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Time Domain Simulation of Jack-up Dynamics With the Extremes of a Gaussian Process

Random simulations are often used to simulate the statistics of storm-driven waves. Work on Gaussian linear random signals has lead to a method for embedding a large wave into a random sequence in such a way that the composite signal is virtually indistinguishable (in a rigorous statistical limit) from a purely random occurrence of a large wave. We demonstrate that this idea can be used to estimate the extreme response of a jack-up in a severe sea-state in a robust and efficient manner. Results are in good agreement with those obtained from a full random time-domain simulation.

1 Introduction

The behavior of a jack-up rig under extreme storm loading is complex. Among offshore engineering specialists, it is widely held that the most accurate methods for estimating structural behaviour are based on extensive random time-domain simulation of the ocean surface to obtain statistics of the extreme response in typically a 3 hour period of a severe storm. A typical example of this approach is given by Rodenbusch (1986). For a jack-up the important responses include deck displacement, bending moment in the legs, forces at the deck/leg connection, reactions at the spud-cans etc. Unfortunately, random time-domain simulation is very time-consuming.

For structures which respond quasi-statically, the extreme response always corresponds to the extreme input sea surface elevation. For these situations, Tromans et al. (1991) have shown that the time-history of the expected surface elevation in the region of an extremum can be derived theoretically, assuming that the surface elevation can be modelled as a Gaussian random process. The resulting NewWave methodology has been successfully used to predict the global response of offshore structures, see for example Tromans and van de Graaf (1992).

For dynamically-responding structures, the extreme response does not always correspond to the extreme input surface elevation. The present structural displacement and the associated stresses are dependent not only on the present value of the applied load, but also on the load history and the structural dynamics. The extreme response might correspond to a combination of a local extreme wave together with unfavourable background "structural memory" response.

To study such systems computationally, it would be advantageous to be able to produce time-series of surface elevation, each of which was constrained to include a maximum of given height, but otherwise was completely random. Thus, time-domain simulations for a small number of specific constrained series would be as informative as a much larger series of unconstrained simulations.

The object of this work is to devise a technique for estimating the probability distribution of the extreme response in a 3 hour period in a robust way without simulating more than ~3 hours of real time. The response distribution should be consistent with that derived by "brute-force" random time-domain simulation (100 × 3 hours). We also require the method to enable the estimation of, say, 6 or 9 hour extremes with no more computation than the 3 hour case.

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Using short O(minute) constrained simulations, we can estimate the distribution of largest response associated with a crest of given size. The distribution of large crests within a random sea-state is known to fit the tail of Rayleigh distribution. Convolution of the response distribution for each crest height with the distribution of the largest crests yields an estimate for the distribution of extreme response in a given period (typically 3 hours). Response distributions obtained from the constrained simulations of ~3 hours of real-time compare well with those from "brute-force" random simulations (i.e., we conducted 100 random simulations, each of 3 hour duration, took the maximum response in each simulation and then used the 100 individual maxima to provide an estimate of the complete distribution of the extreme response at the 1 in 3 hour level). Such simulations provide the only accurate benchmark to compare our method against.

This methodology would be applicable to a wide class of problems in offshore engineering with the proviso that the required extreme response should increase on average with individual crest size.

2 Constraining a Random Process

The local shape of large ocean waves is very variable. This variability in the shape of a wave, and the random motion of the structure in response to waves prior to the large wave, produces considerable variability in the peak structural response associated with a large wave of given elevation. For the largest waves in the sea-state, this shape tends to the auto-correlation function, has previously been referred to as NewWave (Tromans et al., 1991, Jonathan, Taylor and Tromans, 1994).

Lindgren (1970) predicts exact forms for average shape of maxima (and minima) of a linear Gaussian random process and the variability around this average shape. We use Lindgren's results to constrain a random time series for surface elevation to have a large crest of a given size at a chosen time in such a way that, in statistical terms, the extreme is effectively indistinguishable (in the sense described below) from a purely random occurrence of a crest of that height. The method is as follows:

Consider a stationary random process:

$$Y(t) = Y_t = \sum_{n=1}^{N} A_n \cos(\omega_n t) + B_n \sin(\omega_n t), \qquad (1)$$

where $\{A_n, B_m\}$ are the usual independent Normal random variables with zero means. The variance of A_n is σ_n^2 equal to the variance of B_n, defined by the spectral density in each frequency band. The total variance of the process is: $\sigma^2 = \sum_{n=1}^{N} \sigma_n^2$. The second and fourth spectral moments are defined as $\lambda^2\sigma^2$ and $\upsilon^4\sigma^2$ respectively. The autocorrelation function is $r_\tau = \sigma^{-2} \sum_{n=1}^{N} \sigma_n^2 \cos(\omega_n \tau)$.

Lindgren (1970) gives the general form of the expected profile around a maximum of height a at $t = 0$. For large crests, this expected profile of $(Y_t | \text{max of } a \text{ at } t = 0)$ can be written as:

$$E(Y_t | \text{max of } a \text{ at } t = 0) = a\left(r_t - \frac{\sigma^2}{a^2}\left(r_t + \frac{\ddot{r}_t}{\lambda^2}\right) + O\left(\frac{e^{-\gamma^2/2}}{\gamma}\right)\right) \rightarrow a r_t \text{ for } a \text{ large} \quad (2)$$

where $\gamma = \lambda^2 a/\sqrt{\sigma^2(\upsilon^4 - \lambda^4)}$ is a measure for narrow-bandedness for each crest. Similarly, the covariance of the conditioned process can be written in asymptotic form as:

$$\text{cov}((Y_t, Y_{t+\tau}) | \text{max of } a \text{ at } t = 0)$$

$$= \sigma^2\left(r_\tau - r_t r_{t+\tau} - \frac{\dot{r}_t \dot{r}_{t+\tau}}{\lambda^2} - \frac{\sigma^2}{a^2}\left(r_t + \frac{\ddot{r}_t}{\lambda^2}\right)\left(r_{t+\tau} + \frac{\ddot{r}_{t+\tau}}{\lambda^2}\right) + O(\gamma e^{-\gamma^2/2})\right) \rightarrow$$

$$\sigma^2\left(r_\tau - r_\tau r_{t+\tau} + \frac{\dot{r}_\tau \dot{r}_{t+\tau}}{\lambda^2}\right) \text{ for } a \text{ large} \quad (3)$$

The NewWave approximation by Tromans et al. (1991) modelled a turning point $Y_t | (Y_0 = a, \dot{Y}_0 = 0)$ in a Gaussian process, obtaining estimates for the mean profile and covariance which are identical to the leading-order behavior of Lindgren solution valid for large crests. For realistic spectra, the NewWave form is adequate for $a > 2\sigma$. Previous comparisons with measured waves confirm this result (Jonathan and Taylor, 1995).

We now present a method to construct realisations of a stationary Gaussian process which exhibit a local turning point of specified height, at a specified time. The method yields constructed realizations with statistical properties identical to those of the approximation due to Tromans et al. (1991), the Borgman et al. (1983) method for conditioned simulations, and the exact asymptotic form due to Lindgren. It should be noted that the statistical quantities are now over ensembles of simulations, each realization can itself be viewed as a mixture of random and deterministic quantities. However, the ensemble statistics of the constrained realization match those of purely random occurrences of large waves.

Suppose that we wish to constrain the random process (1) by the addition of two non-random functions $e(t)$ and $f(t)$ of the form:

$$e(t) = e_t = \sum_{n=1}^{N} c_n \cos(\omega_n t),$$

$$f(t) = f_t = \sum_{n=1}^{N} d_n \sin(\omega_n t) \quad (4)$$

Consider a constrained process:

$$Z(t) = Z_t = Y(t) + Qe(t) + Rf(t) \quad (5)$$

where the coefficients Q and R are random. We select the forms of Q and R, and the values c_n, d_m so that Z_t is constrained to have a value $Z_0 = a$ and gradient $\dot{Z}_0 = g$ (which we can later set to be zero), at time $t = 0$. Then

$$Q = \frac{1}{c}\left(a - \sum_{n=1}^{N} A_n\right), \quad R = \frac{1}{d}\left(g - \sum_{n=1}^{N} \omega_n A_n\right) \quad (6)$$

where $c = \sum_{n=1}^{N} c_n$, $d = \sum_{n=1}^{N} \omega_n d_n$, from which the statistical properties of $t = 0$, can be calculated in terms of those of A_n, B_m. The expected value of the constrained process Z_t is given by:

$$EZ_t = EY_t + e_t EQ + f_t ER$$

$$= \frac{ae_t}{c} + \frac{gf_t}{d} \quad (7)$$

The constrained autocovariance function measured at time t with time lag τ is:

$$\text{cov}(Z_t, Z_{t+\tau}) = \sum_{n=1}^{N} \sigma_n^2 \cos(\omega_n \tau) + \sum_{n=1}^{N}\sum_{m=1}^{N}\left(-\frac{c_n \sigma_m^2}{c} - \frac{c_m \sigma_n^2}{c} + \frac{c_n c_m \sigma^2}{c^2}\right) \times \cos(\omega_n t)\cos(\omega_m(t+\tau)) + \sum_{n=1}^{N}\sum_{m=1}^{N}\left(-\frac{d_n \omega_m \sigma_m^2}{d} - \frac{d_m \omega_n \sigma_n^2}{d} + \frac{d_n d_m \lambda^2 \sigma^2}{d^2}\right) \times \sin(\omega_n t)\sin(\omega_m(t+\tau)) \quad (8)$$

The variance of the process at time t can be found by setting $\tau = 0$ in (33):

$$\text{var}(Z_t) = \sigma^2\left(1 + \frac{e_t}{c}\left(\frac{e_t}{c} - 2r_t\right) + \frac{f_t}{d}\left(\frac{\lambda^2 f_t}{d} + 2\dot{r}_t\right)\right) \quad (9)$$

There are clearly an infinite number of functions e_t and f_t which could be chosen to constrain the process (1) to be consistent with (6). The choices:

$$e_t/c = r_t, \quad f_t/d = -\dot{r}_t/\lambda^2 \quad (10)$$

ensure that the both the mean and the covariance are identical to the leading order terms in the exact Lindgren solution, as well as NewWave, if the zero slope condition $g = 0$ is also applied. Thus, to leading order such a constructed peak is indistinguishable from a purely random occurrence of a peak of this size. There is, also, a strong fundamental argument for these choices of e_t and f_t. We choose to look for the functions e_t, f_t which minimize the variance (9) of the constrained process Z_t, so that it is as deterministic as possible in the region of the constraint. That is, the profile of Z_t, will be as similar as possible to its expected form EZ_t for each realization of the process. Setting the partial derivatives of the constrained variance with respect to e_t/c and f_t/d equal to zero yields the solutions given in (10). It is also simple to show that these choices correspond to a minimum of the variance as well as turning point.

This constraining procedure can be described informally as:

If the original random simulation has a displacement h_0 and slope g_0 at time $t = 0$, say, then we subtract $h_0 \times$ NewWave to reduce the displacement at $t = 0$ to zero, and subtract $g_0 \times$ NewWave slope to make the local gradient zero. The simulation now has a point of zero value and zero gradient at time $t = 0$. The final step is to create the required displacement h, say, of the extreme signal at $t = 0$ by adding back $h \times$ NewWave to the "squashed" signal. This produces an extreme of value h which is virtually indistinguishable from any large crest of height h present in the original signal.

This process is illustrated in Fig. 1, showing the original and constrained signals, for a Pierson-Moskowitz spectrum. Very close to the constraint point (at $t = 0$), the signals are dramati-

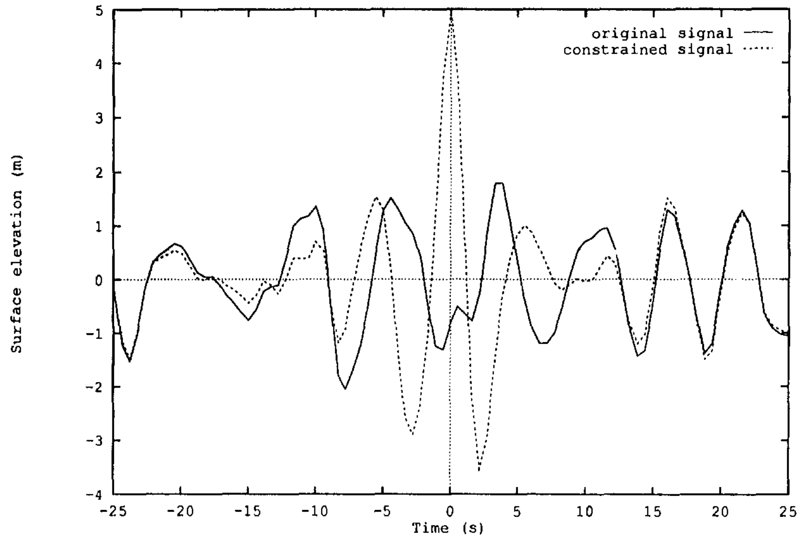

Fig. 1 Original and constrained signals

cally different. However, the modification to the original is strongly localized in time, reflecting the broadbanded nature of the wave field.

Although, the constrained simulation method is illustrated here using linear representations of a random sea-state, the transforms of Creamer et al. (1989, 1994) can be used to modify the linear representations so that they become more than 2nd order accurate in terms of a Stokes-like expansion for the random surface elevation and resulting wave kinematics. This reproduces most of the non-Gaussianity of real sea-states such as the principal vertical trough-crest asymmetry, and is discussed by Jonathan and Taylor (1995).

One important (unresolved) question is the existence of wave groups (localized packets of large waves) and the possibility that the structural response is maximized by the passage of a large group not a single large wave. If the occurrence of wave groups is correctly predicted by linear statistics, then we would expect that the constrained simulation to correctly include their contribution if enough constrained simulations are performed. However, if the occurrence of these groups are more prevalent or long lasting than expected based on linear random models for the ocean surface, these would invalid any approach based on linear Gaussian statistics.

3 A Test Example—A Grossly Simplified Jack-up

A grossly simplified model of a jack-up was defined (Fig. 2). The complications of a real three-legged structure were dropped from the analysis but the important non-linear features of the fluid loading were retained: the Morison drag term integrated to the moving free-surface, causing the effective point of application of the load to move up and down in time. These make the whole problem difficult. The structure was replaced by a single massless uniform beam, pinned at the base and supporting a single point mass which is constrained to move horizontally. The only structural parameter discussed in this paper is the displacement of the mass, corresponding to the deck motion. However, the procedure would work just as well for any measure of structural response.

The aim in this initial exercise was not to simulate a real jack-up but, instead, to demonstrate that the constrained simulation procedure can reproduce results compatible with many hours of random simulations. Once the method has been demonstrated, additional complications can be added such as relative motion terms in the Morison fluid loading, a fully non-linear structural model for a real jack-up, spud-can behavior etc.

Figure 3a shows four realizations of a 10 m crest in a random sea-state with $H_S = 10$ m. A completely random occurrence of a linear crest of this level has a return period of greater than 3 hours. The corresponding deck displacement histories are shown in Fig. 3b, for the case where the resonant frequency is 2× the peak in the wave spectrum, chosen so that the structure exhibits very strong dynamics. The structural damping was set at 2 percent of critical; this is low to emphasize the importance of the random background. Note the variability in the peak responses, even in sign, each of these peak responses associated with large crests of the same size. The duration of each short constrained simulation should be sufficiently long so that any starting transients have decayed sufficiently. For the dynamics calculations reported here, the large crests were imposed at 196 sec, although this is longer than necessary.

Figure 4 shows the results of many constrained simulations: the probability structure of the extreme response for a range of different crest elevations. The points correspond to a given probability of exceedance of response from 5 percent up to 95 percent for each crest elevation. Note how broad the extreme response at each crest height actually is, reflecting the importance of the random wave background and the resonant behavior of the structural model. The constant probability lines are constructed by simple curve fitting. These are required for the con-

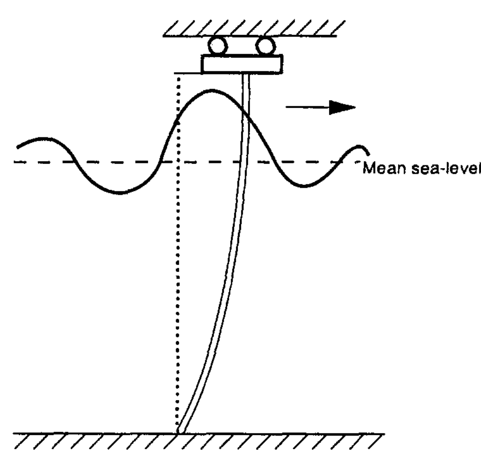

Fig. 2 Single stick model of jack-up

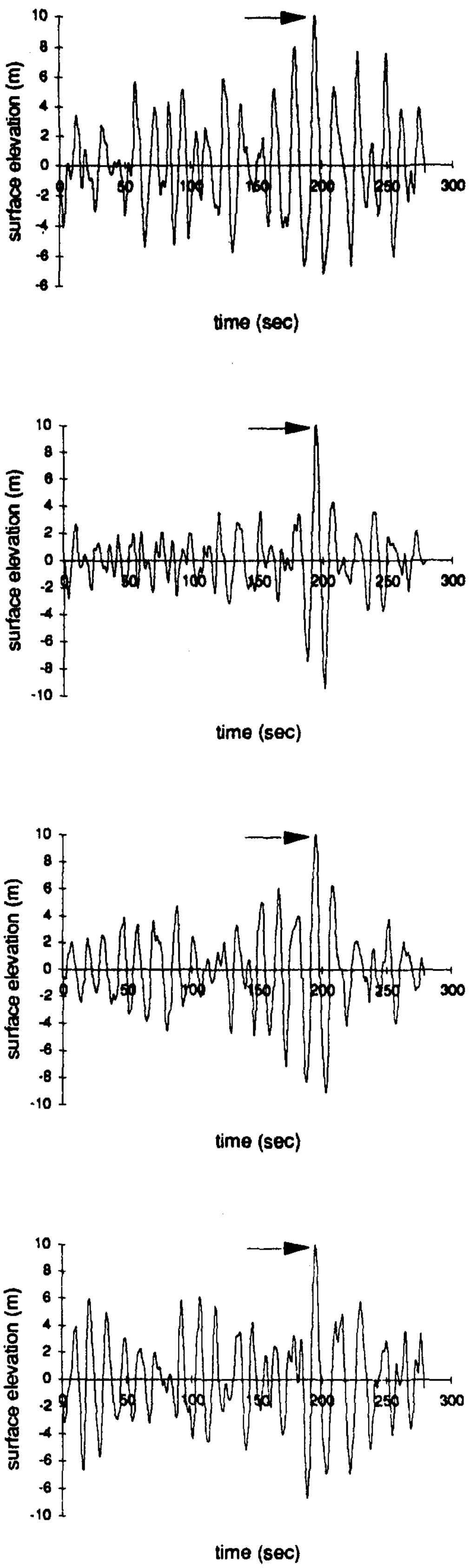

Fig. 3*a* Four realizations of a constrained random wave simulation, all for a P.M. spectrum with H_s = 10 m and ω_p = 0.4 rad/s, constrained to a crest of 10 m at 196 s

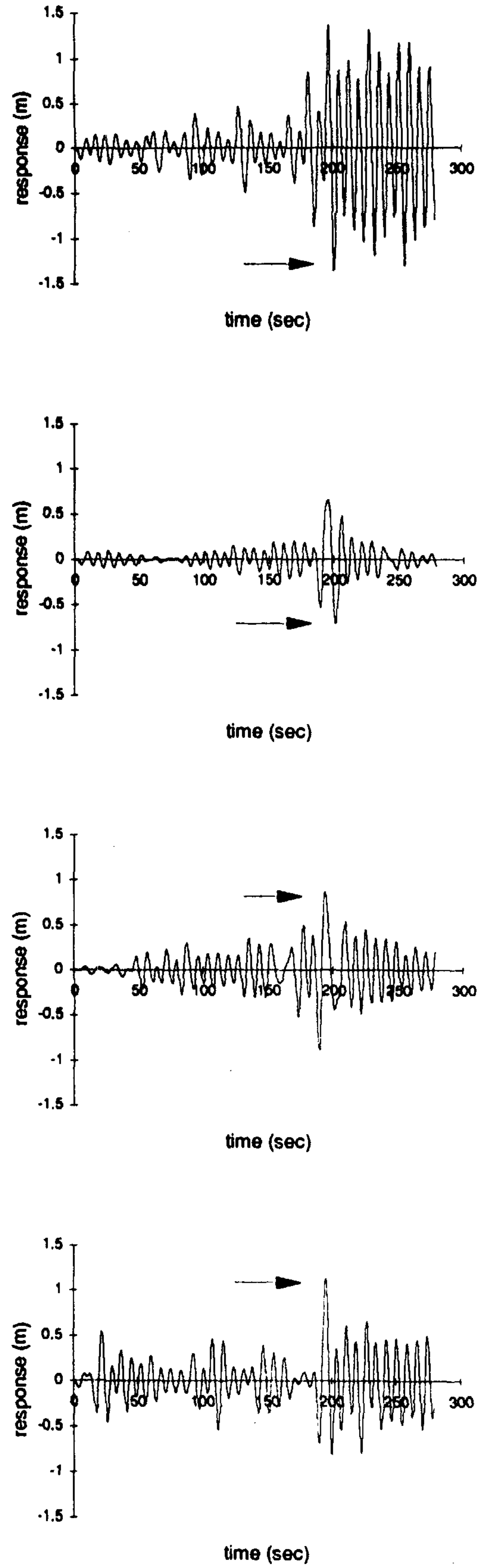

Fig. 3*b* Four examples of response to random wave simulations constrained to a crest of 10 m at 196 s, $\omega_{res}\omega_p$ = 2.0, 2 percent of critical damping

volution with the Rayleigh distribution for crest height to give the complete extreme response distribution in a given period.

Figure 5 shows the probability of non-exceedance of response in a 3 hour period as obtained from the convolution method and via brute-force simulation i.e. 100 realizations, each of 3 hour duration. The degree of agreement is good. Also shown is the peak response due to a deterministic NewWave event in isolation. The amplitude of this NewWave was chosen to correspond to the median (50 percent) level of the extreme crest height distribution for the 3 hour period. The estimate of the extreme response based on an isolated NewWave is in the lower tail of the true distribution for extreme response. Therefore, for dynamically responding structures, NewWave by itself is not directly useful for estimating extreme responses because it neglects the random background. In contrast, convolution of the constrained simulations, which does include the background, reproduces the results of random time-domain simulations well.

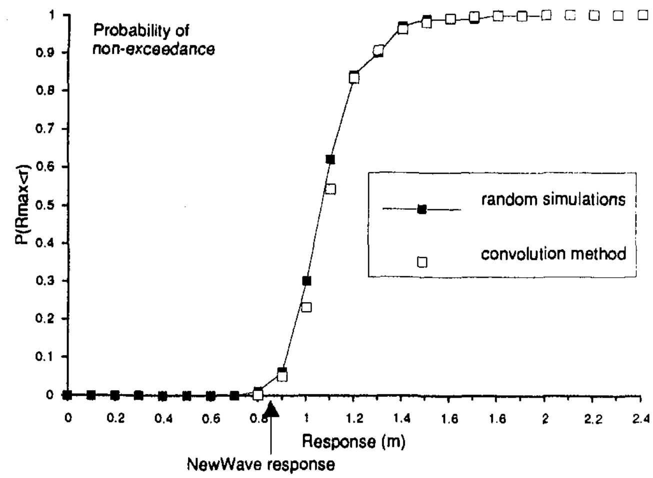

Fig. 5 Probability of non-exceedance estimates for extreme response in 3 hours period. Also shown is the response due to NewWave in isolation, NewWave crest elevation equals median (50 percent level) of extreme crest height distribution in a 3 hour period.

4 Conclusion

The use of constrained time domain simulation represents a significant advance in estimating the statistics of extreme response for dynamically responding structures. Although our first application is aimed at jack-ups, the technique should be equally applicable to other structural types: compliant towers, TLPs, "ringing" of concrete platforms etc. There are only two requirements:

- the required extreme response should be associated, on average, with the occurrence of a large wave within a random sea-state, and
- a relatively simple fluid loading model should be available (such as the Morison equation for drag-dominated structures such as jack-ups and compliant towers).

Having shown that constrained simulation is a viable technique for estimating the extreme responses of structures under nonlinear wave loading, the next step is to demonstrate its effectiveness for the analysis of real structures. A rigorous study of the accuracy and limitations of the method, as applied to real structures using modified commercial software, is currently under way (Harland, Vugts, Jonathan, Taylor, 1996).

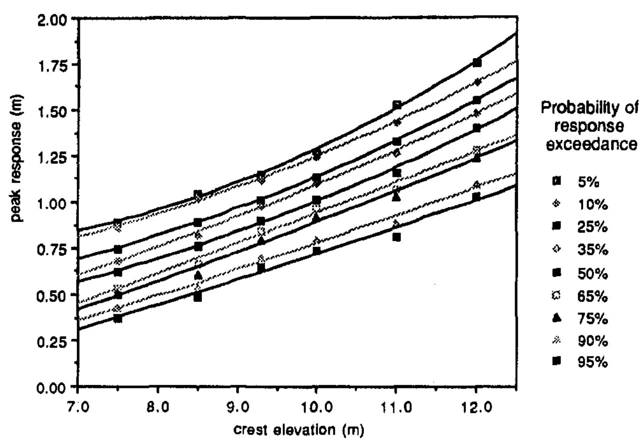

Fig. 4 Fitting to response distributions for a crest of given elevation

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