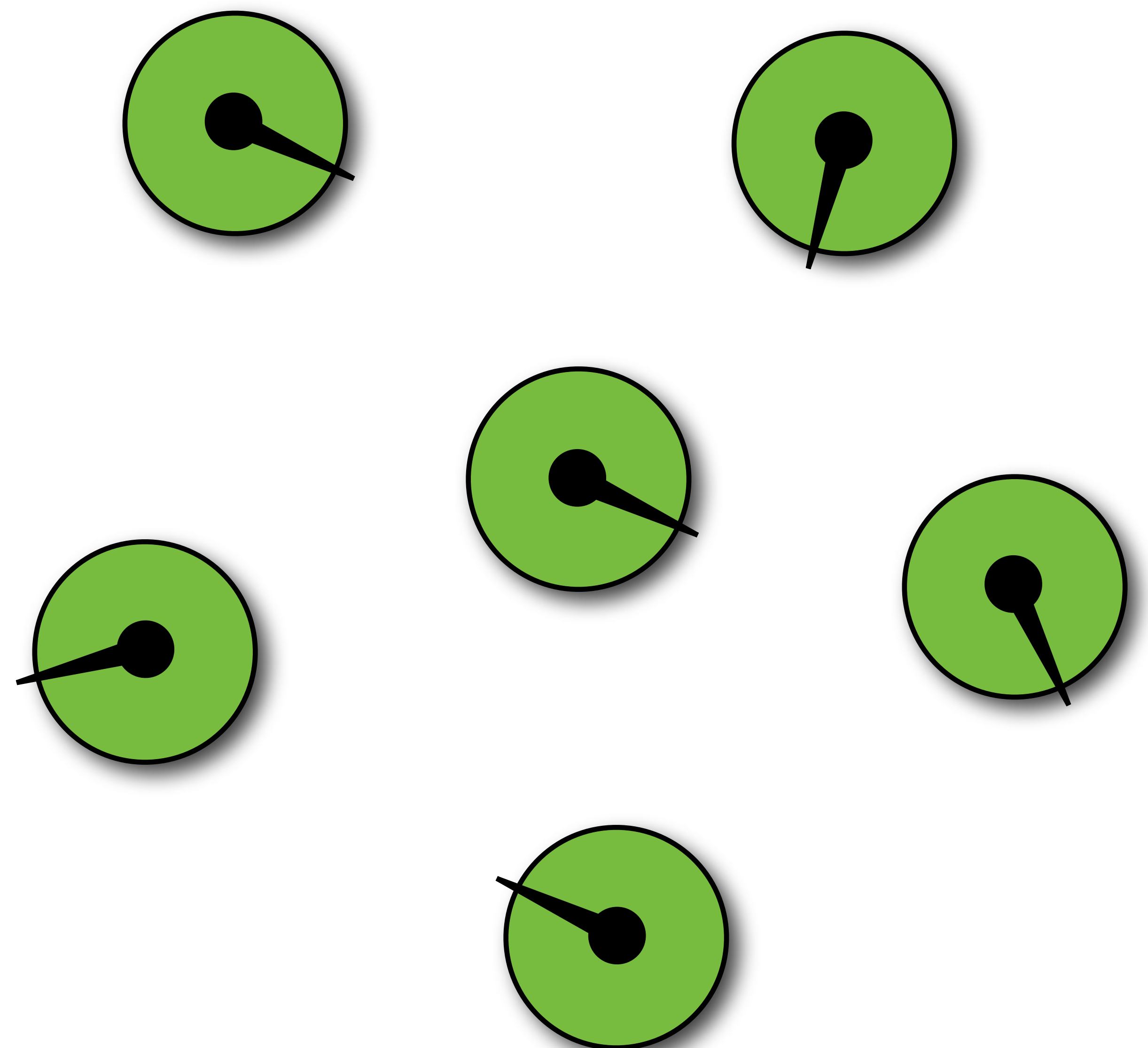

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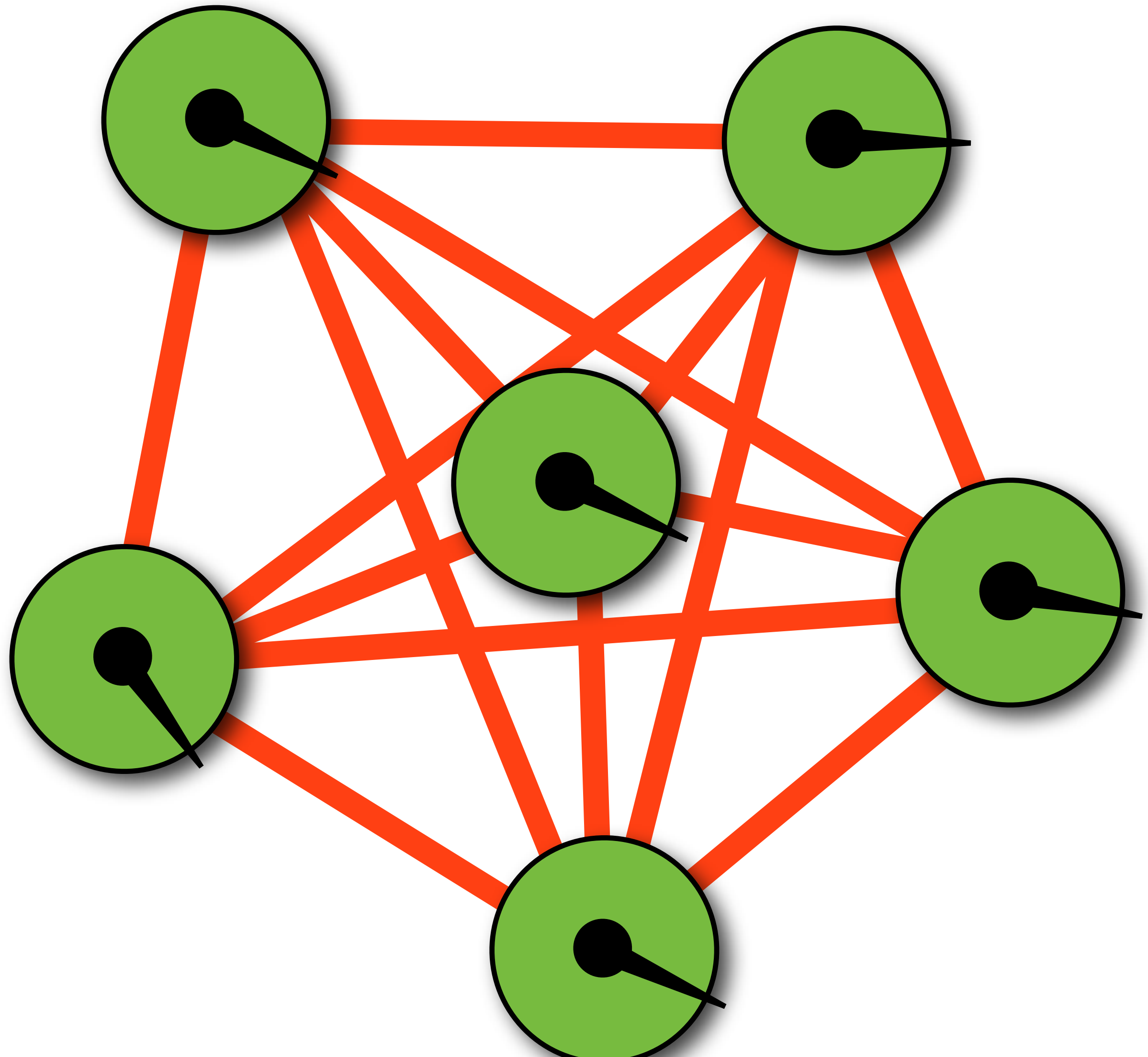

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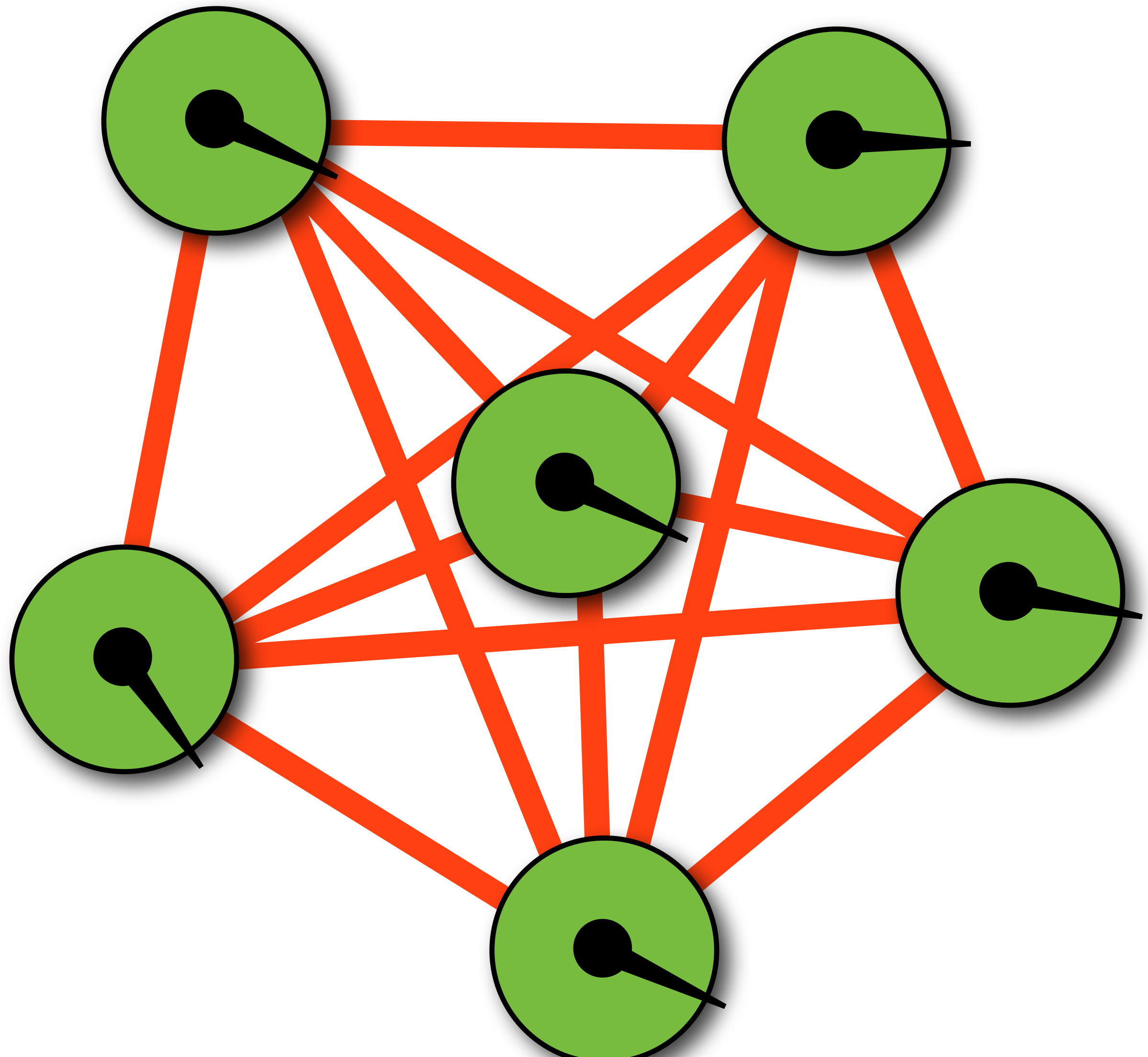
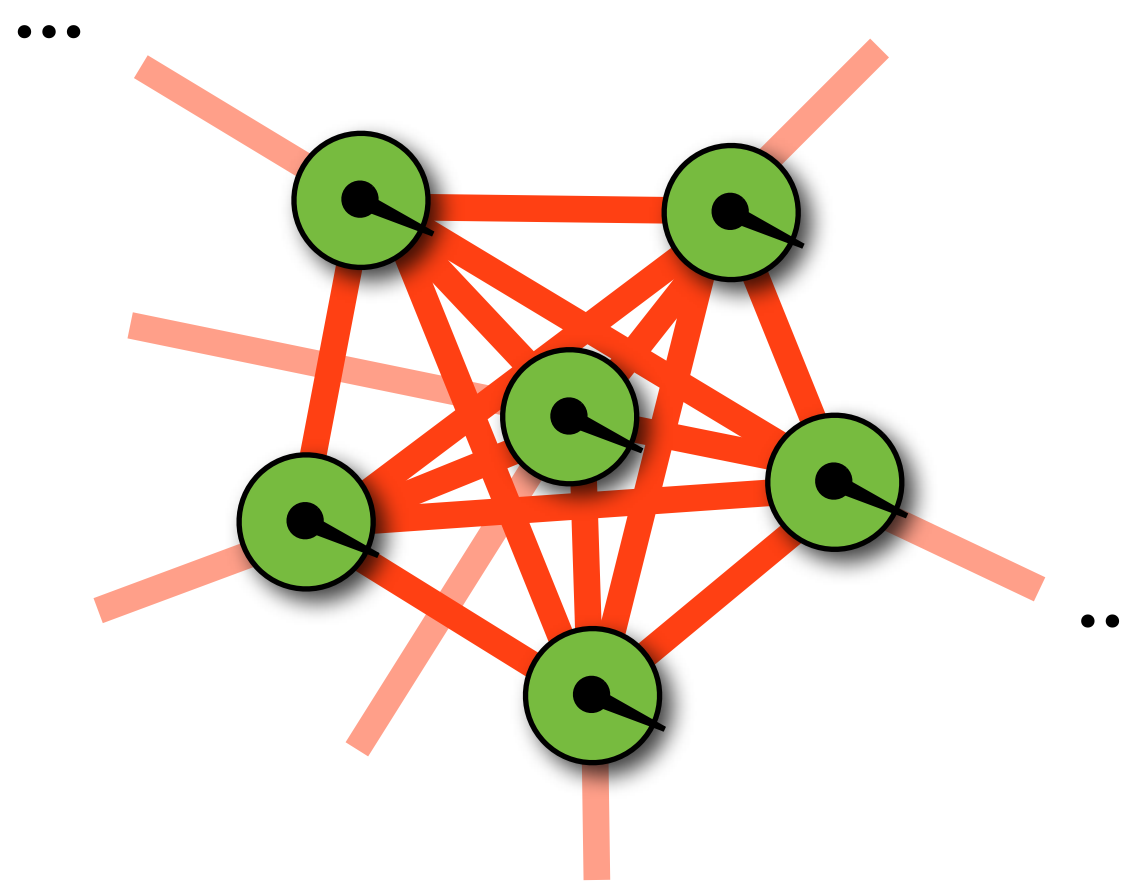
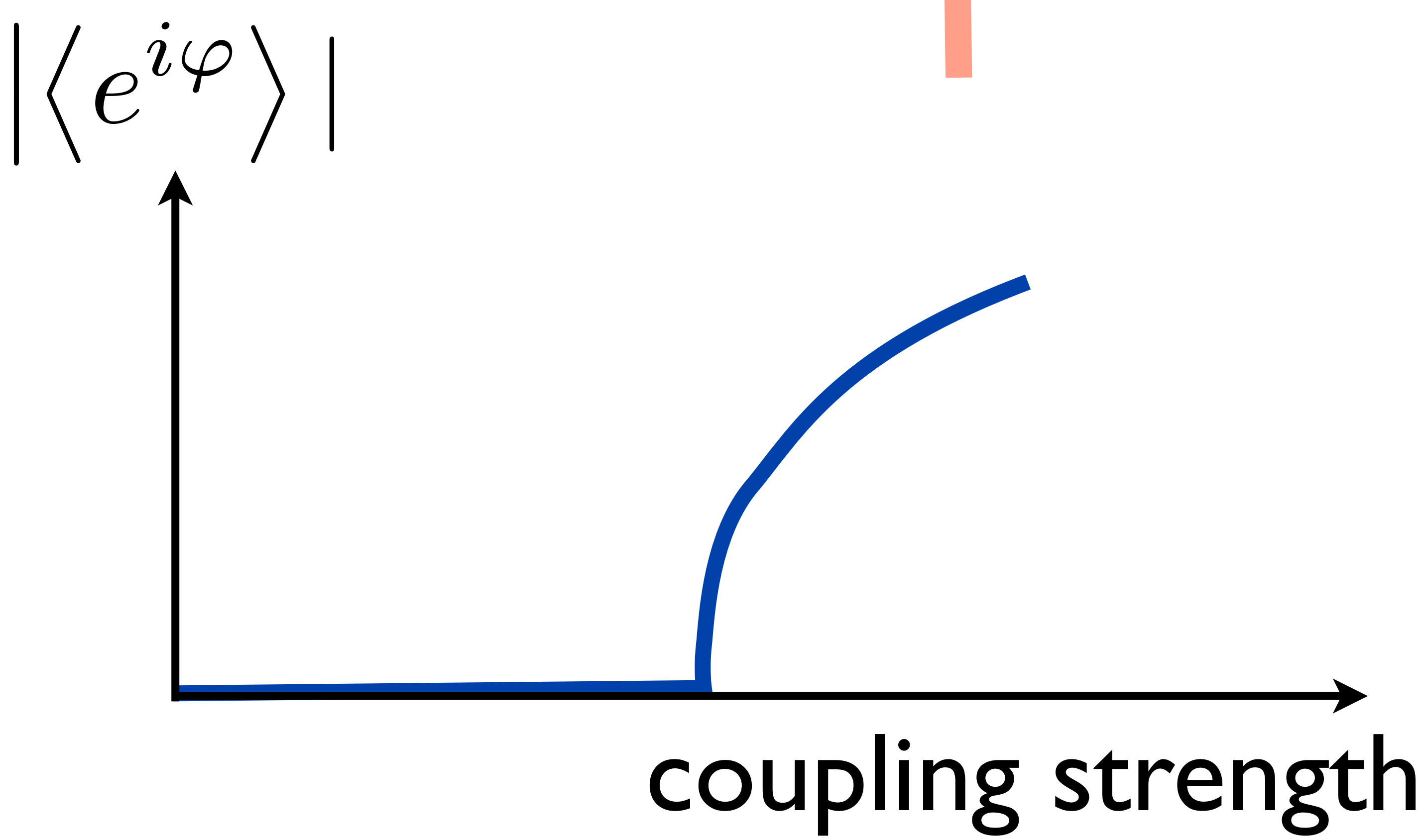


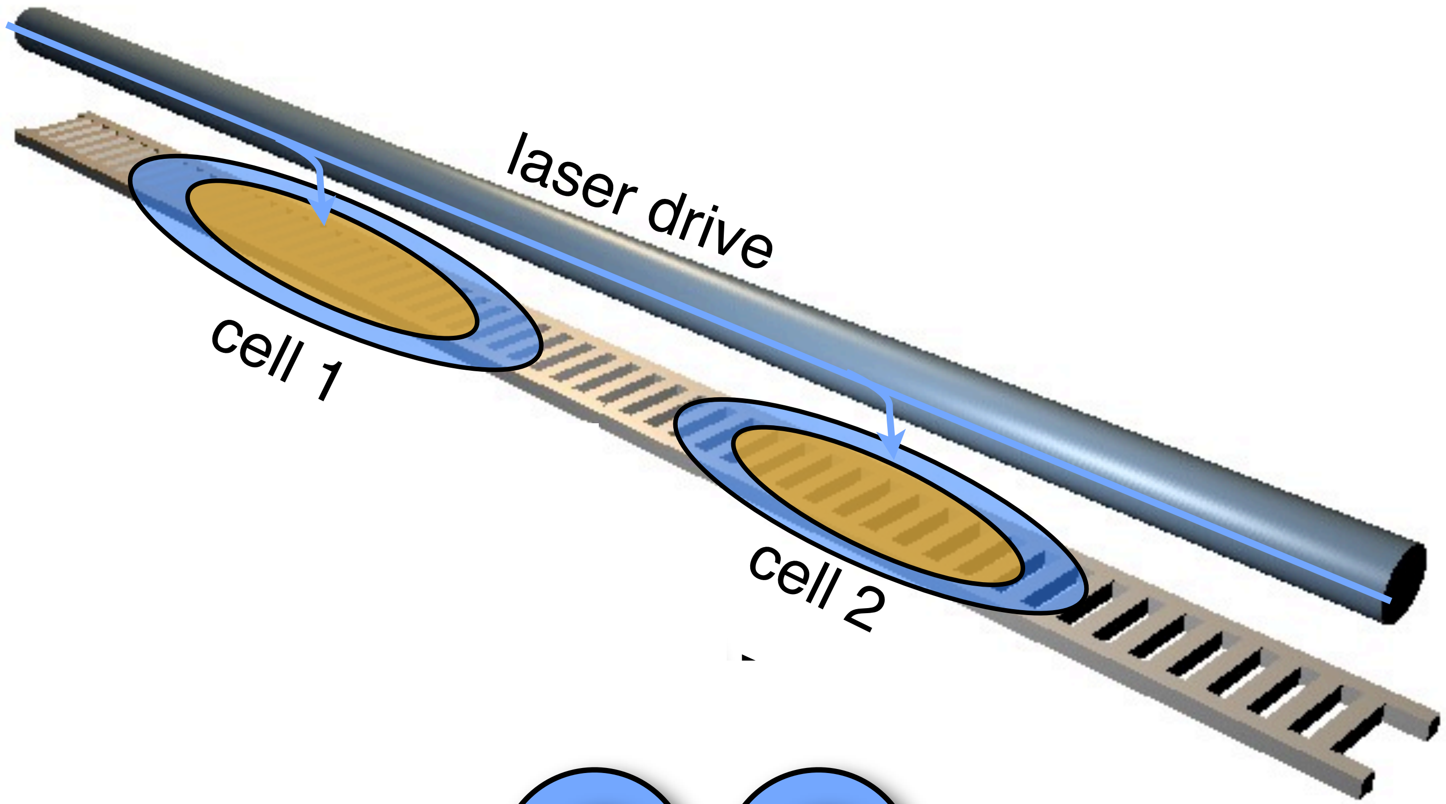
Kuramoto model: Phase-locking transition

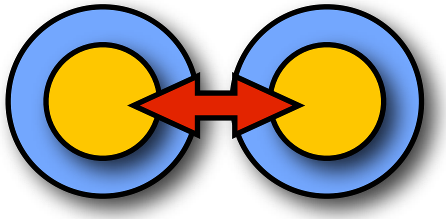
infinite-range coupling
Kuramoto model
displays phase
transition





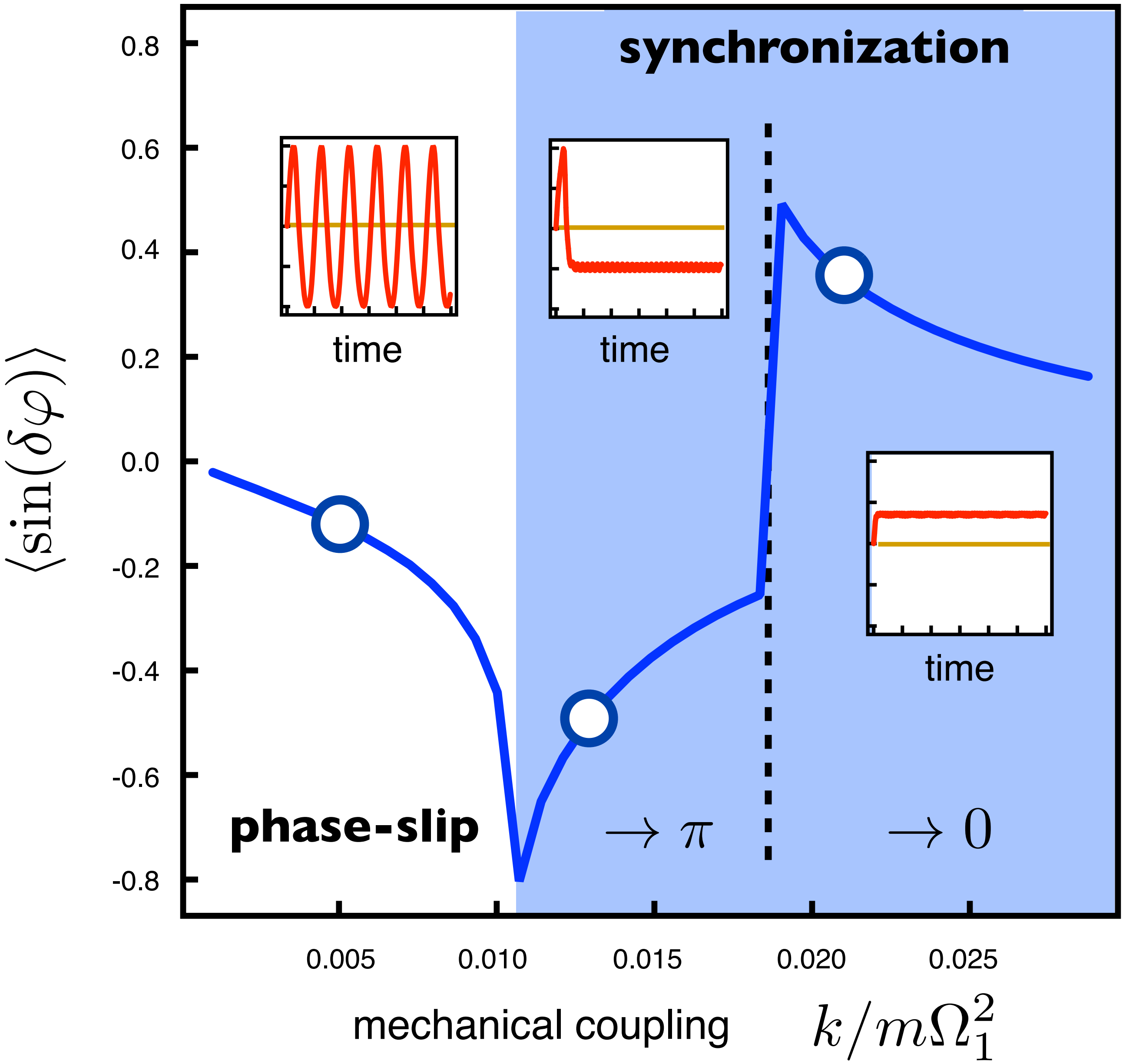
Phase locking of two optomechanical cells



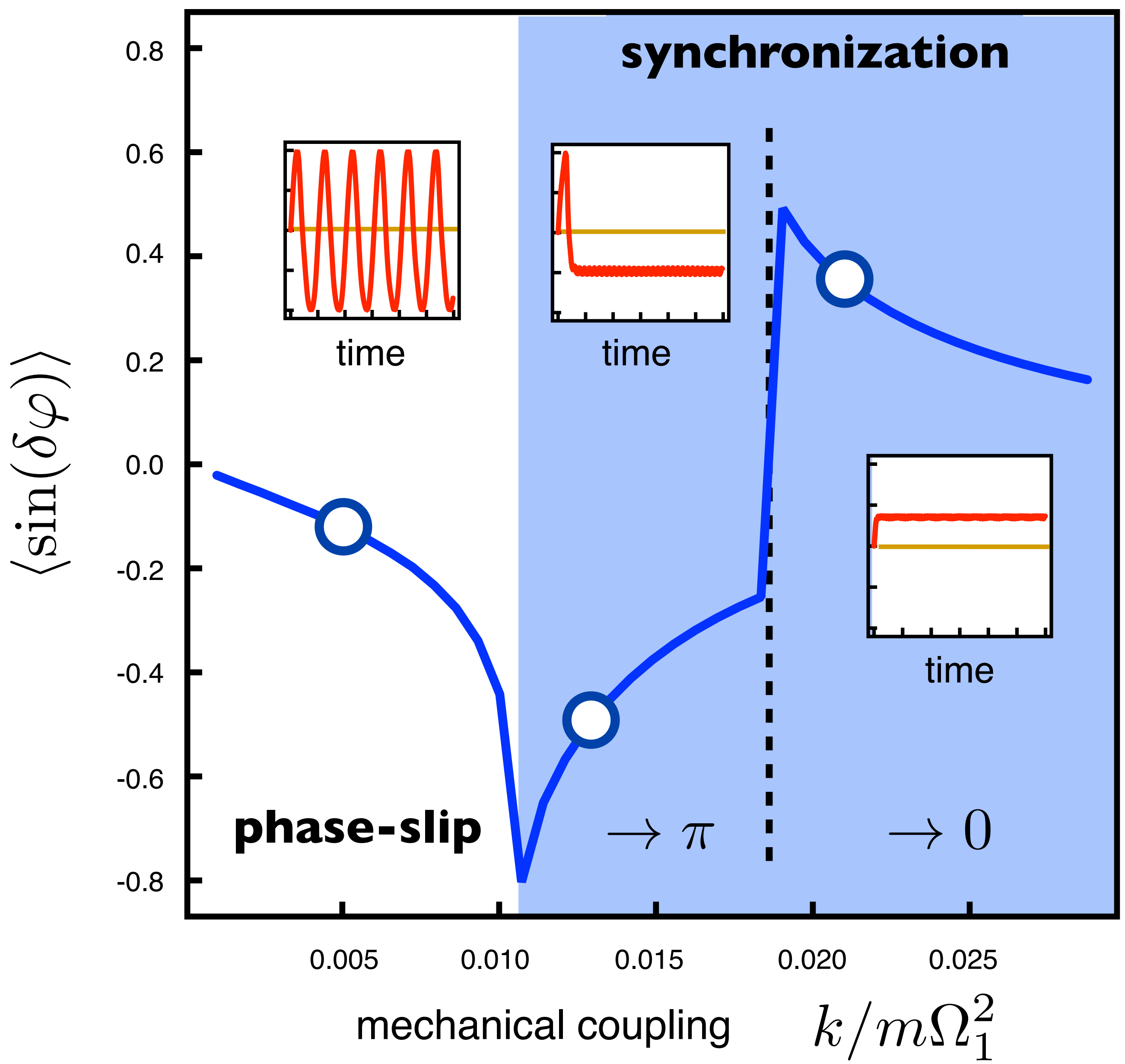


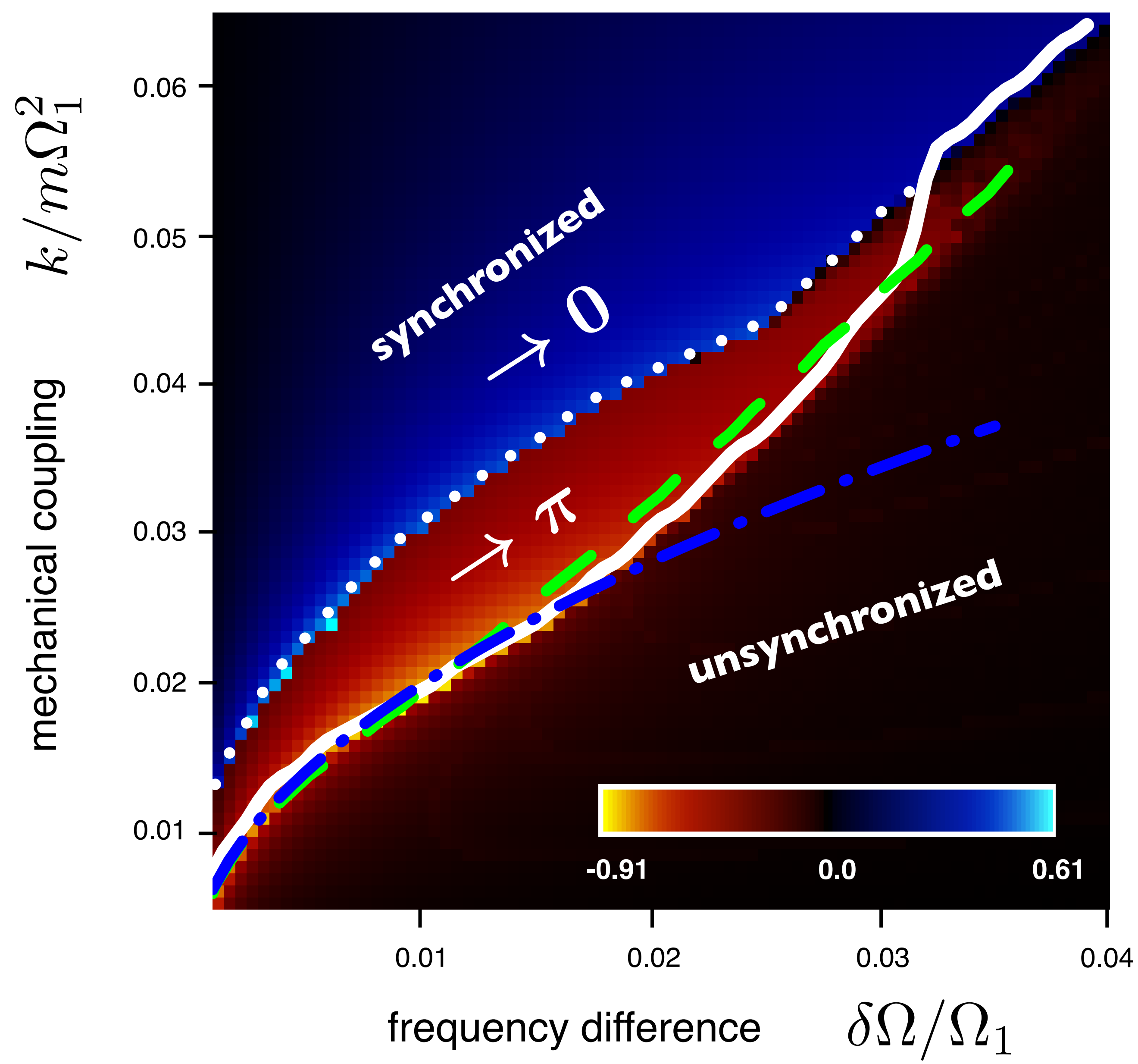
Two optomechanical cells,
fixed laser drive,
increasing mechanical coupling

Phase locking of two optomechanical cells

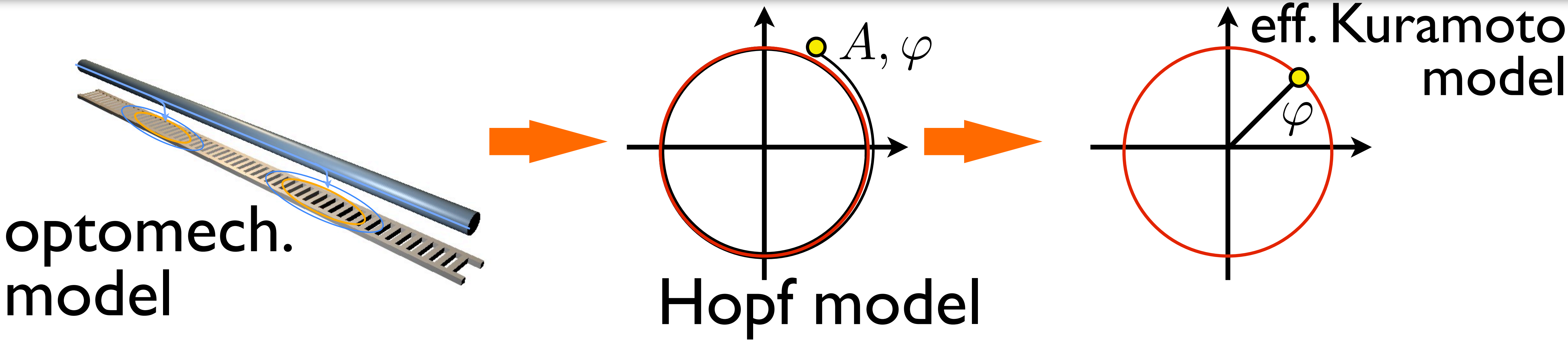


Phase locking of two optomechanical cells



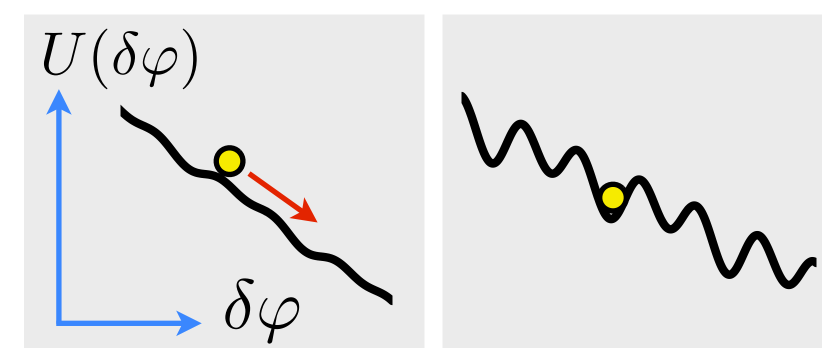


Effective Kuramoto model



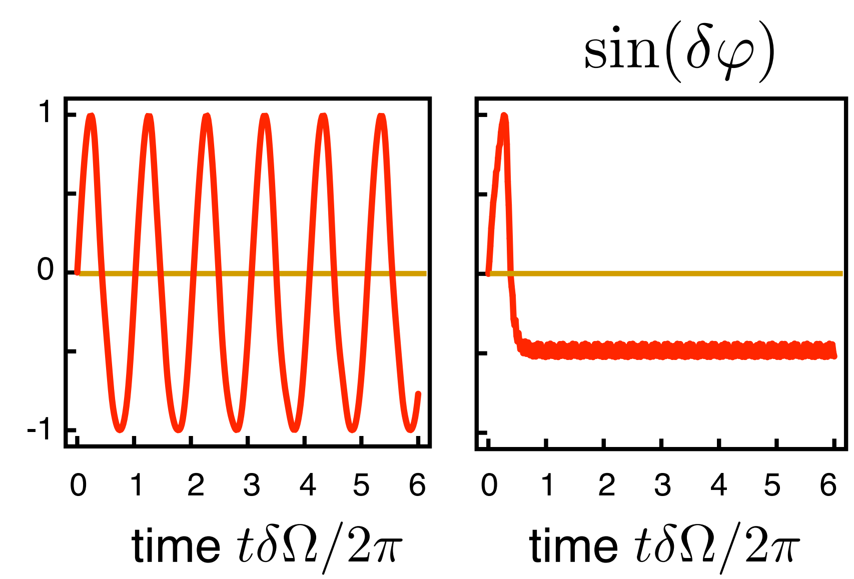
Standard Kuramoto model:

$$\delta\dot{\varphi} = \delta\Omega - 2K\sin(\delta\varphi)$$



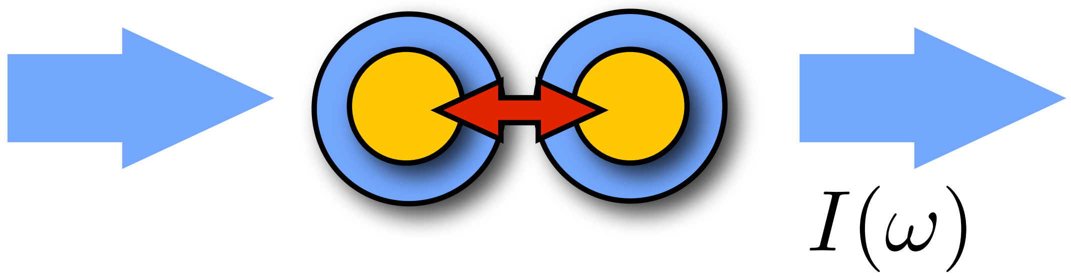
Effective Kuramoto model for coupled Hopf oscillators:

$$\delta\dot{\varphi} = \delta\Omega - 2K_s\sin(2\delta\varphi) - 2K_c\cos(2\delta\varphi)$$

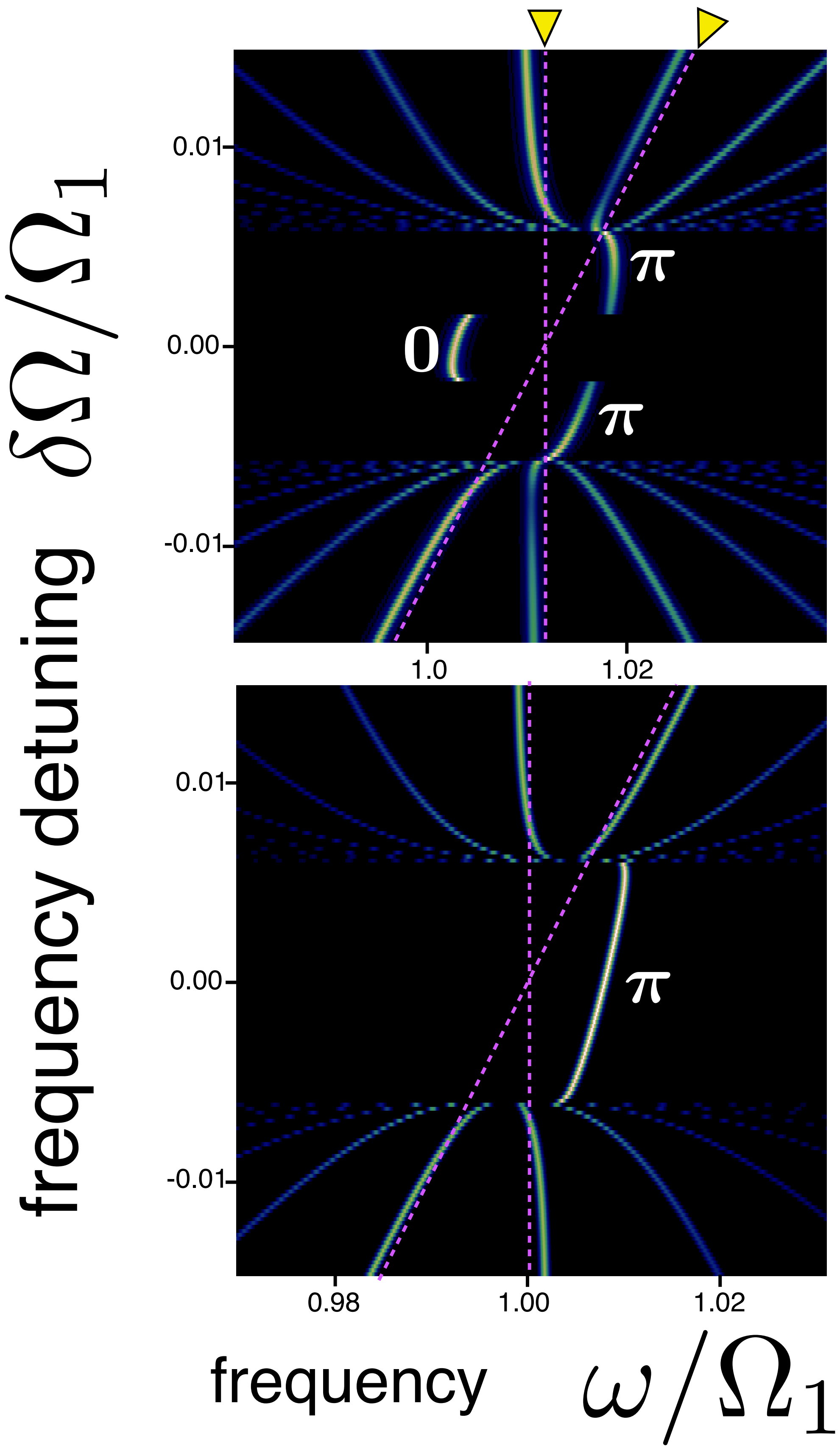


$$K = K(k, \Omega, P_{\mathrm{in}}, \ldots)$$

Frequency locking



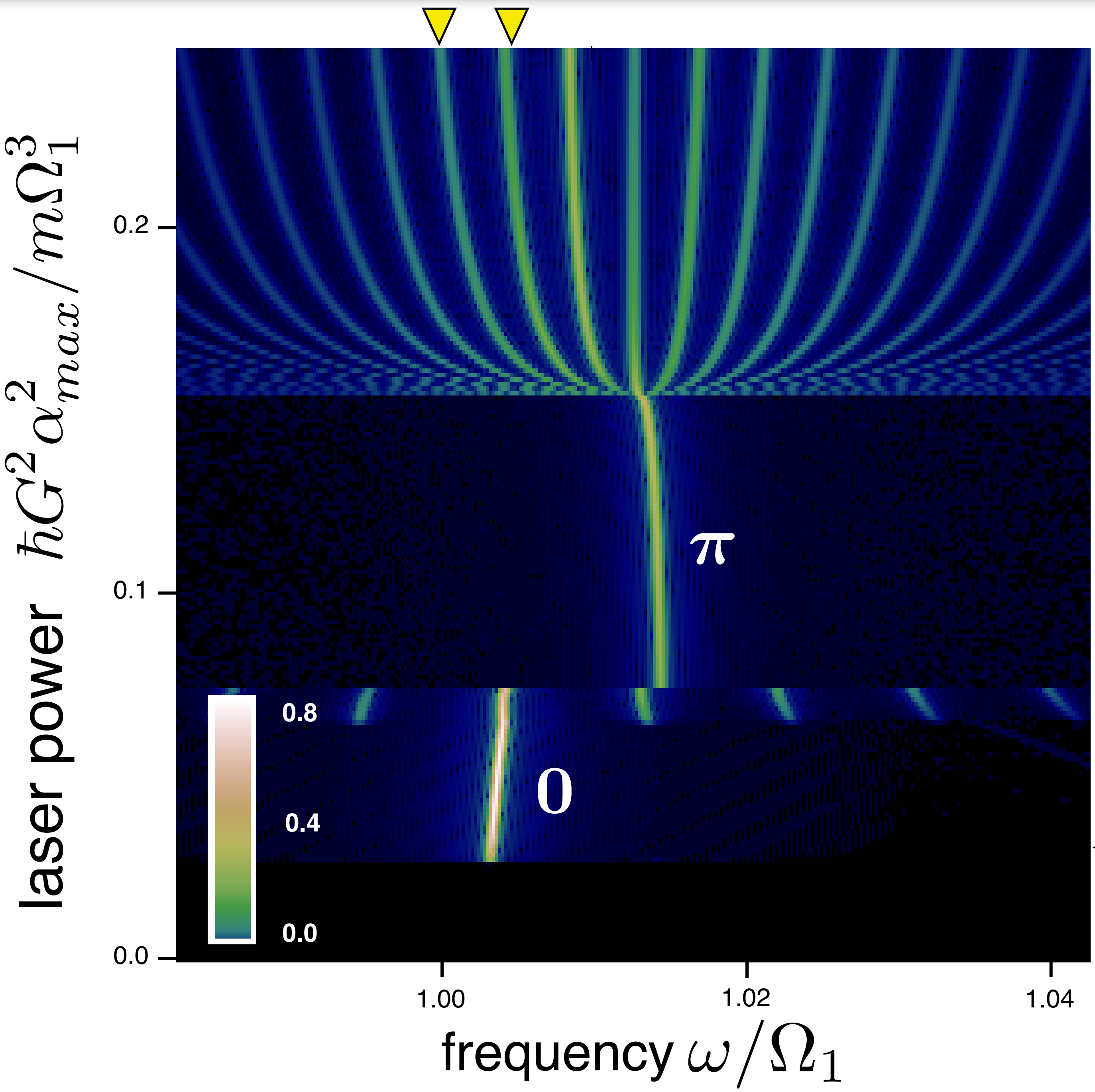
Mechanical spectrum in light field intensity fluctuations



(optom.)

(Hopf)

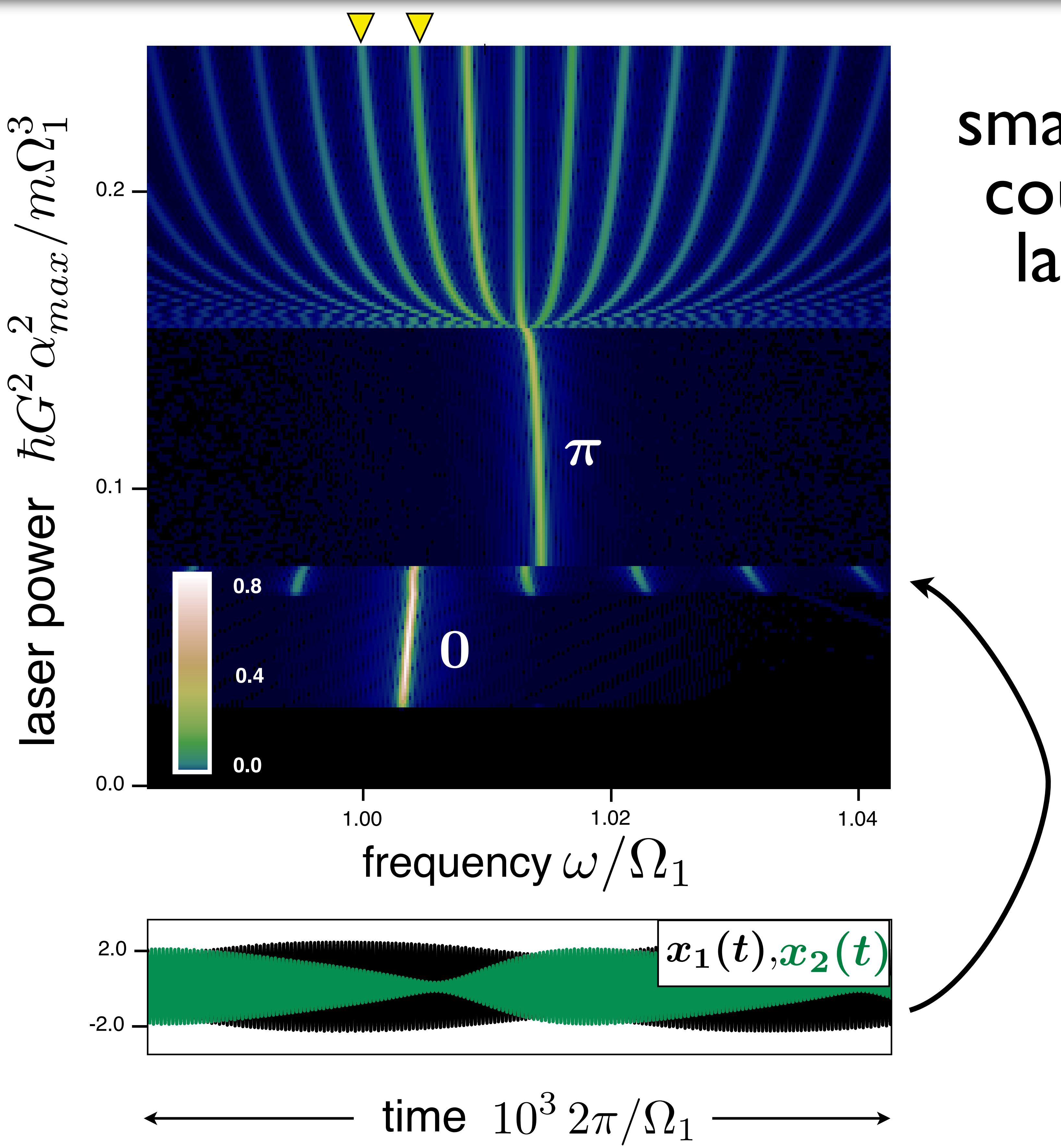
Desynchronization for increasing drive



smaller effective coupling K for larger drive!

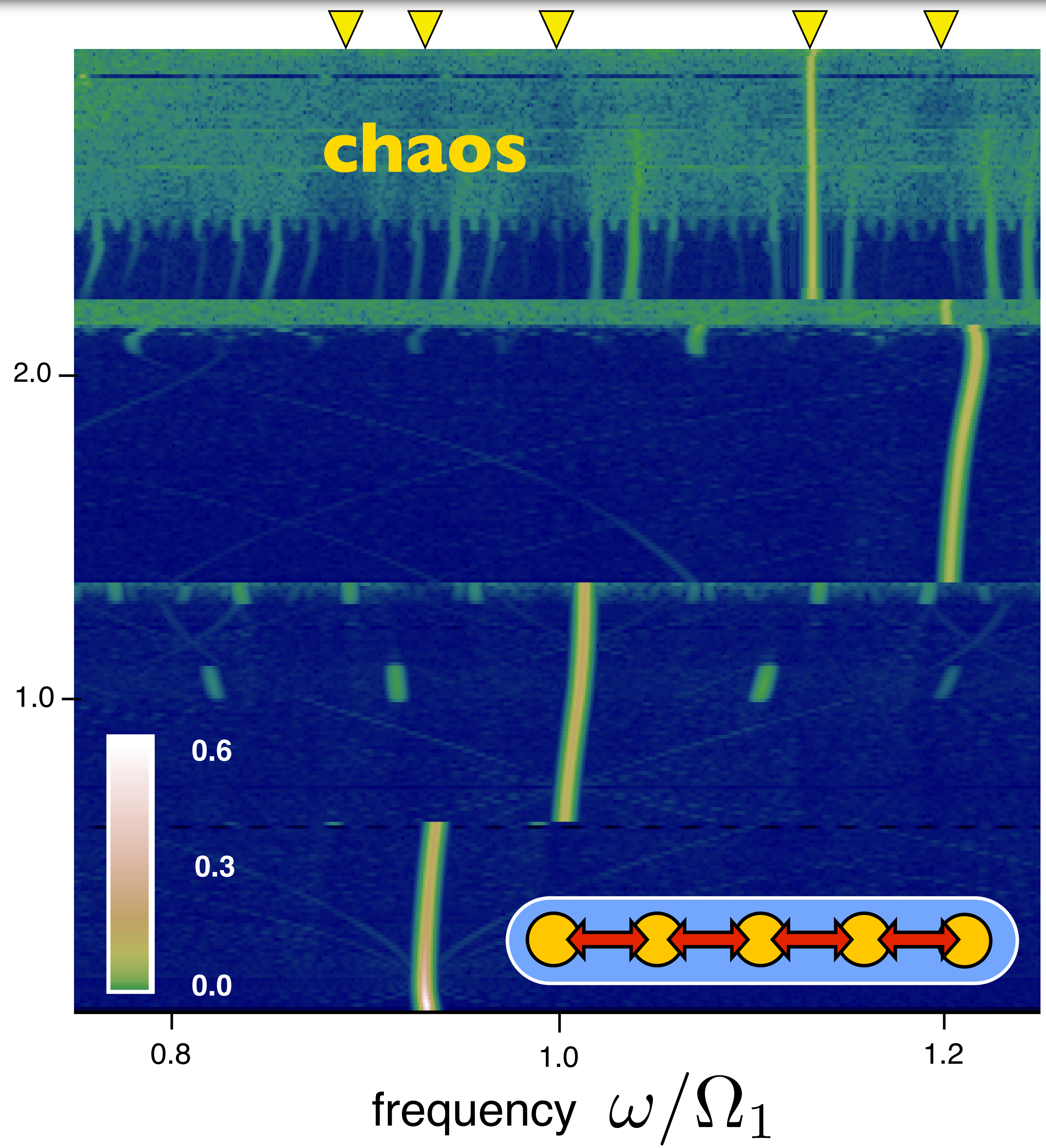
Hopf bifurcation

Desynchronization for increasing drive



smaller effective coupling K for larger drive!

Array with common optical mode

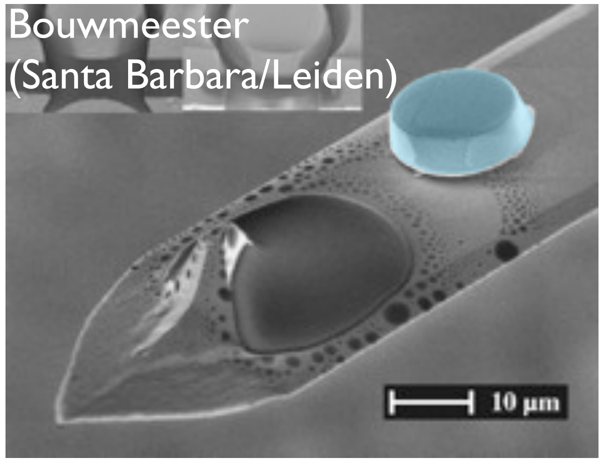


Dynamics in optomechanical arrays

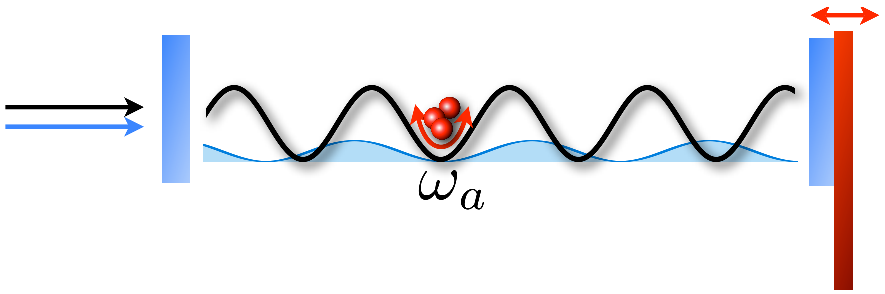
Outlook

- 2D geometries
- Information storage and classical computation
- Dissipative quantum many-body dynamics
- ...

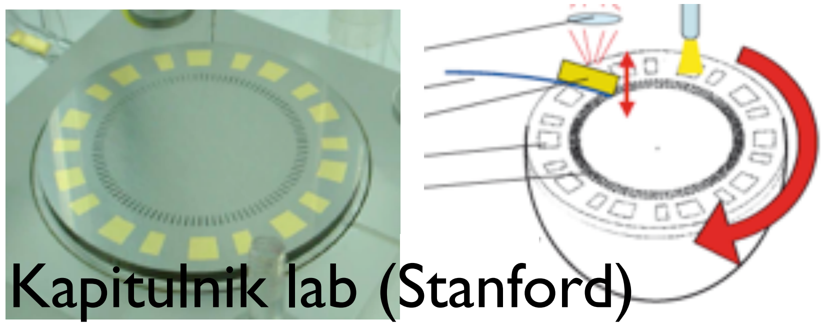
Optomechanics: general outlook



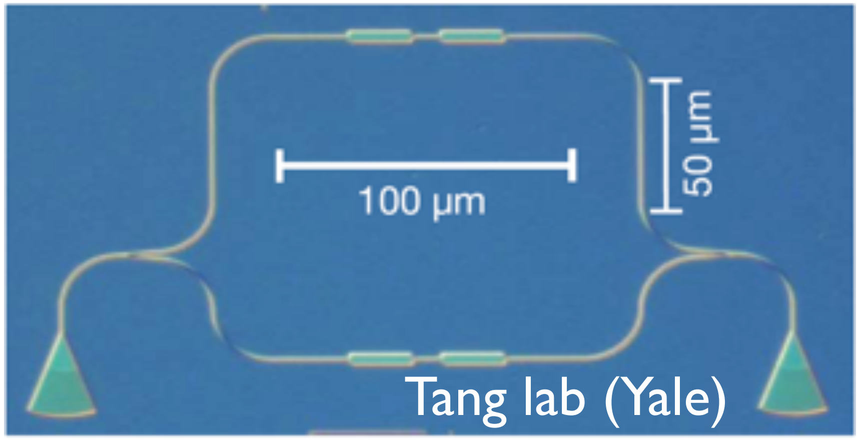
Fundamental tests of quantum mechanics in a new regime: entanglement with 'macroscopic' objects, unconventional decoherence? [e.g.: gravitationally induced?]



Mechanics as a 'bus' for connecting hybrid components: superconducting qubits, spins, photons, cold atoms, ...



Precision measurements
[e.g. testing deviations from Newtonian gravity due to extra dimensions]



Optomechanical circuits & arrays
Exploit nonlinearities for classical and quantum information processing, storage, and amplification; study collective dynamics in arrays

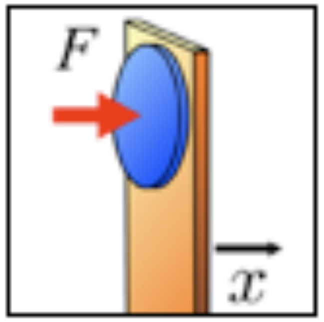
Optomechanics

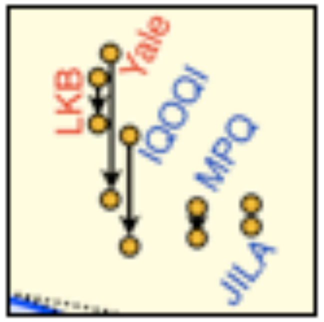
Recent review on optomechanics:
APS Physics 2, 40 (2009)

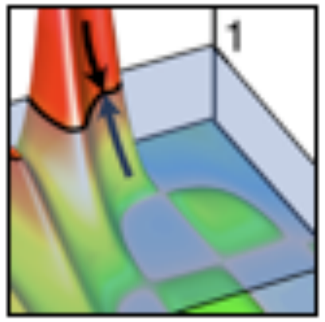
Physics
spotlighting exceptional research

Trends

Optomechanics







Florian Marquardt and Steven M. Girvin, May 18, 2009

Recent review on quantum limits for detection and amplification:
Clerk, Devoret, Girvin, Marquardt, Schoelkopf; RMP 2010