Spin Qubits in Quantum Dots

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Tutorial Review:
R. Zak, B. Röthlisberger, S. Chesi, D. L.,
Riv. Nuovo Cim. 033, 345 (2010); arXiv:0906.4045

$$: Swiss NSF, Nano Center Basel, *EU, ESF,* DARPA, IARPA

Outline

A. Spin qubits in quantum dots

- Basics of quantum computing and quantum dots
- universal gates & entanglement: via interaction or parity measurements

B. Spin decoherence in GaAs quantum dots

- Spin orbit interaction and spin decay
- Alternative spin qubits: holes, graphene, nanotubes,...
- Nuclear spins and hyperfine induced decoherence

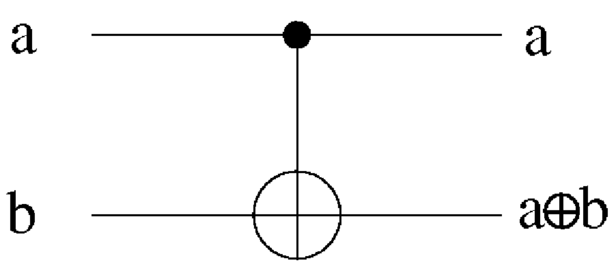
C. Nuclear spin order in 1D (and 2D)

- reduce noise in spin bath by nuclear spin ordering
- Kondo lattice in Luttinger liquids (and marginal Fermi liquid)

Quantum Information

Classical digital computer

network of ‘Boolean logic gates‘, e.g. XOR



b \ a	0	1
0	0	1
1	1	0

- bits: $a, b = 0,1$
- physical implementation: e.g. 2 voltage levels
- ‘gate‘: electronic circuit

Quantum computer

- qubits $|a\rangle, |b\rangle \mathrel{\hat{=}} \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1$
- physical implementation:
  quantum 2-level-system: $|\uparrow\rangle \equiv |0\rangle, \quad |\downarrow\rangle \equiv |1\rangle$
- ‘quantum gate‘: unitary transformation
  (is reversible!)

Quantum Computing (basics)

- basic unit: **qubit** → any state of a quantum two-level system

$$|\Psi\rangle = a|1\rangle + b|0\rangle$$

"natural" candidate: **electron spin**

- quantum computation:

1) prepare N qubits (input)
2) apply unitary transformation in 2^N-dim. Hilbert space
→ computation
3) measure result (output)

- quantum computation faster than classical:

- factoring algorithm (Shor 1994): exp N → N^2
- database search (Grover 1996): N → $N^{1/2}$
- quantum simulations

...

What a quantum computer could do faster:

…search large database (→ biology, climate, physics…)

…break `RSA-Encryption’ (banking, industry, military,…)

…simulate physical und chemical processes
(→ physics, chemical & pharma industry, medicine,…)

…play quantum games

…and many unforseen applications!

Intense search for new quantum algorithms !

Electron qubit: spin better than charge

due to longer relaxation/decoherence* times

$$\tau_{\phi}^{spin} >> \tau_{\phi}^{charge}$$

10ns -1s

1 ns

GaAs

Awschalom et al., ’97
Tarucha et al., ‘02
Kouwenhoven/Vandersypen et al., ’03-’09
Abstreiter et al., ’04/08; Warburton et al., ‘09
Zumbuhl et al., '08
Yacoby/Marcus et al.,’05- ‘09

‘mesoscopics’
Fujisawa et al. ‘03
Marcus et al. ‘01

➔ natural choice for qubit: spin ½ of electron

*) theory: $T_2 \sim T_1$ for single spin in GaAs dot (‘everything optimized’)

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Quantum computation with quantum dots

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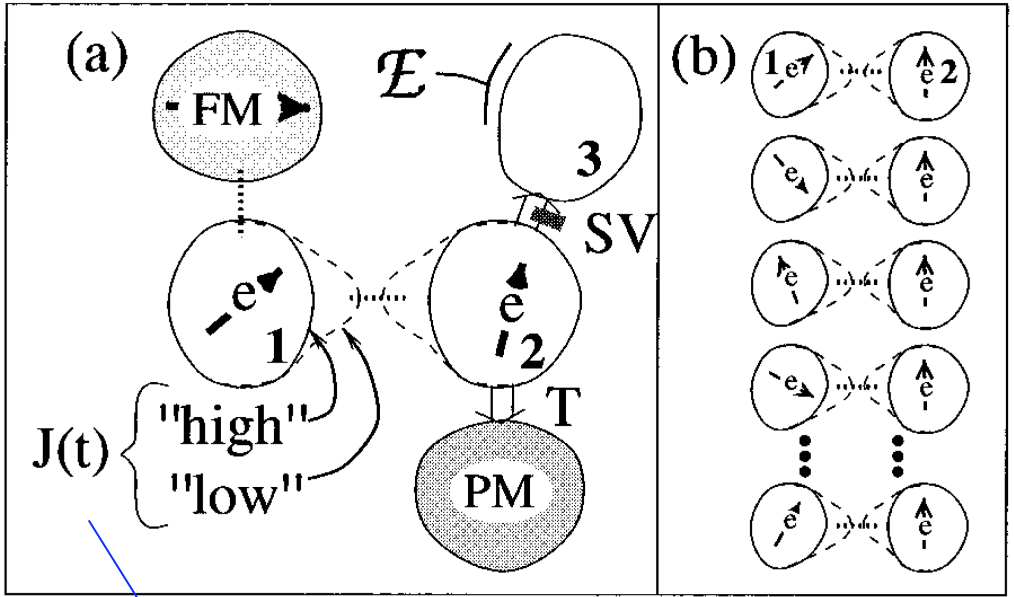
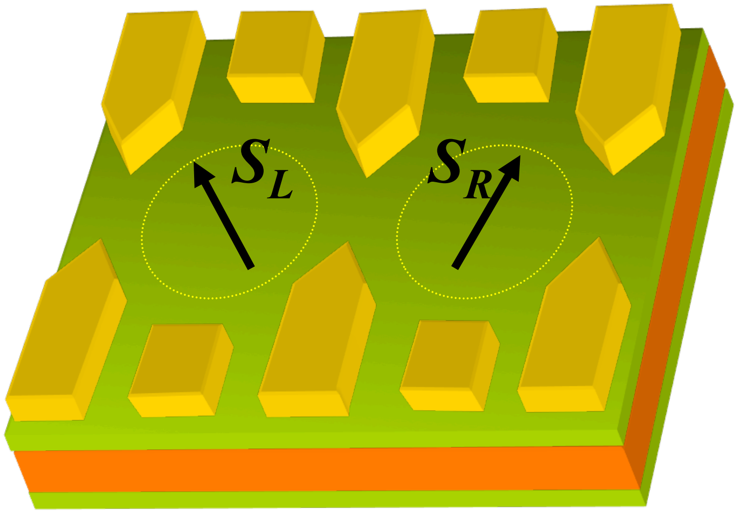


$$\boxed{U_{XOR} = e^{i\frac{\pi}{2}S_1^z}\, e^{-i\frac{\pi}{2}S_2^z}\, U_{SW}^{1/2}\, e^{i\pi S_1^z}\, U_{SW}^{1/2}}$$

'all-electrical scheme'

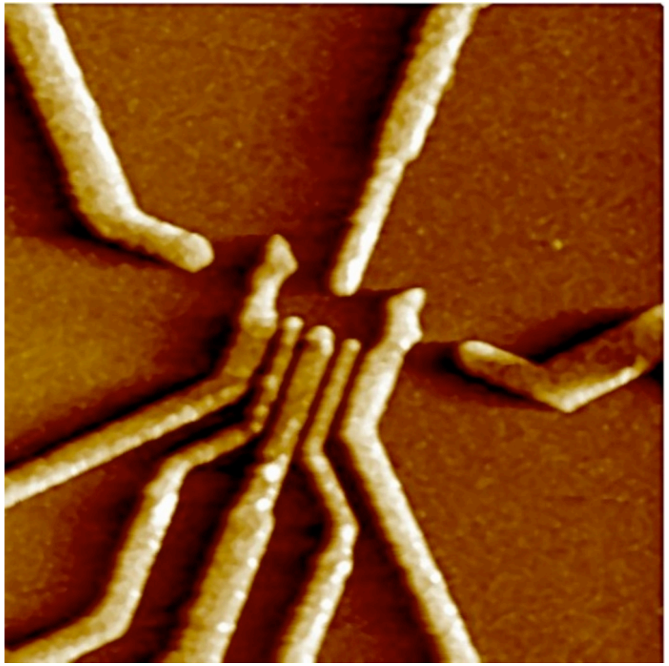
Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



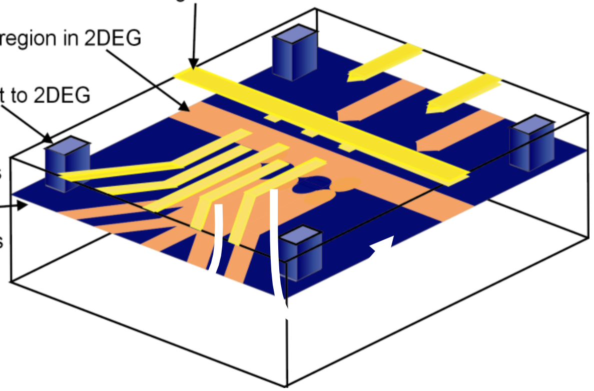
Spin 1/2 of electron = qubit

A quantum dot as a tunable artificial atom



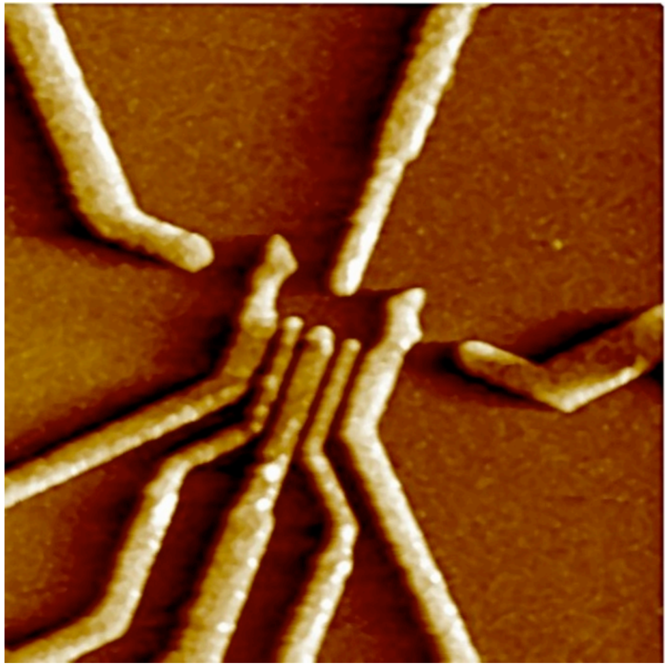
Delft group, since 2003

Confinement
- Discrete # charges
- Discrete orbitals



Electrical control and detection
- Tunable # of electrons
- Tunable tunnel barriers
- Electrical contacts

A quantum dot as a tunable artificial atom



Confinement

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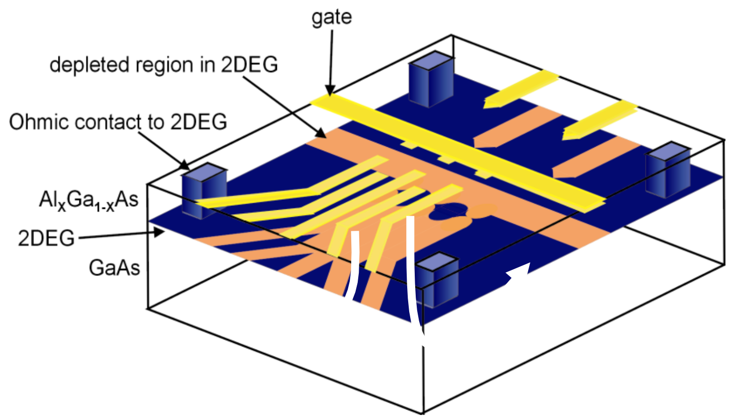
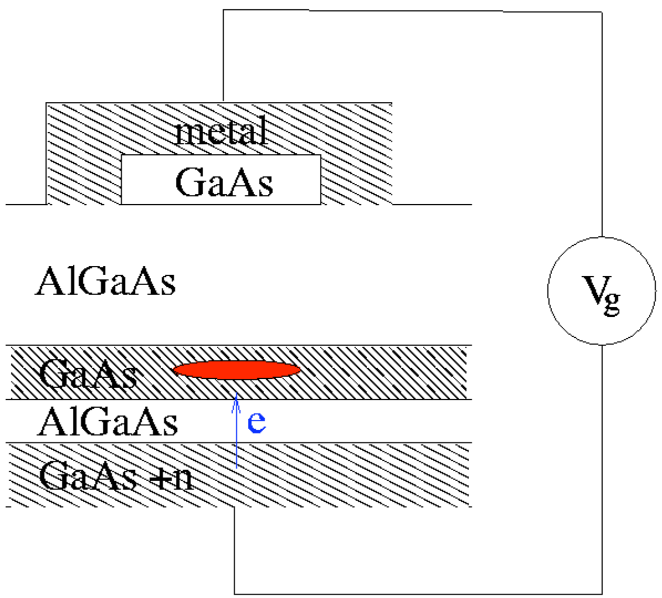


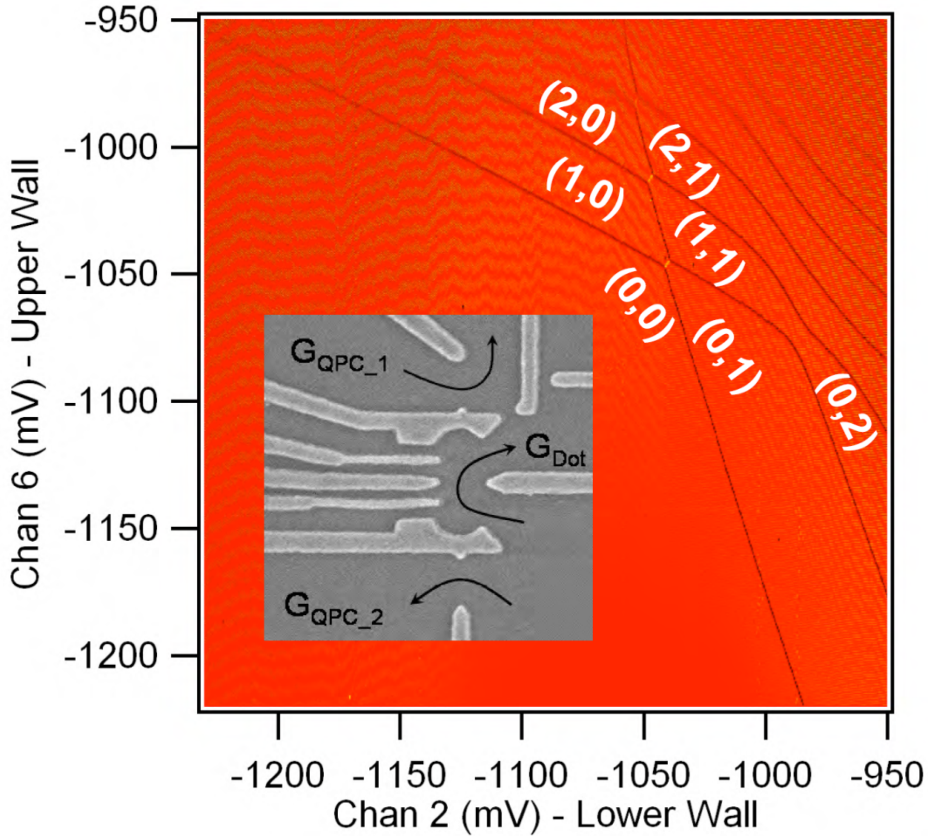
TU Delft, Harvard, Princeton, MIT,
Ottawa, Waterloo, Tokyo,
Sidney, Munich, Aachen, Basel, Zurich,…

GaAs/AlGaAs heterostructure

2DEG 90 nm deep, n_s = 2.9 x 10^{11} cm^{-2}



charging diagram

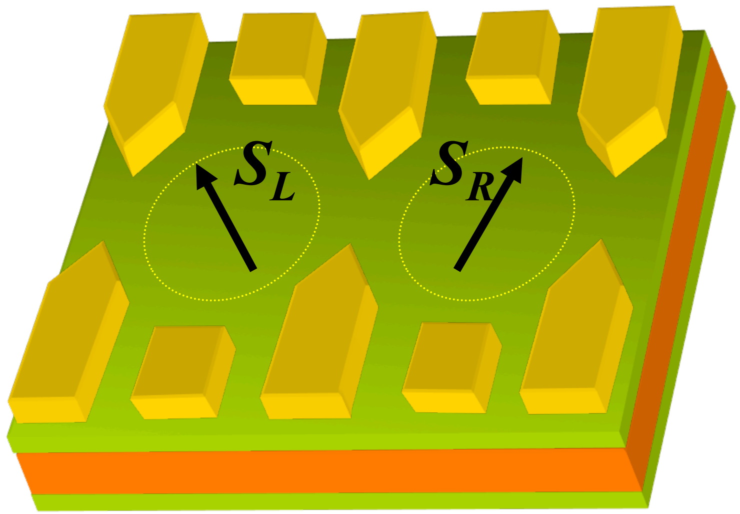


Marcus group (Harvard, 2004)

Temp.: 100 mK

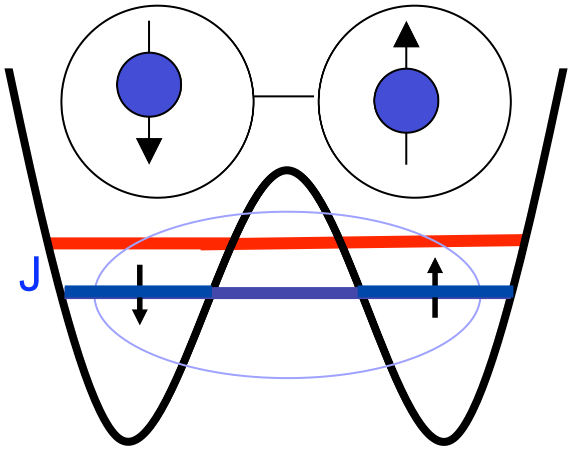
Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



2 quantum dots, each with
1 electron-spin (= qubit)

Key idea:
all-electrical control of spins
➔ spin-qubits are scalable

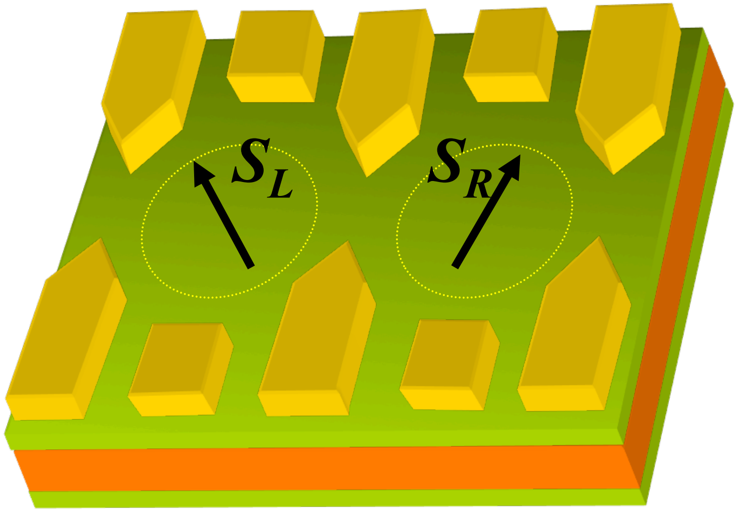


artificial hydrogen molecule ➔ exchange splitting $J \sim t^2/U$

➔ ‘CNOT quantum gate‘

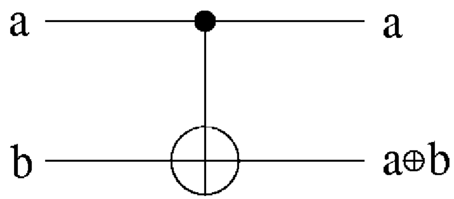
Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



$$H(t) = J(t)\, \boldsymbol{S}_L \cdot \boldsymbol{S}_R$$

➔ CNOT (XOR) gate

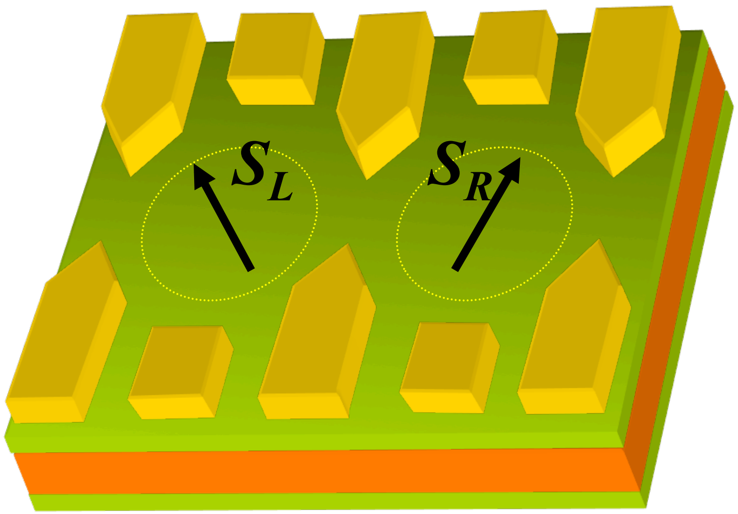


$$\Leftrightarrow \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \sigma_x \end{pmatrix} \begin{array}{c} \text{a b} \\ \left\{ \begin{array}{c} \uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\uparrow \\ \downarrow\downarrow \end{array} \right\} \end{array} = \left\{ \begin{array}{c} \uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\downarrow \\ \downarrow\uparrow \end{array} \right\}$$

$$U(\tau_s) = T e^{-i\int_0^{\tau_s} H'(t)dt}, \quad J \neq 0 \ \ during \ \ \tau_s$$

Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



$$H(t) = J(t)\, \mathbf{S}_L \cdot \mathbf{S}_R$$

➔ CNOT (XOR) gate

$$U_{XOR} = e^{i\frac{\pi}{2}S_1^z}\, e^{-i\frac{\pi}{2}S_2^z}\, U_{SW}^{1/2}\, e^{i\pi S_1^z}\, U_{SW}^{1/2}$$

$$U_{SW} : \uparrow\downarrow \;\Rightarrow\; \downarrow\uparrow$$

sqrt-of-swap: $U_{SW}^{1/2} : \uparrow\downarrow \;\Rightarrow\; \uparrow\downarrow + e^{i\alpha}\downarrow\uparrow$

switching time: 180 ps

Petta *et al.*, Science, 2005

Quantum XOR gate with 'sqrt-of-swap'

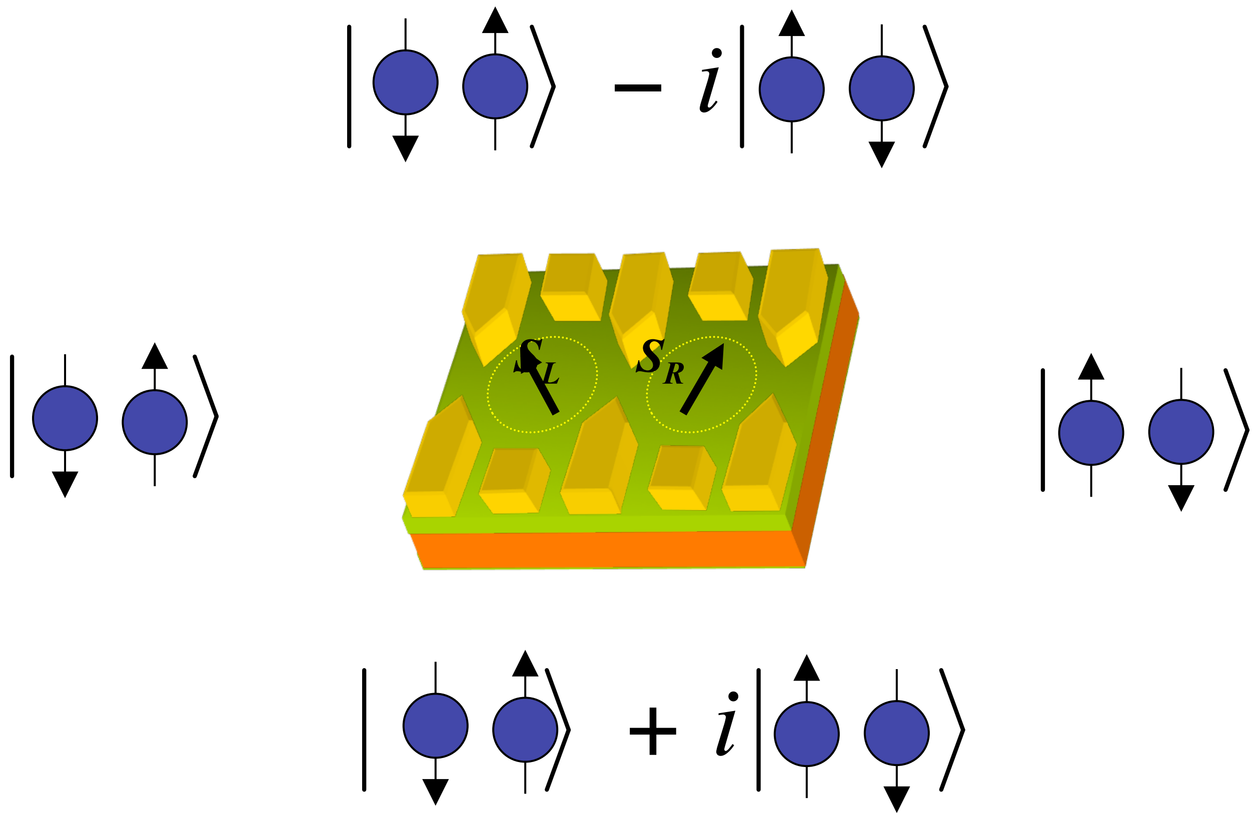
$$U_{XOR} = e^{i(\pi/2)S_1^z} e^{-i(\pi/2)S_2^z} U_{SW}^{1/2} e^{+i\pi S_1^z} U_{SW}^{1/2},$$

Loss & DiVincenzo, PRA 1998

	$\vert\uparrow\uparrow\rangle$	$\vert\uparrow\downarrow\rangle$	$\vert\downarrow\uparrow\rangle$	$\vert\downarrow\downarrow\rangle$
$U_{SW}^{1/2}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$\vert\uparrow\uparrow\rangle$	$\frac{e^{-i\pi/4}}{\sqrt{2}}(\vert\uparrow\downarrow\rangle + i\,\vert\downarrow\uparrow\rangle)$	$\frac{e^{-i\pi/4}}{\sqrt{2}}(i\,\vert\uparrow\downarrow\rangle + \vert\downarrow\uparrow\rangle)$	$\vert\downarrow\downarrow\rangle$
$e^{i\pi S_1^z}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$i\,\vert\uparrow\uparrow\rangle$	$\frac{ie^{-i\pi/4}}{\sqrt{2}}(\vert\uparrow\downarrow\rangle - i\,\vert\downarrow\uparrow\rangle)$	$\frac{ie^{-i\pi/4}}{\sqrt{2}}(i\,\vert\uparrow\downarrow\rangle - \vert\downarrow\uparrow\rangle)$	$-i\,\vert\downarrow\downarrow\rangle$
$U_{SW}^{1/2}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$i\,\vert\uparrow\uparrow\rangle$	$\vert\uparrow\downarrow\rangle$	$-\,\vert\downarrow\uparrow\rangle$	$-i\,\vert\downarrow\downarrow\rangle$
$e^{-i\frac{\pi}{2}S_2^z} \times e^{i\frac{\pi}{2}S_1^z}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$i\,\vert\uparrow\uparrow\rangle$	$i\,\vert\uparrow\downarrow\rangle$	$i\,\vert\downarrow\uparrow\rangle$	$-i\,\vert\downarrow\downarrow\rangle$

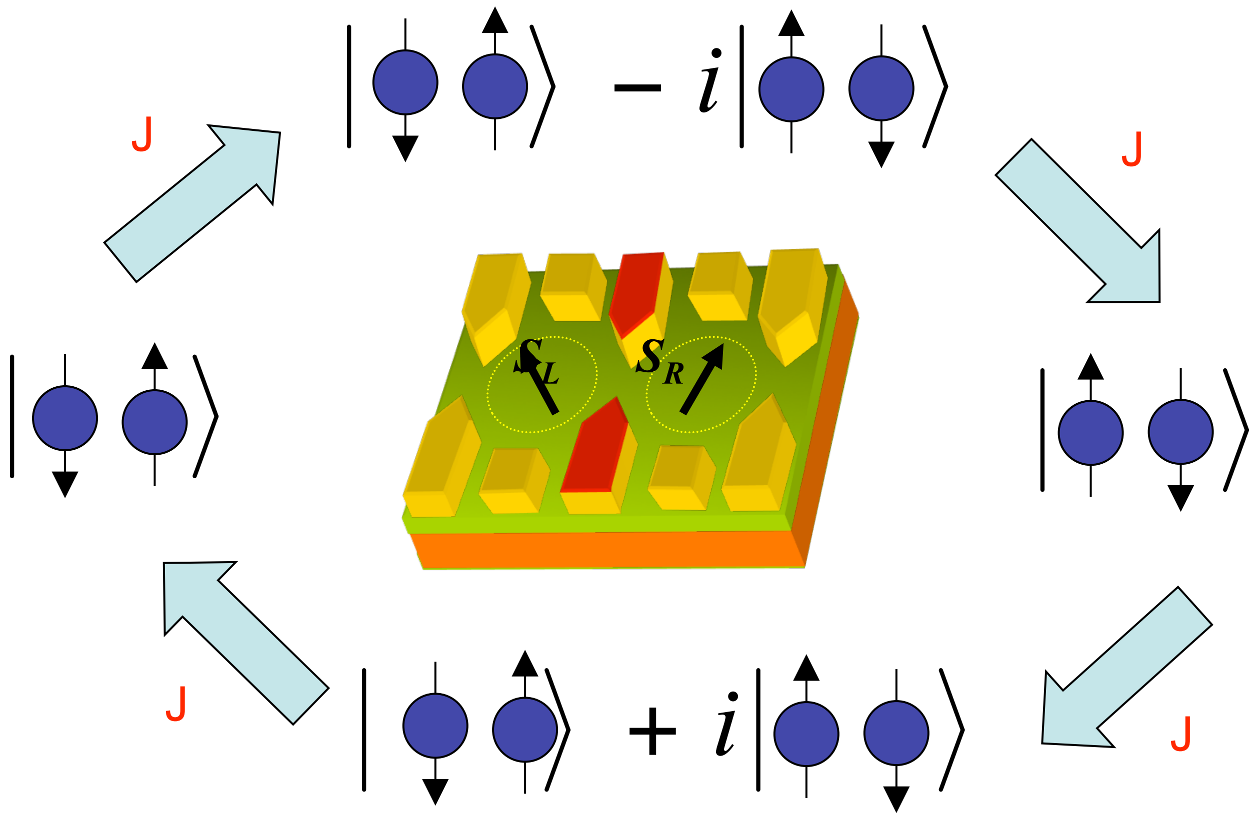
How to make entanglement ‘visible’

Basel theory ’98-’06



How to make entanglement 'visible'

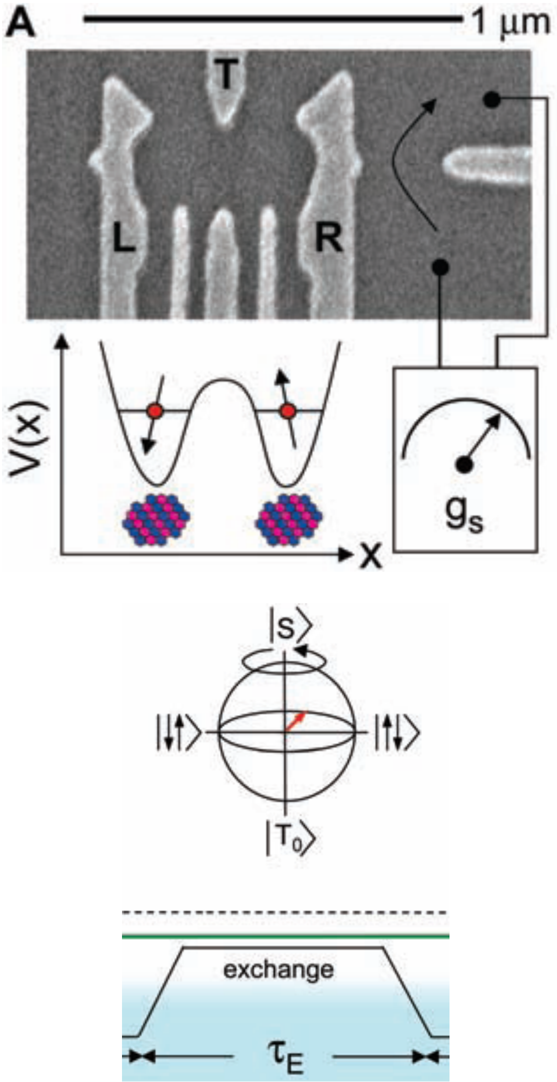
Basel theory '98-'06

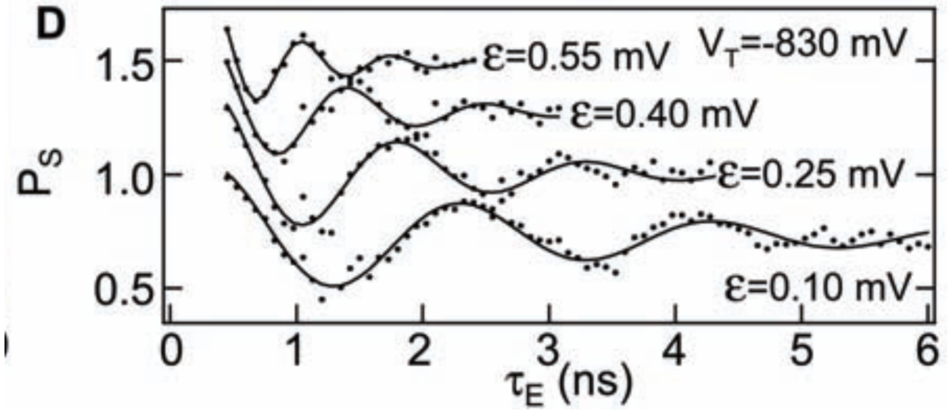


➔ entanglement oscillates !

Entanglement oscillations observed

Petta *et al.*, Science 2005





ultra-fast 'clock speed' to
create entanglement: 180 ps !

Quantum XOR gate with 'sqrt-of-swap'

$$U_{XOR} = e^{i(\pi/2)S_1^z} e^{-i(\pi/2)S_2^z} U_{SW}^{1/2} e^{+i\pi S_1^z} U_{SW}^{1/2},$$

Square-root-of-swap: $|\uparrow\downarrow\rangle \xrightarrow{U_{SW}^{1/2}} |\uparrow\downarrow\rangle + i\,|\downarrow\uparrow\rangle$

$= |\uparrow\rangle_1 \times |\downarrow\rangle_2$

= entangler: product state ➔ entangled state

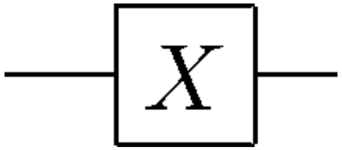
➔ Entanglement is crucial for quantum computing!

Universal set of quantum gates

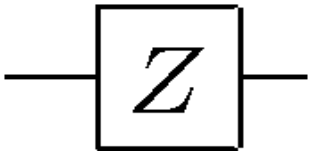
Single-qubit operations and a two-qubit gate that generates entanglement are sufficient for **universal quantum computation**:

Single-qubit gates

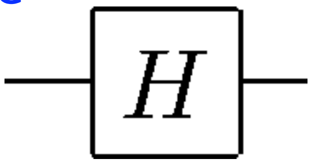
qnot-gate



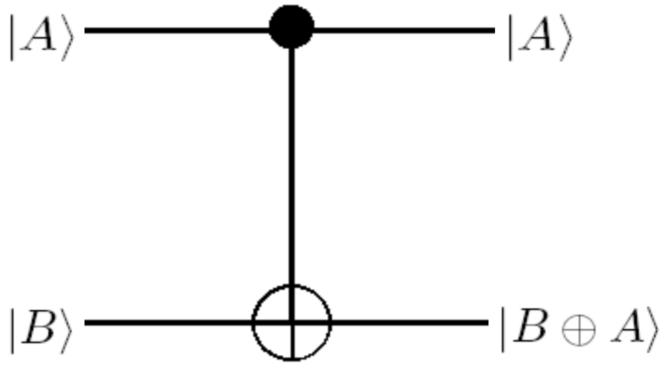
Z-gate



Hadamard-gate



CNOT gate



$\Leftrightarrow$ **entanglement**

A. Barenco et al., Phys. Rev. A 52, 3457 (1995)

Note: Control of exchange interaction J and switching time needs to be very precise ($1:10^4$)

➔ experimental challenge

➔ CNOT gate without interaction?

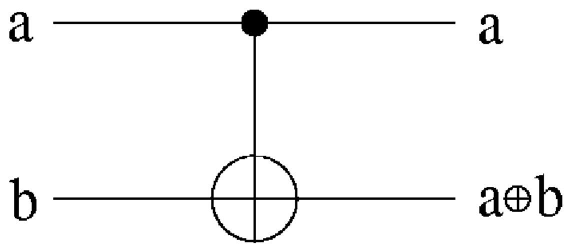
Yes: CNOT gates based on measurement:

Linear optics & single-photon detection	→	conditional sign flip (non-deterministic) [1]
Full Bell state analyzer & GHZ state	→	deterministic quantum computing [2]
Partial Bell-state (parity) measurements	→	deterministic quantum computing [3]

[1] E. Knill, R. Laflamme and G. J. Milburn, Nature 409, **46** (2001).
[2] D. Gottesman and I.L. Chuang, Nature **402**, 390 (1999).
[3] C.W.J. Beenakker *et al.*, Phys. Rev. Lett. **93**, 020501 (2004).

CNOT gate can be implemented with two parity gates

Beenakker, Kindermann & DiVincenzo, PRL ‘04



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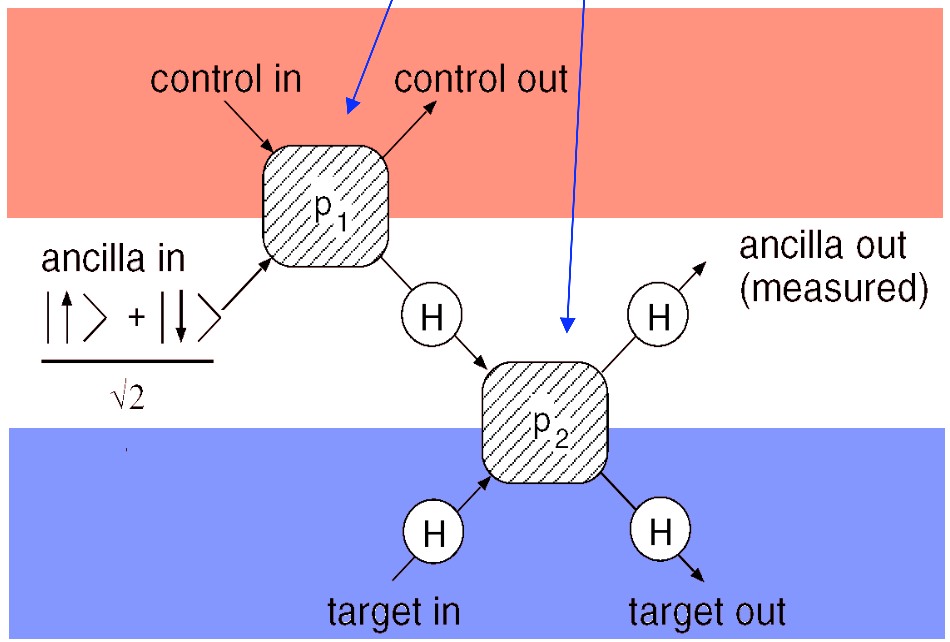
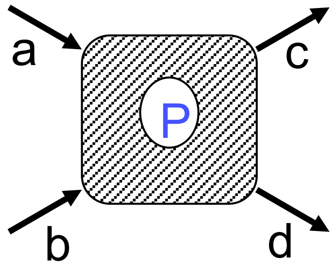


Deterministic entangler:



a,b: input arms
c,d: output arms

$$\underbrace{\left(\alpha\uparrow_a+\beta\downarrow_a\right)}_{\text{input state in arm a}}\otimes\underbrace{\left(\uparrow_b+\downarrow_b\right)}_{\text{input state in arm b (ancilla)}}=\left(\alpha\uparrow_a\uparrow_b+\beta\downarrow_a\downarrow_b\right)+\left(\alpha\uparrow_a\downarrow_b+\beta\downarrow_a\uparrow_b\right)$$

$$\uparrow_a\downarrow_b\equiv\left|\uparrow\right\rangle_a\otimes\left|\downarrow\right\rangle_b$$

$$\Rightarrow\begin{cases}\alpha\uparrow_c\uparrow_d+\beta\downarrow_c\downarrow_d\ ,\ \mathit{if}\ \ p=1\\ \alpha\uparrow_c\downarrow_d+\beta\downarrow_c\uparrow_d\ ,\ \mathit{if}\ \ p=0,\ \Rightarrow\ \alpha\uparrow_c\uparrow_d+\beta\downarrow_c\downarrow_d\end{cases}$$

Projective measurement: measurement of parity p projects input state into either parallel output state (p=1) or antiparallel output state (p=0). If p=0, then apply $\sigma_x^{(d)}$ on output state ➔ get always same final output state in arms c and d.

Thus, we get: $$\boxed{\alpha\uparrow_a+\beta\downarrow_a\ \Rightarrow\ \alpha\uparrow_c\uparrow_d+\beta\downarrow_c\downarrow_d}$$

Beenakker et al., 2004

Measurement-based quantum computing with spin qubits

Engel & DL, Science **309**, 586 (2005)

$$\left.\begin{array}{l} |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \\ |\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle \end{array}\right\}$$ even parity Bell state: parallel spins

$$\left.\begin{array}{l} |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \\ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \end{array}\right\}$$ odd parity Bell state: antiparallel spins

Advantage:
parity measurement is digital (0 or 1) ➔ quantum gate is digital

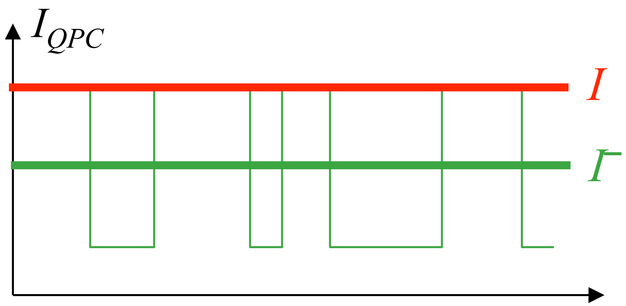
Q: Does scheme exist for electron spins to measure parity of Bell states non-destructively?

Double Quantum Dot and QPC

- Current I_{QPC} depends on charge state[1]

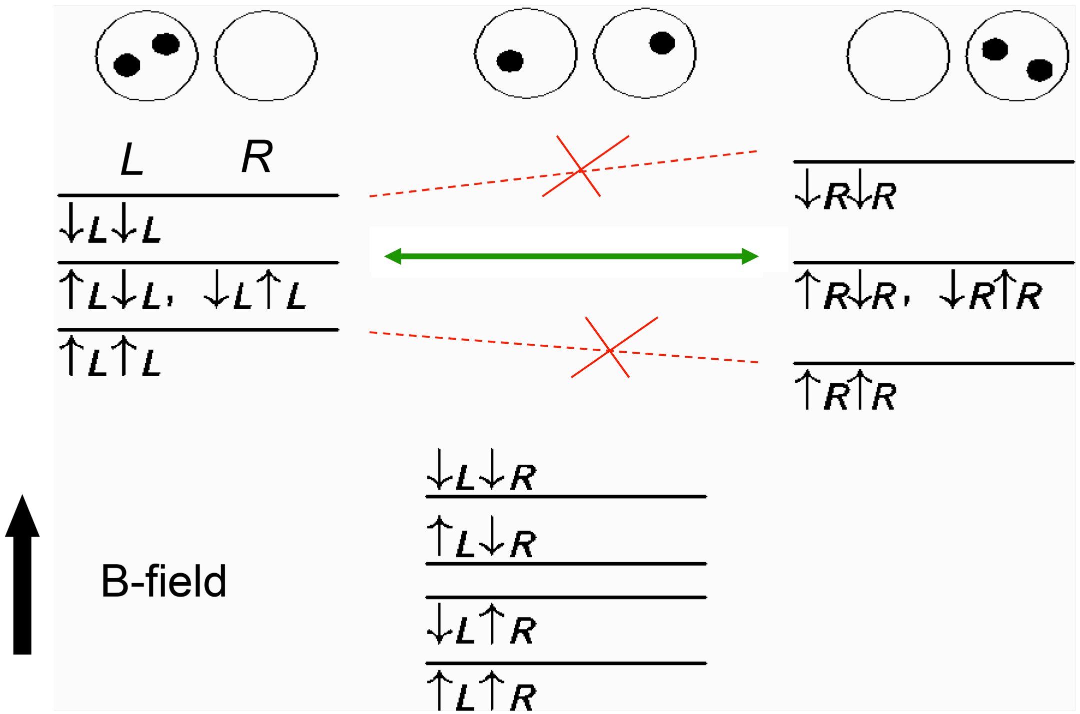
odd parity: tunneling

even parity: no tunneling



[1] J.M. Elzerman *et al.*, Nature **430**, 431 (2004)

Convert spin parity to charge info



Different Zeeman splittings

$\Delta^z{}_L \neq \Delta^z{}_R$

12 dim. Hilbert space
Bloch-Redfield eq.

$$\boxed{W_{L\leftrightarrow R} = \frac{2t_\mathrm{d}^4}{U^2}\frac{\Gamma_\mathrm{d2}}{\varepsilon^2+\Gamma_\mathrm{d2}^2}}$$

- resonant tunneling ($\varepsilon=0$) for antiparallel spins
- but NOT ($\varepsilon >> \Gamma_{d2}$) for parallel spins

➔

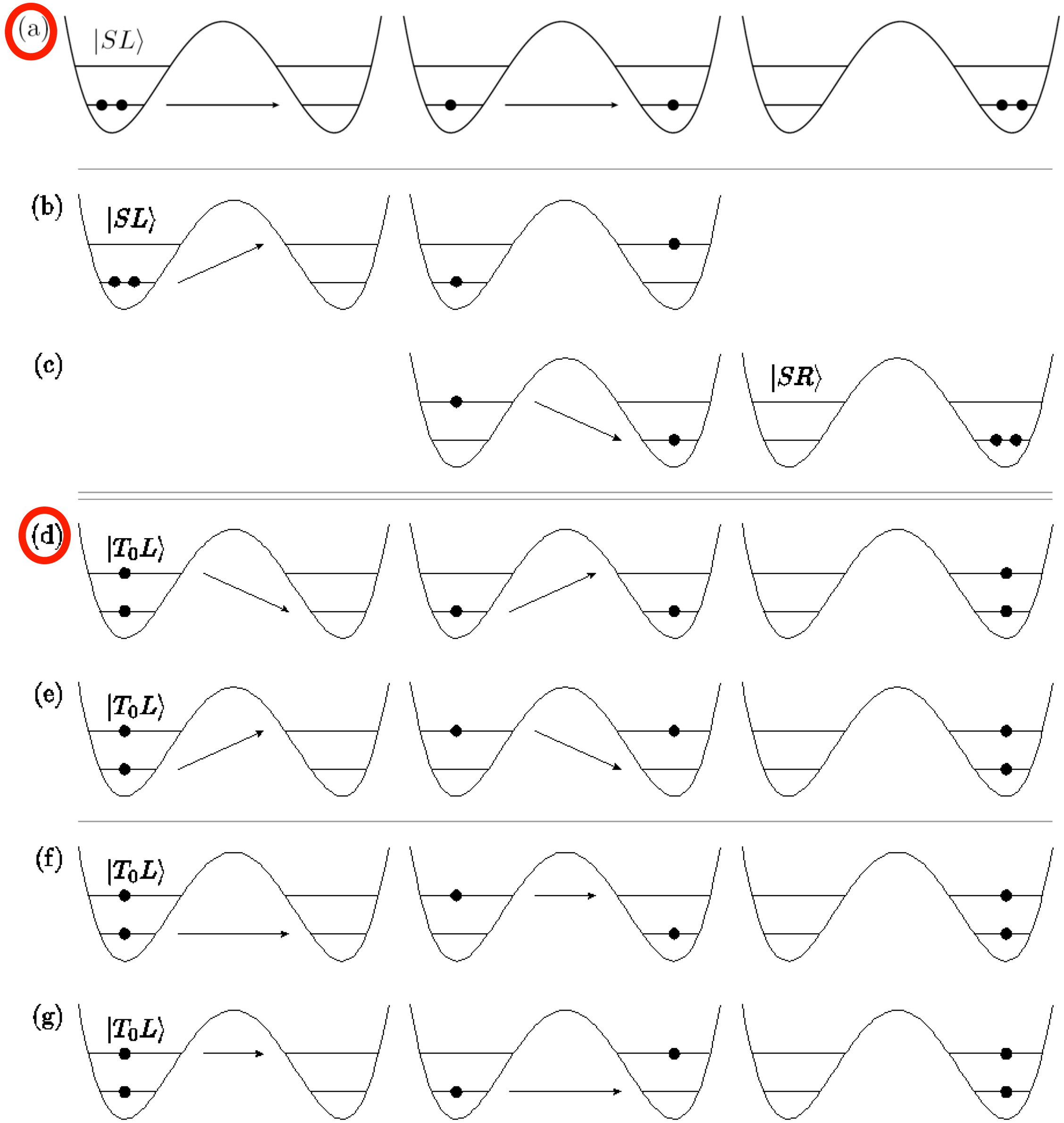
QPC detects charge on right dot ➔ parity of Bell state

1 List of symbols

J	exchange splitting for two electrons on the same dot ($\|LL\rangle$ or $\|RR\rangle$)
J_{LR}	exchange splitting for states $\|LR\rangle$ (taken into account implicitly)
t_S, t_T	tunneling amplitudes for singlet/triplet states
$t_{\rm d}$	tunneling amplitude for simplified model
U	charging energy: $\|LL\rangle$ vs $\|LR\rangle$; $U = E_{SLL} - E_{\uparrow_L\uparrow_R}$
Δ_z^L, Δ_z^R	Zeeman splitting in left/right dot
$\Gamma_{\rm d}$	intrinsic dephasing rate
$\Gamma_{\rm d1}$	intrinsic dephasing rate for a superposition of $\|LR\rangle$ and $\|LL\rangle$ (or for superposition of $\|LR\rangle$ and $\|RR\rangle$)
$\Gamma_{\rm d2}$	intrinsic dephasing rate for a superposition of $\|LL\rangle$ and $\|RR\rangle$
$\Gamma_{\rm r}$	intrinsic relaxation $\|LL\rangle \to \|LR\rangle$ or $\|RR\rangle \to \|LR\rangle$
ε	energy detuning $\|LL\rangle$ vs. $\|RR\rangle$, i.e., $\varepsilon = E_{LL} - E_{RR}$
$W_{L\leftrightarrow R}$	effective tunneling rate from left to right, $\|LL\rangle \leftrightarrow \|RR\rangle$, as function of ε
$W_{\rm relax}$	effective relaxation rate, $\|LL\rangle \to \|LR\rangle$ or $\|RR\rangle \to \|LR\rangle$
$\Gamma_{\rm meas}$	measurement rate, inverse time that is required for partial Bell state measurement
Γ_φ	dephasing rate, rate at which superpositions of states with different parity are decohered
$\|LL\rangle$	charge state with two electrons on the left dot, $\|RR\rangle$ for two electrons on the right dot, $\|LR\rangle$ for one electron on each dot
$\|SLL\rangle$	singlet with both electrons on left dot, $\|SLL\rangle = (\|\uparrow_L\downarrow_L\rangle - \|\downarrow_L\uparrow_L\rangle)/\sqrt{2}$, analogously for $\|SLR\rangle$, and $\|SRR\rangle$, and with "$-$" $\to$ "$+$" for $\|T_0LL\rangle$, $\|T_0LR\rangle$, and $\|T_0RR\rangle$
$H_{\rm d}$	Hamiltonian of double dot in the absence of inter-dot tunneling [Eq. (5)]
H_T	inter-dot tunnel coupling [Eq. (6)]
δ	orbital level spacing
$\rho_n, \rho_{n,m}$	reduced density matrix ρ of double dot, diagonal elements $\rho_n = \langle n\|\rho\|n\rangle$ and off-diagonal elements $\rho_{n,m} = \langle n\|\rho\|m\rangle$
$\hat{n}_{L/R}$	number of electrons on left/right dot
$E^C_{L/R}$	charging energy of left/right dot
n_α	charge induced in single (uncoupled) dot via gate voltages
λ_i	eigenenergies of $H_{\rm d} + H_T$
V	coupling of double dot to QPC, $V = V_{\rm dot}V_{\rm QPC}$
$t_n^{\rm Q}$	tunneling amplitude in QPC for double-dot charge state $\|n\rangle$
$c^\dagger$	creation operator of electrons in incoming/outgoing lead of QPC
$\rho^0_{\rm Q}$	equilibrium density matrix of leads of QPC
$\Delta\mu$	applied bias across QPC
I_n	current in QPC when double dot is in charge state $\|n\rangle$
$\Gamma_{\varphi i}$	dephasing rates $\Gamma_{\varphi 2}$, $\Gamma_{\varphi 1L}$, $\Gamma_{\varphi 1R}$ due to quantum measurement via QPC [Eqs. (47)-(49)]

Tunneling transitions (coherent)

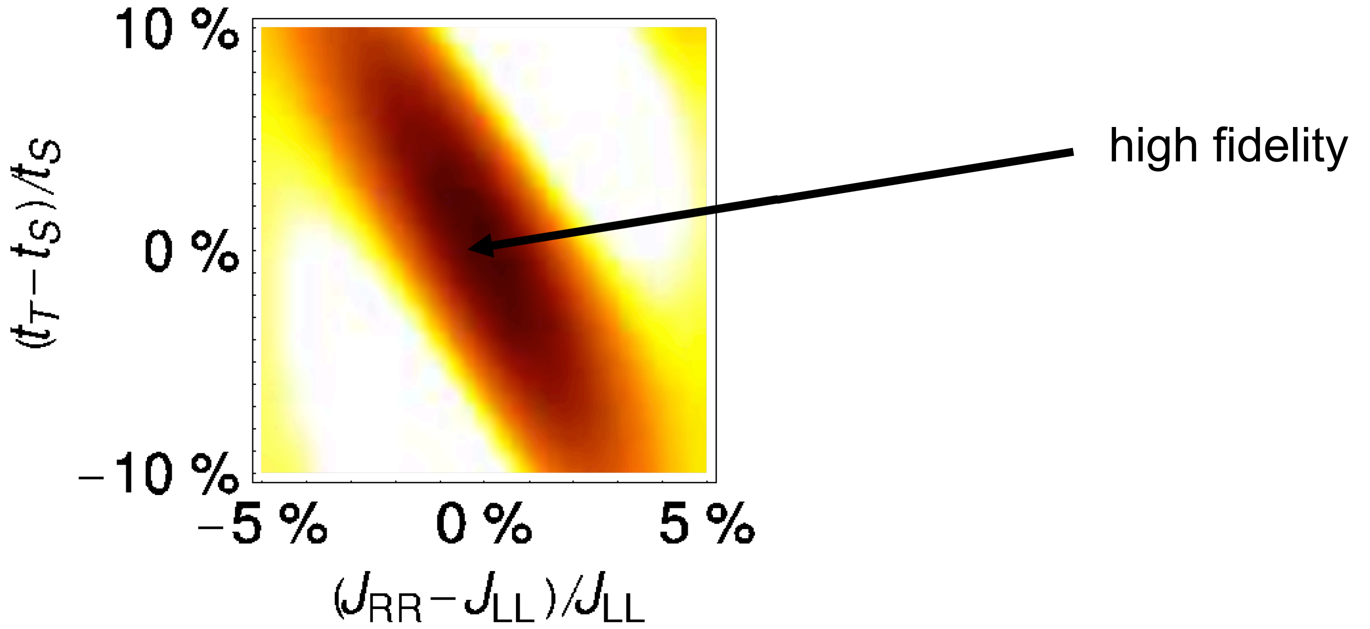
- Single level picture
- Many processes for small level spacing δ
- But for $\delta >> U$ only transitions (a) and (d) are relevant



Imperfections

- Phases due to different Zeeman interaction
  - during virtual occupation of state *LR*
  - $|\uparrow_L\downarrow_R\rangle$ *vs* $|\downarrow_L\uparrow_R\rangle$
  - correctable via one-qubit gates
  - suppression via large t_d and fast read-out
- Detuning from resonance
  - increases measurement time
- Finite exchange J for *LL* and *RR*
  - additional dynamical phase
  - correctable via one-qubit operations
- Tunneling $t_S \neq t_T$ and/or $J_L \neq J_R$
  - $|S\rangle$ and $|T_0\rangle$ are distinguishable $\rightarrow$ decoherence!

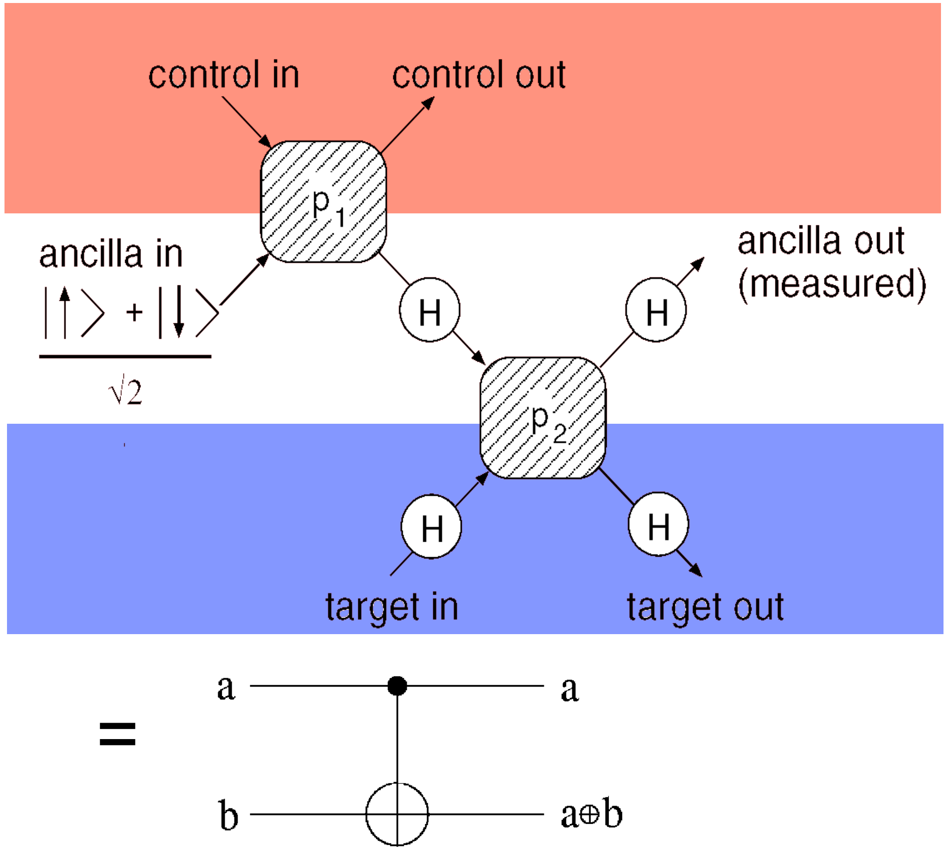
Parity detection is robust against imperfections:



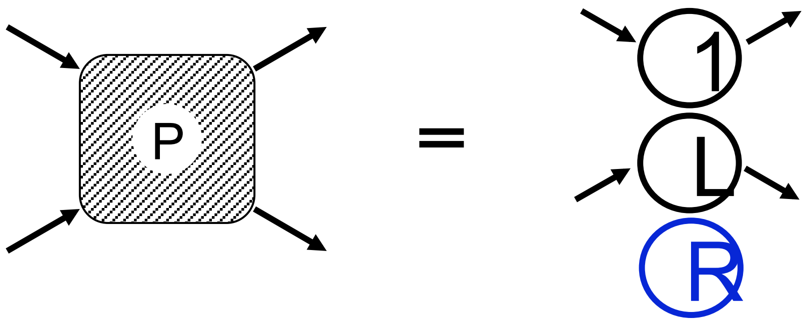
- initial state e.g. $\frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle + \frac{1}{2}\left(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\right)$
- simulation in 144-dimensional Liouville space
- compare ideal result
- quantify with the Uhlmann (square-root) fidelity $F = Tr\sqrt{\rho_f^{1/2}\rho_e\rho_f^{1/2}}$

Universal Quantum Computing with Parity Gates

CNOT gate can be implemented with two parity gates (Beenakker et al., PRL ‘04):



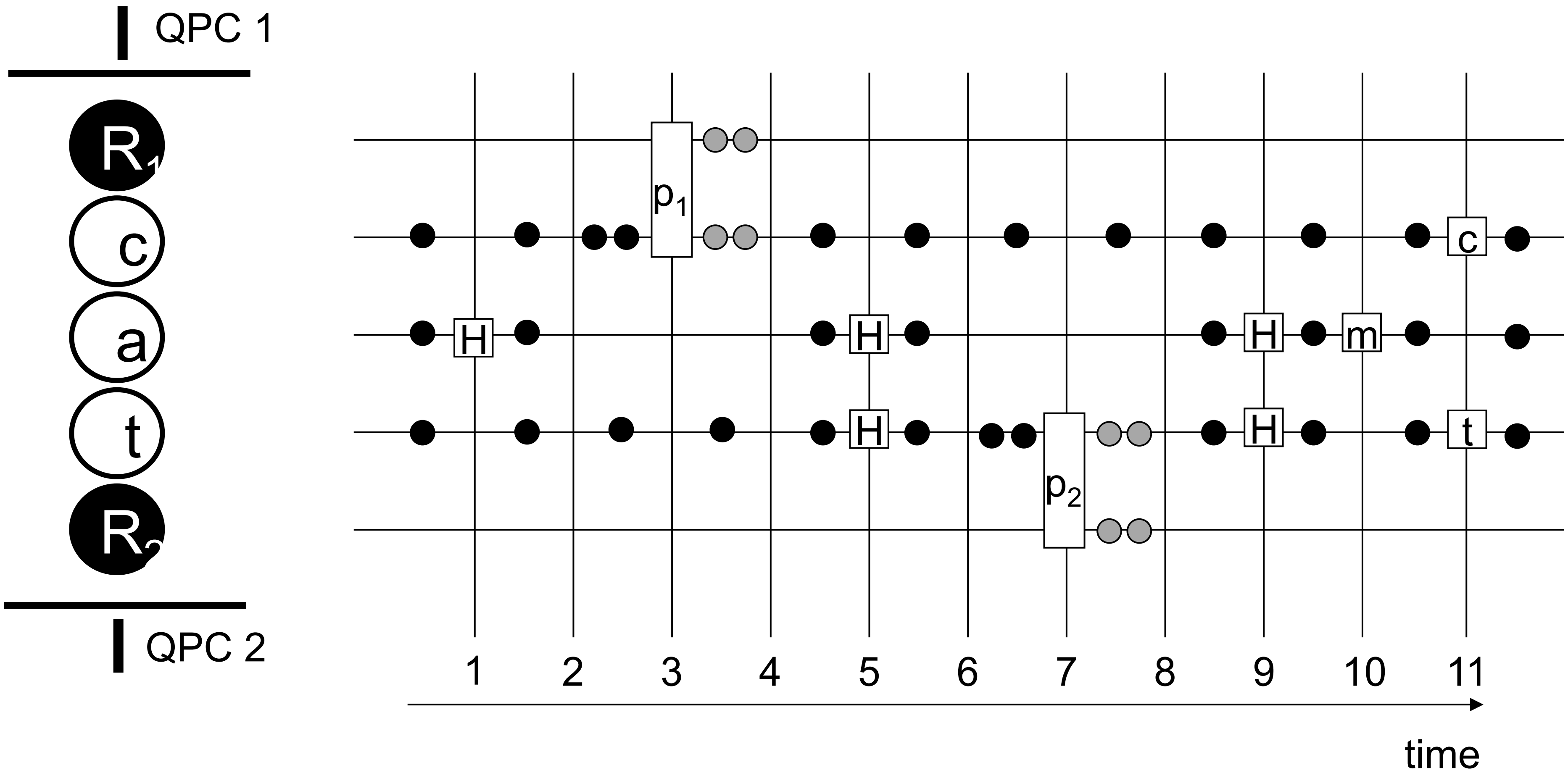
Parity gate for spin ½:
(Engel & Loss, Science ‘05):



Two qubit dots (1 and L), parity measurement using reference dot (R)
(1, 1, 0) → (0, 2, 0) ↔ (0, 0, 2)
Transfer back adiabatically to (1,1,0)

Protocol for CNOT gate (1)

$$U_{CNOT}|c\rangle_1|t\rangle_2 = |c\rangle_1|c \oplus t\rangle_2$$



Engel and DL, Science ‘05

Protocol for CNOT gate (1) $U_{CNOT}|c\rangle_1|t\rangle_2 = |c\rangle_1|c \oplus t\rangle_2$

11 Steps for implementing CNOT gate on qubits "c" (control) and "t" (target):

1. Prepare "a" in state $(|0\rangle+|1\rangle) / \sqrt{2}$
2. Move electron from QD "a" to QD "c"
3. Perform parity measurement p_1 using QDs "c" and "R_1"
4. Move electrons back to QDs "c" and "a"
5. Perform a Hadamard single-qubit gate on "a" and on "t"
6. Move electron from QD "a" to QD "t"
7. Perform parity measurement p_2 using QDs "t" and "R_2"
8. Move electrons back to QD "t" and "a"
9. Perform a Hadamard single-qubits gate on "a" and on "t"
10. Measure qubit "a"
11. Apply conditional operations, according to table

Protocol for CNOT gate (3): Conditional operations

- p_1, p_2, m represent the outcome of the measurements of steps 3, 7, and 10.
- Detection of even parity (superposition of $|00\rangle$ and $|11\rangle$) is labeled as “0”; odd parity as “1”

- In step 11, single-qubit gates listed on the rhs are applied to qubits “c” and “t”
- “I” stands for identity (do nothing), X for σ_X, Z for σ_Z

$\mathbf{p_1}$	$\mathbf{p_2}$	**m**	**“c”**	**“t”**
0	0	0	I	I
0	0	1	I	X
0	1	0	Z	I
0	1	1	Z	X
1	0	0	I	X
1	0	1	I	I
1	1	0	Z	X
1	1	1	Z	I

Topological Quantum Computing via braiding of non-abelian anyons (Kitaev ’03)

E.g. ‘Ising anyons’ (FQHE 5/2, Majorana fermions,..)

Topological Quantum Computing
via braiding of non-abelian anyons (Kitaev '03)

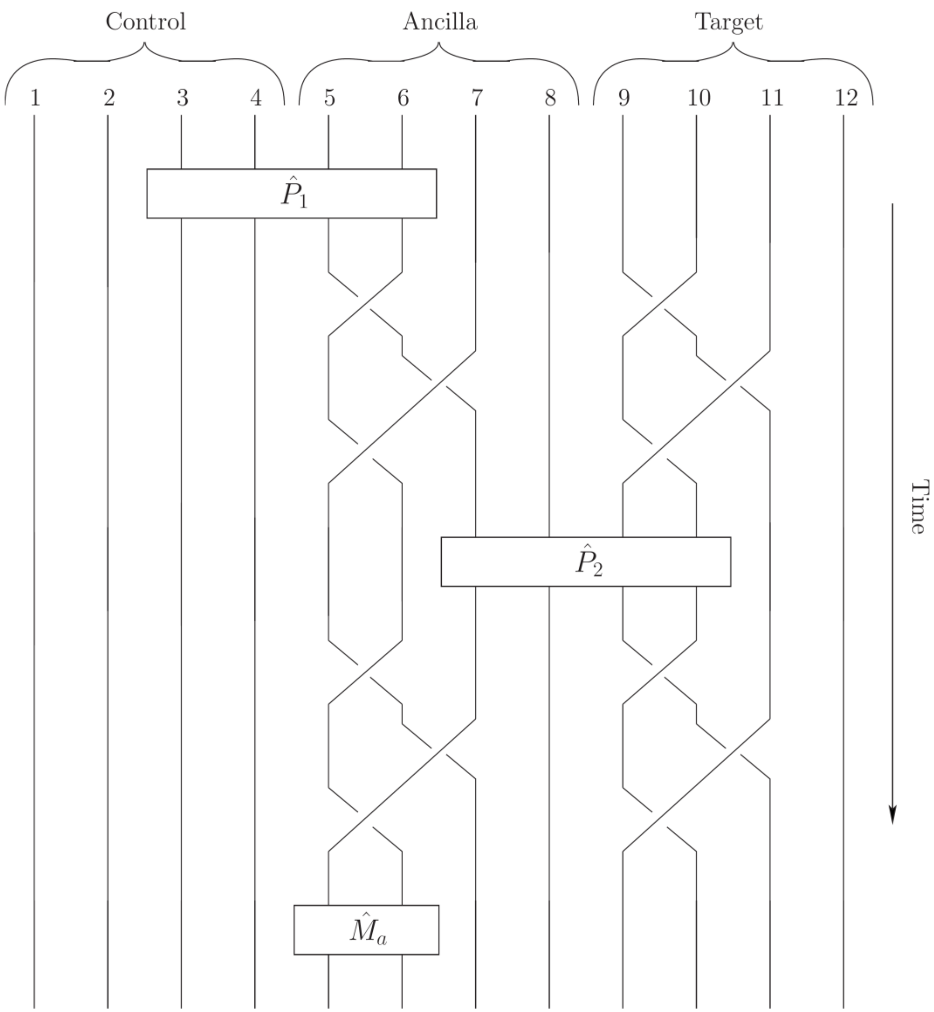
E.g. 'Ising anyons' (FQHE 5/2, Majorana fermions,..)

But: Braiding of 'Ising anyons' <=> classical computing

Solution:
Parity-measurement gates (but they're ***not*** topological…)

Bravyi '06

CNOT-Gate via Parity Measurement & Braiding



e.g. 5/2 FQHE edge states

Bravyi, PRA 73, 042313 (2006)

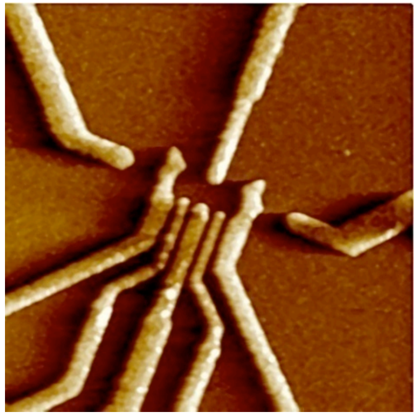
FIG. 6. Measurement-based CNOT gate implemented on $\nu = 5/2$ Ising anyon qubits. The control, ancilla, and target qubits are shown from left to right, e.g., the control qubit is represented by anyons 1,2,3,4. The representative "spin"-parity measurements are shown by the $\hat{P}$ boxes and the ancilla measurement by the box at the bottom. The braiding between the measurements represents Hadamard rotations on the qubits.

Zilberberg, Braunecker, and Loss, PRA 77, 012327 (2008)

(general scheme for composite qubit systems)

Spin qubits in GaAs dots – present status

See also Hanson et al., Rev. Mod. Phys. 2007



All-electrical control and read-out achieved

Initialization 1-electron, low T, high B_0
duration ~ 5 T_1; 99% fidelity ?

Read-out via spin-charge conversion
duration ~ 100 μs; 82-97% fidelity

1-qubit gate electron spin resonance
gate duration ~ 25 ns; observed 8-50 periods

2-qubit gate exchange interaction
gate duration ~ 0.2 ns; observed 3 periods

Energy relaxation
T_1 ~ **1 sec**

Phase coherence
T_2^* ~ **90 ns**
T_2 > **270 μs**

Are Spin-Qubits Scalable ?

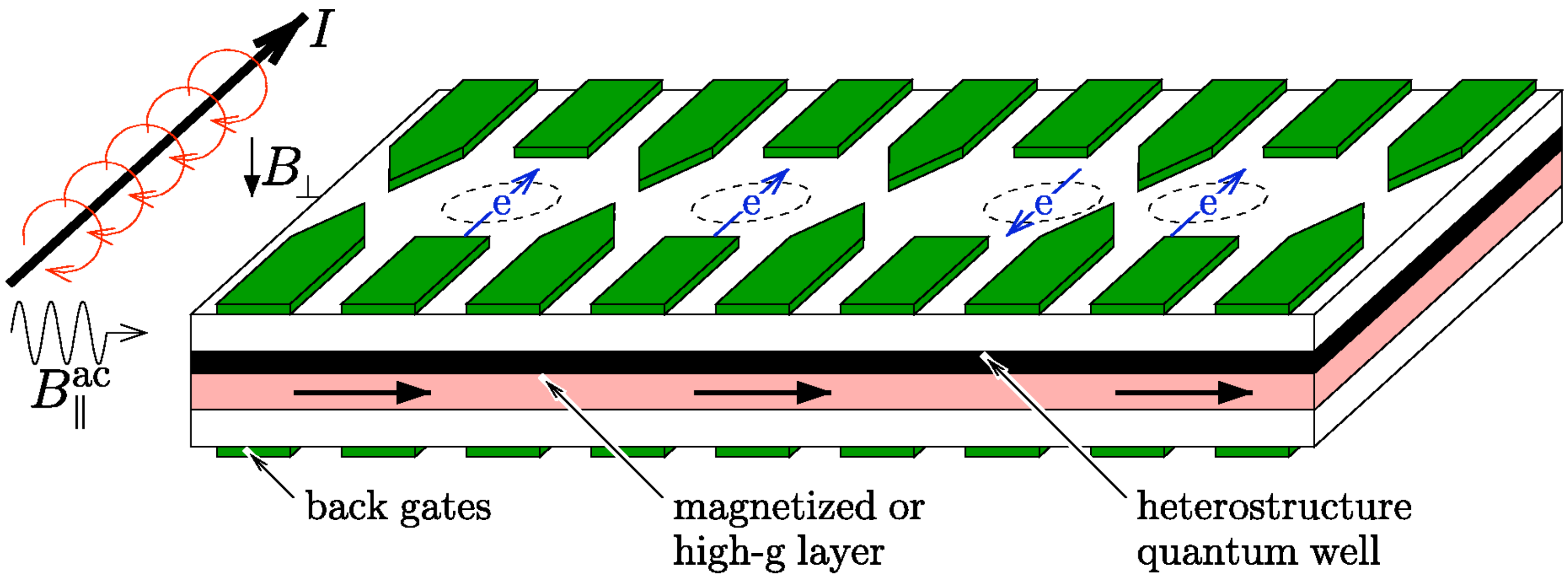
Quantum error correction requires: $T_2/\tau_s \sim 10^4$

GaAs quantum dots: $T_2/\tau_s \sim 10^3 - 10^6$

➔ spin qubits *are* scalable

Spin qubits are **scalable** ➔ quantum dot array

DL & DiVincenzo, PRA **57** (1998) 120

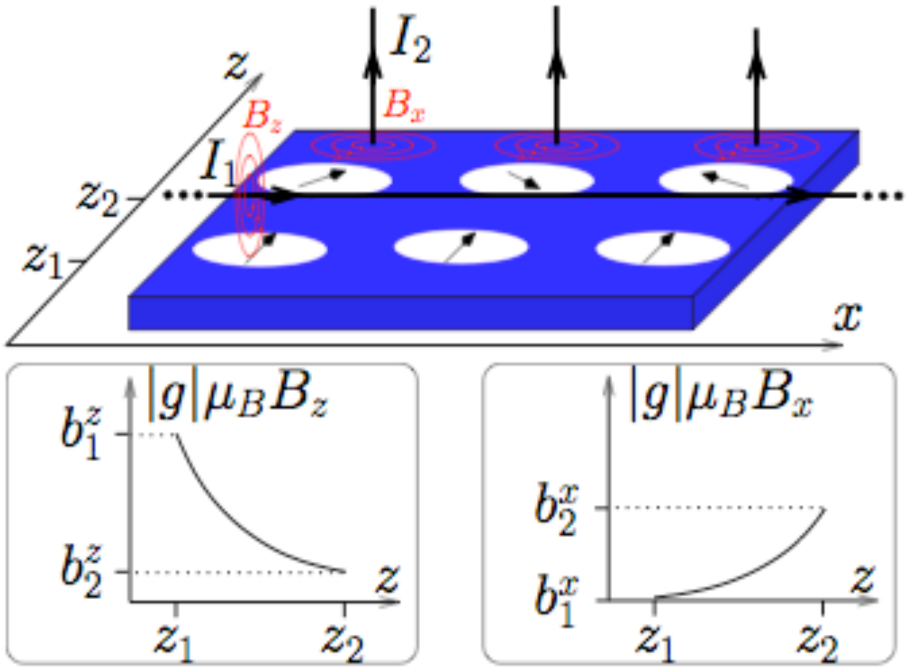


$$H = \sum_{\langle ij \rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i (g_i \mu_B \mathbf{B}_i)(t) \cdot \mathbf{S}_i$$

n.n. exchange local Zeeman

Single-Spin Rotations by Exchange only

Coish & DL, Phys. Rev. B **75**, 161302 (2007)

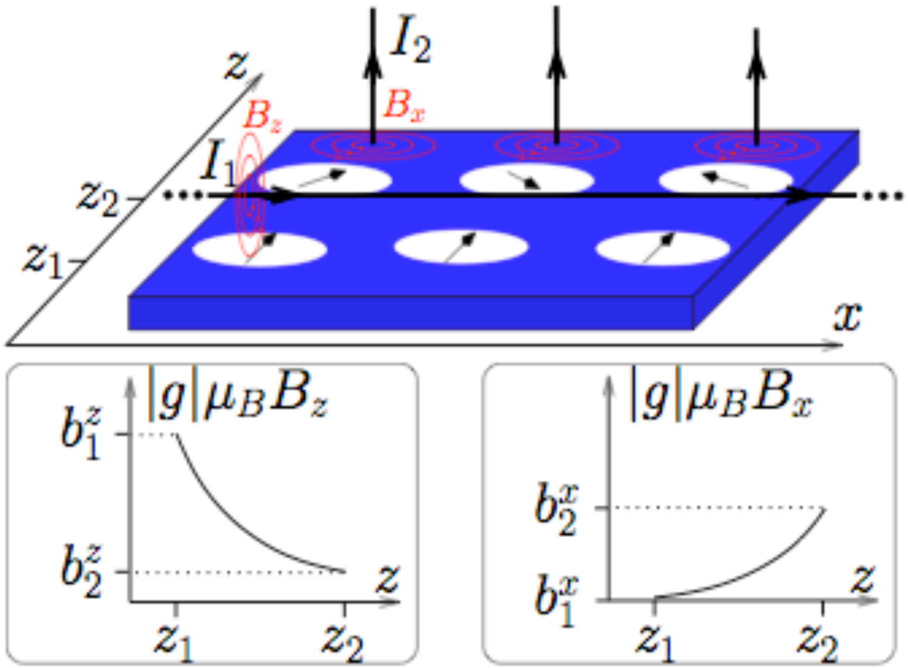


Requires auxiliary spins,
Zeeman gradient & exchange
➔ fast switching times (1ns)
with high fidelity (< 10^{-3})

$$\mathcal{H} = -\sum_{l\sigma} V_l n_{l\sigma} + U_c \sum_l n_{l\uparrow} n_{l\downarrow} + U'_c \prod_l (n_{l\uparrow} + n_{l\downarrow}) + t_{12} \sum_\sigma \left(d^\dagger_{1\sigma} d_{2\sigma} + d^\dagger_{2\sigma} d_{1\sigma} \right) - \sum_l \mathbf{S}_l \cdot \mathbf{b}_l.$$

Single-Spin Rotations by Exchange only

Coish & DL, Phys. Rev. B **75**, 161302 (2007)



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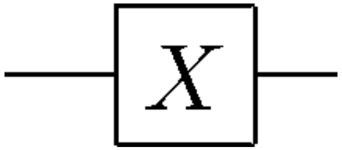
for dot 2: $$\mathcal{H}_{\text{eff}} = -\frac{1}{2} \mathbf{\Delta} \cdot \boldsymbol{\sigma}; \ \mathbf{\Delta} = (b_2^x, 0, b_2^z - J(\epsilon)/2)$$

Universal set of quantum gates

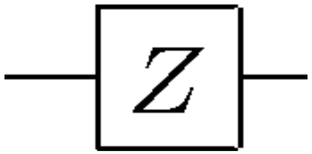
Single-qubit operations and a two-qubit gate that generates entanglement are sufficient for **universal quantum computation**:

Single-qubit gates

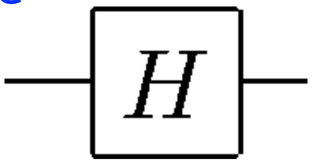
qnot-gate



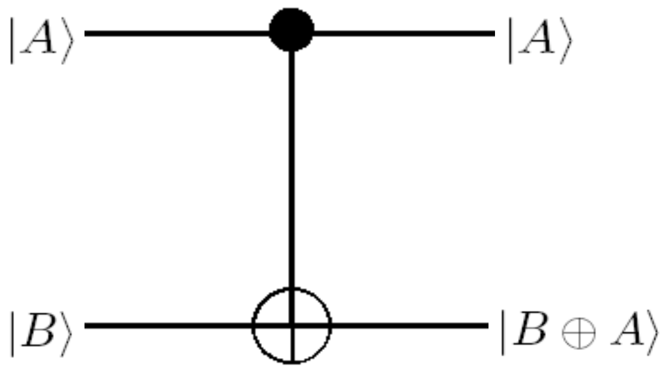
Z-gate



Hadamard-gate



CNOT gate



⇔ **entanglement**

A. Barenco et al., Phys. Rev. A 52, 3457 (1995)

Single-qubit gates via ‘holonomic’ QC*

Golovach, Borhani & DL, PRA 81, 022315 (2010)

- qubit = Kramers doublet in quantum dot (time-reversal invariance)
- spin-orbit interaction provides a non-Abelian gauge field $A_\mu = a_{\mu\nu}\,\sigma^\nu$ which couples to the qubit (even at $B = 0$)
- all single-qubit gates are realized by moving the qubits around close loops (holonomies) in the parameter space of the Hamiltonian

$\Rightarrow$ SU(2) rotation at zero magnetic field

$$U = P \; exp\,\left(\int A_\mu \, d\lambda_\mu\right)$$

Wilson loop

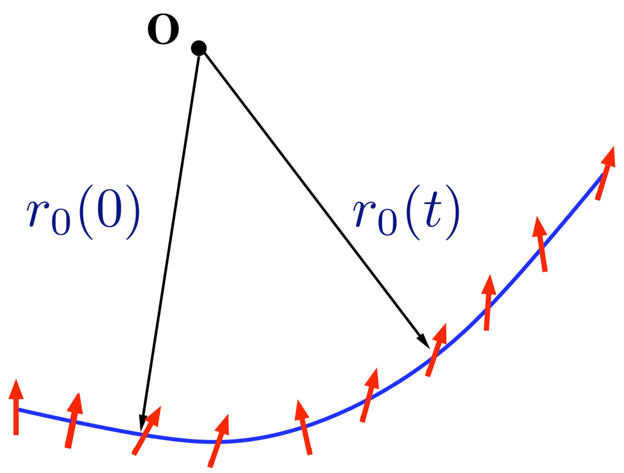
*Holonomic QC: Zanardi & Rasetti, Phys. Lett. A264, 94 (1999)

Spin rotation by moving the quantum dot

$$H(t) = H_d(t) + H_Z + H_{SO}$$

$$H_d(t) = \frac{p^2}{2m_e} + U\left(\vec{r} - \vec{r}_0(t)\right)$$

$$H_{SO} = \alpha(p_x\sigma_y - p_y\sigma_x) + \beta(-p_x\sigma_x + p_y\sigma_y)$$



- The generator of the SU(2) rotation (B = 0):

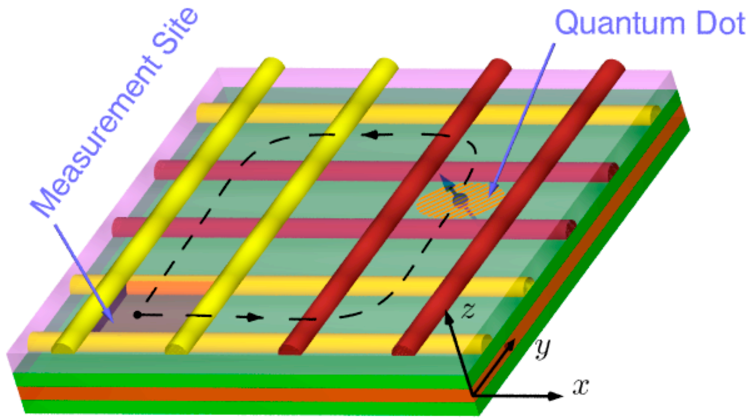
$$\Delta = 1 - i\,\vec{\sigma}\cdot\lambda_{SO}^{-1}\cdot\delta\vec{r}_0$$

$$\lambda_{SO}^{-1} \equiv \begin{pmatrix} 0 & 1/\lambda_- \\ 1/\lambda_+ & 0 \end{pmatrix} \qquad \lambda_\pm = \frac{\hbar}{m_e(\beta \pm \alpha)}$$

Physical implementation

Adiabaticity criterion and decoherence time limit the velocity of the quantum dot:

$$\Gamma\lambda_{SO} \ll |\vec{v}| \ll \lambda_d \omega_0$$



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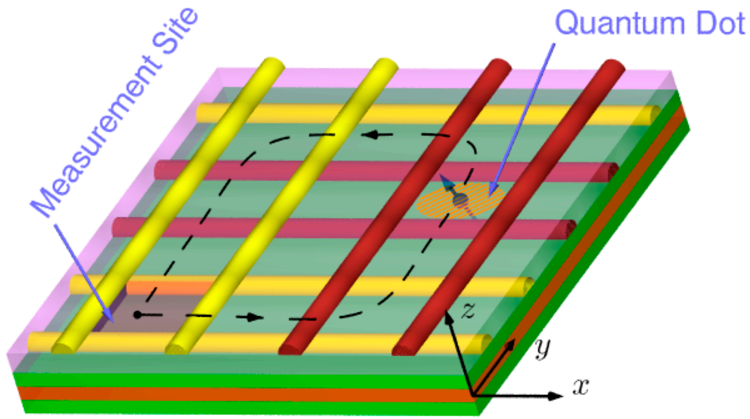
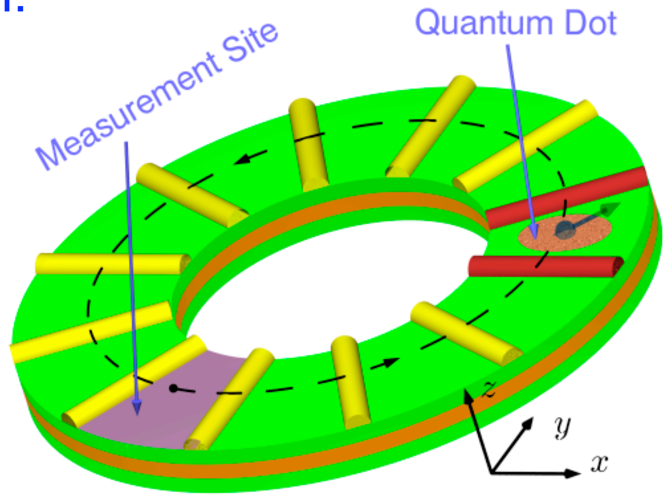


E.g. holonomic rotation matrix for ***circular*** path:

$$\boxed{U(2\pi) = \exp(-\frac{i}{2}\vec{\eta}\cdot\vec{\sigma})}$$

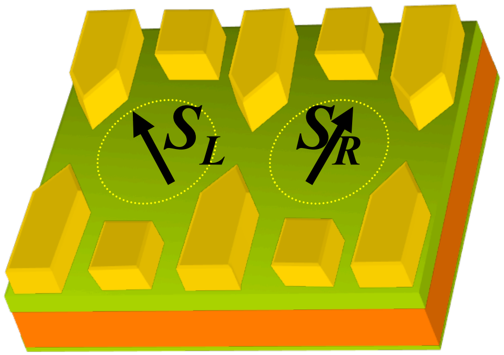
$$\vec{\eta} = 2\pi(1 - \frac{1}{\epsilon})(\frac{2}{\lambda}, 0, 1)$$

$$\epsilon = \sqrt{1 + 4/\lambda^2} \qquad \lambda \equiv \lambda_{\pm}/R$$



Golovach et al., PRA 81 (2010)

Spin-Qubits from Electrons



simplest spin-qubit:
spin-1/2 of 1 electron $|0\rangle = \uparrow, \ |1\rangle = \downarrow$

Many more choices for spin qubits:

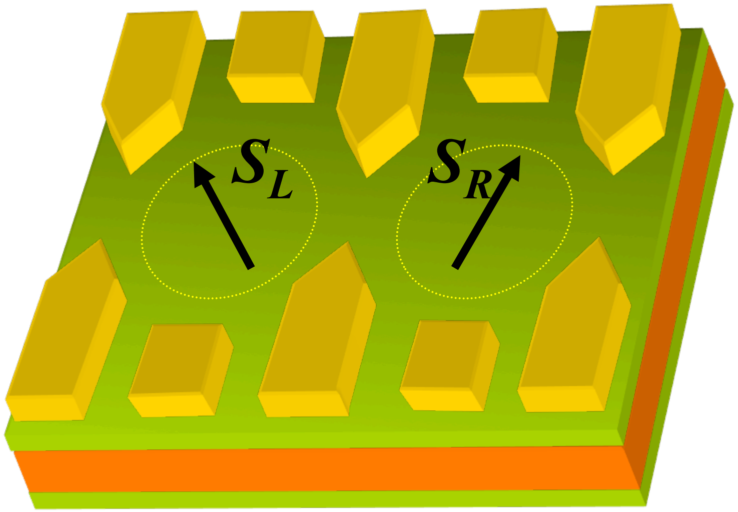
- 'exchange-only qubits' DiVincenzo *et al.*, 2000
  3 electrons: $|0\rangle = S\uparrow, |1\rangle = T_+\downarrow - T_0\uparrow$

- 'singlet-triplet' qubits Levy 2002, Taylor *et al.*, 2005
  2 electrons: $|0\rangle = S, |1\rangle = T_0$

- 'spin-cluster qubits' Meier, Levy & DL, \`03
  N electrons: AF spin chains, ladders, clusters,...

- molecular magnets Leuenberger & DL, ’01; Affronte et al., ’06,
  Lehmann et al., ’07; Trif et al., ’08, ’10

Quantum Computing with Spin-Qubits



CNOT (XOR) gate based on entanglement such as

$$\uparrow\downarrow + e^{i\alpha}\downarrow\uparrow \quad or \quad \uparrow\uparrow + e^{i\alpha}\downarrow\downarrow$$

i.e. **phase coherence** is crucial

➔ Need to understand the dynamics and **decoherence** mechanisms for electron spins in quantum dots

Outline

A. Spin qubits in quantum dots

- Basics of quantum computing and quantum dots
- universal gates & entanglement: via interaction or parity measurements

B. Spin decoherence in GaAs quantum dots

- Spin orbit interaction and spin decay
- Alternative spin qubits: holes, graphene, nanotubes,...
- Nuclear spins and hyperfine induced decoherence

C. Nuclear spin order in 1D (and 2D)

- reduce noise in spin bath by nuclear spin ordering
- Kondo lattice in Luttinger liquids (and marginal Fermi liquid)