A statistical model for the directional evolution of severe ocean storms

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Motivation: non-stationary extremes in time

- Application of the Markov extremal heatwave model of Winter and Tawn (2016, 2017) to evolution of ocean storms
- Evolution of significant wave height is strongly influenced by wave direction → extend heatwave model to include temporal evolution of direction
- Specify and estimate a joint model for wave height Y_t and wave direction Θ_t (for $Y_t > \psi_\theta$)

Data

- Evolution of extreme storm severity (H_S) and associated direction in time

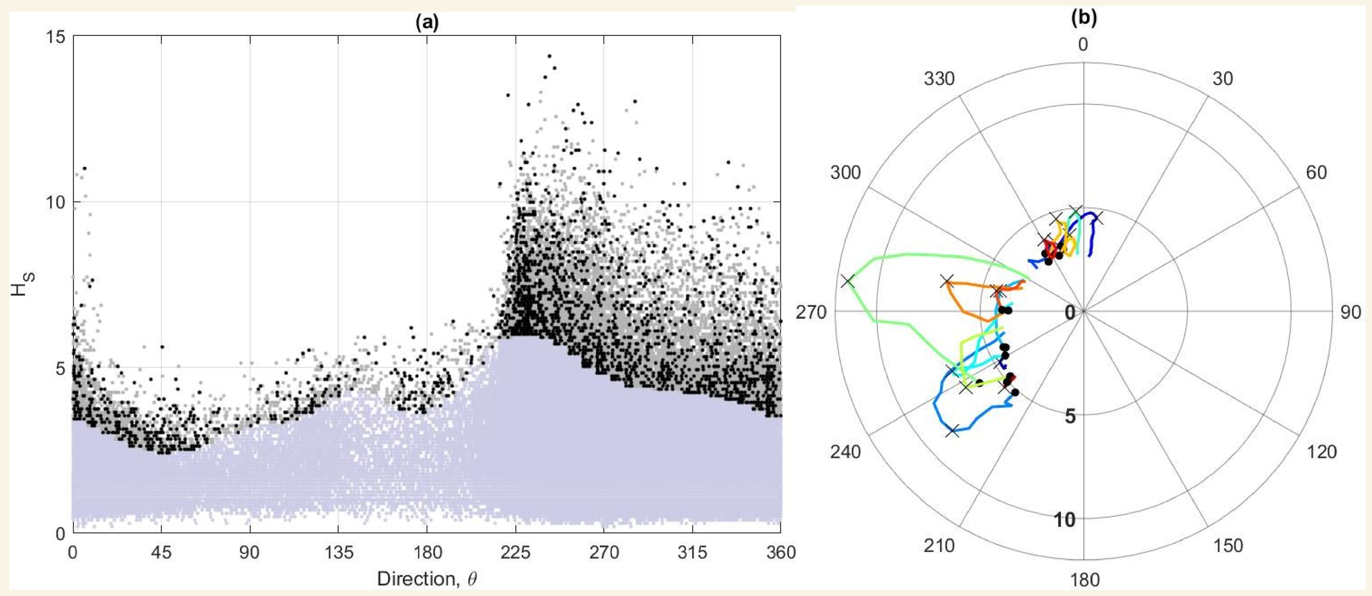
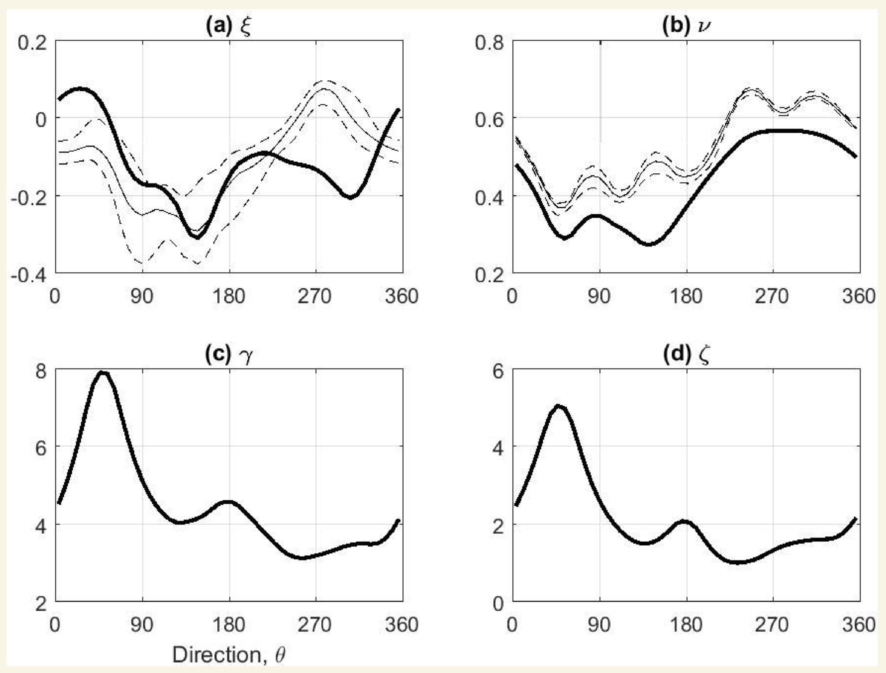


Figure 1: Motivating application. (a) Directional plot of storm peak H_S (black), sea-state H_S (grey) for all storms with directional threshold ψ_θ. (b) Polar plot of H_S with direction for 15 typical storms.

Marginal model

- Estimate marginal non-stationary gamma-GP model for significant wave heights $\{Y_t\}$ with associated wave directions $\{\Theta_t\}$ (using P-splines)
- Transformation to standard Laplace margins: $Y_t \rightarrow X_t$



Parameter estimates ξ, ν, γ and ζ for the directional marginal gamma-GP model for sea state significant wave height H_S and storm peak significant wave height H_S^{sp}. Thick line is median estimate for H_S. Thin solid line is median estimate for H_S^{sp}. Thin dashed lines are 95% uncertainty bands for storm peak analysis. The directional threshold ψ is illustrated in Figure 1.

Markov extremal model (MEM)

Fit

- Split storms into two parts, left and right of the peak
- For each part, fit 2nd-order Markov extremal model (based on conditional extremes model of Heffernan & Tawn):
  $[X_{t+1}, X_{t+2}] = [\alpha_1, \alpha_2]\, X_t + X_t^{[\beta_1,\beta_2]}\, [\mu_1 + \sigma_1 Z_1, \mu_2 + \sigma_2 Z_2]$ for $X_t > \eta$
- Distributions G_1, G_2 of Z_1, Z_2 assumed standard Gaussian for fit only
- Residuals used to estimate distributions G_1, G_2 and $G_{2:1}$ of Z_1, Z_2 and $Z_{2:1}$
- Kernel density estimation used for $G_{2:1}$ in particular

Simulation (starting at peak X_0)

- $X_1 = \alpha_1 X_0 + X_0^{\beta_1}(\mu_1 + \sigma_1 Z_1)$ for $t = 1$
- $X_t = \alpha_2 X_{t-2} + X_{t-2}^{\beta_2}(\mu_2 + \sigma_2 Z_{2|1})$ for $t = 2, 3, \ldots$

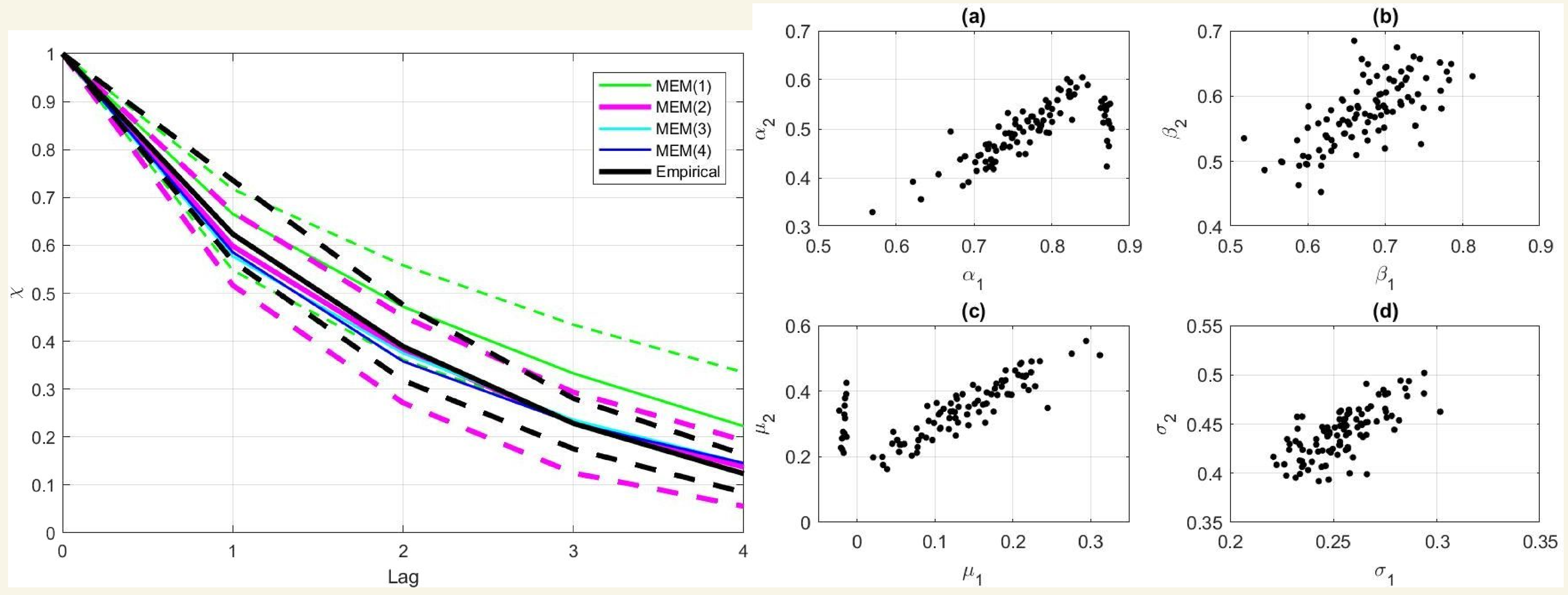


Figure 2: LH: χ extremal diagnostic for different MEM orders: MEM(2) looks good. RH: Scatter plots of MEM α_2 on α_1, β_2 on β_1, μ_2 on μ_1 and σ_2^2 on σ_1^2 for MEM(2) pre-peak model for H_S, using a threshold with non-exceedance probability 0.75 .

Directional model

- Calculate *change* in wave direction ($d\Theta_t/dt$) and transform to approximate Gaussian $\Delta_t = f(d\Theta_t/dt)$.
- Fit AR(k) model: $\Delta_t = \sum_{j=1}^{k} \gamma_j \Delta_{t-j} + \epsilon_t$, $\epsilon_t \sim N\left(0, \sigma^2(X_t)\right)$
- $\sigma^2(x) = \lambda_1 \exp(-\lambda_2 x) + \lambda_3$

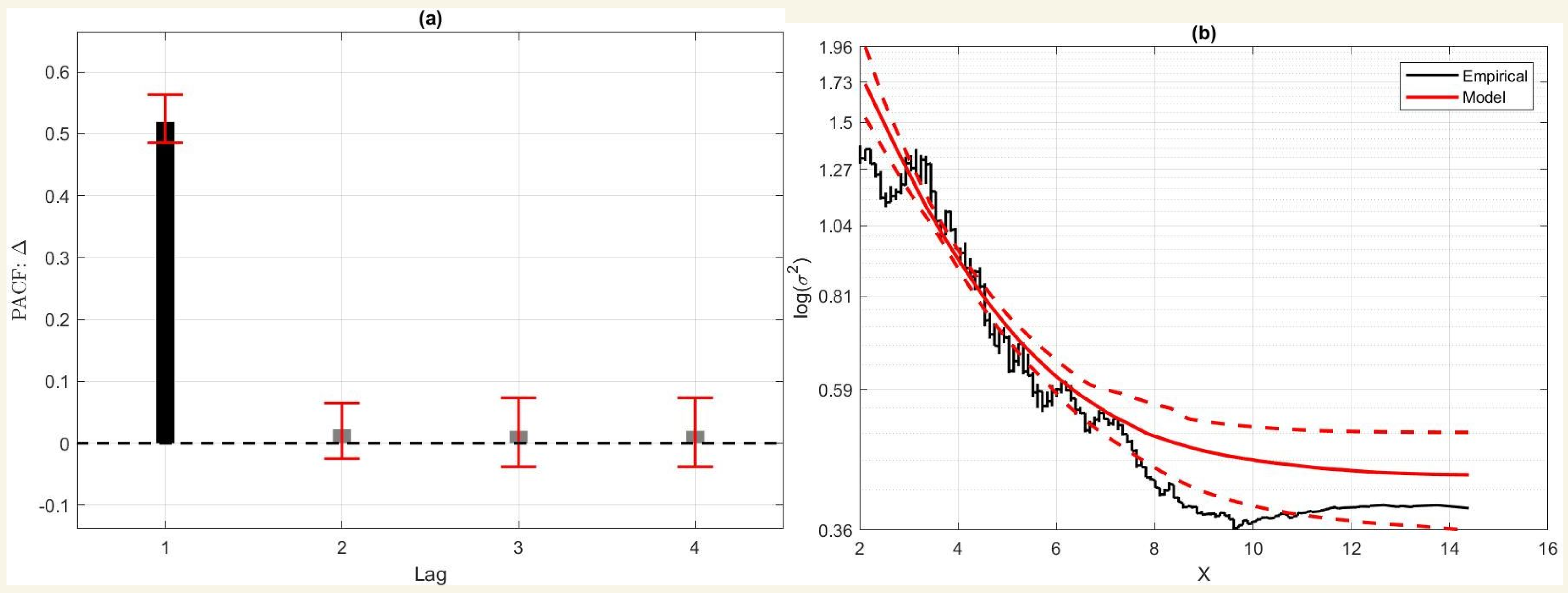


Figure 3: Directional diagnostics. (a) Sample partial autocorrelation function for Δ_t: $k = 1$ looks good. (b) $\sigma^2(X_t)$ on X_t from sample (black) and model (red) with 95% bootstrap uncertainty bands.

Simulation algorithm

simulate storm peak x_0, θ_0 above threshold η **for** $t = 1, 2, \ldots$ **do**

- simulate x_t using MEM;
- **if** $x_t \geq x_0$ *(exceeded storm peak)* **then**
  - reject trajectory and restart;
- **end**
- **if** $x_t < \eta$ *(no longer extreme)* **then**
  - stop and save trajectory;
- **end**
- simulate θ_t using directional model;
- simulate y_t on physical scale using marginal transformation;

end

Key inferences

- Distribution of storm trajectory lengths by direction
- Marginal distribution of H_S by direction

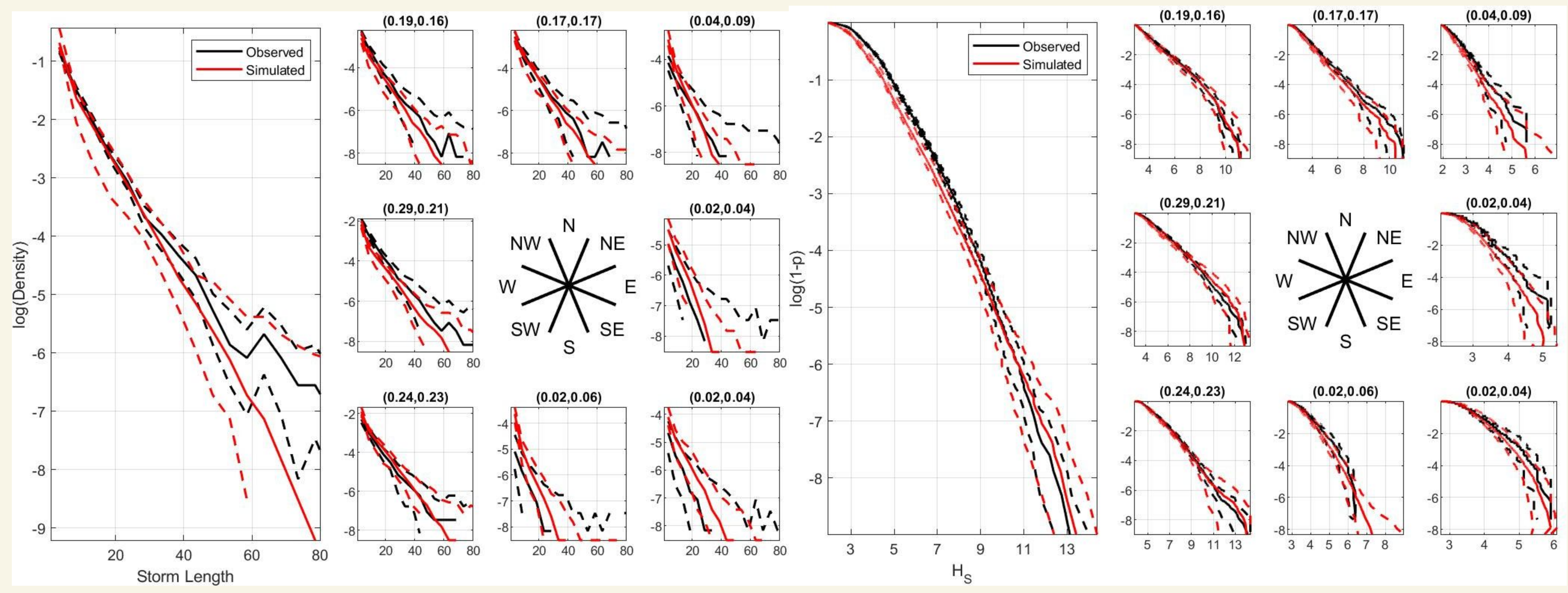


Figure 4: Results using first order directional model and MEM(2), omni-directionally and by directional octant. (a) Logarithm of density of number of sea states in a storm. (b) Tail of distribution of H_S. Black lines give empirical sample estimates. Red lines are estimated by simulation under the model. Dashed lines are 95% bootstrap uncertainty bands.

References

Tendijck, S., Ross, E., Randell, D., Jonathan, P., 2018. A non-stationary statistical model for the evolution of extreme storm events. (In preparation for Environmetrics, draft at www.lancs.ac.uk/∼jonathan).

Winter, H. C., Tawn, J. A., 2016. Modelling heatwaves in central France: a case-study in extremal dependence. J. Roy. Statist. Soc. C 65, 345–365.

Winter, H. C., Tawn, J. A., 2017. kth-order Markov extremal models for assessing heatwave risks. Extremes 20, 393–415.

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