Non-Stationary Estimation of Joint Design Criteria with a Multivariate Conditional Extremes Approach

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Motivation

- **Rational** design and assessment of marine structures to environmental loading [Jonathan and Ewans, 2013]
- Structural response is **combined** effect of *multiple* parameters, e.g., H_S, T_P, W_S, etc.
- Extremal independent and joint characteristics of environmental parameters typically vary with covariates (i.e., wave direction)
- Covariate effects are important to model
- Common approach assume arbitrary extremal dependence or focus on regions where all parameters are extreme
- **Approach**: A framework for estimating joint criteria as a function of multiple covariates

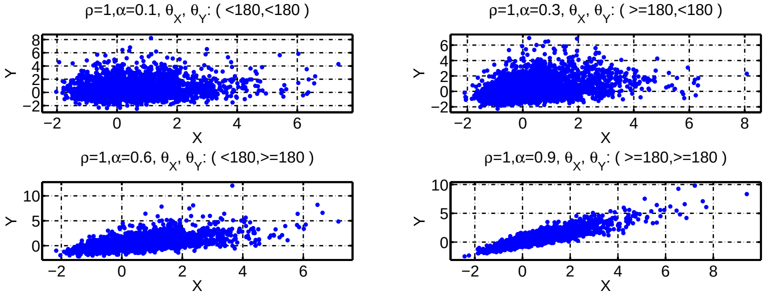
Typical Applications

- Response-based design: Associated T_P corresponding to a H_S with given return period, function of wave direction
- Extreme current profile with depth from different directions for mooring and riser loading
- Weathervaning systems with non-stationary effects for the design of FLNG/FPSO installations, etc.

Presentation Summary

- Non-stationary approach to estimating joint criteria for two or more parameters by extending Heffernan and Tawn [2004]
- Proof of principle using a synthetic T_P given H_S example
- Two practical applications
  - Associated current given significant wave height for northern North Sea
  - Joint current profile for offshore Brazil locations

Problem Illustration - A Simulated 2D Example



Different covariate regions have different correlations, α

Mathematical Outline

1. Estimate marginal extreme value model $\dot{X}_1|\theta_1$,
2. Estimate marginal extreme value model $\dot{X}_2|\theta_2$,
3. Transform $\dot{X}_1$ and $\dot{X}_2$ to random variables X_1 and X_2 respectively with stationary standard Gumbel distributions,
4. For large values of x_1, estimate **conditional extremes** models $X_2|X_1, \theta_1, \theta_2$

$$\begin{aligned}(X_2|X_1 = x_1, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2) &= \alpha_{12}(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)x_1 \\ &+ x_1^{\beta_{12}(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)}(\mu_{12}(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) + \sigma_{12}(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)Z_{12})\end{aligned} \quad (1)$$

5. Using conditional extremes model, simulate realisations of covariates and variates corresponding to a specified return period of interest on Gumbel scale,
6. Transform the realisations from Gumbel to original (physical) scales to get the estimates of associated design criterion for a given return period

Estimating Marginal Parameters

- Marginal EVA parameters $\dot{\psi}_k$, $\dot{\xi}_k$ and $\dot{\zeta}_k$ (threshold and **GP model** estimates) vary smoothly with respect to covariates θ_k,
- Calculate B-spline basis matrix $\mathbf{B}$ $(m_{\theta_k} \times p_{\theta_k})$ for m_{θ_k} covariate values, at p_{θ_k} uniformly spaced knot locations $\in [0, 360)$
- Efficient computation using slick spline algorithms such as GLAMS [Currie et al., 2006] as applied in [Raghupathi et al., 2016]

Estimating Conditional Model Parameters

- For a given threshold exceedance, $x_k^i > \psi_k(\boldsymbol{t}_k^i)$

$$(X_j|X_k = x_k, \boldsymbol{\theta}_j \cup \boldsymbol{\theta}_k = \boldsymbol{t}_{jk}) = \alpha_{jk}(\boldsymbol{t}_{jk})x_k + x_k^{\beta_{jk}(\boldsymbol{t}_{jk})}(\mu_{jk}(\boldsymbol{t}_{jk}) + \sigma_{jk}(\boldsymbol{t}_{jk})Z_{jk})$$

- Parameters $\alpha_{jk} \in [0,1]$, $\beta_{jk} \in (-\infty, 1]$, $\mu_{jk} \in (-\infty, \infty]$ and $\sigma_{jk} \in (\infty, \infty)$ vary smoothly with covariates $\boldsymbol{\theta}_j \cup \boldsymbol{\theta}_k$ with basis

$$\mathbf{B} = \mathbf{B}_{\theta_j} \otimes \mathbf{B}_{\theta_k}$$

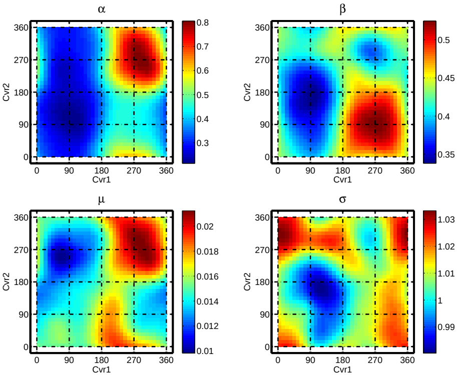
- Estimate model parameters by minimising penalised negative log-likelihood assuming Z_{jk} is standard Gaussian for model fitting only

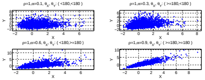
$$\ell^*_{CE,jk} = \ell_{CE,jk} + \lambda_{\alpha_{jk}} R_{\alpha_{jk}} + \lambda_{\beta_{jk}} R_{\beta_{jk}} + \lambda_{\mu_{jk}} R_{\mu_{jk}} + \lambda_{\sigma_{jk}} R_{\sigma_{jk}}$$

Estimation for Simulated 2D Example

- Follow the modelling steps described above
- First, perform marginal fits and estimate diagnostics (Randell et al. 2015) to confirm good performace
- Then, estimate parameters α, β, μ and σ of conditional extremes model

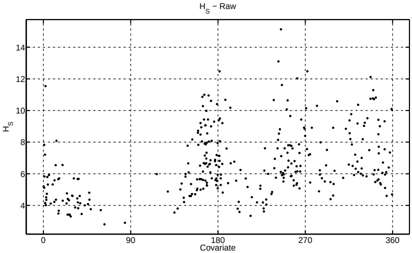
Conditional Model Parameter Estimates



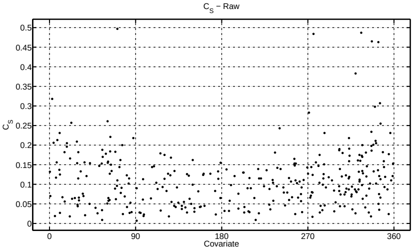


Heffernan and Tawn [2004] model parameters α, β, μ and σ as a function of bi-directional covariates - **agreement** with known dependence. α reflects known values, $\beta \approx 0.5$ (Gaussian dependence), $\mu \approx 0$ and $\sigma \approx 1$ as expected.

North Sea Raw Data

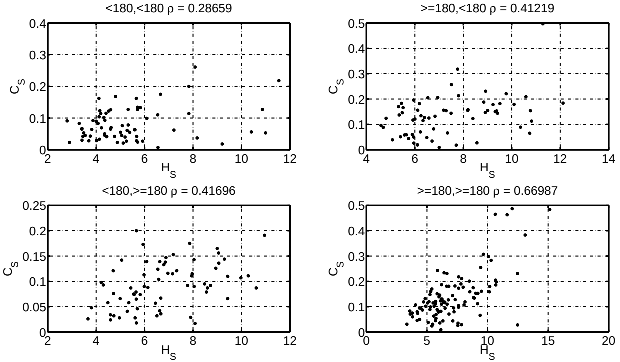


$H_S(m)$, wave direction θ_H "FROM"



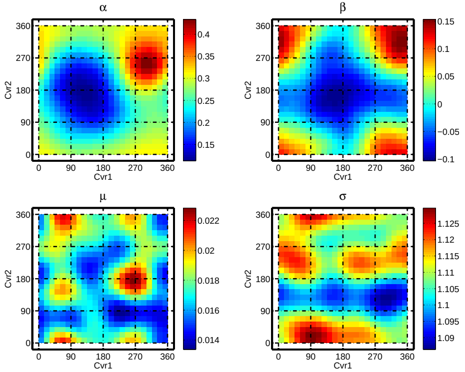
$C_S(m/s)$, current direction θ_C "FROM"

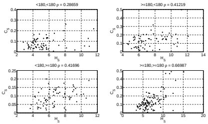
North Sea Dependence



Non-stationary dependence across different covariate sectors

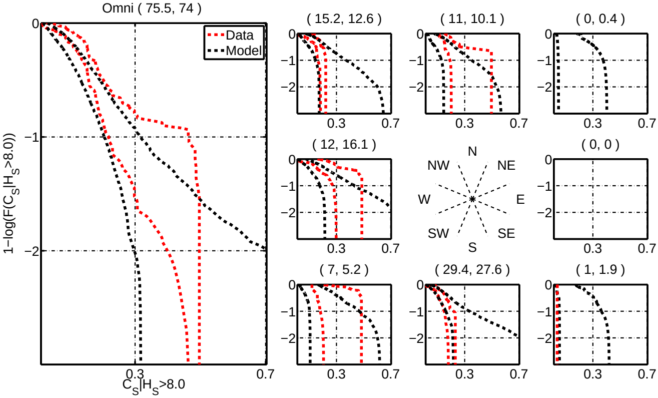
North Sea CE Parameter Estimates





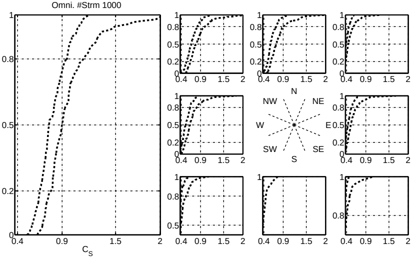
α, β parameters agrees with the scatter. $\mu \approx 0$ $\sigma \approx 1$. Hot spot for α clear.

North Sea Results (1/2)

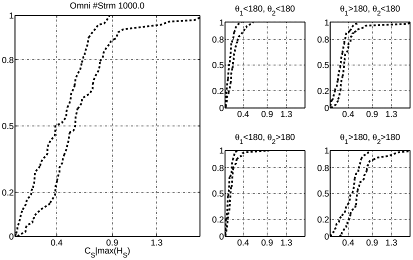


Conditional Model Diagnostics - good agreement

North Sea Results (2/2)



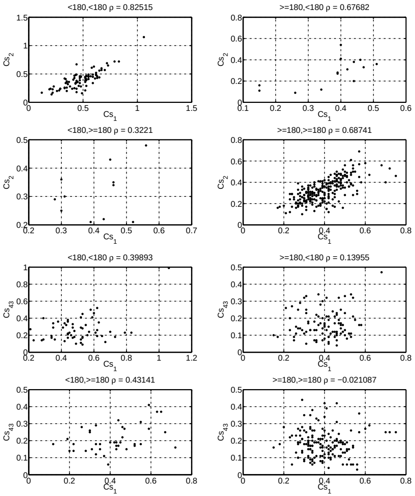
(i) C_{S-100} ~**0.8 m/s**



(ii) $C_S|H_{S-100} \sim$ **0.5 m/s**

Associated design value smaller.

Current Profile - Offshore Brazil Data

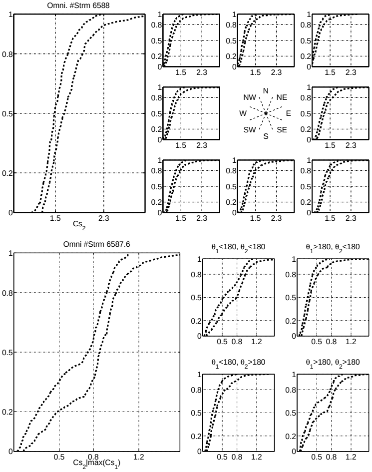


(i) Cs_2 Vs Cs_1 - highly correlated **near surface**

(ii) Cs_{43} Vs Cs_1 - no correlation at **deep water**

Dependence is non-stationary and varies for two pairs of current depth

Current Profile - Offshore Brazil Results (1/2)

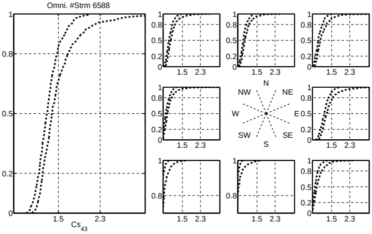


(i) Cs_{2-100} ~**1.6 m/s**

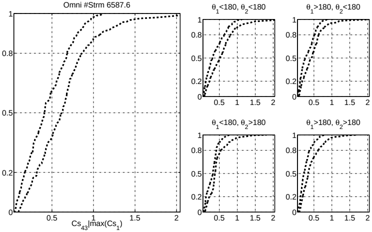
(ii) $Cs_2 | Cs_{1-100}$ ~**0.8 m/s**

Nearer to surface, conditioning **reduces design values** relative to unconditioned.

Current Profile - Offshore Brazil Results (2/2)



(i) $C_{S43-100}$ ~**1.4 m/s**



(ii) $C_{S43}|C_{S1-100}$ ~**0.5 m/s**

At deep water, conditioning reduces design values relative to both unconditioned and surface. **Careful structural design can exploit this.**

Summary

- Non-stationary approach to estimating joint criteria for two or more parameters by extending Heffernan and Tawn [2004]
- Demonstrated two practical applications - North Sea and Brazil
- Dependence structure non-stationary. Conditioning **reduces design values** relative to unconditioned - careful structural design can exploit this

Proposed approach is **data-driven** and **rigorous** for estimating joint criteria

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