School of Computing
and Communications

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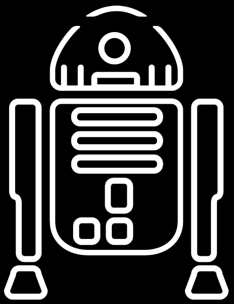
University



# The R2D2 project: Network coding for Rapid and Reliable Data Delivery

A project funded by EPSRC under the First Grant scheme

Web: http://www.lancs.ac.uk/~chatzige/R2D2/index.html

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University of Liverpool, 11th March 2015

# Aims and Aspirations

- 18-month EPSRC research project on network error control mechanisms (February 2014 – July 2015).

- Design novel mathematical frameworks - To identify key relationships between system and channel parameters, understand network dynamics and optimise network-coded architectures.

- Investigate practical aspects - Ultra-reliable communications, delay-constrained applications, energy-efficient architectures

# The R2D2 Team

Ioannis Chatzigeorgiou
*Principal Investigator*

Andrea Tassi
*Postdoctoral Research Associate*

Amjad Saeed Khan
*PhD Candidate*
(Oct. 2014 - present)

Andrew Jones
*MSc by Research*
(Sep. 2013 – Aug. 2014)

# Research Activities

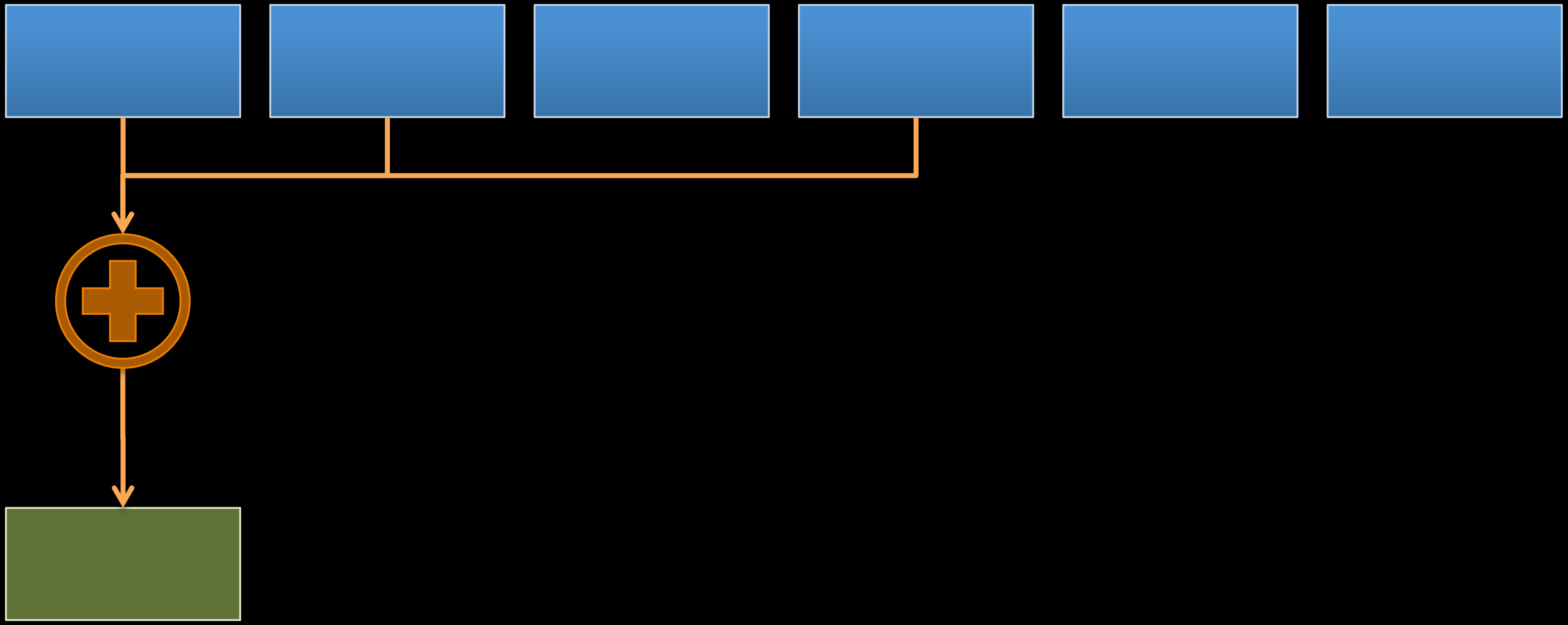
- Performance modelling of network-coded schemes
- System optimisation for layered multicast services
- Design of on-the-fly rateless decoders
- Sparse network coding schemes that trade complexity for delay / energy
- Extension to relay-aided transmission

# 1. Network coding in a nutshell

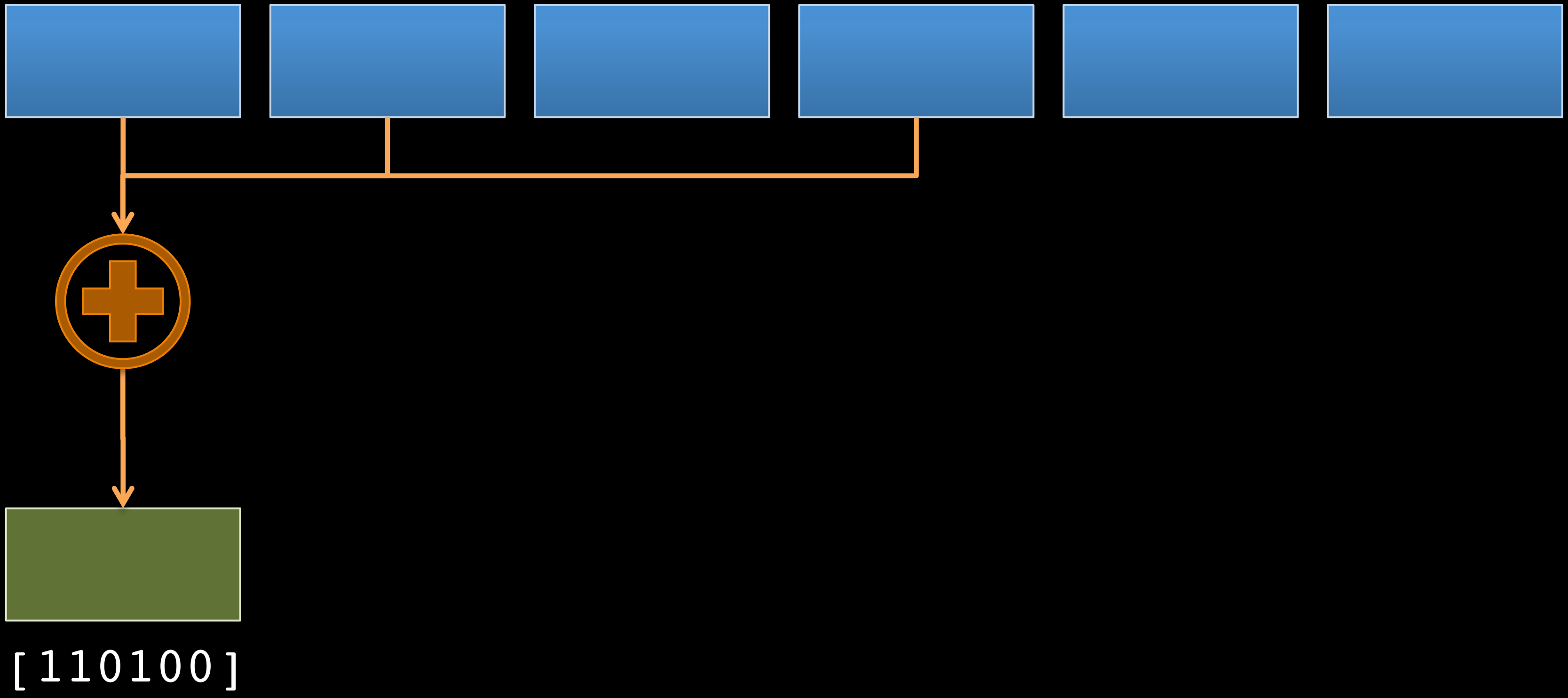
# Random Linear Network Coding

# Random Linear Network Coding

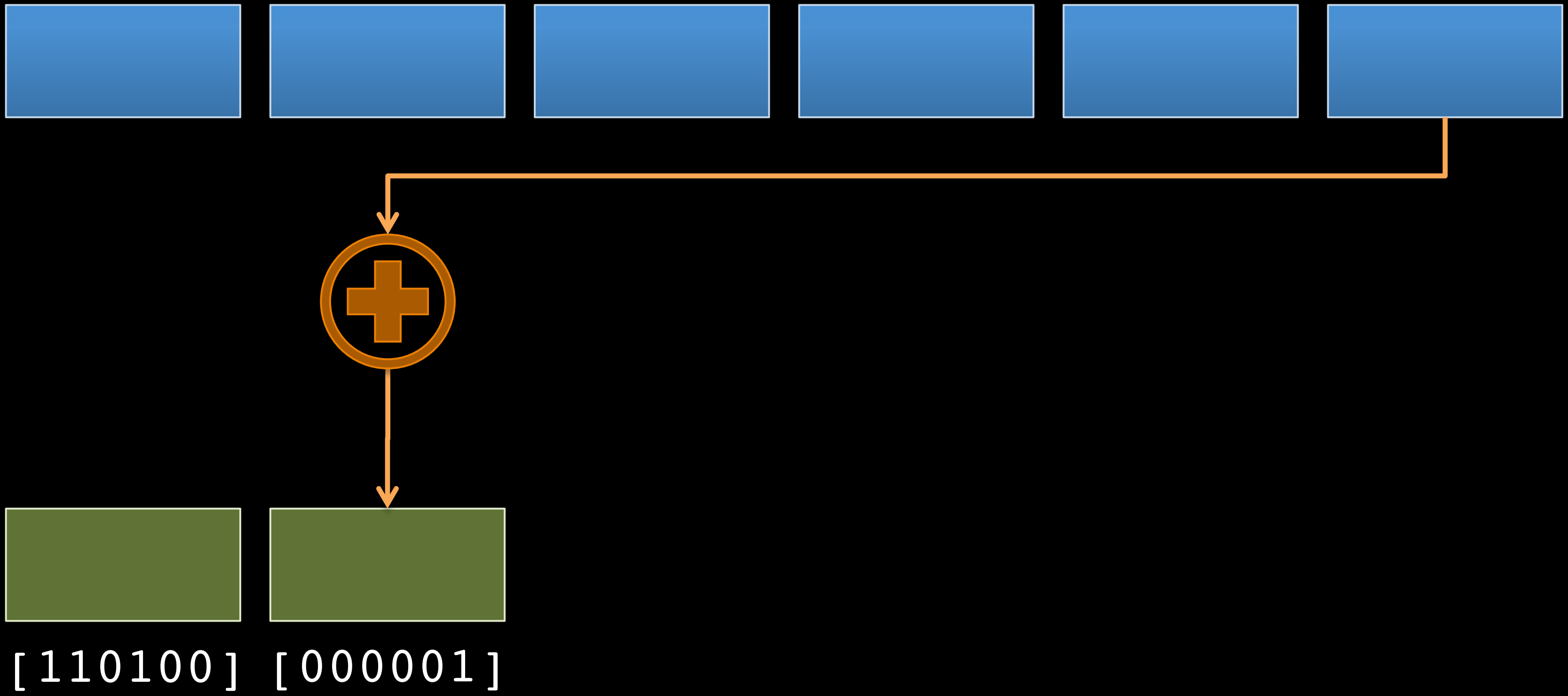
# Random Linear Network Coding



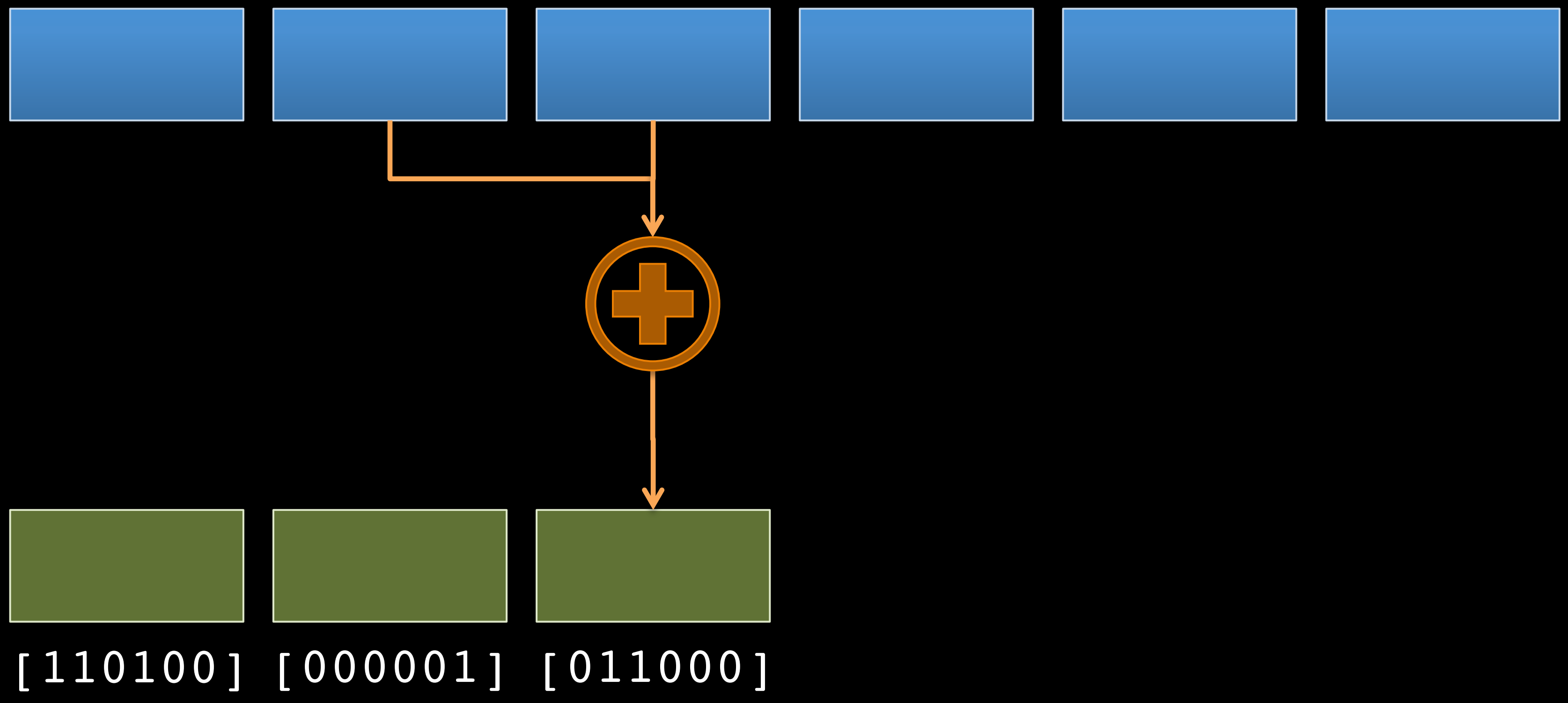
# Random Linear Network Coding



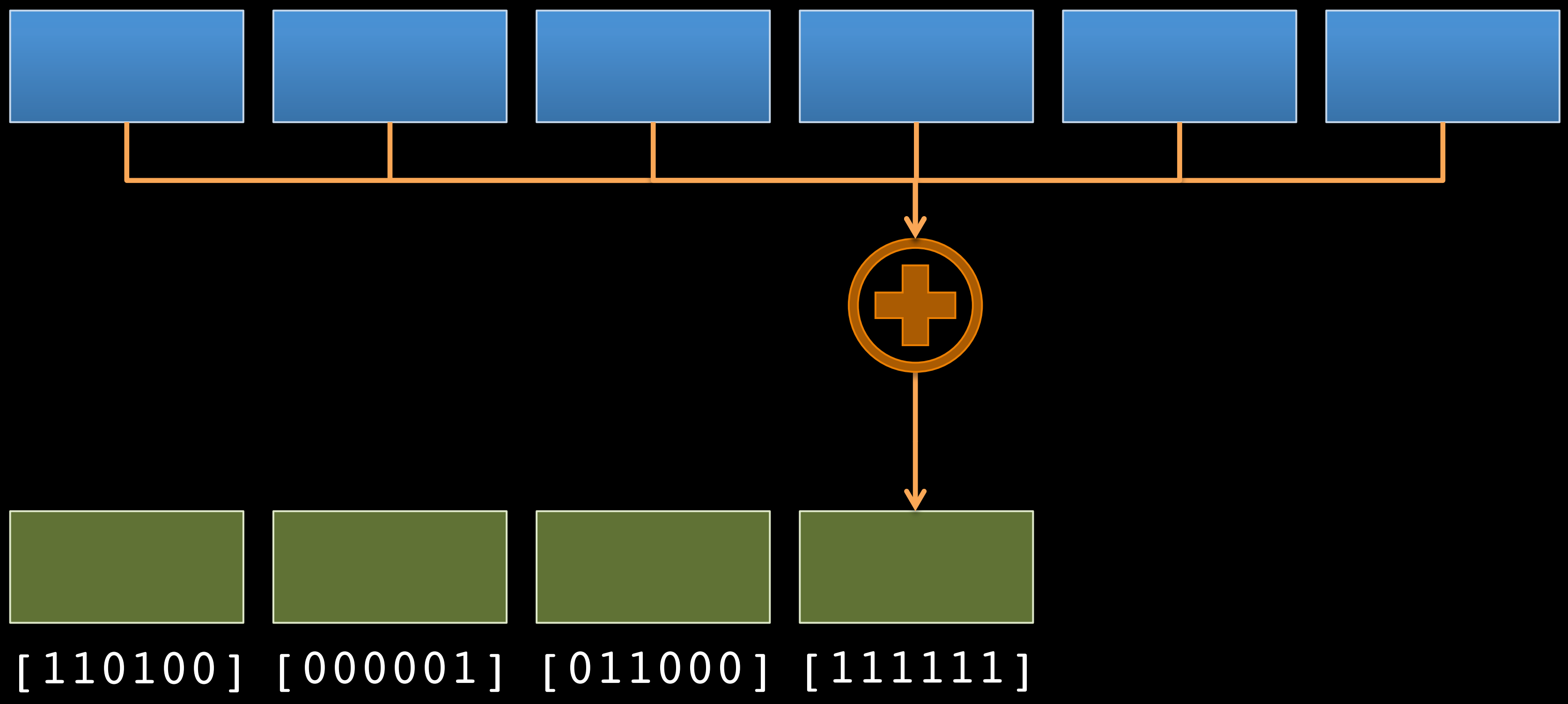
# Random Linear Network Coding



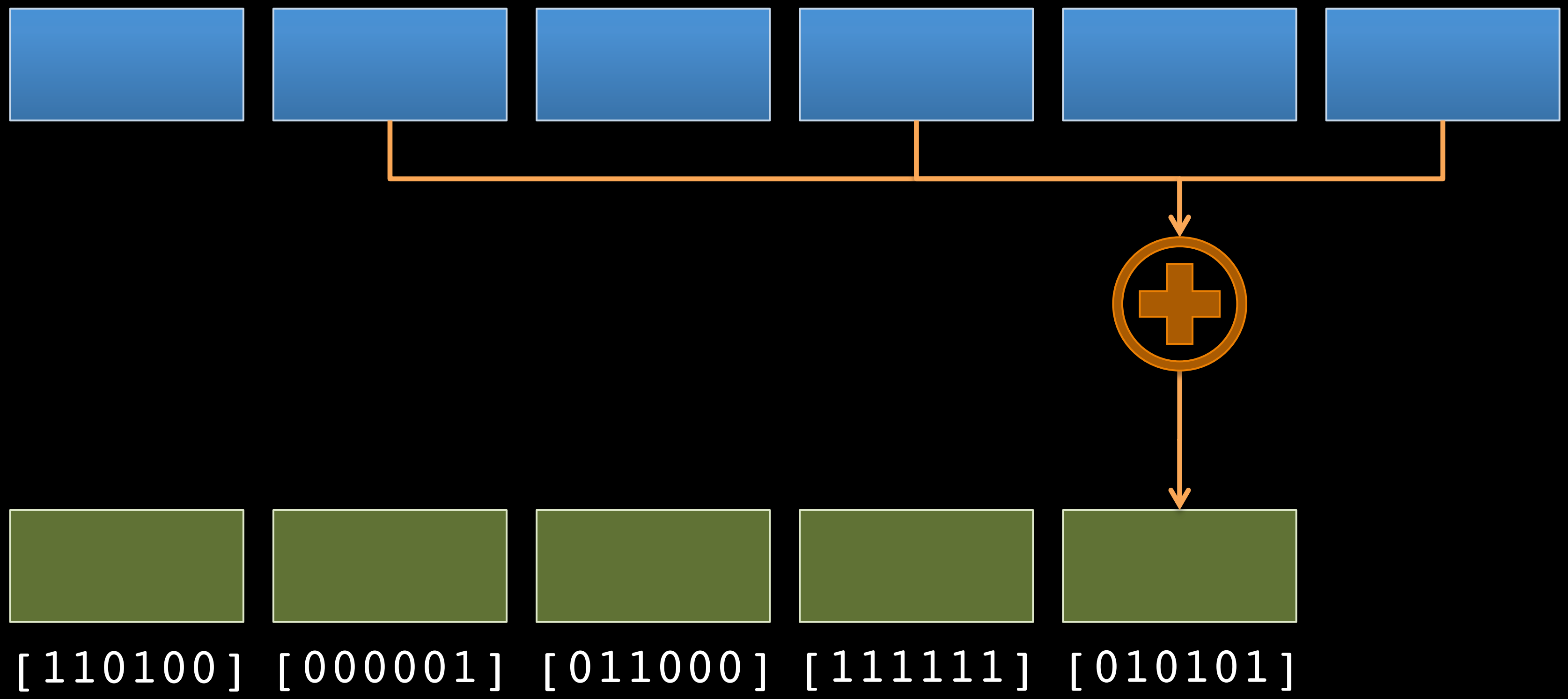
# Random Linear Network Coding



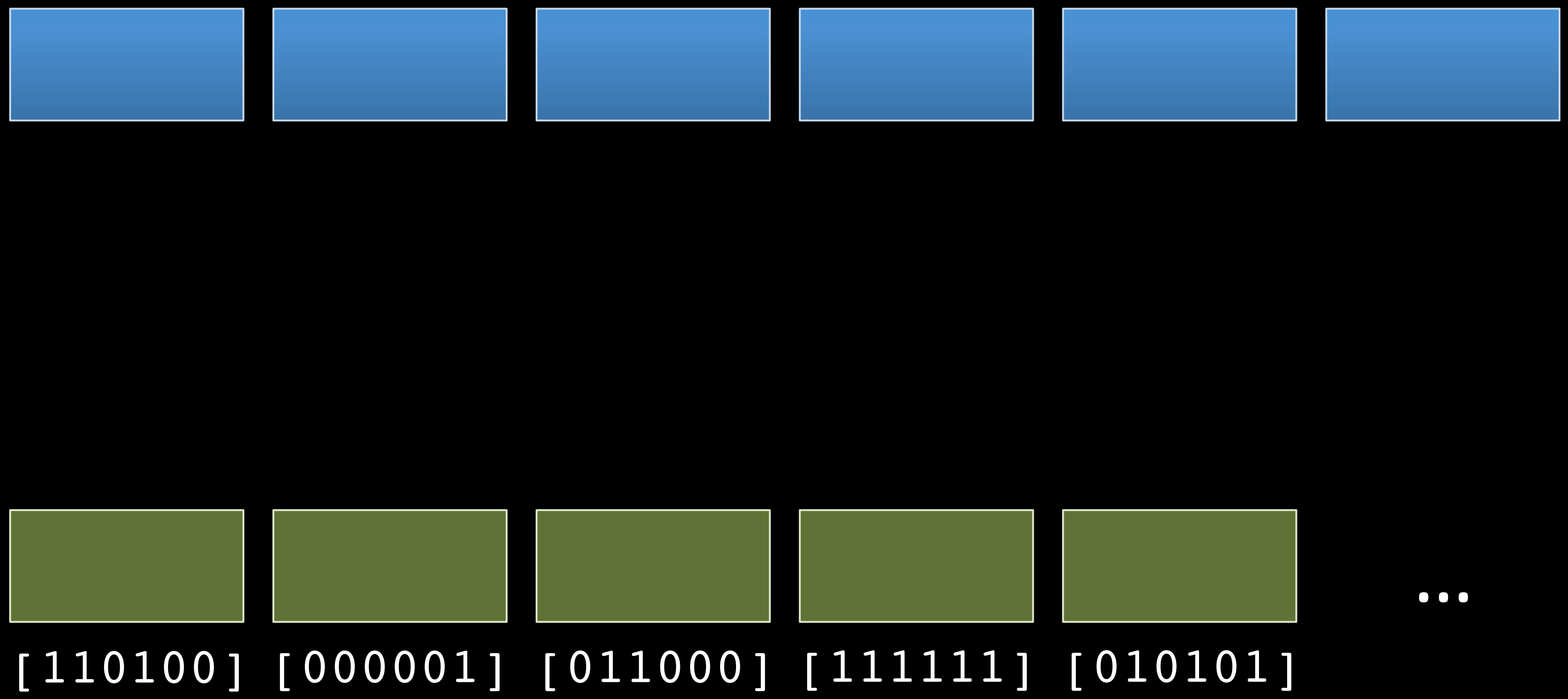
# Random Linear Network Coding



# Random Linear Network Coding



# Random Linear Network Coding



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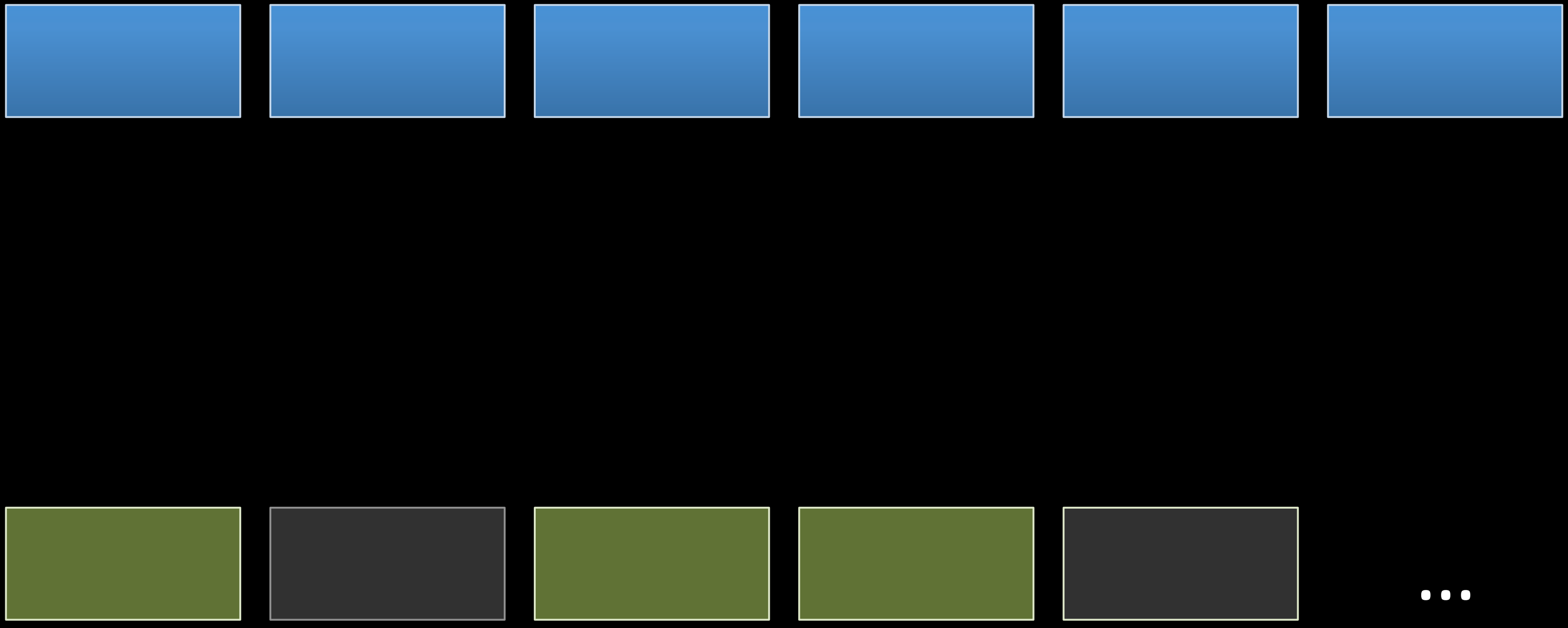
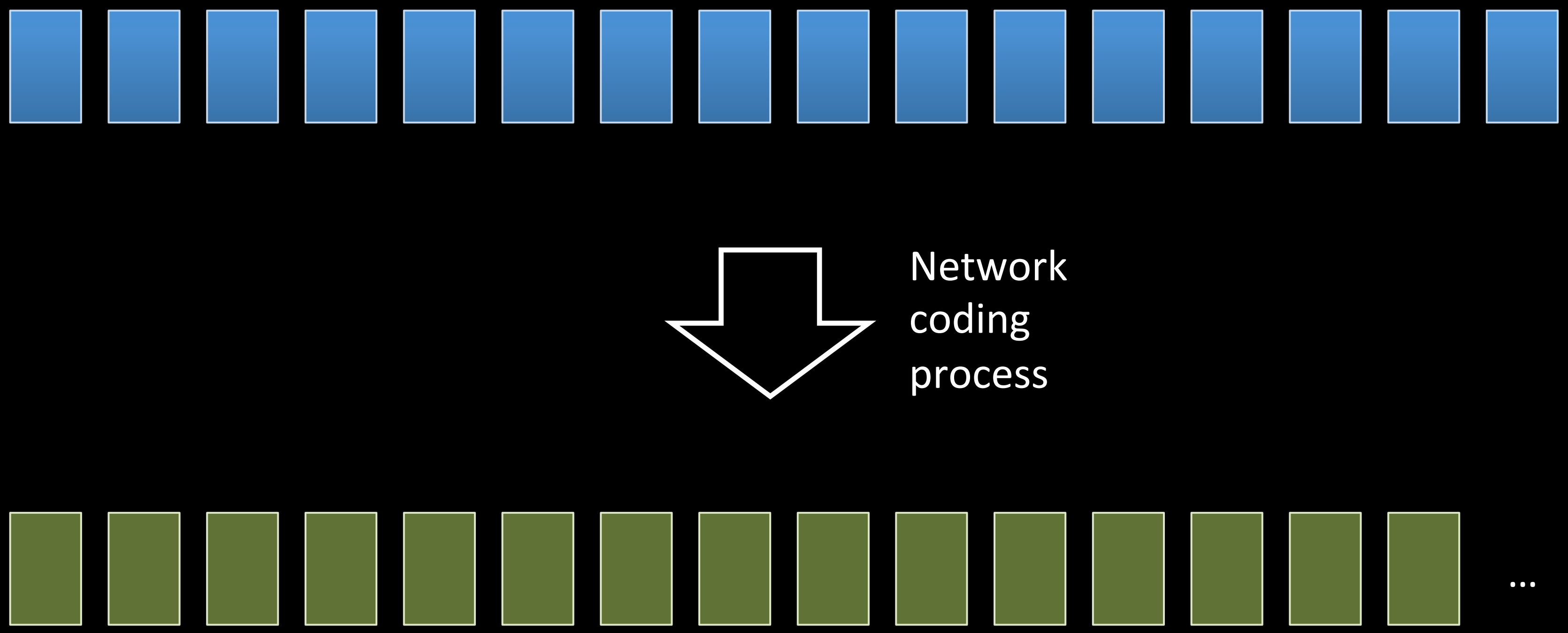
[110100] [000001] [011000] [111111] [010101]

# Random Linear Network Coding

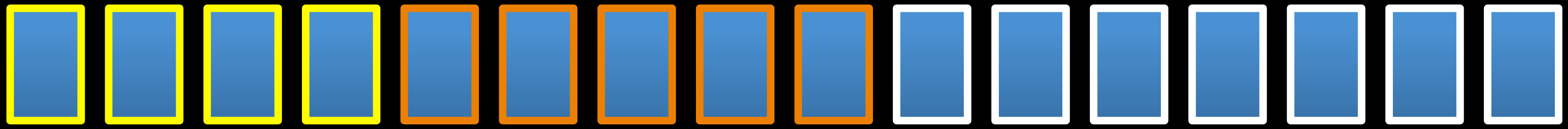


[110100] [000001] [011000] [111111] [010101] …

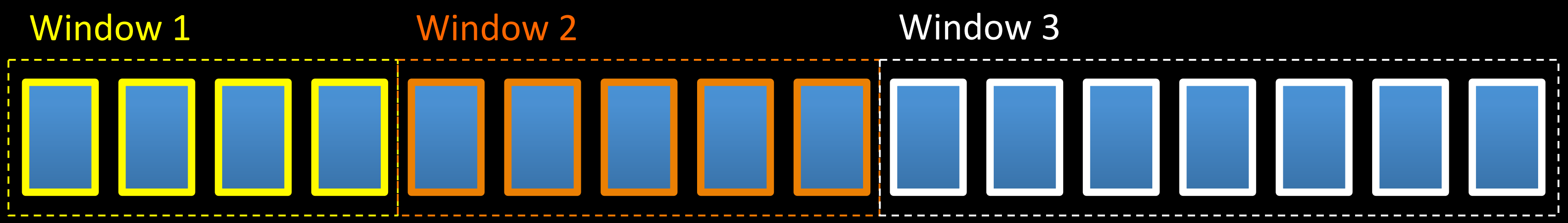
# Random Linear Network Coding



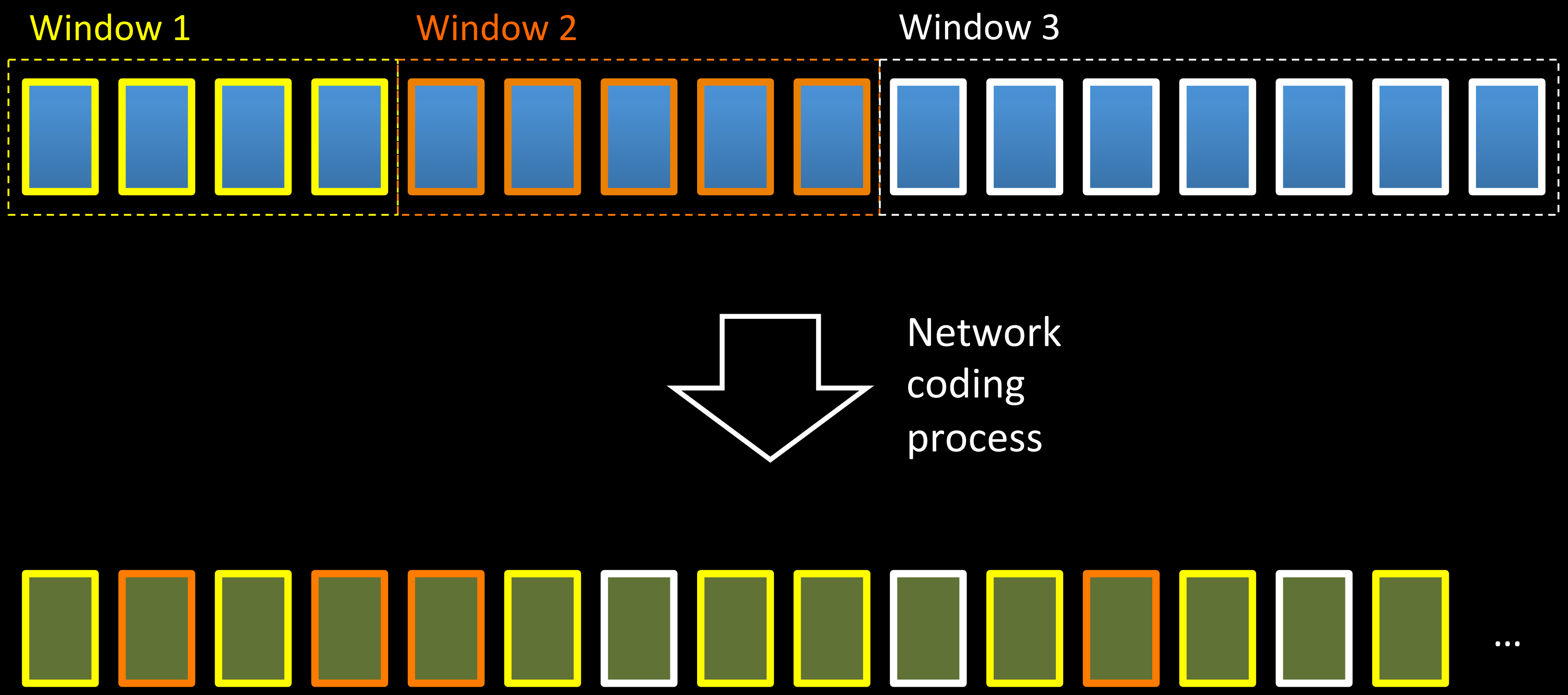
# Non-overlapping window NC



# Non-overlapping window NC

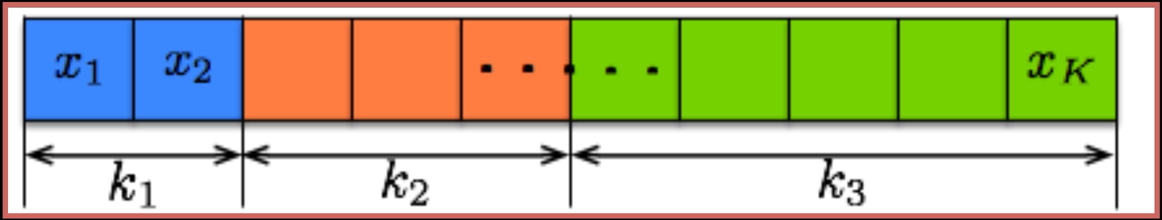


# Non-overlapping window NC



# Non-overlapping window NC

- $\mathbf{x} = \{x_1, x_2, \ldots, x_K\}$ is a layered source message of $K$ source packets, divided into $L$ service layers (here $L$=3).



- Encoding is performed over each service layer independently of the others
- The source linearly combines the $k_l$ data packets composing the $l$-th layer and generates a stream of $n_l \geq k_l$ coded packets, where

$$y_j = \sum_{i=1}^{k_l} g_{j,i} x_i$$

- Coefficients $g_{j,i}$ are selected uniformly at random from a finite field of size $q$, e.g. GF($q$).

# Non-overlapping window NC

- User $u$ can recover the $l$-th layer if $k_l$ linearly independent coded packets from that layer are collected. The probability of this event is

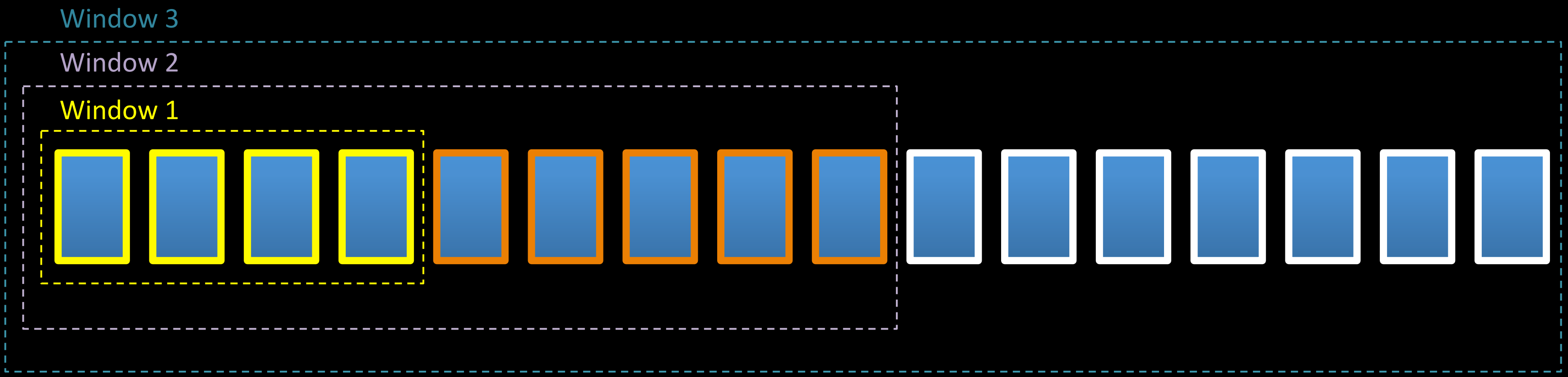
$$P_l(n_{l,u}) = \sum_{r=k_l}^{n_{l,u}} \binom{n_{l,u}}{r} p^{n_{l,u}-r}(1-p)^r \prod_{i=0}^{k_l-1}\left(1-\frac{1}{q^{r-i}}\right)$$

  where $p$ is the packet erasure probability, $n_{l,u}$ is the number of transmitted coded packets of layer $l$, and $r$ is the number of received coded packets.

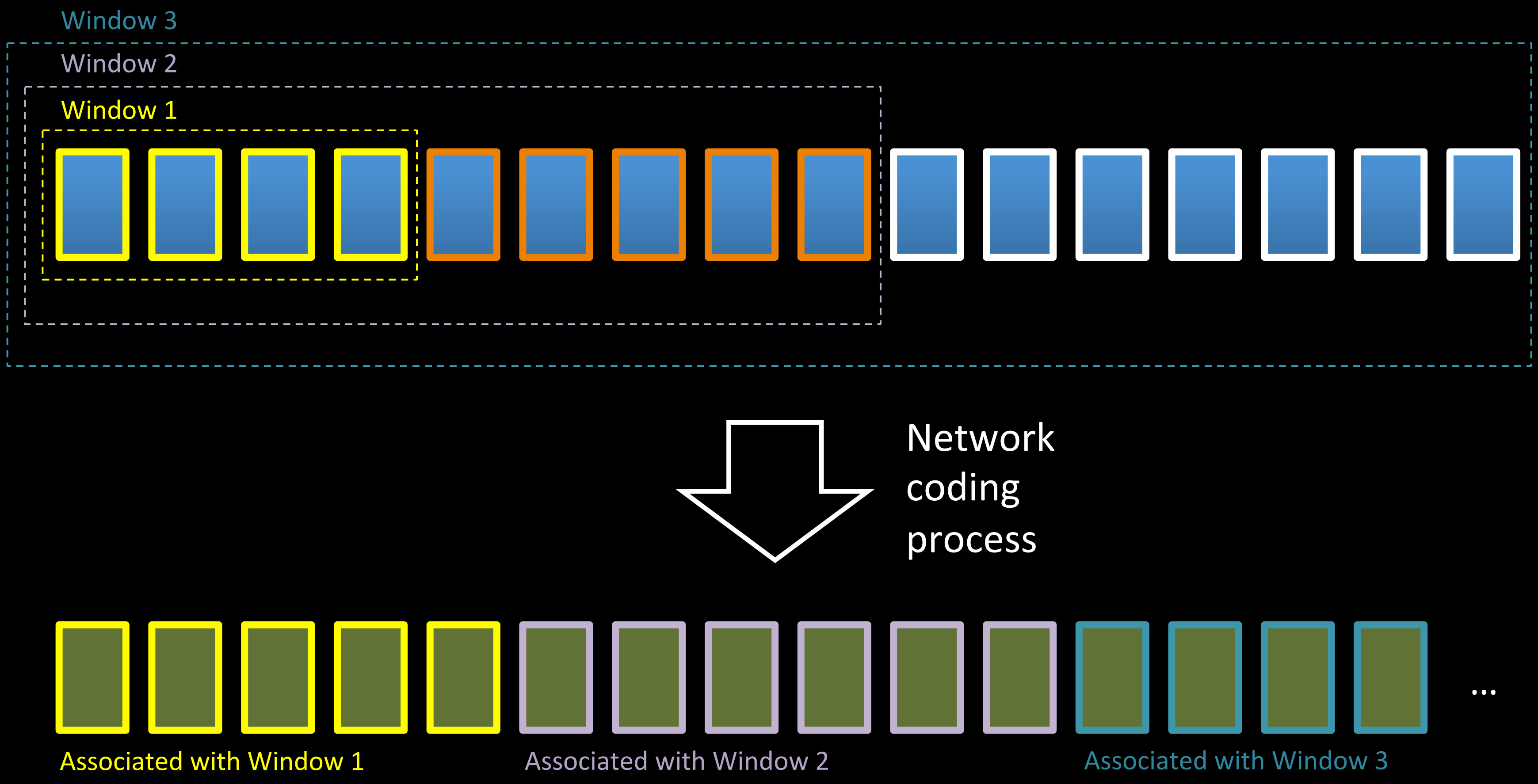
- The probability that user $u$ will recover the first $l$ service layers is

$$D_{\mathrm{NOW}}(n_{1,u},\ldots,n_{l,u}) = \prod_{j=1}^{l} P_j(n_{j,u})$$

# Expanding window NC



# Expanding window NC



Window 3

Window 2

Window 1

Network coding process

…

Associated with Window 1

Associated with Window 2

Associated with Window 3

# Expanding window NC

- Let $K_l = k_1 + k_2 + \ldots + k_l$ denote the size of the $l$-th expanding window, while $N_{l,u}$ denotes the number of coded packets transmitted to user $u$ and associated with expanding window $l$.

- The probability of user $u$ recovering the first $l$ service layers (or, equivalently, the $l$-th expanding window), can be expressed as

$$D_{\mathrm{EW}}(N_{1,u},\ldots,N_{l,u}) =$$

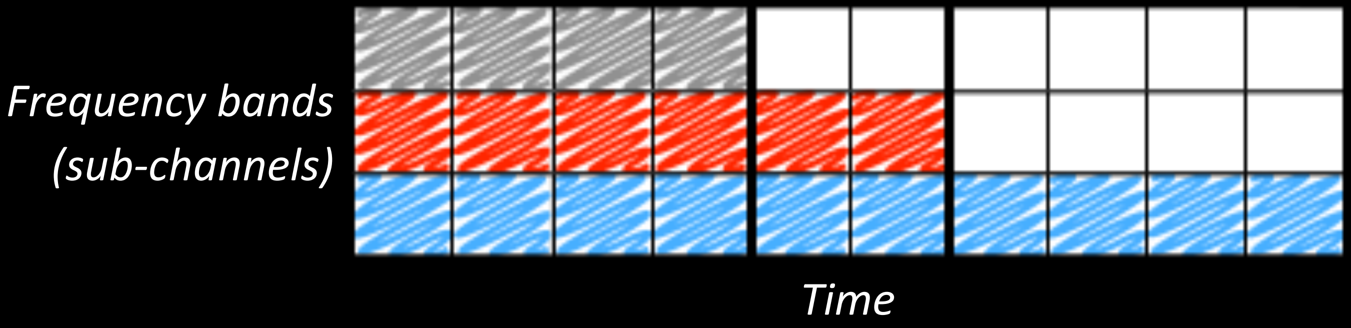
$$= \sum_{r_1=0}^{N_{1,u}} \sum_{r_2=0}^{N_{2,u}} \cdots \sum_{r_l=r_{\min,l}}^{N_{l,u}} \binom{N_{1,u}}{r_1} \cdots \binom{N_{l,u}}{r_l} p^{\sum_{i=1}^{l}(N_{i,u}-r_i)} (1-p)^{\sum_{i=1}^{l} r_i} g_l(r_1,\ldots,r_l)$$

where $r_{\min,\,l} = K_l - K_{l\text{-}1} + \max(r_{\min,\,l\text{-}1} - r_{l\text{-}1}, 0)$ and $r_{\min,1} = K_1$. The probability that $K_l$ out of the $r_1 + r_2 + \ldots + r_l$ received coded packets are linearly independent has been approximated by $g_l(r_1,\ldots,r_l)$.

# 2. Provider-centric optimisation

# Optimisation models

- Structure of transmission medium:



- We consider NOW layered NC and introduce the following indication variable

$$\lambda_{u,l} = I\left(D_{\mathrm{NOW}}(n_{1,u},\ldots,n_{l,u}) \geq \hat{P}\right)$$

- Similarly, we use the following indication variable for EW layered NC

$$\mu_{u,l} = I\left(\underset{j=l}{\overset{L}{\mathrm{OR}}}\left\{D_{\mathrm{EW}}(N_{1,u},\ldots,N_{j,u}) \geq \hat{P}\right\}\right)$$

# Optimisation models

Proposed mixed allocation (MA) strategies:

(NOW-MA)
$$\min_{\substack{m_1,\ldots,m_L \\ n^{(1,c)},\ldots,n^{(L,c)}}} \sum_{l=1}^{L}\sum_{c=1}^{C} n^{(l,c)}$$

subject to:
$$\sum_{u=1}^{U} \lambda_{u,l} \geq U\ \hat{t}_l \quad \text{for } l = 1,\ldots,L$$

$$m_{c-1} \leq m_c \quad \text{for c} = 2,\ldots,L$$

$$0 \leq \sum_{l=1}^{L} n^{(l,c)} \leq \hat{B}_c \quad \text{for c} = 1,\ldots,L$$

(EW-MA)
$$\min_{\substack{m_1,\ldots,m_L \\ N^{(1,c)},\ldots,N^{(L,c)}}} \sum_{l=1}^{L}\sum_{c=1}^{C} N^{(l,c)}$$

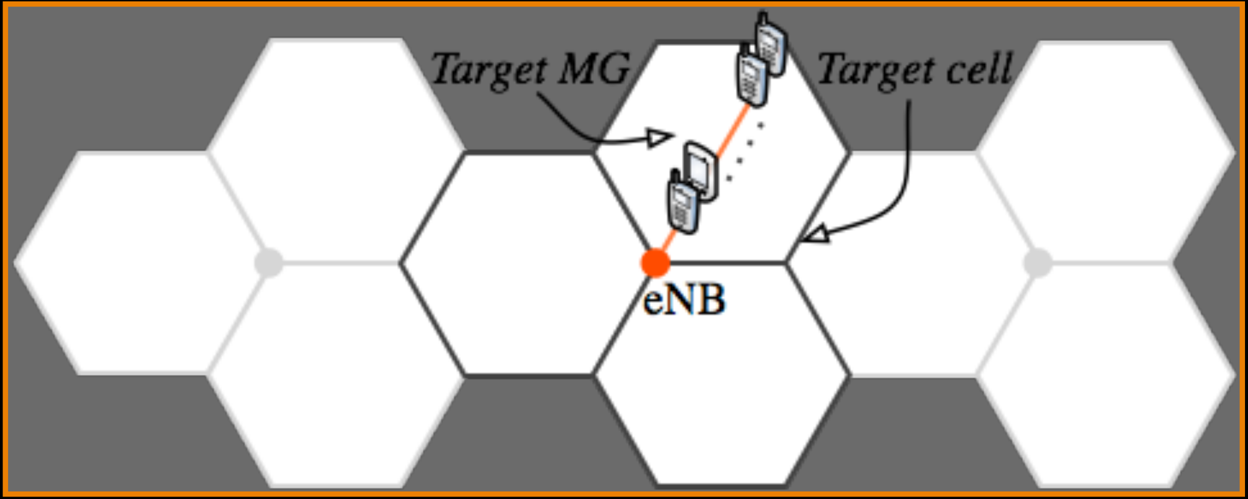
subject to:
$$\sum_{u=1}^{U} \mu_{u,l} \geq U\ \hat{t}_l \quad \text{for } l = 1,\ldots,L$$

$$m_{c-1} \leq m_c \quad \text{for c} = 2,\ldots,L$$

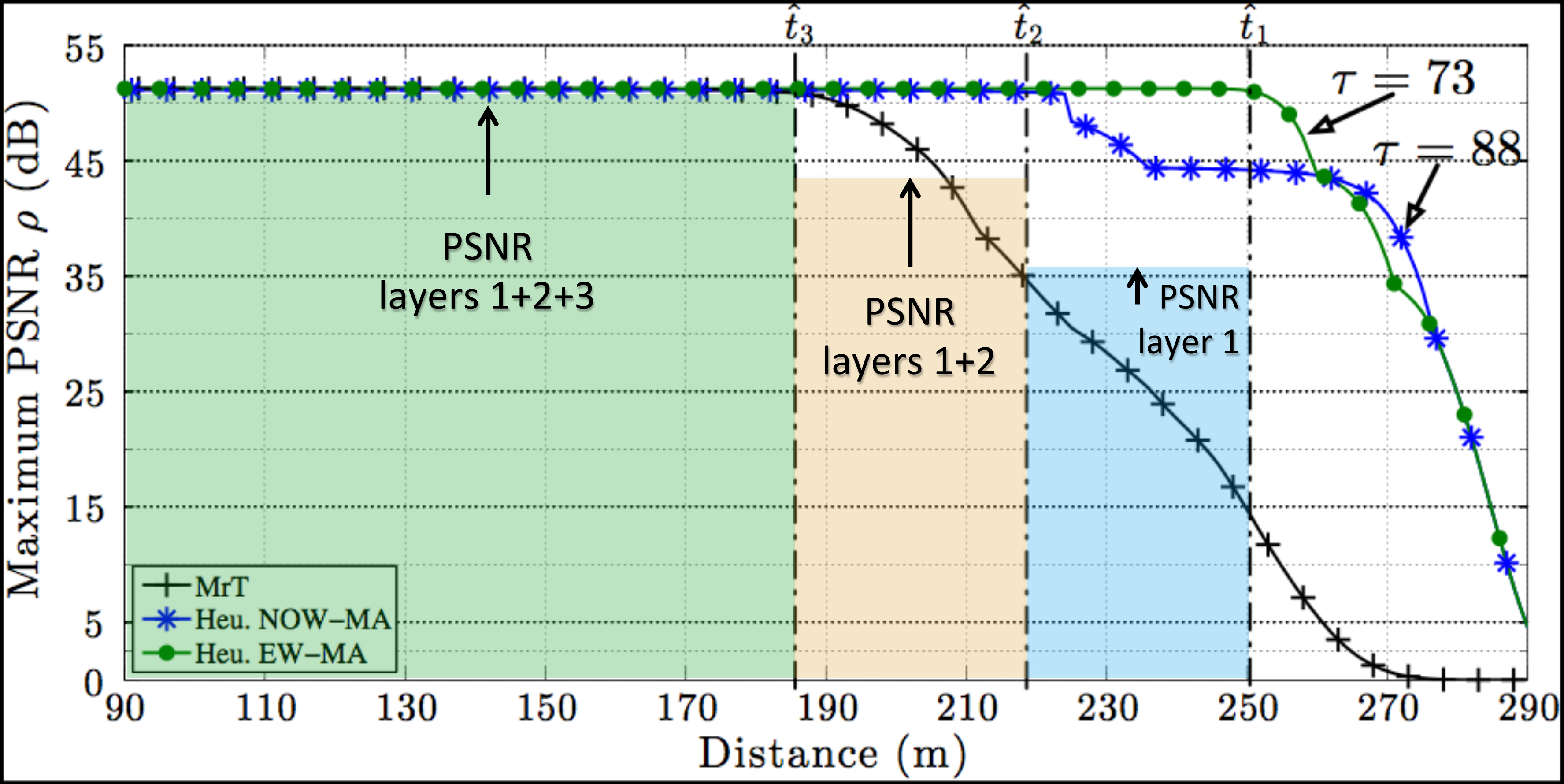
$$0 \leq \sum_{l=1}^{L} N^{(l,c)} \leq \hat{B}_c \quad \text{for c} = 1,\ldots,L$$

# Network configuration

- We have considered 80 users equally spaced and placed along the radial line representing the symmetry axis of one sector of the target cell.
- Node eNB (eNode-B) represents the base station. Users form a Multicast Group (MG).



# Analytical results



The performance of the proposed EW-MA is compared to NOW-MA and the state-of-the-art Multi-rate Transmission (MrT). The focus is on GF(2).

# 3. User-centric optimisation

# Optimisation model

- The objective of the previous allocation method was to minimize the number of transmitted packets, while meeting a minimum set of service level agreements . This is from the point of view of the service provider…
- Best practice for burglars: To steal objects with the maximum value and the minimum weight. Maximising the profit-cost ratio is the objective of the Unequal Error Protection Resource Allocation Model (UEP-RAM) .

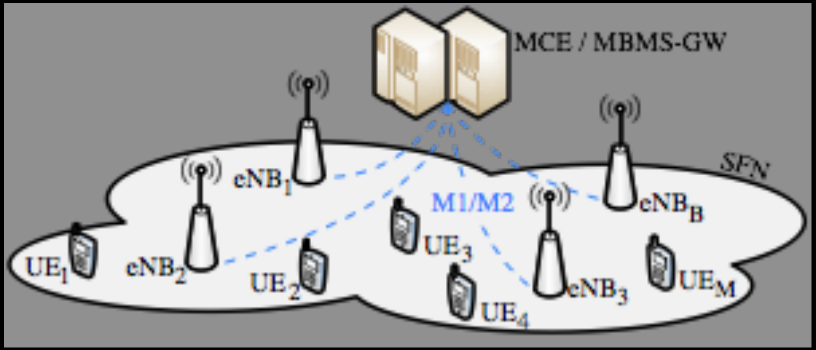
(UEP-RAM)
$$\max_{\substack{m_1,\ldots,m_L \\ N_1,\ldots,N_L}} \sum_{u=1}^{U}\sum_{l=1}^{L}\delta_{u,l} \Big/ \sum_{l=1}^{L} N_l$$

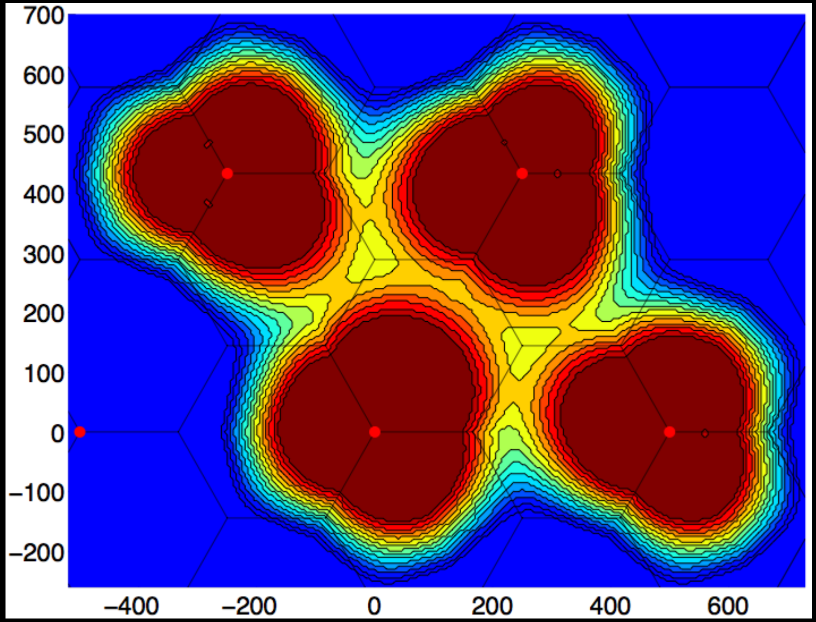
subject to:
$$\sum_{u=1}^{U}\delta_{u,l} \geq U\ \hat{t}_l \qquad l = 1,\ldots,L$$

$$0 \leq N_l \leq \hat{N}_l \qquad l = 1,\ldots,L$$

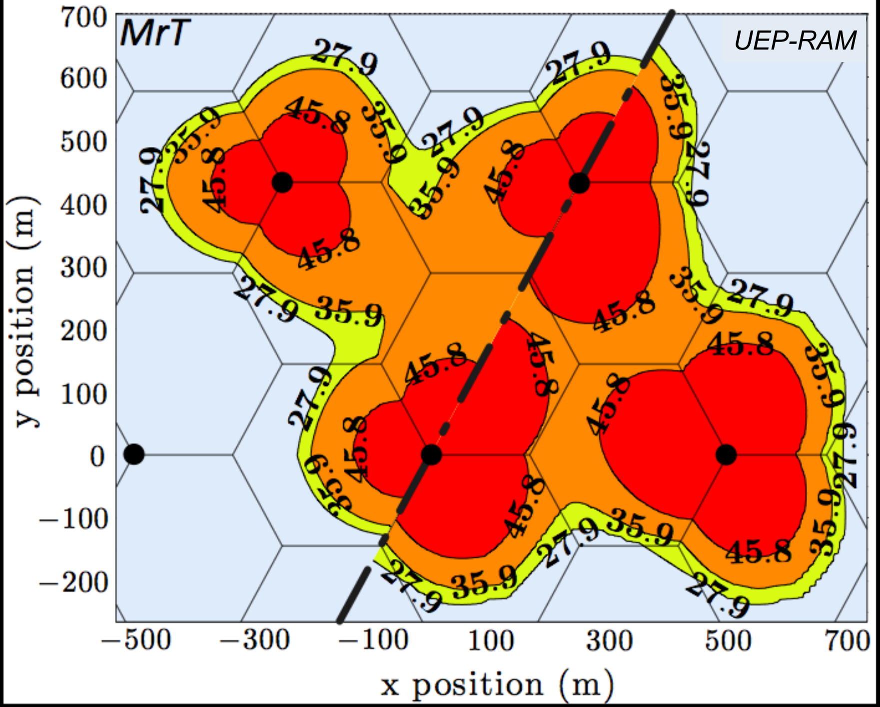
# Network configuration

- LTE-Advanced allows multiple contiguous base stations to deliver (in a synchronous fashion) the same services.
- Top image: we considered a Single-Frequency Network (SFN) comprising 4 base stations and 1700 users.
- Bottom image: The distribution of the Signal-to-Interference-plus-Noise Ratio (SINR) in space.

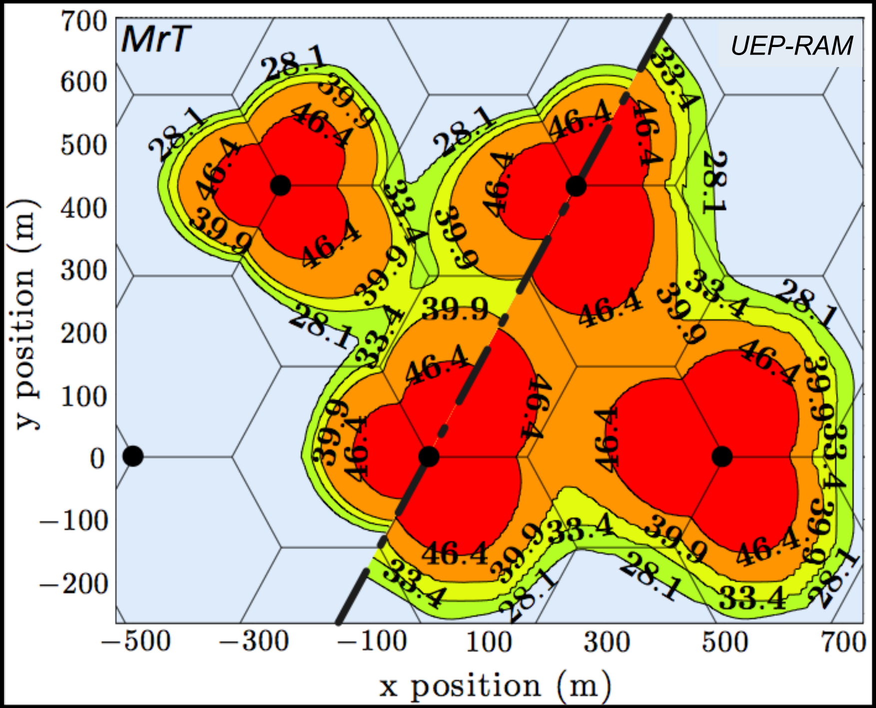




# Analytical results



Three-layered service



Four-layered service

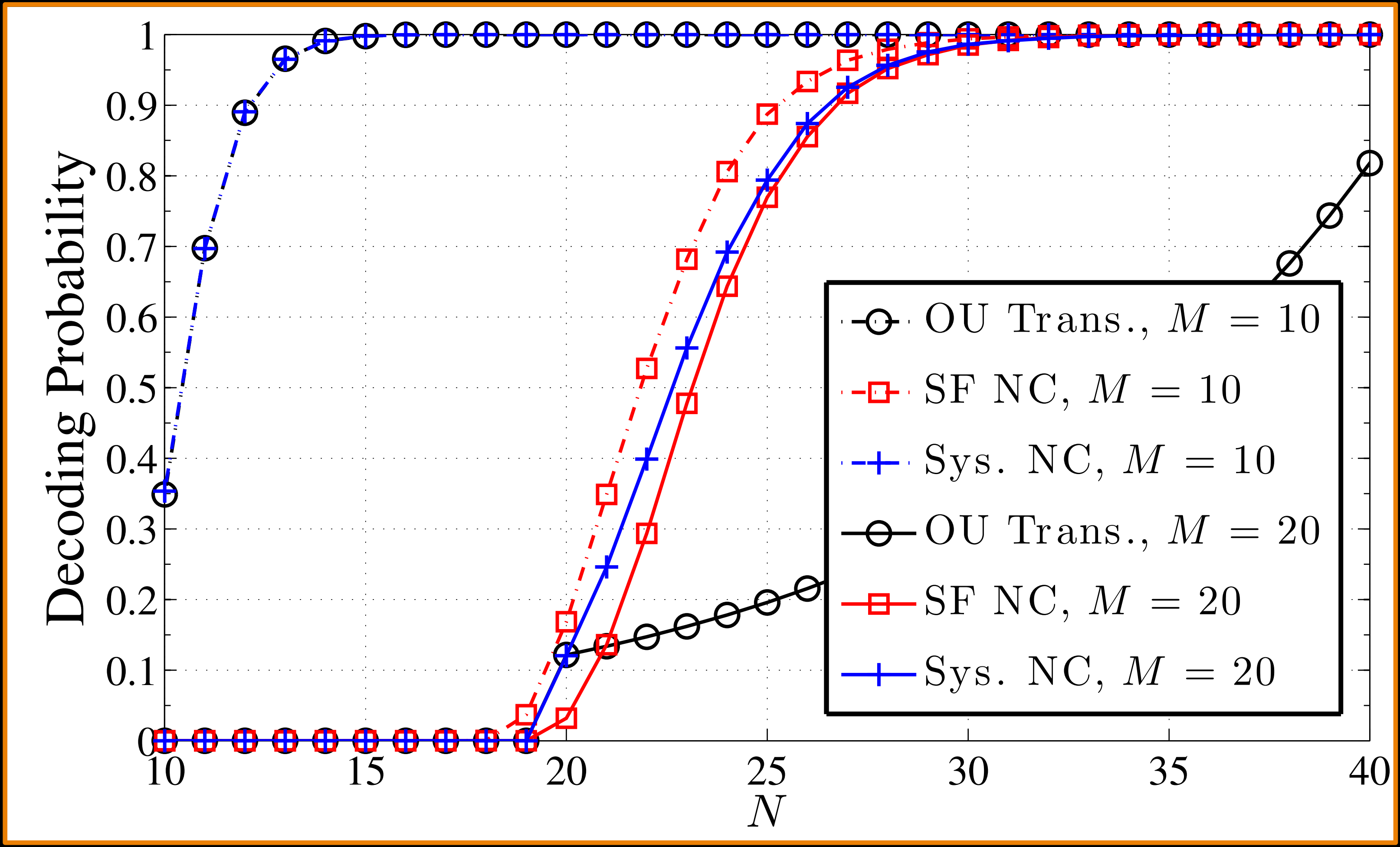
The proposed optimisation framework based on network coding clearly shows an increase in the coverage of service provider (right-hand side of each figure).

# 4. Other key findings

# Systematic vs. straightforward NC

- The transmitter broadcasts *N* packets:
  - They could all be linear combinations of the *K* source packets (straight-forward NC)
  - The first K transmitted packets could be un-coded and the remaining N-K packets could be linear combinations (systematic NC).
- We proved that:
  - The decoding probability of systematic RLNC is lower than the decoding probability of non-systematic RLNC.
  - Systematic RNLC offers the advantage of reduced decoding complexity and progressive packet recovery at the receiver.

# Systematic vs. straightforward NC

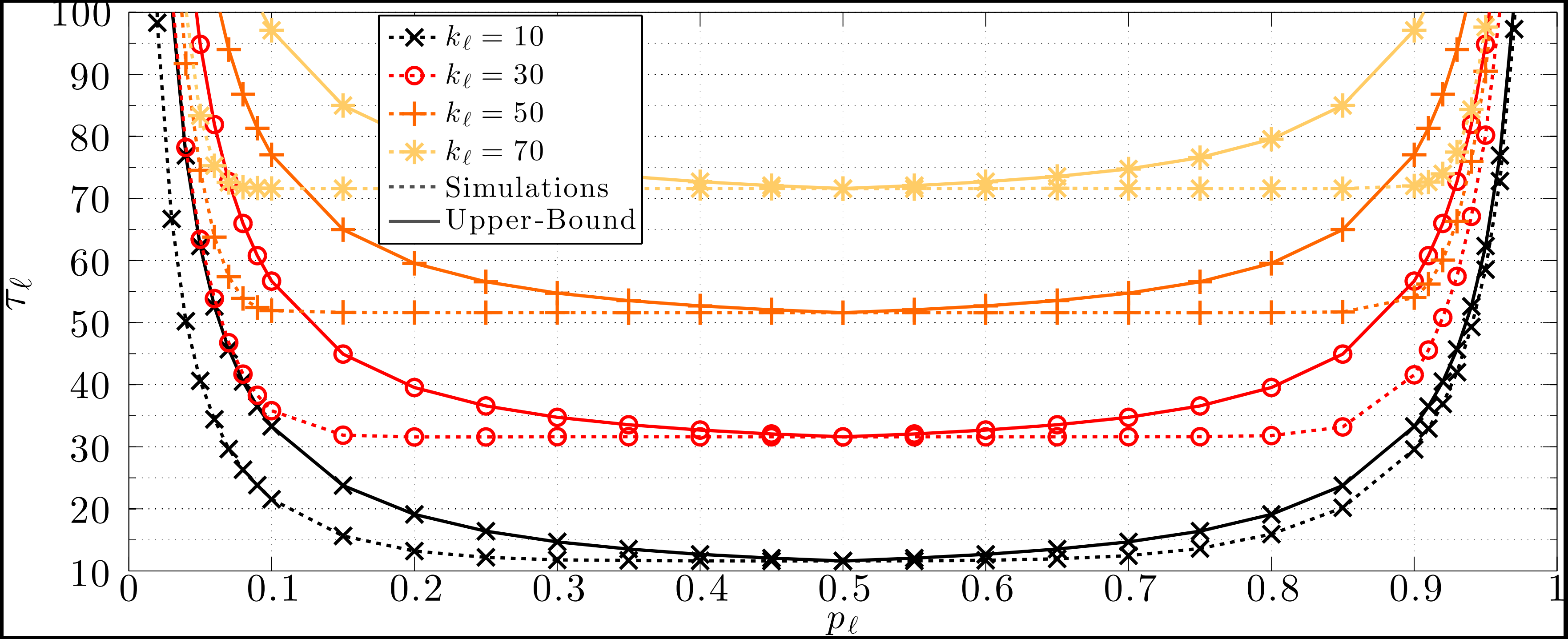


OU: Ordered Un-coded transmission, SF NC: Straightforward Network Coding, Sys. NC: Systematic Network Coding. *The erasure probability is p = 0.1.*

# Sparse Random Network Coding

- So far, the entries of the generator matrix were selected uniformly at random from the elements of a finite field GF(q).
- What if the probability of selecting a non-zero element is uniform but the probability of selecting the zero element ( $p_\ell$ ) is higher than 0.5?
- The service provider can adjust the sparsity of its generator matrix and trade transmit power for reduced decoding complexity.
- Recent results indicate that the performance gains offered by larger finite fields, e.g. GF($2^8$) as opposed to GF(2), do not justify the increase in decoding complexity.

# Sparse Random Network Coding



- Results for different numbers of source packets at the transmitter. Upper bounds (solid lines) are compared to simulations (dashed lines).
- Focus on GF(2). An increase in $p_\ell$ (probability of selecting the zero element) causes an increase in sparsity and a decrease in decoding complexity.
- For 50 source packets and $p_\ell = 0.5$, time to decode=1030 µsec; for $p_\ell = 0.9$, time to decode=352 µsec.

# 5. Concluding remarks

# Random matrices over GF(q)

$$\begin{array}{c|cccccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & \dots & x_{K-1} & x_K \\ \hline y_1 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ y_2 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ y_3 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ y_4 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ y_5 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \vdots & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ y_{N-1} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ y_N & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array}$$

where • is an element of GF(q):

$$\bullet = \begin{cases} 0, & p = 1/q \\ 1, & p = 1/q \\ \dots & \dots \\ q-1, & p = 1/q \end{cases}$$

The decoding probability $P_d$ is the probability that the *N*×*K* matrix (*N* ≥ *K*) is full-rank (Trullols-Cruces *et al.*, 2011):

$$P_d = \prod_{j=0}^{K-1} \left( 1 - \frac{1}{q^{N-j}} \right)$$

O. Trullols-Cruces, J.M. Barcelo-Ordinas,, M. Fiore, "Exact decoding probability under random linear network coding", IEEE Communications Letters, vol.15, no.1, pp .67-69, 2011

# The expanding window principle

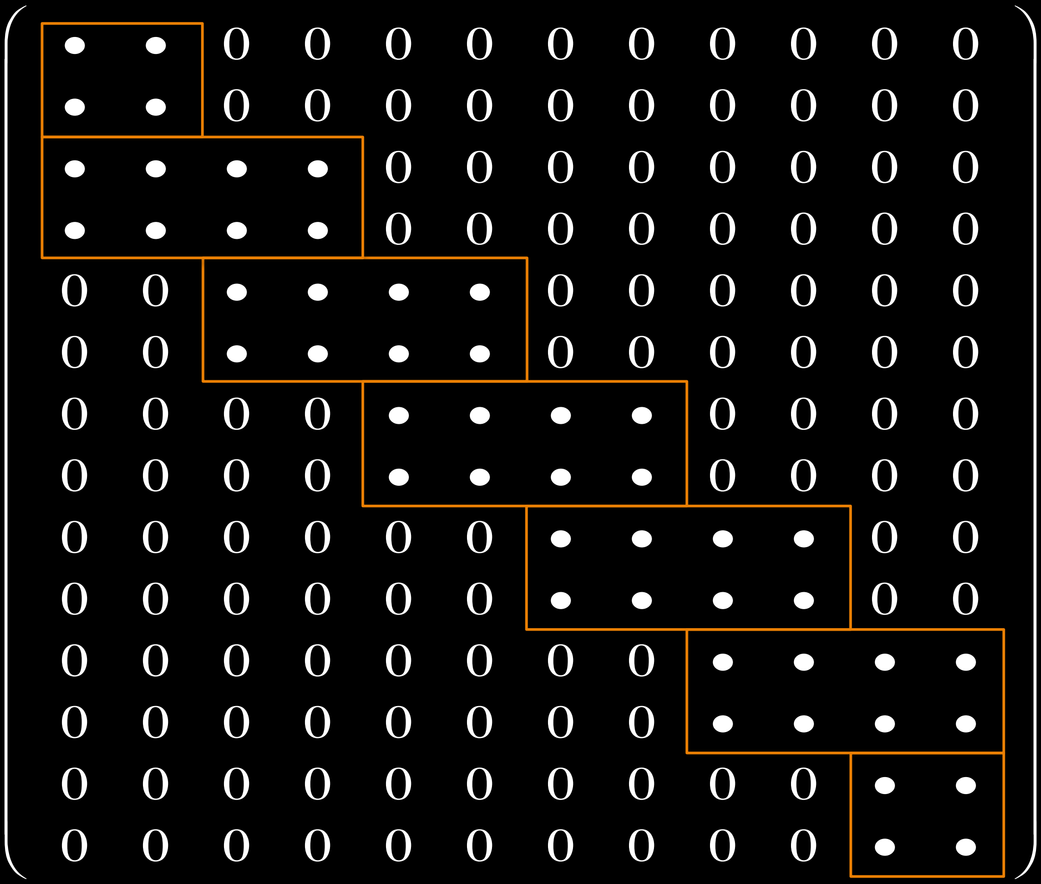
$$\begin{array}{c|cccccccccccc}
 & x_1 & x_2 & x_3 & x_4 & x_5 & \ldots & & & & \ldots & x_{K-1} & x_K \\
\hline
y_1 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_2 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_3 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_4 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_5 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 \\
\vdots & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 \\
 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 \\
 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 \\
 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\vdots & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
y_{N-1} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
y_N & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet
\end{array}$$

**Application:**

✓ Transmission of layered information, where 'nested' windows carry information of higher/ fundamental importance.

The decoding probability $P_d$ has been approximated.

# The sliding window principle



$$\begin{array}{c|cccccccccccc}
 & x_1 & x_2 & x_3 & x_4 & x_5 & \ldots & & & & \ldots & x_{K-1} & x_K \\
\hline
y_1 & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_2 & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_3 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_4 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
y_5 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 \\
\vdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\
y_{N-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet \\
y_N & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet
\end{array}$$

**Applications**:

- ✓ Transmitters with a limited buffer size streaming multimedia content.

The **decoding probability** $P_d$ is under investigation.

# The random block-angular matrix

$$\begin{array}{cc}
 & \begin{array}{cccccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \dots & & & & \dots & x_{K-1} & x_K \end{array} \\
\begin{array}{c} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ \vdots \\ \\ \\ \\ \vdots \\ y_{N-1} \\ y_N \end{array} &
\left(\begin{array}{cccccccccccc}
\bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet
\end{array}\right)
\end{array}$$

**Applications:**

- ✓ Multiple access relay networks (MARC)
- ✓ Distributed data storage.

The decoding probability $P_d$ has been computed (Ferreira *et al.*, 2013).

P. J. S. G. Ferreira, B. Jesus, J. Vieira, A. J. Pinho, “The rank of random binary matrices and distributed storage applications”, *IEEE Communications Letters*, vol.17, no.1, pp.151-154, 2013

# Sparse random matrices

$$\begin{array}{c|cccccccc|}
 & x_1 & x_2 & x_3 & x_4 & x_5 & \dots & x_{K-1} & x_K \\
y_1 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
y_2 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
y_3 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
y_4 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
y_5 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\vdots & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
y_{N-1} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
y_N & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet
\end{array}$$

where • is an element of GF(q):

$$\bullet = \begin{cases} 0, & p_\ell \\ 1, & p = (1 - p_\ell)/(q-1) \\ \cdots & \cdots \\ q-1, & p = (1 - p_\ell)/(q-1) \end{cases}$$

The decoding probability $P_d$ has been bound from above (Blömer *et al.*, 1997).
We are not aware of exact expressions.

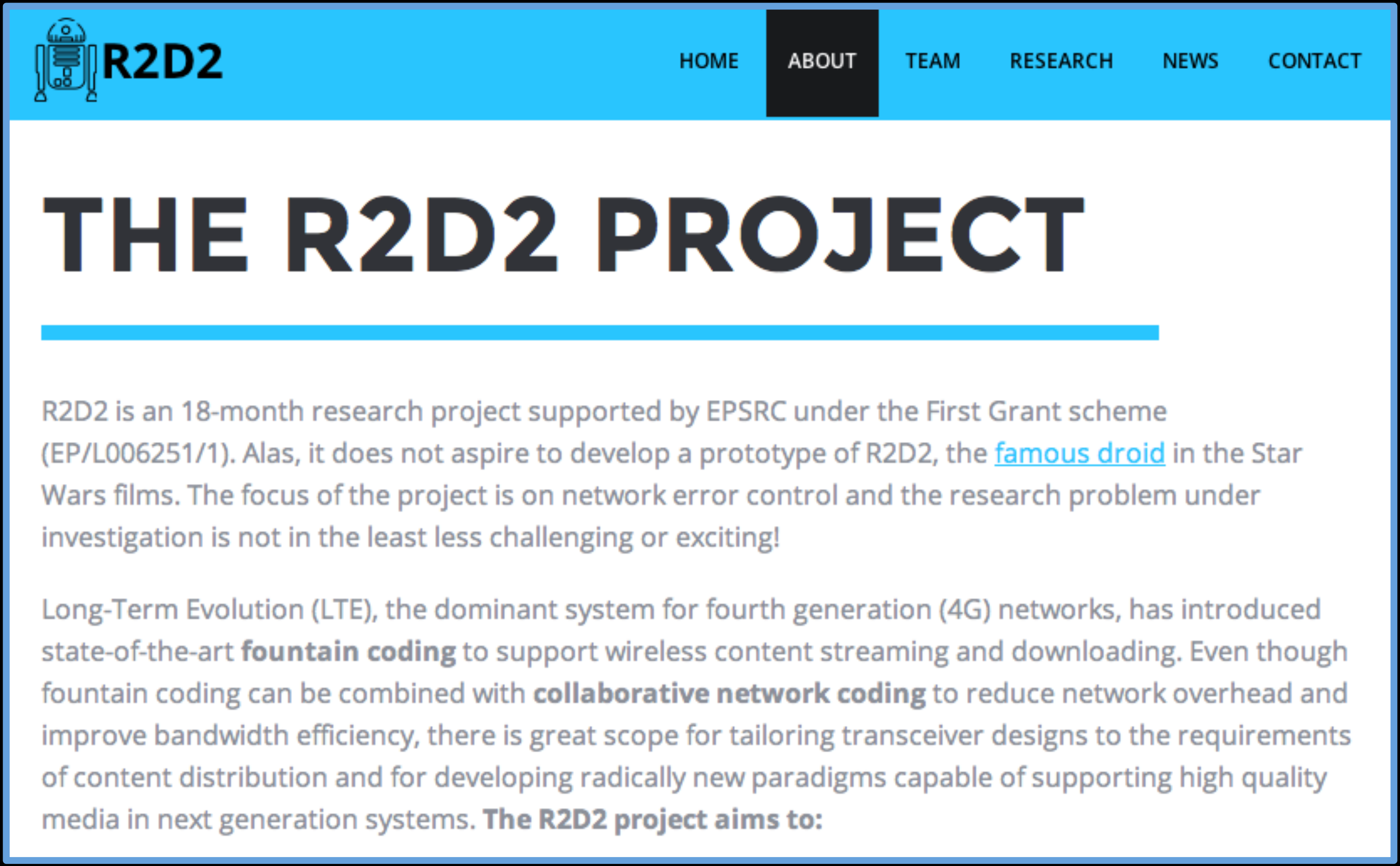
J. Blömer, R. Karp, E. Welzl, "The rank of sparse random matrices over finite fields", *Random Struct. Algorithms*, 10 (1997), pp. 407-419.

# Publications – Project outcomes

- A. Tassi, I. Chatzigeorgiou and D. Vukobratović, “Resource allocation framework for network-coded layered multimedia multicast services”, to appear in *IEEE Journal on Selected Areas in Communications* (JSAC).
- A. Khan and I. Chatzigeorgiou, “Performance analysis of random linear network coding in two-source single-relay networks”, to appear in *Proc. IEEE Int. Conf. on Commun.* (ICC), Workshop on Cooperative and Cognitive Mobile Networks, London, UK, June 2015.
- A. Tassi, I. Chatzigeorgiou, D. Vukobratović and A. L. Jones, “Optimized network-coded scalable video multicasting over eMBMS networks”, to appear in Proc. *IEEE Int. Conf. on Commun.* (ICC) , London, UK, June 2015.
- A. L. Jones, I. Chatzigeorgiou and A. Tassi, “Binary systematic network coding for progressive packet decoding”, to appear in Proc. *IEEE Int. Conf. on Comm.* (ICC) , London, UK, June 2015.
- A. L. Jones, I. Chatzigeorgiou and A. Tassi, “Performance assessment of fountain-coded schemes for progressive packet recovery”, in Proc. *9th Int. Symp. on Comm. Systems, Networks & Dig. Sig. Proc.* (CSNDSP), Manchester, UK, July 2014.
- I. Chatzigeorgiou, “Conditions for cooperative transmission on Rayleigh fading channels”, in Proc. *80th IEEE Vehicular Tech. Conference* (VTC), Vancouver, Canada, September 2014.

# Project Website

http://www.lancs.ac.uk/~chatzige/R2D2/index.html



# THE R2D2 PROJECT

R2D2 is an 18-month research project supported by EPSRC under the First Grant scheme (EP/L006251/1). Alas, it does not aspire to develop a prototype of R2D2, the famous droid in the Star Wars films. The focus of the project is on network error control and the research problem under investigation is not in the least less challenging or exciting!

Long-Term Evolution (LTE), the dominant system for fourth generation (4G) networks, has introduced state-of-the-art **fountain coding** to support wireless content streaming and downloading. Even though fountain coding can be combined with **collaborative network coding** to reduce network overhead and improve bandwidth efficiency, there is great scope for tailoring transceiver designs to the requirements of content distribution and for developing radically new paradigms capable of supporting high quality media in next generation systems. **The R2D2 project aims to:**

# The saga continues…