Modelling Risk in Hazardous Material Transport

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Motivation

Whilst any company transporting goods is looking to make a profit, it is only ethical that the risk of transporting their goods is considered.

In order to best fulfil this responsibility companies must consider a model that minimises risk but also minimises the time and cost of material transportation.

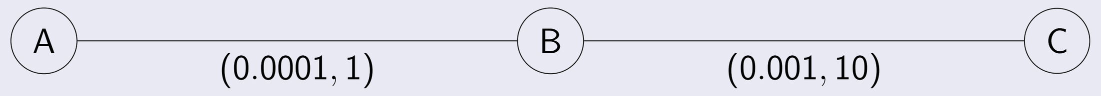
Routes are modelled as networks to facilitate this.

Modelling Risk

The most commonly used model is Traditional Risk [1]:

$$TR = \sum_{i=1}^{n} p_i c_i$$

Here p_i is the probability of an accident on a link i and c_i is the consequence of this accident. An example of a two link network is shown below:



Selecting the Consequence Vectors

Selecting the consequence values requires the impact radius for a hazard and the density of the population within this radius:

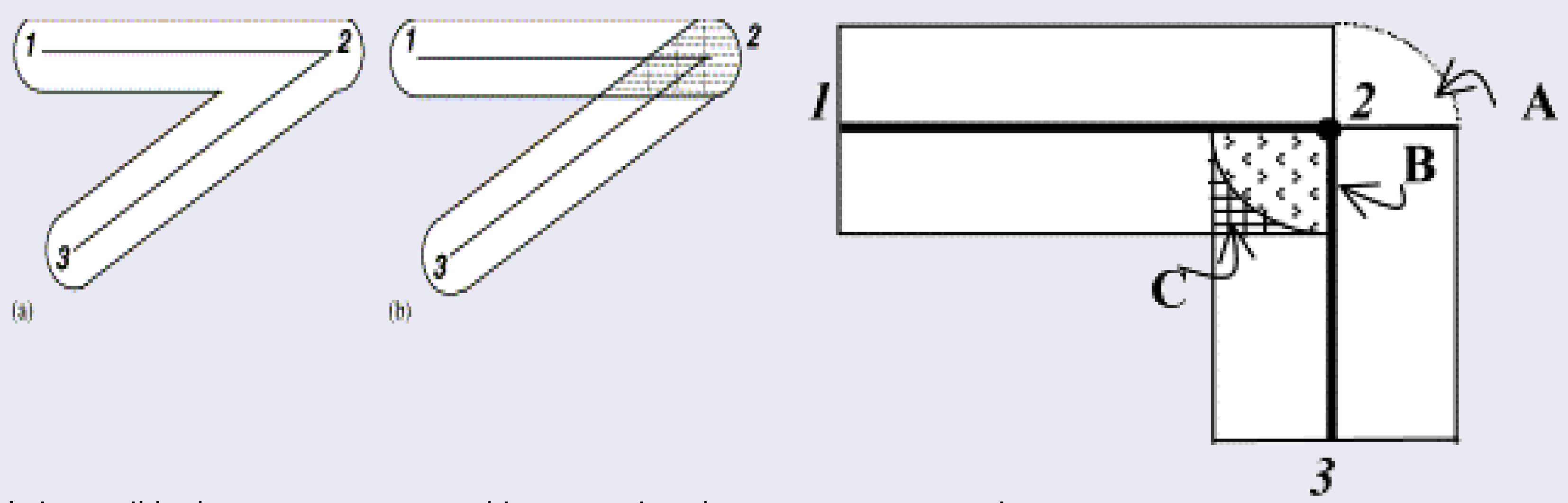
Hazard	Impact Radius (Miles)
Fire	0.071
Explosion	0.186
Nuclear	0.772

Table 1: Three common hazards and their impact radius

In order to cover the area accurately we propose the use of a rectangular area with semi-circles modelling the population coverage at the nodes, hence:

$$c_i = \rho(i)\left(2rl + \pi r^2\right)$$

This, however, leads to double counting or over estimation, as seen in the shaded area below:



It is possible, however, to remove this error using the exact error expression:

$$\rho(i)\left[\frac{r^2}{\tan\left(\frac{\alpha}{2}\right)} - \frac{(180-\alpha)r^2\pi}{360} + r^2\pi\right]$$

where $\rho(i)$ is the population density, r is the radius and α the joining angle.

Perceived Risk

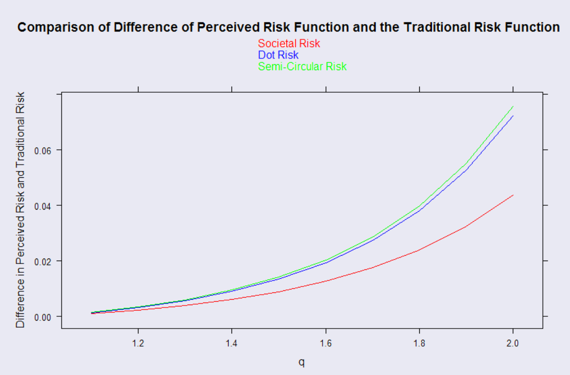
Population Density is not the only measure of a severe accident.

How it will be perceived can also be considered.

For this there is the Perceived Risk model[2]:

$$TR = \sum_{i=1}^{n} p_i c_i^{q_i}$$

The image to the right illustrates the difference between these two risk models.



Modelling Profitability

It is also possible to model the profitability, P, of a network [3]. Taking the parameters:

p_s	Probability of successful delivery of goods
p_f	Probability of successful delivery of goods but sale is not as profitable
p_c	Probability of a catastrophe occurring on the route
ΔG	Profit from a good sale
ΔB	Profit / loss from a non-viable sale
ΔC	Loss resulting from a catastrophe

and applying the CLT gives $\mathrm{P} \sim \mathrm{N}(\mu, \sigma^2)$ with parameters:

$$\mu = p_s \Delta G + p_f \Delta B + p_c \Delta C$$

$$\sigma^2 = p_s p_f (\Delta G - \Delta B)^2 + p_c\left[p_s(\Delta G + \Delta C)^2 + p_f(\Delta G - \Delta)^2\right]$$

From this it is possible to calculate the expected profit from a route.

Optimal Solutions

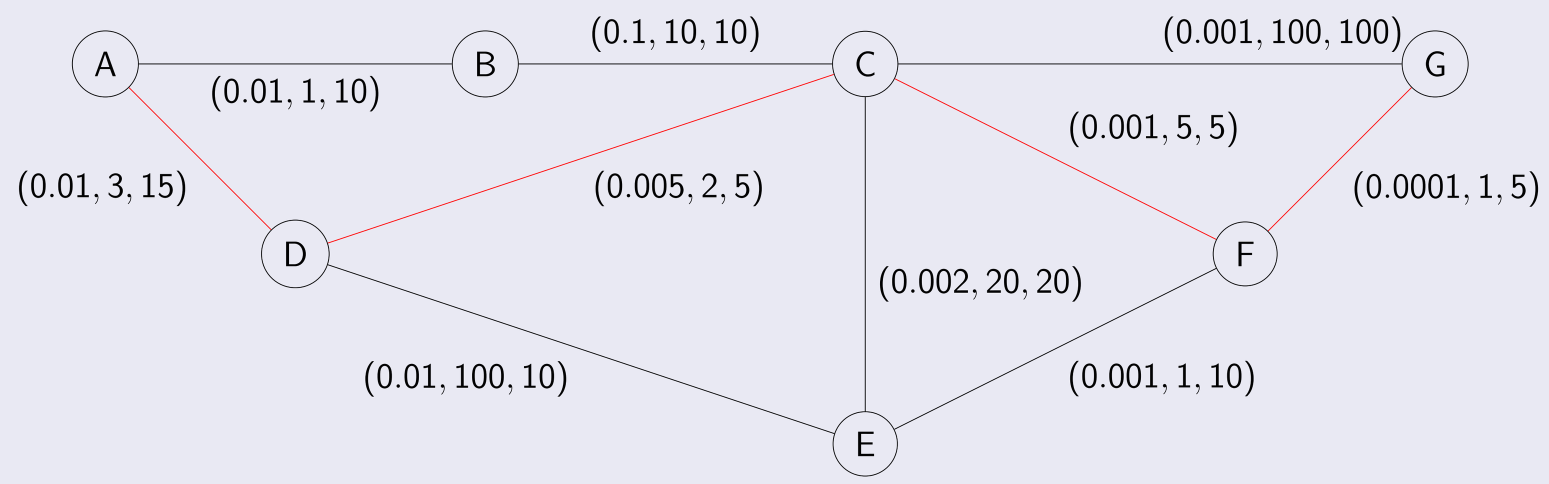
One widely used optimisation method is policy iteration[1].
This allows for both risk and route time, τ_i, to be minimized together.

$$\nu(k) = \min_{u \in \mathcal{P}}\left\{\sum_{i=1}^{n} \tau_i + J_\mu(u)\right\}$$

where u is the path in the set, $\mathcal{P}$ of all paths. To find a minimizer we follow these steps:

- Find a minimum path using Djikstra's Algorithm
- Evaluate the policy given by $\nu(k)$
- For each edge $i \in \mathcal{P}\backslash U$ see if altering the path reduces the value of $\nu(k)$.
- Once the new edges have been added see if $\nu(1) = \nu(2)$. If so, then the optimal policy has been found. If not, repeat the process for $\nu(2)$.

We can display a network using the ordered triple (p_i, c_i, τ_i):



Accident Locations

It is possible to model the location of the accident by linking the travel time to a Poisson Process with rate λ.
We can derive the probability of an accident after time t as:

$$P(N=0) = e^{-\lambda t}$$

$$P(N>t) = P(\text{no accident up until point t}) = e^{-\lambda t}$$

We are able to write this as an Exponential CDF:

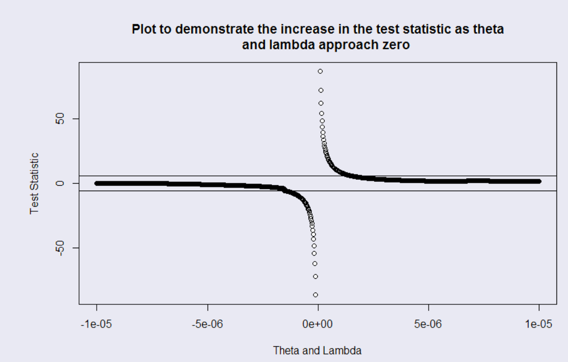
$$F_X(t) = \begin{cases} 1 - e^{-\lambda t} & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

Convolution then enables the accident probability on an entire route to be modelled as $T_n \sim \text{Gamma}(n, \lambda)$ where n is the number of links.
It is also possible to fit a Generalized Poisson Distribution to the process[4].

$$P_t(\theta, \lambda) = \frac{\lambda(\lambda + t\theta)^{t-1} e^{-\lambda - t\theta}}{t!}$$

Confidence intervals can then be generated for θ and λ using accident data and a χ^2_2 test.

The figure on the right shows asymptotic confidence interval behaviour, using accident data from the USA for 1990 to 2016.



Comparing the Fit of the Processes

Two tests can be used to assess the quality of the fit of the Poisson Process:

- Likelihood Ratio Test

$$c = \frac{L(\hat{\theta}_1)}{L(\hat{\theta}_0)} \quad \text{with test statistic} \quad -2\log(c) \sim \chi^2_k$$

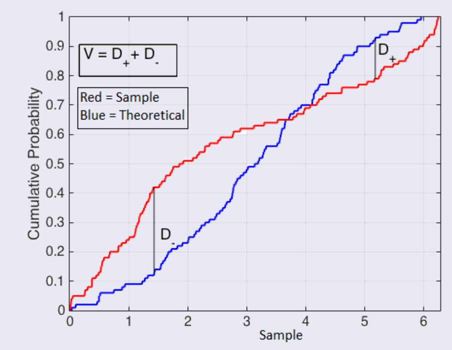
- Kuiper Test[5]

$$z_i = F(x_i) \quad \text{with F a distribution}$$

$$D^+ = \max\left(\frac{i}{n} - z_i\right)$$

$$D^- = \max\left(z_i - \frac{i-1}{n}\right)$$

$$V = D^+ + D^-$$



Here the Likelihood Ratio Test declares which process fits the data better and the Kuiper Test verifies which distribution better represents the data.

For the USA data both tests conclude that the GPD is the more appropriate model.

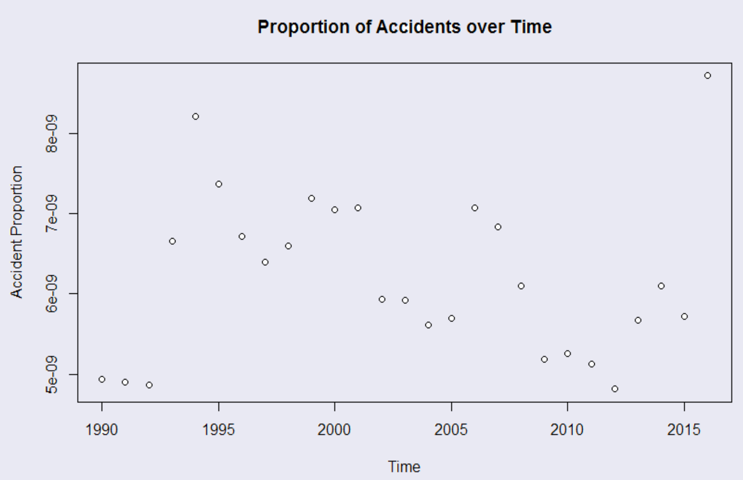
Moving Towards a New Risk Model

Several issues with the Traditional Risk model exist:

- The consequence value may not be solely linked to the population density.
- The risk model may overstate the risk through the assumption that an accident will be severe.
- The accident probability fails to then account for particular accident blackspots.

From this we argue that the Traditional model can be too inflexible.

Figure 1: Plot showing proportion of accidents resulting in injury in the USA between 1990 and 2016, Source: phmsa.dot.gov/hazmat



The Modern Risk Model

In order to create a more flexible risk model we introduce three new parameters:

q_i	The perceived risk value of an accident on a link
s_i	The probability of an accident being severe on the link
l_i	An adjustment parameter for blackspots or other circumstances

This allows for the proposal of the new model:

$$MR = \sum_{i=1}^{n} p_i c_i^{q_i} s_i^{l_i}$$

References

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