On Optimization of Network-coded Scalable Multimedia Service Multicasting

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Starting Point and Goals

๏ Delivery of multimedia broadcast/multicast services over 4G/5G networks is a challenging task. This has propelled research into delivery schemes.

๏ Multi-rate Transmission (MrT) strategies have been proposed as a means of delivering layered services to users experiencing different downlink channel conditions.

๏ Layered service consists of a basic layer and multiple enhancement layers.

Goals

๏ Error control - Ensure that a predetermined fraction of users achieves a certain service level with at least a given probability

๏ Resource optimisation - Reduce the total amount of radio resources needed to deliver a layered service.
Index

1. System Parameters and Performance Analysis
2. Multi-Channel Resource Allocation Models and Heuristic Strategies
3. Analytical Results
4. Concluding Remarks
1. System Parameters and Performance Analysis
System Model

- One-hop wireless communication system composed of one source node and $U$ users

- Each PtM layered service is delivered through $C$ orthogonal broadcast erasure subchannels

- Each subchannel delivers streams of (en)coded packets (according to the RLNC principle).
Non-Overlapping Layered RNC

- $\mathbf{x} = \{x_1, \ldots, x_K\}$ is a layered source message of $K$ source packets, classified into $L$ service layers.

- Encoding performed over each service layer independently from the others.

- The source node will linearly combine the $k_l$ data packets composing the $l$-th layer $\mathbf{x}_l = \{x_i\}_{i=1}^{k_l}$ and will generate a stream of $n_l \geq k_l$ coded packets $\mathbf{y} = \{y_j\}_{j=1}^{n_l}$, where

$$
y_j = \sum_{i=1}^{k_l} g_{j,i} x_i
$$

Coefficients of the linear combination are selected over a finite field of size $q$. 
Non-Overlapping Layered RNC

User $u$ recovers layer $l$ if it will collect $k_l$ linearly independent coded packets. The prob. of this event is

$$P_l(n_{l,u}) = \sum_{r=k_l}^{n_{l,u}} \binom{n_{l,u}}{r} p^{n_{l,u}-r} (1-p)^r h(r)$$

$$= \sum_{r=k_l}^{n_{l,u}} \binom{n_{l,u}}{r} p^{n_{l,u}-r} (1-p)^r \prod_{i=0}^{k_l-1} \left[ 1 - \frac{1}{q^{r-i}} \right]$$

The probability that user $u$ recover the first $l$ service layers is

$$D_{NO,l}(n_1,u, \ldots, n_L,u) = D_{NO,l}(n_u) = \prod_{i=1}^{l} P_i(n_{i,u})$$
Expanding Window Layered RNC

- We define the $l$-th window $X_l$ as the set of source packets belonging to the first $l$ service layers. Namely, $X_l = \{x_j\}_{j=1}^{K_l}$ where $K_l = \sum_{i=1}^{l} k_i$

- The source node (i) linearly combines data packets belonging to the same window, (ii) repeats this process for all windows, and (iii) broadcasts each stream of coded packets over one or more subchannels
Expanding Window Layered RNC

The probability $D_{EW,l}$ of user $u$ recovering the first $l$ layers (namely, the $l$-th window) can be written as

$$D_{EW,l}(N_1,u, \ldots, N_L,u) = D_{EW,l}(N_u) = \sum_{r_1=0}^{N_1,u} \sum_{r_{l-1}=0}^{N_{l-1},u} \sum_{r_l=r_{\text{min}},l}^{N_l,u} \binom{N_1,u}{r_1} \ldots \binom{N_l,u}{r_l} p \sum_{i=1}^{l} (N_{i,u} - r_i) (1 - p) \sum_{i=1}^{l} r_i$$

- Sums allow us to consider all the possible combinations of received coded packets
2. Multi-Channel Resource Allocation Models
Allocation Patterns

Subchannel 1
Subchannel 2
Subchannel 3

Separated Allocation Pattern

Coded packets from $x_1$
Coded packets from $x_2$
Coded packets from $x_3$

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Allocation Patterns

Mixed Allocation Pattern

coded packets from $x_1$ or $X_1$
coded packets from $x_2$ or $X_2$
coded packets from $x_3$ or $X_3$
NO-MA Model

Consider the variable \( \lambda_{u,l} = I\left(D_{NO,l}(n_u) \geq \hat{D}\right) \). It is 1, if \( u \) can recover the first \( l \) layers with a probability value, otherwise it is 0.

\[
\min_{m_1, \ldots, m_C} \sum_{l=1}^{L} \sum_{c=1}^{C} n^{(l,c)}
\]

subject to

\[
\sum_{l=1}^{L} \sum_{c=1}^{C} n^{(l,c)} = 1
\]

Minimization of resource footprint

No. of packets of layer \( l \) delivered over \( c \)
Consider the variable \( \lambda_{u,l} = I \left( D_{\text{NO},l}(n_u) \geq \hat{D} \right) \). It is 1, if \( u \) can recover the first \( l \) layers with a probability value \( \geq \hat{D} \), otherwise it is 0.

\[
\text{(NO-MA)} \quad \min_{m_1, \ldots, m_C} \sum_{l=1}^{L} \sum_{c=1}^{C} n^{(l,c)}
\]

subject to \( \sum_{u=1}^{U} \lambda_{u,l} \geq U \hat{t}_l \) \( l = 1, \ldots, L \)

Each service level shall be achieved by a predetermined fraction of users

Target fraction of users

No. of users
NO-MA Model

Consider the variable $\lambda_{u,l} = I\left(D_{NO,l}(n_u) \geq \hat{D}\right)$. It is 1, if $u$ can recover the first $l$ layers with a probability value $\geq \hat{D}$, otherwise it is 0.

\[
\begin{align*}
\text{(NO-MA)} & \quad \min_{m_1,\ldots,m_C} \sum_{l=1}^{L} \sum_{c=1}^{C} n^{(l,c)} \\
\text{subject to} & \quad \sum_{u=1}^{U} \lambda_{u,l} \geq U \hat{t}_l \quad l = 1, \ldots, L \\
& \quad m_{c-1} < m_c \quad c = 2, \ldots, L \\
& \quad 0 \leq \sum_{l=1}^{L} n^{(l,c)} \leq \hat{B}_c \quad c = 1, \ldots, C
\end{align*}
\]
NO-MA Heuristic

- The NO-MA is an hard integer optimisation problem because of the coupling constraints among variables

- We propose a two-step heuristic strategy
  1. MCSs optimisation \((m_1, \ldots, m_C)\)
  2. No. of coded packet per-subchannel optimization \((n^{(1,c)}, \ldots, n^{(L,c)})\)

- The first step selects the value of \(m_c\) such that packets delivered through subch. \(c\) are received (at least with a target prob.) by \(U \cdot \hat{t}_c\) users.

<table>
<thead>
<tr>
<th>Step 1 Subchannel MCSs optimization.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: (c \leftarrow C) |</td>
</tr>
<tr>
<td>2: (v \leftarrow m_{\text{MAX}}) and</td>
</tr>
<tr>
<td>3: \textbf{while } c \geq 1 \textbf{ do}</td>
</tr>
<tr>
<td>4: \hspace{1em} \textbf{repeat} |</td>
</tr>
<tr>
<td>5: \hspace{2em} m_c \leftarrow v |</td>
</tr>
<tr>
<td>6: \hspace{2em} v \leftarrow v - 1 |</td>
</tr>
<tr>
<td>7: \hspace{1em} \textbf{until }</td>
</tr>
<tr>
<td>8: \hspace{1em} c \leftarrow c - 1 |</td>
</tr>
<tr>
<td>9: \hspace{1em} \textbf{end while}</td>
</tr>
</tbody>
</table>
The idea behind the second step can be summarised as follows:

\[ D_{NO,1}(\bar{n}^{(1)}) \geq \hat{D} \quad D_{NO,2}(\bar{n}^{(1)}, \bar{n}^{(2)}) \geq \hat{D} \]

\[ D_{NO,3}(\bar{n}^{(1)}, \bar{n}^{(2)}, \bar{n}^{(3)}) \geq \hat{D} \]
NO-MA Heuristic

The idea behind the second step can be summarised as follows.

\textbf{Step 2} Coded packet allocation for a the NO-MA case.

1: \( c \leftarrow 1 \)
2: \( \bar{n}^{(l,c)} \leftarrow 1 \) for any \( l = 1, \ldots, L \) and \( c = 1, \ldots, C \)
3: \( \bar{n} = \{ \bar{n}^{(l)} \}_{l=1}^L \), where \( \bar{n}^{(l)} \leftarrow 1 \) for any \( l = 1, \ldots, L \)
4: \textbf{for} \( l \leftarrow 1, \ldots, L \ \textbf{do} \)
5: \textbf{while} \( D_{\text{NO},l}(\bar{n}) < \hat{D} \) and \( c \leq C \) \textbf{do}
6: \( \bar{n}^{(l,c)} \leftarrow \bar{n}^{(l,c)} + 1 \)
7: \( \bar{n}^{(l)} \leftarrow \sum_{t=1}^{C} \bar{n}^{(l,t)} \) for any \( l = 1, \ldots, L \)
8: \textbf{if} \( \sum_{t=1}^{L} \bar{n}^{(t,c)} = \hat{B}_c \) \textbf{then}
9: \( c \leftarrow c + 1 \)
10: \textbf{end if}
11: \textbf{end while}
12: \textbf{if} \( D_{\text{NO},l}(\bar{n}) < \hat{D} \) and \( c > C \) \textbf{then}
13: \textit{no solution can be found.}
14: \textbf{end if}
15: \textbf{end for}
§ Consider the EW delivery mode

We define the indicator variable

\[
\mu_{u,l} = I \left( \bigvee_{t=l}^{L} \left\{ D_{EW,t}(N_u) \geq \hat{D} \right\} \right)
\]

User \( u \) will recover the first \( l \) service layers (at least) with probability \( \hat{D} \) if any of the windows \( l, l+1, \ldots, L \) are recovered (at least) with probability \( \hat{D} \)
The RA problem for the EW-MA case is

**(EW-MA)** \[
\min_{m_1,\ldots,m_C} \sum_{l=1}^{L} \sum_{c=1}^{C} N^{(l,c)}
\]

subject to

\[
\sum_{u=1}^{U} \mu_{u,l} \geq U \hat{t}_l \quad l = 1,\ldots,L
\]

\[
m_{c-1} < m_c \quad c = 2,\ldots,L
\]

\[
0 \leq \sum_{l=1}^{L} N^{(l,c)} \leq \hat{B}_c \quad c = 1,\ldots,C
\]

It is still an **hard integer optimisation problem** but the previously proposed heuristic strategy can be still applied.
“Egalitarian” Model

- Previous strategies ensure minimum SLA and minimize the resource footprint. **Point of view of the ISP**...

- **Best practice for burglars** - To still object with the maximum value and the minimum weight. The profit-cost ratio is maximized.

Model for a SA pattern as:

\[
(E-SA) \text{ minimize } \frac{\sum_{u=1}^{U} \sum_{l=1}^{L} Q_{u,l}}{\sum_{l=1}^{L} N^{(l)}} \quad \text{subject to } \sum_{u=1}^{U} Q_{u,l} \geq U \hat{t}_l \quad l = 1, \ldots, L
\]

- **Profit** - No. of video layers recovered by any of the users
- **Cost** - No. of transmissions needed
- **SLA-related constraint**

We can refer to the previous heuristics.
3. Analytical Results
Analytical Results (part 1)

- LTE-A eMBMS scenarios
- We compared the proposed strategies with a classic Multi-rate Transmission strategy

\[
\max_{m_1, \ldots, m_L} \sum_{u=1}^{U} \text{PSNR}_u
\]

It is a maximization of the sum of the user QoS

No error control strategies are allowed (ARQ, RLNC, etc.)

- System performance was evaluated in terms of

\[
\sigma = \begin{cases} 
\sum_{l=1}^{L} \sum_{c=1}^{C} n^{(l,c)}, & \text{for NO-RNC} \\
L \sum_{c=1}^{C} \sum_{l=1}^{L} N^{(l,c)}, & \text{for EW-RNC}
\end{cases}
\]

Resource footprint
Analytical Results (part 1)

- LTE-A eMBMS scenarios
- We compared the proposed strategies with a classic Multi-rate Transmission strategy

\[
\max_{m_1, \ldots, m_L} \sum_{u=1}^{U} \text{PSNR}_u
\]

It is a maximization of the sum of the user QoS

- No error control strategies are allowed (ARQ, RLNC, etc.)

System performance was evaluated in terms of

\[
\rho(u) = \begin{cases} 
\max_{l=1, \ldots, L} \left\{ \text{PSNR}_l \ D_{NO,l}^{(u)} \right\}, & \text{for NO-RNC} \\
\max_{l=1, \ldots, L} \left\{ \text{PSNR}_l \ D_{EW,l}^{(u)} \right\}, & \text{for EW-RNC}
\end{cases}
\]

PSNR after recovery of the basic and the first l enhancement layers
Scenario with a high heterogeneity. 80 UEs equally spaced and placed along the radial line representing the symmetry axis of one sector of the target cell.

We considered Stream A and B which have 3 layers, bitrate of A is smaller than that of B.

Analytical Results (part 1)

We considered Stream A and B which have 3 layers, bitrate of A is smaller than that of B.
Analytical Results *(part 1)*

All the proposed strategies meet the coverage constraints.

Stream A

- MrT
- Heu. NOW–MA
- Heu. EW–MA

Maximum PSNR $\rho$ (dB)

Distance (m)
Analytical Results (part 1)

All the proposed strategies meet the coverage constraints.
Analytical Results (part 2)

- LTE-A allows multiple contiguous BS to deliver (in a synchronous fashion) the same services by means of the same signals.

  4-BS SFN, 1700 users placed at the vertices of a regular square grid placed on the playground.

Single Frequency Network

Spacial SINR distribution
Also in this case MrT cannot ensure the desired coverage!
Also in this case MrT cannot ensure the desired coverage!
4. Concluding Remarks
Concluding Remarks

- **Definition of a **generic system model** that can be easily adapted to practical scenarios and different viewpoints (ISP vs users).**

- Derivation of the **theoretical framework to assess user QoS**

- **Definition of efficient resource allocation frameworks**, that can jointly optimise both system parameters and the error control strategy in use

- **Development of efficient heuristic strategies** that can derive good quality solutions in a finite number of steps.

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